# COUPLED ROTOR-BODY EQUATIONS OF MOTION HOVER FLIGHT

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# ABSTRACT

A set of linearized equations of motion to predict the linearized dynamic response of a single rotor helicopter in a hover trim condition to cyclic pitch control inputs is described. The equations of motion assume four fuselage degrees of freedom; lateral and longitudinal translation, roll angle, pitch angle, four rotor degrees of freedom; flapping (lateral and longitudinal tilt of the tip path plane), lagging (lateral and longitudinal displacement of the rotor plane center of mass) and dynamic inflow (harmonic components). These ten degrees of freedom correspond to a system with eighteen dynamic states. In addition to examination of the full system dynamics, the computer code supplied with this report permits the examination of various reduced order models. The code is presented in a specific form such that the dynamic response of a helicopter in flight can be investigated. As described in the report, with minor modifications to the code the dynamics of a rotor mounted on a flexible support can also be studied.

## INTRODUCTION

The equations of motion for the linearized small perturbation motion of a single rotor helicopter about a hovering trim condition were formulated using a Lagrangian approach. For a linearized investigation of the hovering flight dynamics it can be assumed that the fuselage center of mass remains in an horizontal plane during the disturbed motion if collective inputs are not of interest. Vertical translation will be uncoupled from horizontal translation and fuselage roll and pitch. Thus the collective pitch is assumed constant at its trim value. Also as a consequence of this assumption, the coning angle and the average value of the induced velocity are also assumed to be constant in the code. The influence of the tail rotor is neglected and the yaw angle is assumed to be constant at its trim value, as coupling of the degree of freedom will be relatively weak near hover. Also the steady value of the lag angle is assumed constant at its trim value. The documentation of a larger code for hover and translational flight trim conditions which includes the vertical and yaw degrees of freedom as well as the tail rotor effects is nearly complete and will be available in the near future.

The fuselage center of mass is assumed to lie on the rotor shaft and no fuselage aerodynamics are included. Thus the hover trim condition corresponds to zero values of longitudinal and lateral cyclic pitch. Only the specific formulation for a helicopter in flight is considered in this discussion. See the

Discussion section for a description of modifications to the code to examine other problems such as ground resonance.

The rotor dynamics are included in the code through the use of multiblade coordinates. The longitudinal and lateral tilt of the rotor plane relative to the shaft are described by the coordinates $a_{1s}$, $b_{1s}$. The lag motion is described by $\gamma_1$ and $\gamma_2$ corresponding to lateral and longitudinal displacement of the rotor center of mass. These are the first four state variable in the computer program. Four body coordinates are fuselage pitch ($\theta_F$), fuselage roll ($\phi_F$), lateral translation ($y_F$) and longitudinal translation ($x_F$). Note that longitudinal translation is positive to the rear. This particular choice of coordinate depends on the selection of the elements in the T matrix as described below. Two state variables $v_c$ and $v_s$ describe the dynamic inflow. Thus the displacement variables for the full system are

$$\{a_{1s},\ b_{1s},\ \gamma_1,\ \gamma_2,\ \theta_F,\ \phi_F,\ y_F,\ x_F,\ v_c,\ v_s\}$$

The dynamic inflow variables are first order while the other variables are second order so that there are a total of eighteen state variables. The system equations are ordered such that the rotor and body displacement coordinates are followed by the rotor and body velocities and then the dynamic inflow components. Thus for the full system the state variables appear in the following order in the computer code output,

$$\{x_i\}^T = \{a_{1s},\ b_{1s},\ \gamma_1,\ \gamma_2,\ \theta_F,\ \phi_F,\ y_F,\ x_F,\ \dot{a}_{1s},\ \dot{b}_{1s},\ \dot{\gamma}_1,\ \dot{\gamma}_2,$$
$$\dot{\theta}_F,\ \dot{\phi}_F,\ \dot{y}_F,\ \dot{x}_F,\ v_c,\ v_s\}^T$$

The control variables are,

$$\{U_i\}^T = \{A_{1s},\ B_{1s}\}$$

For the reduced order system option without dynamic inflow, the last two state variables ($v_c, v_s$) are dropped and there are sixteen state variables.

For the quasi static option, both the rotor blade motion and the dynamic inflow become algebraic variables and are eliminated from the equations of motion to yield eight state variables, describing the fuselage motion

$$\{x_{qs}\} = \{\theta_F,\ \phi_F,\ y_F,\ x_F,\ \dot{\theta}_F,\ \dot{\phi}_F,\ \dot{y}_F,\ \dot{x}_F\}^T$$

The quasi-static case has two options, with and without dynamic inflow.

Note that since there is no dependence in the equations of motion on fuselage displacement ($y_F$, $x_F$) so in all of these cases there will be two zero eigenvalues.

While the quasi-static model is useful for developing physical insight, it has been shown in Reference 1 by comparison of this theory with flight test that the rotor-body coupling and dynamic inflow are significant in predicting the actual response of a helicopter to cyclic pitch.

The equation of motion are formulated for an articulated rotor helicopter with equal flap and lag hinge offset. Hingeless rotor helicopter dynamics can be approximated by the addition of springs about the flap and lag hinges. These spring constants appear in the input file.

The rotor blade element aerodynamics are assumed linear, i.e., the blade element lift coefficient is assumed to be a linear function of blade element angle of attack and the blade profile drag coefficient is assume constant. With these assumptions the blade aerodynamic forces can be integrated along the blade span to the flapping hinge and multiblade coordinates introduced to describe the rotor motion. The dynamic inflow model used in the computer code is described in Appendix I and is due to Peters. Rotor angular velocity is assumed constant.

## NOMENCLATURE

### Rotor Geometry

$\theta_B = \theta_o - A_{1s} \sin\psi - B_{1s} \cos\psi + \Delta\theta_H + \Delta\theta_E$

$\psi$ = rotor blade azimuth angle measured from longitudinal axis ($x_F$)

$\theta_o$ = collective pitch assumed constant, calculated in code (TNOT), rad.

$A_{1s}, B_{1s}$ = lateral and longitudinal cyclic pitch, control input terms corresponding to lateral and longitudinal stick deflection, radians

$\Delta\theta_H$ = blade pitch change due to hinge coupling

$\Delta\theta_H = d^* \Delta\beta + e^* \Delta\zeta$

$d^*$ corresponds to a $\delta_3$ hinge

$e^*$ corresponds to $\alpha_2$ hinge

$\Delta\theta_E$ = blade pitch change due to elastic deformation of swash plate

$\Delta\theta_E = \Delta\theta_c \cos\psi + \Delta\theta_s \sin\psi$

$\Delta\theta_c = (A - 1)\ \theta x_H + B\theta y_H + Cx_H + Dy_H$

$\Delta\theta_s = E\theta x_H + (F - 1)\ \theta y_H + Gx_H + Hy_H$

For rigid shaft A = 1, F = 1, B=C=D=E=G=H=0

$\beta = a_o - a_{1s} \cos\psi - b_{1s} \sin\psi$, flap angle, positive up. rad

$a_o$ = coning angle assumed constant, calculated in code (BETA), rad

$a_{1s}$, $b_{1s}$ = multiblade flapping coordinates (state variables) $a_{1s}$ longitudinal tilt, positive for flap back, rad

