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Modeling of Near-Wall Turbulence

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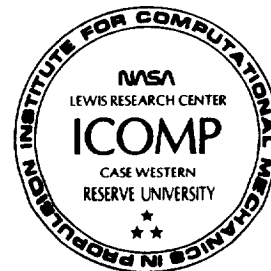*Lewis Research Center*
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ABSTRACT

This paper presents an improved k-ϵ model and a second order closure model for low-Reynolds number turbulence near a wall. For the k-ϵ model, a modified form of the eddy viscosity having correct asymptotic near-wall behavior is suggested, and a model for the pressure diffusion term in the turbulent kinetic energy equation is proposed. For the second order closure model, we modify the existing models for the Reynolds-stress equations to have proper near-wall behavior. A dissipation rate equation for the turbulent kinetic energy is also reformulated. The proposed models satisfy realizability and will not produce unphysical behavior. Fully developed channel flows are used for model testing. The calculations are compared with direct numerical simulations. It is shown that the present models, both the k-ϵ model and the second order closure model, perform well in predicting the behavior of the near wall turbulence. Significant improvements over previous models are obtained.

1. INTRODUCTION

Many flow problems of practical importance such as diffusion controlled transition, drag-reduction, and flow separation and reattachment, are the result of low-Reynolds number turbulence near a wall. Accurate predictions of these flows require better understanding and modeling of turbulence near a wall. Most practical closure models (including k-ϵ models and second order closure models) do not resolve a rather thin, viscosity-affected sublayer. They are usually incorporated with wall functions (Launder et al.[1], Shih and Lumley[2]). However, wall functions are not suitable for many flows of practical importance where the equilibrium assumptions built in the wall functions are not valid. It is therefore necessary to extend and develop schemes for modeling turbulence directly down to the wall. There exist several low-Reynolds number k-ϵ models which were discussed in detail by Patel et al.[3]. Patel et al. pointed out that the damping functions used in those models, especially the one for the eddy viscosity, need to be modified in order to improve model performance. Hanjalic and Launder[4] proposed a Reynolds-stress closure model for near wall turbulence with no wall functions. However, they did not use the full set of transport equations for the Reynolds-stress tensor in their implementation of the model. Most of the existing models that do not use wall functions do not properly predict the level of the normal components of the Reynolds-stress. For more discussions about the development of models for low-Reynolds number near wall turbulence, see Rodi[5] and Launder and Tselepidakis[6]

In this paper, the k-ϵ model is presented in section 2. The near-wall asymptotic behavior of the eddy viscosity and the pressure transport term in the k-equation is analyzed. According to that near-wall behavior, we proposed new models for the eddy viscosity and the pressure transport term. In addition, a modified dissipation rate equation is proposed following an argument similar to that of Lumley[7]. In section 3, we present the second order closure model.

The near-wall behavior of turbulence (see e.g. Mansour, Kim & Moin[8], MKM hereafter) is used to form a set of model transport equations for the Reynolds-stress tensor and the dissipation rate of turbulent kinetic energy. We developed a near wall model for the combination of the velocity pressure-gradient correlation and dissipation rate tensor, and constructed a modeled dissipation rate equation. Near the wall, viscous effects become important and the reduction of velocity fluctuations normal to the wall becomes much more significant comparing to other velocity components. This wall effect makes the viscous diffusion terms (which are usually neglected in high-Reynolds number turbulence closures away from the wall) become one of the largest terms that must be properly balanced by the other terms in the Reynolds-stress equations. The proposed model equations satisfy the realizability condition which ensure no unphysical behavior. In section 4, we test the proposed models using a fully developed channel flow. For this flow, direct numerical simulation (Kim et al.[9]) data is available. This data was recently verified using three-dimensional particle tracking velocimeter measurements (Nishino and Kasagi[10]). The modeled transport equations in this flow are one-dimensional and steady, hence model testing will be accurate. The results show that the k-ϵ model proposed in this paper obtains an improvement over existing k-ϵ model. The present second order closure model is capable of predicting wall behavior of various turbulent quantities. The mean velocity, shear stress and normal stresses calculated from the second order closure model are in a good agreement with the direct numerical simulation data.

