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DYNAMIC ANALYSIS OF SPACE-RELATED LINEAR AND NON-LINEAR STRUCTURES

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Abstract

In order to be cost-effective, space structures must be extremely light-weight, and subsequently, very flexible structures. The power system for Space Station Freedom is such a structure. Each array consists of a deployable truss mast and a split "blanket" of photo-voltaic solar collectors. The solar arrays are deployed in orbit, and the blanket is stretched into position as the mast is extended. Geometric stiffness due to the preload make this an interesting non-linear problem.

The space station will be subjected to various dynamic loads, during shuttle docking, solar tracking, attitude adjustment, etc. Accurate prediction of the natural frequencies and mode shapes of the space station components, including the solar arrays, is critical for determining the structural adequacy of the components, and for designing a dynamic controls system.

~~This~~ paper chronicles the process used in developing and verifying the finite element dynamic model of the photo-voltaic arrays. Various problems were identified ~~in the~~ investigation, such as grounding effects due to geometric stiffness, large displacement effects, and pseudo-stiffness (grounding) due to lack of required rigid body modes. ~~Various~~ analysis techniques, such as development of rigorous solutions using continuum mechanics, finite element solution sequence altering, equivalent systems using a curvature basis, Craig-Bampton superelement approach, and modal ordering schemes were utilized. ~~This~~ paper emphasizes the grounding problems associated with the geometric stiffness ARE EMPHASIZED

Nomenclature

a	factor defined by Eq.(13)
D_i	arbitrary constants in Eq. (10)

d/dx, or $'$	differential operator with respect to position
d/dt, or $\dot{}$	differential operator with respect to time
E	modulus of elasticity
e_a	axial strain
$\{F\}$	input force vector at the beginning of a step
$F(x,t)$	applied transverse force
g	factor defined by Eq.(14)
I	moment of inertia
$[K]$	stiffness matrix
$[K_e]$	elastic stiffness matrix
$[K_g]$	geometric stiffness matrix
L	length
M	moment
dM	change in moment
m	mass per unit length
P	axial force
P'	pseudo-force necessary for equilibrium
$\{R\}$	force vector, output force vector at the end of a step
T	kinetic energy
$\{u\}$	displacements at the node points
u	longitudinal displacement
U_A	strain energy due to axial load
U_B	strain energy due to bending
v	transverse displacement
V	shear
dV	change in shear
$\underline{V}$	potential of the external loads
$dVol$	change in volume
w	natural frequency
x	axis defined by Figure 1
y	axis defined by Figure 1
β	1/2 the angle of rotation
δ	factor defined by Eq.(11)
ϵ	factor defined by Eq.(12)
θ	angle of rotation
σ	stress

Introduction

NASA's Space Station Freedom consists of various modules supported by a space truss. Power for the space station will be provided by a deployable system of split blanket photo-voltaic arrays, which will have two degree of freedom rotational capabilities in order to track the sun during its orbit. The arrays are designed to be operated in a zero-gravity environment.

NASA Lewis Research Center, along with its contractors, have the responsibility for developing a verified finite element dynamics model of the solar arrays, which could be combined with the other space station substructures for both structural and dynamic control studies. The development of the model necessitated the use of unique procedures, and rigorous analytical checks.

The procedure included the following:

1. Development of an idealized model of the solar arrays, and derivation of a unique solution for the response frequencies for the idealized array cantilevered from the space truss, using equations developed from continuum mechanics.[1]

2. Comparison of the frequencies from the MSC/NASTRAN finite element dynamic model of the idealized array with the rigorous solution from continuum mechanics.[2]

3. Refinement of the finite element mesh.

4. Rigid body mode checks of the finite element models.

5. Various parameter studies involving the amount of tension in the blanket, rigidity of the blanket tip beam, type of elements used, etc..