$b_{1s}$ lateral tilt, positive for tilt down on right side, rad

$\zeta = \zeta_o - \gamma_1 \cos\psi - \gamma_2 \sin\psi$, lag angle, positive for lag opposite to rotation, rad

$\zeta_o$ = steady lag angle assumed constant, calculated in code (RNOT), rad. This quantity does not appear elsewhere in the code due to the use of multiblade coordinates

$\gamma_1, \gamma_2$ = multiblade lag coordinates $\gamma_1$ corresponds to lateral displacement of the rotor center of mass, positive to the right, and $\gamma_2$ correspond to longitudinal displacement, positive forward, rad

Inflow

$v = v_o + v_c \times \cos\psi + v_s \times \sin\psi$, positive downard, fps

$v_o$ = average induced velocity, assumed constant, calculated in code (VNNT), fps

$v_c$, $v_s$ = harmonic components, tip amplitude, calculated as described in Appendix I

Fuselage

h = distance between fuselage CG and hub, ft

Hub Motion

$x_H$ = longitudinal displacement of hub, positive aft, ft

$y_H$ = lateral displacement of hub, positive to the right, ft

$\theta y_H$ = pitch angle, positive nose up, rad

$\theta x_H$ = roll angle, positive left side down, rad

<u>Generalized Coordinates</u>

$x_H \;\; = \Sigma \; tx_i \; q_i$

$y_H \;\; = \Sigma \; ty_i \; q_i$

$\theta y_H = \Sigma \; t_{\theta y_i} \; q_i$

$\theta x_H = \Sigma \; t_{\theta x_i} \; q_i$

$tx_i$, $ty_i$, $t_{\theta y_i}$, $t_{\theta x_i}$ = elements of T matrix relating choice of generalized coordinates ($q_i$) to hub motion. These are selected in the code as

$q_1 = \theta_F$ , fuselage pitch angle, positive nose up, rad

$q_2 = \phi_F$ , fuselage roll angle, positive right side down, rad

$q_3 = y_F$ , fuselage center of mass lateral displacement, positive to the right, ft.

$q_4 = x_F$ , fuselage center of mass longitudinal displacement, positive to the rear, ft

$T = \{tx_i, ty_i, t_{\theta x_i}, t_{\theta y_i}\}$, transformation matrix

$$T = \begin{bmatrix} h & 0 & 0 & 1 \\ 0 & h & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Note that the sign of the roll angle coordinate has been reversed in the transformation matrix.

## DISCUSSION

In order to investigate various coupled rotor-body problems such as: 1) The dynamics of a helicopter disturbed from hovering flight; 2) The dynamics of a helicopter resting on the ground (ground resonance); 3) The dynamics of a rotor on a flexible mount, the effect of hub motion is treated in the following way. Hub motion is assumed to be horizontal displacement in two directions (x,y), and rotation in pitch and roll ($\theta_y$, $\theta_x$),

$$\{x_H\} = \{x,\ y,\ \theta_y,\ \theta_x\}^T.$$

Generalized coordinates ($q_i$) are used to describe the support motion and are related to the hub motion by a transformation matrix T:

$$\{x_H\} = [T]\ \{q_i\}$$

The matrix T can be selected to represent a variety of supports.

The equations of motion are of the general form:

Rotor Equations:

$$\Delta_{RR}\{x_R\} \quad + \Delta_{RH}\{x_H\} = 0$$

Support Equations:

$$\Delta_{SRR}\{x_R\} + \Delta_{SHR}\{x_H\} + \Delta_{SHS}\{x_H\} = 0$$

The $\Delta$'s are second order operators with the following interpretation

$\Delta_{RR}$ = rotor forces and moments due to rotor motion

$\Delta_{RH}$ = rotor forces and moments due to hub motion

$\Delta_{SRR}$ = support forces and moments from rotor due to rotor motion

$\Delta_{SHS}$ = support forces and moments from support due to hub motion

$\Delta_{SHR}$ = support forces and moments from rotor due to hub motion

Now to express the equations of motion in terms of support motion $\{q_i\}$ the transformation

$$\{x_H\} = [T] \{q_i\}$$

is introduced and the equations of motion expressed as:

$$\Delta_{RR}\{x_R\} + (\Delta_{RH}T)\{q_i\} = 0$$

$$\Delta_{SRR}\{x_R\} + (\Delta_{SHS}T + \Delta_{SHR}T) \{q_i\} = 0$$

This is the general form of the coupled rotor-support equations of motion. Thus, the equations of motion of a rotor on a rigid support with no hub motion ($q_i = 0$) are:

$$\Delta_{RR}\{x_R\} = 0$$

and the equations of motion of the support without the rotor are

$$\Delta_{SHS}T \{q_i\} = 0$$

It is assumed in the equations of motion given below that the matrix of operators ($\Delta_{SHS}T$) describing the support system motion is diagonal and is of the form:

$$\Delta_{SHS}T = \frac{1}{2} [\bar{M}_{ii} S^2 + \bar{C}_{ii} S + \bar{K}_{ii}]$$

Thus the $q_i$'s are generalized coordinates describing the support degrees of freedom. Care must be taken in the selection of the support motion coordinates, such that the matrix ($\Delta_{SHS}T$) is diagonal. The factor 1/2 appears due to the normalization of of support equations of motion.

$$\frac{1}{2}\,\bar{M}_{ii} = \frac{2}{bI_B}\,(M_{ii})$$

The system matrices are of the general form,

$$[M]_1 = \begin{bmatrix} M_{RR} & M_{RH} \\ M_{SRR} & (M_{SHS} + M_{SHR}) \end{bmatrix}$$

etc.

Potential Energy and Virtual Work Due to Thrust

The potential energy terms and the virtual work due to the rotor thrust must be treated carefully since second order terms are involved which are sensitive to the assumptions made regarding support motion. The rotor terms are calculated based on the assumption that the hub motion takes place in horizontal plane. The linearized equations of motion assuming either horizontal motion of the fuselage support or horizontal motion of the hub, are the same as shown below. The potential energy and virtual work due to thrust must be treated carefully. These terms will be different in the two cases but their sum will be the same. It is necessary to include in the evaluation of these terms the effect of a vertical displacement (z). It is desirable to consider the formulation with the fuselage or support center of mass translational displacement in a horizontal plane. This will also be a suitable approximation if the rotor is on a flexible mount.

## Rigid Fuselage/Shaft - Rotorcraft in Flight

First consider the case in which the equations of motion are used to examine the dynamics of a helicopter in flight where the shaft is assumed to be rigid. Both vertical motion of the hub and fuselage are permitted at this point. The potential energy of the system is (Fig. 1):

$$V = M_F g\ z_F + bM_B g\ z_H \tag{1}$$

and the virtual work due to the thrust is:

$$\begin{aligned} \delta W_T &= T\cos\theta_x \sin\theta_y\ \delta x_H - T\sin\theta_x \delta y_H \\ &\quad + T\cos\theta_x \cos\theta_y\ \delta z_H \end{aligned} \tag{2}$$

The thrust, T, is assumed constant since the trim condition is hovering flight, however since it has a non-zero value in trim the virtual work due to the second order displacement $\delta z_H$ must be included since it will produce a first order term in the virtual work.