2. k-ϵ MODEL

2.1 Near-Wall Asymptotic Behavior

To analyze the near-wall asymptotic behavior of the eddy viscosity and other turbulent quantities, we expand the fluctuating velocity and pressure in Taylor series about a normal distance y from the wall as follows:

$$
\begin{aligned}
u_1 &= b_1 y + c_1 y^2 + d_1 y^3 + \ldots \\
u_2 &= c_2 y^2 + d_2 y^3 + \ldots \\
u_3 &= b_3 y + c_3 y^2 + d_3 y^3 + \ldots \\
p &= a_p + b_p y + c_p y^2 + d_p y^3 + \ldots
\end{aligned}
\tag{2.1}
$$

where the coefficients $a_p, b_1, c_2, \ldots$ are functions of x, z and t. Based on the continuity and momentum equations, MKM show that the following relations between the coefficients hold:

$$
\begin{aligned}
2c_2 &= -(b_{1,1} + b_{3,3}) \\
a_{p,1} &= 2\nu c_1 \\
a_{p,3} &= 2\nu c_3 \\
b_p &= 2\nu c_2
\end{aligned}
$$

where $(\ )_{,i}$ represents a derivative with respect to x_i. The eddy viscosity is defined by

$$
-\langle u_i u_j \rangle = \nu_T (U_{i,j} + U_{j,i}) - \frac{2}{3} k \delta_{ij}
\tag{2.2}
$$

where $\langle\ \rangle$ stands for ensemble average and $k \equiv \langle u_i u_i \rangle / 2 \equiv \langle q^2 \rangle / 2$ is the turbulent kinetic energy. For plane shear flows, $\nu_T = -\langle uv \rangle / \frac{\partial U}{\partial y}$ from Eq. (2.2). Using Eq. (2.1), we obtain the

2

near-wall asymptotic behavior of the eddy viscosity:

$$\nu_T \frac{\partial U}{\partial y} = -\langle b_1 c_2 \rangle y^3 + (-\langle b_1 d_2 + c_1 c_2 \rangle + 2\langle b_1 c_2 \rangle \langle c_1 \rangle)y^4 + O(y^5) \tag{2.3}$$

That is, near the wall ν_T is $O(y^3)$, because $\partial U/\partial y$ is usually $O(1)$. We shall see later that some existing models do not have this wall behavior. The near-wall asymptotic behavior of the turbulent kinetic energy k and its dissipation rate $\epsilon \equiv \nu \langle u_{i,j} u_{i,j} \rangle$ are from Eq. (2.1)

$$k = \frac{\langle b_1^2 \rangle + \langle b_3^2 \rangle}{2} y^2 + (\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle)y^3 + O(y^4) \tag{2.4}$$

$$\frac{\epsilon}{\nu} = \langle b_1^2 \rangle + \langle b_3^2 \rangle + 4(\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle)y + O(y^2) \tag{2.5}$$

In the k-equation, the pressure transport term $\Pi \equiv -\frac{1}{\rho}\langle u_i p_{,i} \rangle$ becomes

$$\Pi = -2\nu(\langle b_1 c_1 \rangle + \langle b_3 c_3 \rangle)y + O(y^2) \tag{2.6}$$

while the turbulent transport term, $-\langle k u_i \rangle_{,i}$, is $O(y^3)$. Therefore, the pressure transport term becomes much more important than the turbulent transport term near the wall and must be properly modeled.

2.2 Eddy Viscosity and Pressure Transport Term

The eddy viscosity model can be in general written as $\nu_T = c\, u' \ell'$, where u' and ℓ' are the turbulent characteristic velocity and length scale, respectively. In two-equation models, e.g. k-ϵ models, $k^{1/2}$ is used as the characteristic velocity, and the length scale is characterized by $k^{3/2}/\epsilon$. Hence, the eddy viscosity is written as[13]

$$\nu_T = C_\mu f_\mu \frac{k^2}{\epsilon} \tag{2.7}$$

where $C_\mu = 0.07$, f_μ is a damping function. The form of the damping function is quite critical to the prediction of the mean flow field[3]. In fact, the prediction of the mean velocity depends on the eddy viscosity model. Therefore it is important for an eddy viscosity model to have proper near-wall behavior. We have examined the near-wall behavior of various eddy viscosity models which are listed in table 1. Table 1 shows that some of the k-ϵ models do not have the correct near-wall behavior of the eddy viscosity.