6. Craig-Bampton approach for appending rigid body modes to substructures (superelements) [3].

7. Modal ordering schemes for identifying "important" modes.

8. Study of grounding effects due to lack of rigid body mode capabilities.[4]

A detailed summary of the project was presented [5]. It should be noted that this study is ongoing at the present time.

This paper will be restricted to the grounding problems associated with the geometric stiffness due to blanket pre-load.

Grounding

The space station solar arrays were modeled utilizing MSC/NASTRAN. As a routine check, the stiffness matrices generated by the model were multiplied by a matrix of rigid body modes, and large pseudo-forces were developed (grounding). The cause of this "grounding" phenomenum was examined.

Finite element solves non-linear problems of the form

$$[[Ke] + [Kg]] * \{u\} = \{R\} - \{F\}$$

where [Ke] is the elastic stiffness matrix, and [Kg] is the geometric, or initial stress stiffness matrix.

[Kg] is a function of the pre-load. Thus, it equals zero for a linear problem. [Ke] possesses the required rigid body modes. However, [Kg] lacks the capacity for rigid body rotation. Hence, an erroneous stiffening, or "grounding", occurs when a pre-loaded beam deforms.

The traditional, or consistent geometric stiffness matrix, developed by Martin [6] and others, is

$$K_g = P \begin{bmatrix} 6/5L & 1/10 & -6/5L & 1/10 \\ 1/10 & 2L/15 & -1/10 & -L/30 \\ -6/5L & -1/10 & 6/5L & -1/10 \\ 1/10 & -L/30 & -1/10 & 2L/15 \end{bmatrix}$$

This matrix does not possess rigid body rotation capabilities. Various refinements to the geometric stiffness have been developed which contain higher order terms [6,7,8] . However, none of these possess all the rigid body modes. Bosela [4] developed a modified [Kg] with complete rigid body modes when used with an exact rigid body rotation matrix, but [Kg] lost some of its rigid body capabilities.

Closer examination of the traditional formulation of [Kg] indicated that there is a load imbalance in the representation, and that pseudo-forces occur to maintain equilibrium. (Fig. 1)

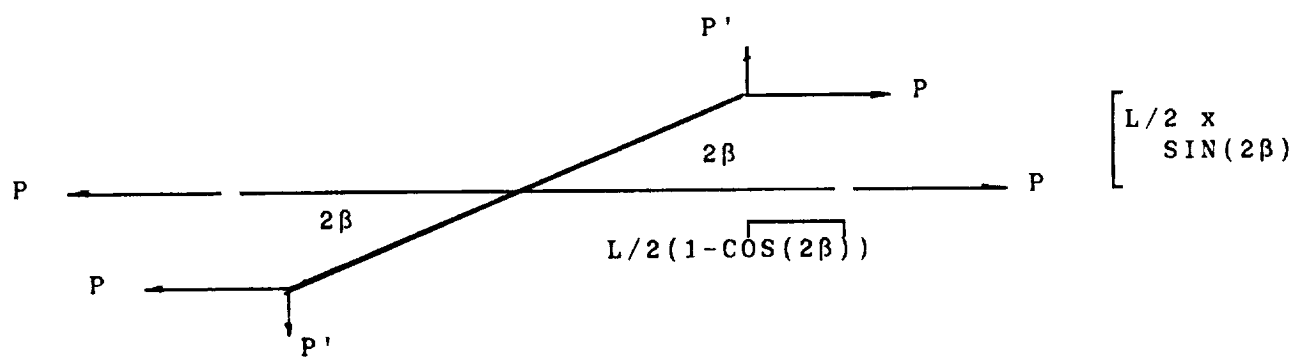

Fig 1. P' Represents Pseudo-forces Required for Equilibrium

In Reference [9], Collar and Simpson indicate that the lack of rigid body rotation capabilities for [Kg] is not a problem, because the energy representation is correct. It can be shown that it is correct to β^2 terms, but error does occur, as a function of β^4. For large rigid body rotation, as will occur with the solar arrays, this is significant.