The relationship between the fuselage displacements and the hub displacements are:

$$\begin{aligned} x_H &= x_F + h' \cos\theta_x \sin\theta_y \\ y_H &= y_F - h' \sin\theta_x \\ z_H &= z_F + h' \cos\theta_x \cos\theta_y \end{aligned} \tag{3}$$

h' is the height of the rotor center of mass above the fuselage center of mass.

$$h' = (h + \frac{S_B}{M_B}\beta_o)$$

It can be shown that the linearized equations of motion that result from assuming either hub motion constrained to a horizontal plane ($z_H = 0$) or fuselage motion constrained to a horizontal plane ($z_F = 0$) will be the same if the potential energy and virtual work terms are evaluated consistent with either of these approximations. The vertical velocity of the hub ($\dot{z}_H$) does not appear in the rotor kinetic energy terms due to the hub motion assumption used in deriving the rotor terms in the equations of motion. Using the alternate assumption that the fuselage center of mass motion is constrained to the horizontal plane ($z_F = 0$), there is no linearized contribution to the kinetic energy from $\dot{z}_H$ which arises only due to rotation of the shaft. Therefore, it is only in the potential energy and virtual work terms which arise due to the thrust vector that the effect of this different assumption must be considered. The effect of rotation on vertical displacement must be included in the evaluation of these terms.

Horizontal Fuselage Motion

First consider the evaluation of the potential energy and virtual work based on the assumption that the fuselage center of mass remains in a horizontal plane ($z_F = 0$). In this case, potential energy terms will arise due to the vertical displacement of the rotor mass.

Equation (1) becomes,

$$V = bM_B g\ z_H = bM_B\ gh'\cos\theta_x\ \cos\theta_y$$

$$\frac{\partial V}{\partial q_i} = (-bM_B gh')(\sin\theta_x \cos\theta_y \frac{\partial\theta_x}{\partial q_i} + \cos\theta_x \sin\theta_y \frac{\partial\theta_y}{\partial q_i})$$

$$\cong - bM_B gh' \ (\theta_x \frac{\partial\theta_x}{\partial q_i} + \theta_y \frac{\partial\theta_y}{\partial q_i})$$

These potential energy terms are considered apparent spring terms in the computer code since they depend upon the support model assumption. They are entered directly in the input file as $K_{ii}$ terms. They take the following form,

$$\frac{\partial V}{\partial q_i} = - bM_B gh'((\Sigma \ t_{\theta xi} q_i) \ t_{\theta xi} + (\Sigma \ t_{\theta yi} q_i) \ t_{\theta yi})$$

For the coordinate selection given below,

$$\frac{\partial V}{\partial q_1} = \frac{\partial V}{\partial \theta_F} = - bM_B gh' \ \theta_F \tag{4}$$

$$\frac{\partial V}{\partial q_2} = \frac{\partial V}{\partial \phi_F} = - bM_B gh' \ \phi_F$$

The virtual work terms due to rotor thrust simplify for this assumption of ($z_F = 0$) by noting that since the rotor thrust lies along the shaft, angular motion of the shaft does not contribute to the virtual work. Using Equations (3), with $z_F = 0$, equation (2) becomes,

$$\delta W_T = T \cos\theta_x \sin\theta_y \ \delta x_F - T \sin\theta_x \ \delta y_F$$

$$= Q_{x_F} \ \delta x_F + Q_{y_F} \ \delta y_F$$

Consequently the linearized terms are

$x_F$ equation

$$Q_{x_F} = T\ \theta_y = T\ \Sigma\ t_{\theta yi}\ q_i$$

$y_F$ equation (5)

$$Q_{y_F} = -T\ \theta_x = -\ T\ \Sigma\ t_{\theta xi}\ q_i$$

These terms are entered directly in the spring matrix $[K]_1$ for for the specific form of the T matrix given below, i.e.,

$$t_{\theta y1} = 1$$

$$t_{\theta x2} = -1$$

In the computer code these are terms RK1(8,5) and RK1(7,6).

The transformation matrix is selected in this case to identify the $q_i$ as follows:

$$q_1 = \{\theta_F,\ \Phi_F,\ y_F,\ x_F\}^T$$

where

$\theta_F$ fuselage pitch, positive nose up

$\phi_F$ fuselage roll, positive right side down

$y_F$ fuselage lateral translation, positive to the right

$x_F$ fuselage longitudinal translation, positive to the rear

and the matrix T is

$$T_{ij} = [t_{xi} \quad t_{yi} \quad t_{\theta xi} \quad t_{\theta yi}]$$

$$T = \begin{bmatrix} h & 0 & 0 & 1.0 \\ 0 & h & -1.0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \end{bmatrix}$$

Note that the sign convention for the roll angle has been reversed ($t_{2,3} = -1.0$).

With this selection of coordinates the support terms are of the form

$$(M_{ii} \ddot{q}_i + K_{ii} q_i)$$

specifically

$q_1$ $(\theta_F)$

$$(I_y \ddot{\theta}_F - K_B \theta_F)$$

$q_2$ $(\phi_F)$

$$(I_x \ddot{\phi}_F - K_B \theta_F)$$

$q_3$ $(y_F)$

$$(M_F \ddot{y}_F)$$

$q_4$ $(x_F)$

$$(M_F \ddot{x}_F)$$

where $K_B$ is obtained from the potential energy terms (equations (4)) above.

$$K_B = - bM_B gh'$$

The spring constant $K_B$ is negative, representing the fact that the rotor mass is above the fuselage mass.

<u>Horizontal Hub Motion</u>

It will now be shown that if the alternate assumption ($z_H = 0$) is employed the same terms will result. In this case equations (1) and (2) become:

$$V = M_F g \; z_F$$

$$\delta W_T = T \cos\theta_x \sin\theta_y \; \delta x_H - T \sin\theta_x \; \delta y_H$$

and from Eq. (3)

$$\delta z_F = + h' \sin\theta_x \cos\theta_y \; \delta\theta_x + h' \cos\theta_x \sin\theta_y \; \delta\theta_y$$

$$\delta x_H = \delta x_F - h' \sin\theta_x \sin\theta_y \; \delta\theta_x + h' \cos\theta_x \cos\theta_y \; \delta\theta_y$$

$$\delta y_H = \delta y_F - h' \cos\theta_x \; \delta\theta_x$$

Assuming small angles and combining

$$\begin{aligned} \delta V - \delta W_T &= (M_F g - T)\, h' \,\theta_x \,\delta\theta_x \\ &\quad + (M_F g - T)\, h' \,\theta_y \,\delta\theta_y \\ &\quad + T \,\theta_y \,\delta x_F - T \,\theta_x \,\delta y_F \end{aligned} \tag{6}$$

From vertical equilibrium

$$T = M_F g + b \; M_B g \tag{7}$$

it can be seen that equations (6) and (7) give the same result for the two terms given by equations (4) and (5). The terms are:

$$\begin{aligned} \delta V - \delta W_T &= -bM_B gh' \,\theta_x \,\delta\theta_x \\ &\quad - bM_B \, gh' \,\theta_y \,\delta\theta_y \\ &\quad + T \,\theta_y \,\delta x_F - T \,\theta_x \,\delta y_F \end{aligned} \tag{8}$$