One expects that near the wall, the size of the large eddies (or energy containing eddies) should be order of the wall distance $O(y)$. Equations (2.4) and (2.5) show that $k^{3/2}/\epsilon$ is $O(y^3)$. Hence, $k^{3/2}/\epsilon$ is not an appropriate quantity to represent the length scale of the large eddies near the wall. However, we can define a variable $\tilde{\epsilon} \equiv \epsilon - \nu k_{,i} k_{,i}/2k$ which has a nice property: $\tilde{\epsilon}$ approaches ϵ away from the wall and is $O(y^2)$ near the wall. Therefore $k^{3/2}/\tilde{\epsilon}$ is $O(y)$ and is a proper quantity to characterize the length scale of the large eddies. With this length scale, the eddy viscosity model should be written as

$$\nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\epsilon}} \tag{2.8}$$

Now, in order for ν_T to have correct near-wall behavior, the damping function f_μ must be $O(y)$ near the wall and approach 1 away from the wall. The damping functions used in various k-ϵ

models are also listed in table 1. If we consider the presence of the wall as the main effect on the eddy viscosity, then we can assume f_μ is mainly a function of y^+ (defined as $u_\tau y/\nu$, where u_τ is the friction velocity). The form of f_μ can be determined quite accurately if we know ν_T, k and $\tilde\epsilon$, for example, from the direct numerical simulations. We may also optimize the following simple form by numerical experiments:

$$f_\mu = 1 - \exp(-a_1 y^+ - a_2 {y^+}^2 - a_3 {y^+}^3 - a_4 {y^+}^4) \tag{2.9}$$

The optimal values for channel flows are $a_1 = 6 \times 10^{-3}, a_2 = 4 \times 10^{-4}, a_3 = -2.5 \times 10^{-6}, a_4 = 4 \times 10^{-9}$. They can be further tuned using the direct numerical simulation data.

For the term of the pressure transport of the turbulent kinetic energy, we propose:

$$\Pi = \left\{ \frac{0.05}{f_\mu[1 - \exp(-y^+)]} \frac{\nu_T}{\sigma_k} k_{,j} \right\}_{,j} \tag{2.10}$$

which has a similar form to the standard turbulent transport model, but with a coefficient to ensure the near-wall behavior, Eq. (2.6). Here the coefficient 0.05 is an optimal model constant for the channel flows.

2.3 Modeled k-ε Equation

To complete the eddy viscosity model, we need the modeled equations for the turbulent kinetic energy and for its dissipation rate. In general one may write the modeled k -equation as follows:

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}[(\nu + \frac{\nu_T}{\sigma_k})\frac{\partial k}{\partial x_j}] + \Pi + \nu_T(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})\frac{\partial U_i}{\partial x_j} - \epsilon + D \tag{2.11}$$

where D is zero in the present k-ϵ model, because ϵ is finite at the wall. In k-ϵ models that assume $\epsilon = 0$ at the wall, an artificial term D must be added in order to balance the viscous diffusion term at the wall. The form of D for various k-ϵ models is listed in table 1.

For the modeled dissipation rate equation, we write

$$\epsilon_{,t} + U_j \epsilon_{,j} = [(1 + \nu_T/\sigma_\epsilon)\epsilon_{,j}]_{,j} - \frac{\epsilon\tilde\epsilon}{k}\Psi \tag{2.12}$$

where Ψ stands for the entire mechanism of the production and destruction of the dissipation rate ϵ. At the level of the k-ϵ model, we assume that Ψ is a function of ν, ν_T, k, ϵ, $\tilde\epsilon$, $U_{i,j}$ and $U_{i,jk}$. Because Ψ is an invariant, it must be a function of the invariants that can be constructed from at least following quantities: R_t, $\nu_T U_{i,j} U_{i,j}/\tilde\epsilon$ and $\nu\nu_T U_{i,jk} U_{i,jk} k/\epsilon\tilde\epsilon$, where R_t is the turbulent Reynolds number $k^2/\nu\epsilon$. We now expand Ψ in a Taylor series about these invariants and take only the linear terms. We obtain

