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FINITE ELEMENT FORMULATIONS FOR COMPRESSIBLE FLOWS

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ABSTRACT

We have started our studies related to the development and application of computational methods for compressible flows. Particular attention is given to proper numerical treatment of sharp layers occuring in such problems and to general mesh generation capabilities for intricate computational geometries. Mainly finite element methods enhanced with several state-of-the-art techniques (such as the streamline-upwind/Petrov-Galerkin, discontinuity capturing, adaptive implicit-explicit, and grouped element-by-element approximate factorization schemes) will be employed.

INTRODUCTION

The physics and dynamics of the problems involving compressible flows in aerospace applications are not yet fully understood. Particularly, better understanding of dynamical, thermal and chemical aspects of the renetry conditions is needed. Prediction of aerodynamic and heating loads is very important for designing space vehicles. The conditions such as altitude and speed under which such maneuvering is to be performed makes the simulation by ground test facilities impossible Therefore, computational predictions based on the solution of appropriate governing equations become very important and imperative.

The main objective of this project is to investigate the development and application of computational tools for aerospace problems with special reference to compressible flows. These tools need to be not only accurate and reliable but also efficient and easy to apply to complicated geometries. Unrestricted geometric generality of a numerical analysis tool will become more and more important and indispensible as research in this area targets more and more flow simulations for computational geometries which give realistic representations of the actual configurations. To this end we plan to base our efforts on the finite element method (FEM) which is widely acknowledged to be the most geometrically flexible computational fluid dynamics (CFD) tool available.

Among the difficulties associated with the numerical simulation of compressible flows is the treatment of shocks and sharp layers. For hypersonic flows the magnitude of such sharp variations in the flow field becomes very large and this makes the computation even more challenging. We plan to use streamline-upwind/Petrov-Galerkin (SUPG) formulations which are well-known to result in stable and accurate results for convection-dominated problems in the presence of discontinuites and sharp layers at moderately high wave numbers. The SUPG method essentially introduces a disspative effect in the direction of the flow (or in the direction of the characteristic vector). For very strog discontinuities and sharp layers (i.e. very high wave numbers) we plan to use the discontinuity capturing (DC) method which introduces a dissipative effect in the direction of the vector normal to the discontinuity front. This approach produced very satisfactory results for the model problems studied. Furthermore, with our adaptive implicit-explicit (AIE) method we will be able to have an implicit refinement around sharp layers. This will not only increase the efficiency of the computations substantially, but also, and the most importantly, will result in stable and accurate results near sharp layers.

PROBLEM STATEMENT

Let Ω be an open region $\Re^{nsd}$, where nsd is the number of space dimensions. The boundary of Ω is denoted by Γ. Spatial coordinates are denoted by $\mathbf{x} \in \Omega$.

Assuming real gas behavior with equilibrium chemistry, mass, momentum and energy balance equations are given as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\mathbf{u}\rho) = 0 \qquad \text{(mass balance)} \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \bullet (\mathbf{u}\rho\mathbf{u} - \sigma) = 0 \qquad \text{(momentum balance)} \qquad (2)$$

$$\frac{\partial}{\partial t}(\rho e) + \nabla \bullet (\mathbf{u}\rho e - \sigma \mathbf{u} + \mathbf{q}) = 0 \qquad \text{(energy balance)} \qquad (3)$$

where ρ, $\mathbf{u}$, σ, $\mathbf{q}$ and e are the density, velocity vector, stress tensor, heat flux vector and total specific energy, respectively.

Assuming Newtonian fluids which obey the Fourier's law of heat conduction we can express the constitutive relations as follows:

$$\sigma = -p\mathbf{I} + \tau \qquad (4)$$

$$\tau = \lambda(\nabla \bullet \mathbf{u})\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \qquad (5)$$

$$\mathbf{q} = -\kappa \nabla T \qquad (6)$$

In the above expressions p is the pressure, τ is the viscous stress tensor, λ and μ are the two coefficients of viscosity, T is the temperature, and κ is the conductivity.