Rotor Mounted on Support - Fixed Support Point

Now consider the case in which the rotor is mounted on a flexible transmission/shaft system. There is assumed to be no vertical motion of the support point ($z_F = 0$). The potential energy is due only to the blade mass,

$$V = bM_B g\ z_H$$

The virtual work due to the thrust is given by equation (2). If we assume that the flexibility of the mounting is described by a number of normal modes of amplitude $q_i$, then the hub motion is related to the $q_i$ by the transformation matrix $\dot{T}$. The support equation of motion would then be of the form,

$$M_{ii}\{\ddot{q}_i\} + C_{ii}\{\dot{q}_i\} + K_{ii}\{q_i\} + \{\frac{\partial V}{\partial q_i} - Q_i\} + \Delta_{SRR}\{x_R\} + \Delta_{SHR}{}^T\ \{q_i\} = 0$$

where the effect of hub vertical motion ($z_H$) should be included in the evaluation of the potential energy term due to the blade mass and the virtual work term due to the thrust. In general as noted above it is not necessary in the linearized formulation to include the effect of $\dot{z}_H$ in the remaining terms due to the rotor. $M_{ii}$, $C_{ii}$, $K_{ii}$ are the generalized mass, damping and stiffness associated with the ith mode. In general $z_H$ will be a nonlinear function of shaft deflection, i.e., a function of the lateral deflection of the shaft squared. Physically this means that the $z_H$ deflection terms associated with thrust component and the potential energy term will give rise to apparent spring terms.

## Rigid Shaft

Consider first the rigid shaft case as discussed above. For small deflections,

$$
\begin{aligned}
\delta W_T &= T\ \theta_y\ \delta x_H - T\ \theta_y\ \delta y_H + T\ \delta z_H \\
\delta V &= b M_B h'\ \delta z_H
\end{aligned}
\tag{9}
$$

For rigid rotation of the shaft with no translation of the support motion ($\delta x_F = 0$, $\delta y_F = 0$)

$$
\begin{aligned}
\delta x_H &= h'\ \delta\theta_y \\
\delta y_H &= -h'\ \delta\theta_x \\
\delta z_H &= -h'\ \theta_x\ \delta\theta_x - h'\ \theta_y\ \delta\theta_y
\end{aligned}
\tag{10}
$$

Substitution of equations (10) into (9) shows that the virtual work is zero. An apparent spring is produced due to the potential energy term associated with blade mass. That is, physically, if the shaft is rigid the thrust does not produce a moment about the support. Thus to examine this problem the terms corresponding to the virtual work in the computer code should be removed (RK1(8,5), RK1(7,6)).

## Flexible Shaft

Consider now a simple case in which the shaft has one deflection mode, in longitudinal plane only. The shaft deflection is represented by a mode shape $\eta_1$. The local deflection of the shaft is (Figure 2):

$$x_s = \eta_1(z)\ q_1$$

The local slope is

$$\theta_s = \frac{d\eta_1}{dz}(z)\ q_1$$

At the hub location, the mode shapes are represented by the T matrix coefficients.

$$\eta_1(h) = T_{x1}$$

$$\frac{d\eta_1}{dz}(h) = T_{\theta y1}$$

The z deflection of the hub is

$$\delta z_H = -\frac{\partial}{\partial q_1}\left(\frac{1}{2}\int_o^L \left(\frac{dx}{dz}\right)^2 dz\right)$$

where

$$\frac{\partial x}{\partial z} = \frac{d\eta_1}{dz}\ q_1$$

Thus

$$\delta z_H = -\left(\int_o^L \left(\frac{d\eta_1}{dz}\right)^2 dz\right) q_i\ \delta q_i$$

This is an apparent spring terms or an effective change in stiffness due to the potential energy and virtual work terms.

Note that the thrust is a "follower" force so that the apparent change in stiffness depends upon the mode shape. The virtual work due to the thrust is ($\delta y_H = 0$)

$$\delta W_T \equiv T\ \theta_y\ \delta x_H + T\ \delta z_H$$

Therefore,

$$\delta W_T \equiv T\ \left[T_{\theta y1}\ T_{x1} - \int_o^L \left(\frac{d\eta_1}{dz}\right)^2 dz\right] q_1\ \delta q_1$$

$$\equiv T\ \left[\frac{d\eta_1}{dz}(L)\ \eta_1(L) - \int_o^L \left(\frac{d\eta_1}{dz}\right)^2 dz\right] q_1\ \delta q_1$$

For a rigid shaft

$$\eta_1 = \frac{z}{L}$$

and

$$\delta W_T = 0$$

For a flexible shaft the sign of the apparent spring depends upon the sign of the term in brackets above. This term can be also expressed as,

$$\delta W_T \equiv T \left[ \int_o^L \eta_1 \frac{d^2\eta_1}{dz^2} dz \right] q_1 \ \delta q_1$$

by integrating by parts in the case $\eta_1(0) = 0$.

Thus the thrust will often tend to act as a negative spring even though a tension is produced due to the fact that it is a "follower" force (Figure 2).

These terms will be small, for a typical flexible shaft on which a rotor is mounted. That is, the thrust and blade weight will not have a strong effect on the shaft frequencies.

In the general case, with more complex shaft/transmission modes, the T coefficients correspond to the deflection and slope at the end of the shaft or support. If the mode characteristics are only available numerically then an approximation must be made to determine the $z_H$ dependence with deflection which should be of the general form indicated above. Note also that if the shaft is assumed rigid with a rotation about the base, the above equations for $\delta z_H$ will recover the correct result for a rigid shaft.

The generalized stiffness and mass are:

$$K_{11} = \int_{o}^{L} EI(\eta_1'')^2 \, dz$$

$$M_{11} = \int_{o}^{L} m(\eta_1)^2 \, dz$$

## SUMMARY

A linearized set of equations of motion for a coupled rotor-body system for the trim condition of hovering flight have been described. The rotor kinetic energy terms are derived based on the assumption the hub motion takes place in a horizontal plane. The formulation permits the analysis of various coupled rotor-body problems, including a rotor mounted on various supports and free flight. Support motion can be assumed to lie in a horizontal plane such that vertical displacement of the hub arises only from rotation (rigid shaft) or support flexibility if the potential energy and virtual work due to the thrust force account for the vertical displacement of the shaft. For this assumption, vertical displacement of the hub will be of second order and will not contribute to the kinetic energy, but will give rise to first order term in the potential energy and virtual work due to thrust. Other virtual work terms due to inplane forces and hub moments are not affected by this assumption.

The terms to be evaluated for the specific support assumption are:

$$V = M_F g \; z_F + bM_B g \; z_H \tag{1}$$

and

$$\begin{aligned} \delta W_T = & \; T \cos\theta_x \; \sin\theta_y \; \delta x_H - T \sin\theta_x \; \delta y_H \\ & + T \cos\theta_x \; \cos\theta_y \; \delta z_H \end{aligned} \tag{2}$$

The equations of motion as described in detail below and

currently in the computer program assume rigid shaft motion and fuselage motion in a horizontal plane ($z_F = 0$). This assumption yields the relationships in equation (8)

$$\begin{aligned} \delta V &= bM_B gh'(-\ \theta_x\ \delta\theta_x - \theta_y\ \delta\theta_y) \\ \delta W_T &= T\ \theta_y\ \delta x_F - T\theta_x\ \delta y_F \end{aligned} \tag{8}$$

The terms due to the virtual work of the thrust are entered directly in the matrices (RK1(8,5), RK1(7,6)) and their placement is reflected by selection of the $q_i$'s as noted. The potential energy terms give rise to apparent spring terms that are entered in the input file ($K_{11}$,$K_{22}$). Again their placement is reflected by the selection of the $q_i$'s.