It should be noted that as long as the pre-load P is assumed to remain horizontal during rotation, work will be done by the force. Thus, true rigid body rotation cannot occur. In order for the strain energy to equal zero, the force P must change its orientation as the beam rotates (ie. a follower force).

Rigorous Solution Of Pre-Loaded Beam

Suppose we have an axially loaded beam in space subjected to a time varying transverse loading (Figure 2).

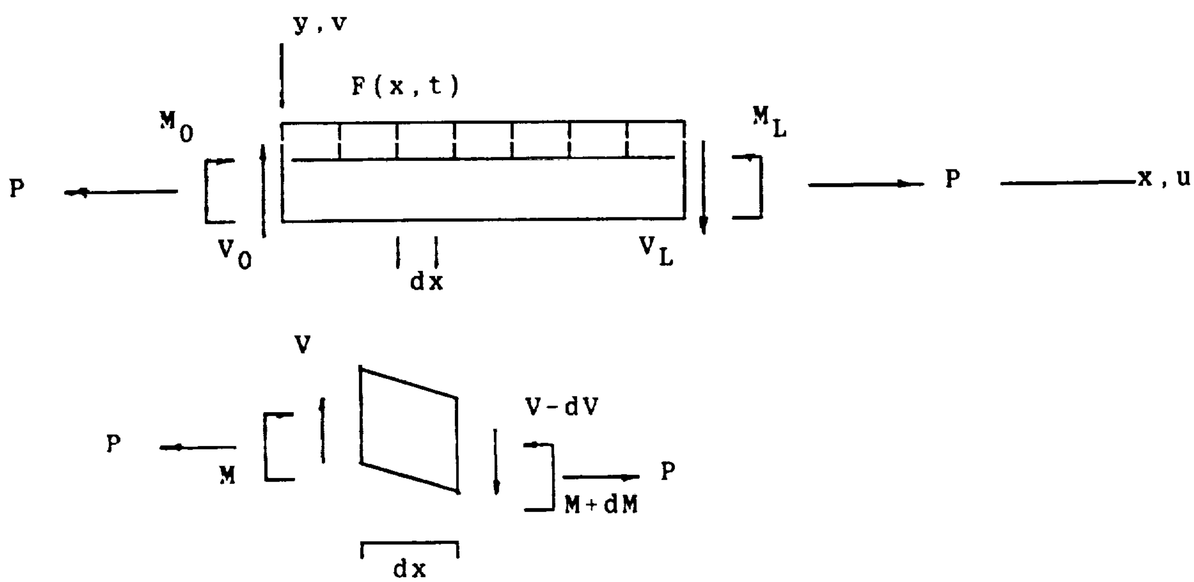

Figure 2   Beam in Tension and Differential Element

The kinetic energy is

$$T = \int_0^L \frac{m\ (v')^2}{2} dx \quad . \tag{1}$$

The strain energy due to bending is

$$U_B = \int \frac{E\ I}{2} (v'')^2\ dx \quad . \tag{2}$$

The strain energy due to axial load is

$$U_A = \frac{1}{2} \int \sigma\ e_a\ dVol \quad . \tag{3}$$

Letting dVol = dA dx and applying non-linear elasticity yields

$$U_A = \int \frac{EA}{2} \left[(du/dx)^2 + du/dx(dv/dx)^2 + 1/4(dv/dx)^4 \right] dx \quad . \tag{4}$$

Neglecting axial displacement and higher order terms yields

$$U_A = \int_0^L \frac{P}{2}\left[(v')^2\right] dx \qquad (5)$$

The potential of the external loads is

$$\underline{V} = -\int F(x,t)\ v\ dx + V_0\ v(0,t) + M_0\ v'(0,t) - V_L\ v(L,t) - M_L\ v'(L,t) \qquad (6)$$