The total energy e can be written as

$$e = i + (1/2)\ \|u\|^2 \tag{7}$$

where i is the specific internal energy. We assume that for real gases, pressure and temperature data is available in a form which can be expressed as follows:

$$p = p(\rho, i) \tag{8}$$

$$T = T(\rho, i) \tag{9}$$

Remark

In a more compact vector form, equations(1) - (3) can be written as follows:

$$\frac{\partial U}{\partial t} + \nabla \bullet \mathbf{F}(U) = \mathbf{0} \tag{10}$$

where $\mathbf{U}(x,t)$ is the vector of unknown variables and $\mathbf{F}(U)$ is the flux vector.

We assume that associated with (10) the following boundary and initial conditions are given:

$$\mathbf{B}\,\mathbf{U} = \mathbf{G} \qquad \text{on } \Gamma \tag{11}$$

$$\mathbf{U}(x,0) = \mathbf{U_0}(x) \qquad \text{on } \Omega \tag{12}$$

where $\mathbf{B}$ (U) is a boundary operator and G and $\mathbf{U_0}(x)$ are given functions.

THE FINITE ELEMENT FORMULATION

Consider a discretization of Ω into element subdomains Ω^e, e = 1,2,..., n_{el} where n_{el} is the number of elements. Let Γ^e denote the boundary of Ω^e. We assume

$$\bar{\Omega} = \bigcup_{e=1}^{n_{el}} \bar{\Omega}^e \tag{13}$$

$$\varnothing = \bigcap_{e=1}^{n_{el}} \Omega^e \tag{14}$$

Throughout, we shall assume that trial solutions, **U**, satisfy **BU** = **G** on Γ and weighting functions, **W** , satisfy **BW** = **0** on Γ.

The weak(variational) form of (10) -(12) is given as follows:

$$\int_{\Omega} \mathbf{W} \bullet \left(\partial \mathbf{U}/\partial t + \nabla \bullet \mathbf{F} \right) d\Omega = 0 \tag{15}$$

$$\int_{\Omega} \mathbf{W} \bullet \left(\mathbf{U}(x,0) - \mathbf{U}_0(x) \right) d\Omega = 0 \tag{16}$$

Remark

If the weighting functions are selected from the same set of functions as the trial solutions then (15) is a regular (Bubnov-) Galerkin formulation; else, it is a Petrov-Galerkin formulation. The significance of the Petrov-Galerkin formulation will be explained in the sequel.

Spatial discretization of (15-16) is carried out by expanding **U** and **W** in terms of a set of finite element basis functions corresponding to (13-14). For example the expression for **U** might take the form

$$U(x,t) = \sum_{B} N_B(x)\ U_B(t) \tag{17}$$

where B is the nodal index, N_B is the finite element basis function associated with node B, and U_B is the nodal value of U.

The spatial discretization leads to a set of semi-discrete, ordinary differential equations:

$$\mathbf{M}\ \dot{\mathbf{V}} + \mathbf{N}\ (\mathbf{V}) = \mathbf{F} \tag{18}$$

with the initial condition:

$$\mathbf{V}(0) = \mathbf{V_0} \tag{19}$$

where V is the vector of nodal values of U, $\mathbf{V_0}$ is its given initial value, M is a generalized "inertia" matrix, N is a nonlinear vector function of V, and F is a "force" vector. A superposed dot denotes time differentiation.

Given the initial condition (19), equation (18) can be integrated by an appropriate temporal discretization scheme.

CONCLUSIONS

We have started with our development efforts and preliminary computations for finite element formulations applied to compressible flows.We are particularly interested in the proper numerical treatment of sharp layers occuring in such problems We also target the most general mesh generation capabilities for intricate computational geometries. Our methods will be enhanced with several state-of-the-art techniques such as the streamline-upwind/Petrov-Galerkin, discontinuity capturing, adaptive implicit-explicit, and grouped element-by-element approximate factorization schemes. Our preliminary computations show that these methods are stable, have minimal numerical dissipation, and are very reliable.