Thus to study dynamic problems of a rotor on a support with other selections of generalized coordinates, the potential energy and virtual work due to the thrust must be calculated for the specific problem and entered suitably in the computer program. The virtual work terms are in a form corresponding to free flight and are due to thrust. In general they should be removed from the computer code (RK1(8,5), RK1(7,6)), for other problems.

## PROGRAM INPUT FILE AND OPTIONS

The computer program is set up to provide the equations of motion for a hovering helicopter in four cases:

1. Full system with dynamic inflow
2. Full system without dynamic inflow
3. Quasi-static system with dynamic inflow
4. Quasi-static system without dynamic inflow

The full system includes blade flap and lag degrees of freedom, body degrees of freedom and dynamic inflow. The dynamic inflow model is described in Appendix I of this report and the quasi-static formulation is given in Appendix II. For the full system with dynamic inflow, the state variables are,

$$\{x\} = \{a_1, b_1, \gamma_1, \gamma_2, q_1, q_2, q_3, q_4, v_c, v_s\}^T$$

The selection of the T matrix yields for the q's,

$$q_1 = \theta_F$$

$$q_2 = \phi_F$$

$$q_3 = y_F$$

$$q_4 = x_F$$

For the full system without dynamic inflow, $v_c$ and $v_s$ are dropped as state variables. For the quasi-static system,

$$\{x_{qs}\} = \{q_1, q_2, q_3, q_4\}^T$$

The rotor blade motion and the dynamic inflow are not state variables and are eliminated as explained in Appendix II. Note however that in the quasi-static case there is the option to include quasi-static inflow terms which will of course influence the results.

The data input file describes the physical characteristics of the helicopter. The sample included is for the case of a helicopter in flight with a rigid shaft.

The first three lines of the data file are the generalized mass, spring and damping associated with the $q_i$ degrees of freedom. These depend upon the choice of the T matrix (the next two lines of the data file). For the selection of the T matrix elements in the sample data file,

$$q_1 = \theta_F \quad , \; M_{11} = I_{yy}$$

$$q_2 = \phi_F \quad , \; M_{22} = I_{xx}$$

$$q_3 = y_F \quad , \; M_{33} = m_F$$

$$q_4 = x_F \quad , \; M_{44} = m_F$$

Note that these are inertial characteristics of the helicopter without rotor blades. The generalized spring terms arise from considerations discussed above and are equal to

$$K_{11} = K_{22} = - bM_B g \; (h + \frac{S_B}{M_B} \beta_o)$$

The generalized damping terms are equal to zero in this case.

The next two lines are the elements in the T matrix,

$$tx_i \quad , \; ty_i$$

$$t\theta x_i \; , \; t\theta y_i$$

The selection in the example is explained in the introduction. The sixth line in the data file contains

$C_\zeta$ = mechanical lag damping ft-lb/rad/sec

$K_{HL}$ = equivalent lag spring ft-lb/rad

$K_{HB}$ = equivalent flap spring ft-lb/rad

$\Omega$ = rotor rpm, rad/sec

The seventh line is,

$M_B$ = blade mass, slugs

$M_s$ = blade first mass moment, slug-ft

$I_B$ = blade flapping moment of inertia, slug-ft$^2$ (flap and lag moments of inertia assumed equal)

The eighth line is,

R = blade radius, ft

e = flap/lag hinge offset, ft

c = blade chord, ft

$\sigma$ = blade solidity (number of blades appears through solidity)

a = blade lift curve slope

The ninth line is,

$\rho$ = ambient air density, slug/ft$^3$

$\delta$ = blade average profile drag coefficient

The tenth line is,

d* = pitch change due to flap ($\delta_3$ hinge)

e* = pitch change due to lag

The eleventh and twelfth lines permit introduction of swash plate deflections with shaft bending. In the form given there is no swash plate deflection relative to the shaft with shaft motion. The thirteenth line is:

$T$ = trim thrust, lbs

$\bar{h}$ = dynamic inflow parameter

WRF = wake rigidity factor

$\bar{h}$ is the height of the equivalent cylinder of air to be accelerated and WRF is the wake rigidity factor which should be set equal to 2.0 as explained in Appendix I..

## APPENDIX I

### DYNAMIC INFLOW

The equations of motion also permit addition of dynamic inflow components. Since the trim condition is hovering flight only variable harmonic components are introduced. The harmonic components are assumed to vary linearly with radius.

$$\Delta v = v_c \times \cos\psi + v_s \times \sin\psi$$

The equations describing the harmonic inflow component amplitudes at the blade tip are:

$$\tau_I \dot{v}_c + v_c = - k_I \left[\frac{4C_M}{a\sigma}\right]$$

$$\tau_I \dot{v}_s + v_s = - k_I \left[\frac{4C_\ell}{a\sigma}\right]$$

The harmonic components are positive for downward flow through the rotor. $\tau_I$ is the time constant associated with response time of the inflow to changes in aerodynamic moments on the rotor ($C_M$, $C_\ell$) and $k_I$ is the proportionality between the aerodynamic moments and the inflow in the steady state. These are expressed as

$$\tau_I = \frac{\bar{h}}{2\bar{v}_o \Omega \, f_w}$$

$$k_I = \frac{a\sigma R\Omega}{2\bar{v}_o \, f_w}$$

There are two variable parameters in these expressions which are input parameters to the computer program

$\bar{h} = \frac{H}{R}$ where H is the height of an equivalent cylinder of air which must be accelerated. The theoretical value is H = .453R.

$f_w$ is the wake rigidity factor. The correct value of this parameter is 2.0 (non-rigid wake) although in some places in the literature it is taken as 1.0 (rigid-wake). This terminology is from Miller. ($f_w$ = (WRF)).

In addition the harmonic induced velocity terms will change the aerodynamic forces on the rotor blades. Denoting $v = \{v_c, v_s\}^T$.

The complete equations of motion with dynamic inflow are:

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{v} \end{Bmatrix} + \begin{bmatrix} C & 0 \\ -DYB2 & 1 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{v} \end{Bmatrix} + \begin{bmatrix} K & DYE \\ -DYB1 & -DYC \end{bmatrix} \begin{Bmatrix} x \\ v \end{Bmatrix}$$

$$= \begin{bmatrix} B \\ DF \end{bmatrix} \{u\}$$

Introducing the notation

$$C^*_{MB} = \frac{4C_{MB}}{a\sigma}$$

where $C^*_{MB}$ the nondimensional blade flapping moment has two parts, one due to the rotor and body motion, denoted $C^*_{MBo}$ , and one due to the harmonic inflow. Therefore,

$$C^*_{MB} = C^*_{MBo} + \frac{\partial C^*_{MB}}{\partial \bar{v}_c} \bar{v}_c$$

Note that in the equations above v is dimensional. In the notation of the computer program.