Applying Hamilton's principle, and performing the variation, yields

$$\int_{t_1}^{t_2}\left[\int_0^L \left[EIv''\delta(v'')+Pv'\delta(v')-m\dot{v}\delta(\dot{v})-F(x,t)\delta(v)\right]dx + V_0\delta v(0,t)+M_0\delta v'(0,t)-V_L\delta v(L,t)-M_L\delta v'(L,t)\right]dt = 0 \quad . \qquad (7)$$

Integrating by parts yields the differential equation

$$d^2/dx^2(EId^2v/dx^2) - P\ d^2v/dx^2 + m\ d^2v/dt^2 = F(x,t) \quad , \qquad (8)$$

which agrees with Clough in reference [10], after a sign change required to express the axial force in tension instead of compression. This is also in agreement with Shaker in Reference [11].

For a beam in space, the moment and shear at the end points must equal zero. Thus, the boundary conditions are

$$EIv''(0,t)=EIv''(L,t)=v'''(0,t)-\frac{P}{EI}\ v'(0,t)=v'''(L,t)-\frac{P}{EI}\ v'(L,t)=0 \quad . \qquad (9)$$

Choose a solution of the form

$$v(x) = D_1\sin(\delta x) + D_2\cos(\delta x) + D_3\sinh(\epsilon x) + D_4\cosh(\epsilon x) \quad . \qquad (10)$$

where $$\delta = \left[(a^4+g^4/4)^{1/2}-g^2/2\right]^{1/2} \qquad (11)$$

$$\epsilon = \left[(a^4+g^4/4)^{1/2}+g^2/2\right]^{1/2} \qquad (12)$$

$$a4 = mw^2/EI \qquad (13)$$

$$g^2 = P/EI \qquad . \qquad (14)$$

Applying the boundary conditions at x=0, and after much mathematical manipulation, yields

$$v(x) = D_3\left[\frac{\delta \sin\delta x + \sinh\epsilon x}{\epsilon}\right] + D_4\left[\frac{\epsilon^2 \cos\delta x + \cosh\epsilon x}{\delta^2}\right] \qquad . \qquad (15)$$

Applying the boundary conditions at x=L, and after more mathematical manipulations, yields

$$D_3\left[\delta^3 \cosh\epsilon L - \delta^3 \cos\delta L\right] + D_4\left[\epsilon^3 \sin\delta L + \delta^3 \sinh\epsilon L\right] \qquad . \qquad (16)$$

Expressing Eq.(15) and Eq.(16) into matrix form, setting the determinant equal to zero, and after more mathematical manipulations, the following characteristic equation is obtained

$$\pm 2a^6(\cosh\epsilon L\cos\delta L - 1) + (\epsilon^6 - \delta^6)\sinh\epsilon L\sin\delta L = 0 \qquad . \qquad (17)$$

Using Eq.(13), this can be expressed as

$$\pm w^3(m/EI)^{3/2}(\cosh\epsilon L\cos\delta L - 1) + (\epsilon^6 - \delta^6)\sinh\epsilon L\sin\delta L = 0 \ . \qquad (18)$$

By observation, when w=0, a=0, and δ=0. Letting sin(0)=0 yields

$$w^3(m/EI)^{3/2}(\cosh\epsilon L\cos\delta L - 1) = 0 \qquad . \qquad (19)$$

The w^3 term indicates that there must be three zero roots of "w", which suggests the three required rigid body modes.

Conclusion

Lack of complete rigid body mode capabilities is inherent in the physical representation of the pre-tensioned beam problem currently used to formulate the geometric stiffness matrix. This lack of complete rigid body mode capabilities invalidates the rigid body mode check for non-linear problems, and adversely impacts the use of traditional finite element techniques to predict dynamic response of pre-loaded structures unless the missing rigid body modes are somehow apppended on to the structure, such as by the Craig-Bampton technique.

The rigorous solution of the axially-loaded beam with free/free boundary conditions developed in this paper may lend itself to the development of a new geometric stiffness matrix for a beam element with full rigid body capabilities.

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