$$DYE\ v_c = -\frac{\gamma\Omega^2}{2} \frac{\partial C^*_{MB}}{\partial \bar{v}_c} \bar{v}_c = -\frac{\gamma\Omega}{2R} \frac{\partial C^*_{MB}}{\partial \bar{v}_c} v_c$$

$$DYC(1,1) = -\left[\frac{1}{\tau_I} + \frac{k_I}{\tau_I}\frac{1}{\Omega R}\frac{\partial C^*_{MH}}{\partial \bar{v}_c}\right]$$

$$DYC(1,1) = \frac{(-VNNT)(WRF)2}{HH} + \frac{PP}{\Omega R}(RMFVC + EB\ FNVC)$$

$$DYB = -PP\ C^*_{MB_o} = DYB2\ \dot{x} + DYB1\ x$$

$$PP = \frac{a\sigma R\Omega^2}{\bar{h}} = \frac{k_I}{\tau_I}$$

where

$WRF = f_w$

$VNNT = v_o$ average induced velocity, dimensional

$HH = h = \bar{h}\ R$, dimensional

$\bar{h}$ and WRF are input constants in computer program

$$\frac{\partial C^*_{MB}}{\partial \bar{v}_c} = -\ RMFVC$$

$$\frac{\partial C^*_{MH}}{\partial \bar{v}_c} = -\ [RMFVC + (EB)(FNVC)]$$

For the quasi-static inflow assumption,

$$DYCv = -\ DYB$$

and v can be eliminated from the force and moment equations. The additional terms in the force and moment equations accounting for inflow effects are,

$$-\ (DYE)(DYC)^{-1}\ DYB$$

The term due to $v_c$ equals,

$$\left\{\frac{\frac{\gamma\Omega}{2R}\left(\frac{\partial C^*_{MB}}{\partial \bar{v}_c}\right)\quad(-PP)}{\frac{1}{\tau_I}+\frac{k_I}{\tau_I}\frac{1}{\Omega R}\left(\frac{\partial C^*_{MH}}{\partial \bar{v}_c}\right)}\right\} C^*_{MB_o}$$

Substituting for PP

$$\frac{\gamma\Omega^2}{2}\left\{\frac{\frac{k_I}{\Omega R}\frac{\partial C^*_{MB}}{\partial \bar{v}_c}}{\frac{1}{\tau_I}+\frac{k_I}{\Omega R}\frac{\partial C^*_{MH}}{\partial \bar{v}_c}}\right\} C^*_{MB_o}$$

$$\frac{k_I}{\Omega R}=\frac{a\sigma}{2\bar{v}_o f_w}\quad,\qquad \frac{\partial C^*_{MB}}{\partial \bar{v}_c}\cong\frac{\partial C^*_{MH}}{\partial \bar{v}_c}\cong -\ \mathrm{RMFVC}\cong\frac{1}{4}$$

$$\frac{\gamma\Omega^2}{2}\qquad\frac{\left\{\frac{a\sigma}{8\bar{v}_o\ f_w}\right\}}{1+\left\{\frac{a\sigma}{8\bar{v}_o f_w}\right\}}\Bigg\}\ C^*_{MB_o}$$

This term can be combined with other aerodynamic terms in the equations of motion which are of the form

$$-\ \frac{\gamma\Omega^2}{2}\ C^*_{MB_o}$$

to yield,

$$-\ \gamma\ \left\{\frac{1}{1+\frac{a\sigma}{8\bar{v}_o f_w}}\right\}\ \frac{\Omega^2}{2}\ C^*_{MB_o}$$

Thus the effect of the inflow in the quasi-static case can be viewed as a reduction in Lock number. The effective Lock number is,

$$Y_E = \frac{Y}{1 + \frac{a\sigma}{8\bar{v}_o f_w}}$$

## APPENDIX II

## QUASI-STATIC FORMULATION

The quasi-static formulation is obtained by first separating the rotor and body degrees of freedom.

$$x_r = \{a_1,\ b_1,\ \gamma_1,\ \gamma_2\}^T \qquad\qquad v = \{v_c,\ v_s\}^T$$

$$x_b = \{q_1,\ q_2,\ q_3,\ q_4\}^T$$

The equations of motion are written as:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_r \\ \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_r \\ \dot{x}_b \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} x_r \\ x_b \end{Bmatrix}$$

$$+ [DYE]\ \{v\} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \{u\}$$

The equations of motion for the inflow components are:

$$\{\dot{v}\} = [DYC]\{v\} + [DYB1]\{x\} + \{DYB2\}\{\dot{x}\} + [DF]\ \{u\}$$

The quasi-static assumption is obtained by setting:

$$M_{11} = M_{21} = C_{11} = C_{21} = 0$$

$$\dot{v} = 0$$

solving the inflow equation,

$$\{v\} = -\ [DYC]^{-1}\ [DYB1]\{x\} - [DYC]^{-1}\ [DYB2]\{\dot{x}\} - [DYC]^{-1}\ [DF]\ \{u\}$$

This is substituted into the first set of equations:

$$[M]\ \{x\} + [C-DYE\ DYC^{-1}\ DYB2]\{\dot{x}\} + [K-DYE\ DYC^{-1}\ DYB1]\ \{x\}$$

$$= [F + DYE\ DYC^{-1}\ DF]\ \{u\}$$

Rewriting the equations:

$$[\tilde{M}]\ \{\ddot{x}\} + [\tilde{C}]\ \{\dot{x}\} + [\tilde{K}]\ \{x\} = [\tilde{F}]\ \{u\}$$

or

$$\begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_r \\ \ddot{x}_b \end{Bmatrix} \quad \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} \\ \tilde{C}_{21} & \tilde{C}_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_r \\ \dot{x}_b \end{Bmatrix} \quad \begin{bmatrix} \tilde{K}_{11} & \tilde{K}_{12} \\ \tilde{K}_{21} & \tilde{K}_{22} \end{bmatrix} \begin{Bmatrix} x_r \\ x_b \end{Bmatrix}$$

$$= \begin{bmatrix} \tilde{F}_1 \\ \tilde{F}_2 \end{bmatrix} \{u\}$$

The quasi-static assumption is:

$$\tilde{M}_{11} = \tilde{M}_{21} = \tilde{C}_{11} = \tilde{C}_{21} = 0$$

Solving the first equation for the rotor motion,

$$\{x_r\} = -\ \tilde{K}_{11}^{-1}\ \tilde{M}_{12}\ \{\ddot{x}_b\} - \tilde{K}_{11}^{-1}\ \tilde{C}_{12}\ \{\dot{x}_b\} - \tilde{K}_{11}^{-1}\ \tilde{K}_{12}\ \{x_b\}$$
$$+ \tilde{K}_{11}^{-1}\ \tilde{F}_1\ \{u\}$$

This is substituted into the second equation,

$$[\tilde{M}_{22} - \tilde{K}_{21}\ \tilde{K}_{11}^{-1}\ \tilde{M}_{12}]\ \{\ddot{x}_b\} + [\tilde{C}_{22} - \tilde{K}_{21}\ \tilde{K}_{11}^{-1}\ \tilde{C}_{12}]\ \{\dot{x}_b\}$$
$$+ [\tilde{K}_{22} - \tilde{K}_{21}\ \tilde{K}_{11}^{-1}\ \tilde{K}_{12}]\ \{x_b\}$$
$$= [\tilde{F}_2 - \tilde{K}_{21}\ \tilde{K}_{11}^{-1}\ \tilde{F}_1]\ \{u\}$$

These are the equations of motion for the quasi-static system

$$[QM]\ \{\ddot{x}\ \} + [QC]\ \{\dot{x}\ \} + [QK]\ \{X\ \} = [QF]\ \{u\}$$

These matrices are calculated in the subroutine QUASI.

COMPUTER PROGRAM

Subroutine Matrix 2 calculates the following matrices

[AX] [BX] [CX] [FX]

[DF] [DYB] [DYC] [DYE]

The complete system equations are:

$$[AX]\ \{\ddot{x}\} + [B]\ \{\dot{x}\} + [CX]\ \{x\} + [DYE]\ \{v\} = \{FX\}\ u$$

$$\{\dot{v}\} = [DYC]\ \{v\} + + [DYB1]\ \{x\} + [DYB2]\ \{\dot{x}\} + [DF]\ \{u\}$$

THE VARIABLES ARE

$$\{x\} = \{a_1,\ b_1,\ \gamma_1,\ \gamma_2,\ q_1,\ q_2,\ q_3,\ q_4\}^T$$

$$\{v\} = \{v_c,\ v_s\}$$

NOTE:

AX, BX, CX, FX are renamed M, C, K, F

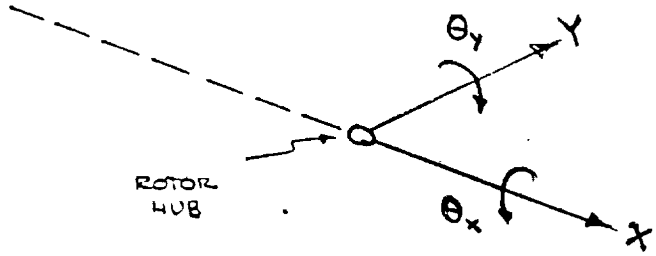
when Matrix 2 is called.

Complete system equations of motion

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \\ \dot{v} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -M^{-1}K & -M^{-1}C & -M^{-1}DYE \\ DYB1 & DYB2 & DYC \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \\ v \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1}F \\ DF \end{bmatrix} \{u\}$$

# REFERENCES

1. Studies in Interactive System Identification of Helicopter Rotor/Body Dynamics Using an Analytically-Based Linear Model, (with R. M. McKillip, Jr.). Paper presented at the Helicopter Handling Qualities and Control International Conference, London, England, November 15-17, 1988.




ORDERED ROTATION

$\theta_y$ ABOUT $Y_G$

$\theta_x$ ABOUT $X'$

$$\{X'\} = \begin{bmatrix} c\theta_y & 0 & -s\theta_y \\ 0 & 1 & 0 \\ s\theta_y & 0 & c\theta_y \end{bmatrix} \{X_G\}$$

TO SHAFT

$$\{X_S\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_x & s\theta_x \\ 0 & -s\theta_x & c\theta_x \end{bmatrix} \qquad \{X_S\} = \begin{bmatrix} c\theta_y & 0 & -s\theta_y \\ s\theta_x s\theta_y & c\theta_x & c\theta_y s\theta_x \\ c\theta_x s\theta_y & -s\theta_x & c\theta_x c\theta_y \end{bmatrix} \{X_G\}$$

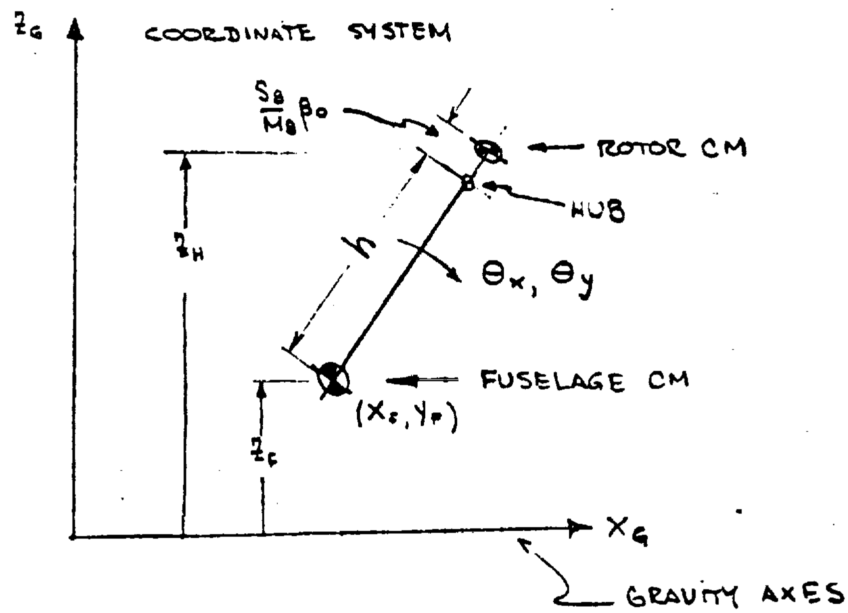
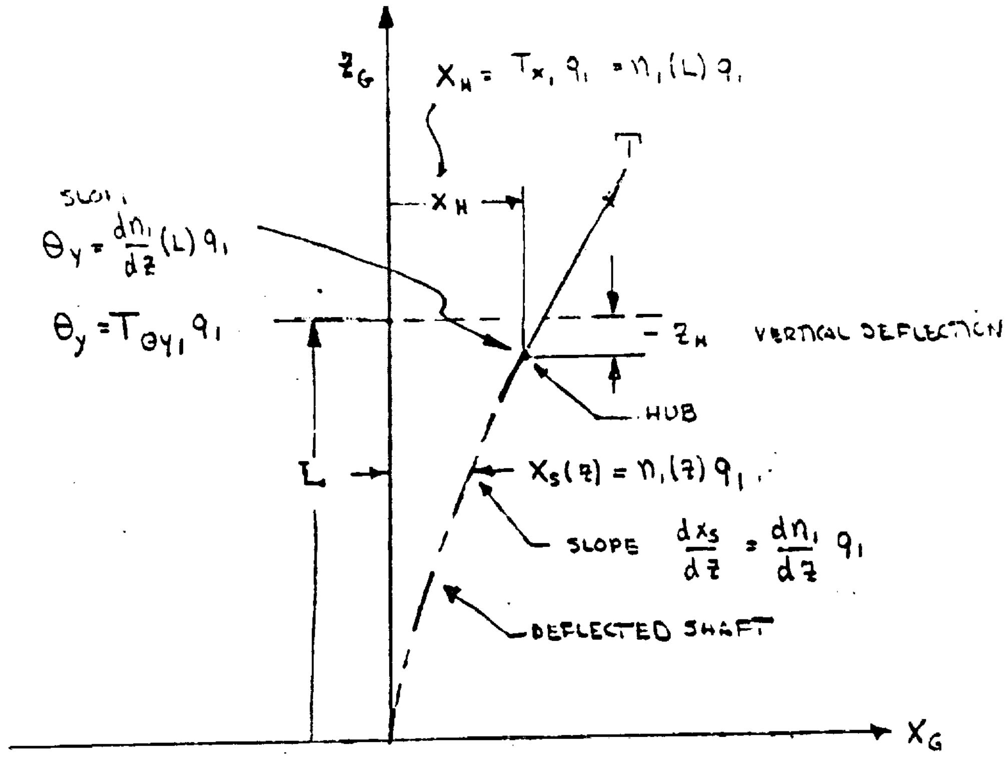



FIGURE 1 COORDINATE DEFINITIONS




$$Z_H \cong -\frac{1}{2}\int_0^L \left(\frac{dx_1}{dz}\right)^2 dz = -\frac{1}{2} q_1^2 \int_0^L \left(\frac{d\eta_1}{dz}\right)^2 dz$$

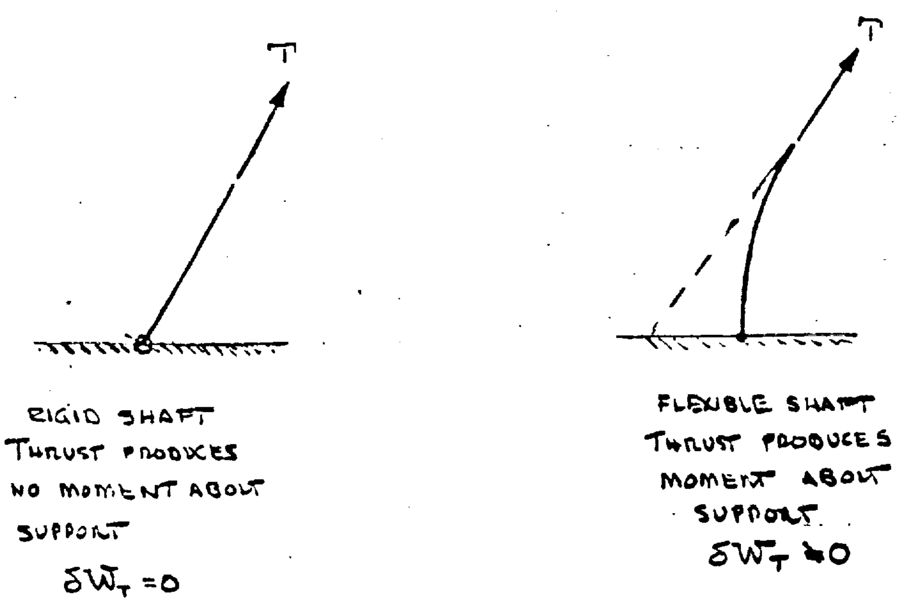



FIGURE 2 FLEXIBLE SHAFT SUPPORT

## SAMPLE DATA FILE
## (BHEFA.DAT)

## UH-60A BLACKHAWK PARAMETERS

```
38512.0, 4659.0, 460.9, 460.9
-7959.0, -7959.0, 0.0, 0.0
0.0, 0.0, 0.0, 0.0
6.87, 0.0, 0.0, 1.0, 0.0, 6.87, 1.0, 0.0
0.0, -1.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.0
4600,0.0,0.0,27.0
7.98, 86.70, 1512.6
26.83, 1.25, 1.73, 0.0821, 5.73
1.95E-03, 0.015
0.0, 0.0
1.0, 0.0, 0.0, 0.0
0.0, 1.0, 0.0 ,0.0
15870.0,0.46,2.00
```

BLACKHAWK data for flight test. Includes two additional parameters following the thrust: HH, the height of the equivalent cylinder of air; and WRF, the wake rigidity factor, equal to 1.0 for a rigid wake, and 2.0 for non-rigid wake.

===[ OPEN-LOOP EIGENVALUES ]===

| EIGENVALUE | REAL | IMAG |
|---|---|---|
| 1 | .000E+00 | .000E+00 |
| 2 | .000E+00 | .000E+00 |
| 3 | -9.095E+00 | 5.203E+01 |
| 4 | -9.095E+00 | -5.203E+01 |
| 5 | -1.983E+00 | 3.911E+01 |
| 6 | -1.983E+00 | -3.911E+01 |
| 7 | -2.576E+01 | 2.464E+00 |
| 8 | -2.576E+01 | -2.464E+00 |
| 9 | -1.353E+00 | 1.828E+01 |
| 10 | -1.353E+00 | -1.828E+01 |
| 11 | -2.997E+00 | 4.940E+00 |
| 12 | -2.997E+00 | -4.940E+00 |
| 13 | -4.263E+00 | .000E+00 |
| 14 | -1.511E+00 | .000E+00 |
| 15 | 5.173E-02 | 3.275E-01 |
| 16 | 5.173E-02 | -3.275E-01 |
| 17 | 6.505E-03 | 3.539E-01 |
| 18 | 6.505E-03 | -3.539E-01 |

SAMPLE PROGRAM OUTPUT

FULL SYSTEM WITH DYNAMIC INFLOW

INPUT DATA FILE (BHEFA.DAT)

Eigenvalue Identification

| | |
|---|---|
| (1,2) | Associated with lack of dependence on $x_F$, $y_F$ |
| (3,4) | Advancing flap mode |
| (5,6) | Advancing lag mode |
| (7,8) | Inflow mode |
| (9,10) | Regressing lag mode |
| (11,12) | Coupled roll/body flap mode |
| (13,14) | Coupled pitch/body flap modes |
| (15,16,17,18) | Coupled roll, pitch, body translation modes |

Notes on the usage and compilation of H5NEW:

H5NEW is an updated version of a program for calculation of helicopter rotor/body linearized dynamic equations of motion. The helicopter is assumed to be in hover; formulation is sufficiently general to accommodate a wide variety of helicopter and rotortypes, as explained in the detailed documentation. These paragraphs are meant to aid installation of the program on a machine other than that on which it was developed, namely, an IBM-PC with an 8087 numeric coprocessor.

Files on this distribution diskette include source code for the generation of matrices in the equations of motion, as well as executable code that should run, unmodified, on an IBM-PC class machine with an 8087 math chip installed. Scenarios are presented below if this is not your current installation.

If you will be running the program on another machine, the source must be recompiled and relinked into an executable module. One of the notable problems that will be immediately encountered is that the source for the eigenvalue/eigenvector routines is NOT on this diskette, due to space constraints. These routines, called from the subroutine "EIGSYS", are part of the standard EISPACK matrix subroutine package for real general matrices, and should (hopefully) be available at your installation. Note that the eigenanalysis portion of the main program H5NEW is really a post-processing function, and not intrinsic to the generation of the system matrices in the first place. Thus, one could just comment out the lines in the main program to eliminate the eigenanalysis, and then recompile and relink the code.

The file "LNK" contains a "batch"-like file for the linking process, following the successful recompilation of the program and subroutines on the disk. Linking of the object code (*.OBJ) can be done via:

link @LNK

provided that the EISPACK routines have been compiled and grouped into their own library. Compilation of the programs on another computer (under a UNIX or VMS environment) will of course depend upon the appropriate incantations necessary for the resident FORTRAN compiler, and are best formulated by a quick study of the contents of the listings.

The main routine is called "H5NEW", which reads in both controlling information and names of input data files. This main executive then passes control to "MATRIX2", which is given the task of computing all of the system matrices for the problem. Then, according to the wishes of the user, one or more support routines are used to combine the matrices produced from "MATRIX2". These include: FRST, a subroutine that reduces the mass-spring-damping formulation into 2-N sets of first-order ordinary differential equations; MINV, a matrix inversion routine, required in several matrix combination operations; QUASI, a subroutine that generates a "quasi-static" model based upon the full rotor+body dynamics equations; and EIGSYS, a subroutine that in turn calls EISPACK routines to compute system eigenvalues and eigenvectors for either the full or quasi-static systems.

Also on the disk is a file called OPTSYS.EXE, which is an executable program (again, requiring an 8087 chip) for computing system poles for matrix-vector representations of linear systems. This routine allows one to investigate effects of certain feedback on the pole locations of the closed-loop system.

BHEFA.DAT is an example data file for input to the program, with some additional commenting after the last line of numeric input.

Should installation questions arise, you may contact me for further help:

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