$$\Psi = \psi_0 + \psi_1 \frac{\nu_T U_{i,j} U_{i,j}}{\tilde\epsilon} + \psi_2 \nu\nu_T U_{i,jk} U_{i,jk} \frac{k}{\epsilon\tilde\epsilon} \tag{2.13}$$

where the coefficients ψ_0, ψ_1 and ψ_2 are in general functions of R_t. Finally, we write the dissipation rate equation as

$$\frac{\partial \epsilon}{\partial t} + U_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_j}[(\nu + \frac{\nu_T}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial x_j}] + C_1 f_1 \frac{\epsilon}{k}\nu_T U_{i,j} U_{i,j} - C_2 f_2 \frac{\epsilon\tilde\epsilon}{k} + E \tag{2.14}$$

4

where C_1 and C_2 are model constants, and f_1 and f_2 are functions of R_t. The term E in the present model comes from the last term in Eq. (2.13): $E = \nu\nu_T U_{i,jk} U_{i,jk}$, where we have taken $\psi_2 = -1$. The forms of E and C_1, C_2, f_1 and f_2 for various k-ϵ models are listed in table 1.

3. SECOND ORDER CLOSURE MODEL

3.1 Reynolds-Stress Equation

For an incompressible flow, the Reynolds-stress equations can be written

$$\frac{D}{Dt}\langle u_i u_j\rangle = P_{ij} + T_{ij} + D_{ij} + \Pi_{ij} - \varepsilon_{ij} \tag{3.1}$$

where $\langle\ \rangle$ stands for an ensemble average, $D/Dt = \partial/\partial t + U_k \partial/\partial x_k$, and the terms on the right hand side of the above equations are identified as follows: $P_{ij} = -\langle u_i u_k\rangle U_{j,k} - \langle u_j u_k\rangle U_{i,k}$, $T_{ij} = -\langle u_i u_j u_k\rangle_{,k}$, $D_{ij} = \nu\langle u_i u_j\rangle_{,kk}$, $\Pi_{ij} = -\frac{1}{\rho}\langle u_i p_{,j} + u_j p_{,i}\rangle$, and $\varepsilon_{ij} = 2\nu\langle u_{i,k} u_{j,k}\rangle$. In Eq. (3.1), the turbulent transport T_{ij}, the velocity pressure-gradient correlation Π_{ij} and the dissipation rate ε_{ij} must be modeled. Near-wall analysis of existing models shows that they do not exhibit a proper near-wall behavior, which indicates that the modeled transport equations are not properly balanced near the wall. This may explain the poor quantitative predictions from existing models for near wall turbulence, specifically, in predicting the normal stresses.[5],[6]

To analyze the near-wall asymptotic behavior of the different terms in the Reynolds-stress equations, we use (2.1) to estimate each term in the Reynolds-stress equations. For example, for the $\langle u_1 u_2\rangle$ component,

$$\frac{D\langle u_1 u_2\rangle}{Dt} = O(y^3)$$
$$P_{12} = O(y^4)$$
$$T_{12} = O(y^4)$$
$$D_{12} = 6\nu\langle b_1 c_2\rangle y + O(y^2) \tag{3.2}$$
$$\Pi_{12} = -2\nu\langle b_1 c_2\rangle y + O(y^2)$$
$$\varepsilon_{12} = 4\nu\langle b_1 c_2\rangle y + O(y^2)$$

The near-wall budget for other components can be expressed similarly. We see from these budgets that the viscous diffusion term D_{ij} is always the largest term. Its leading term is balanced by the dissipation rate ε_{ij} or by the combination of the dissipation rate and the velocity pressure-gradient correlation, $-\varepsilon_{ij} + \Pi_{ij}$. In this paper, we derive a near wall model for ε_{ij} and Π_{ij} in a rational way, using the wall behavior as a model constraint. Away from the wall, we adopt the high-Reynolds number realizable model of Shih and Lumley.[11] Near the wall, we write the combination velocity pressure-gradient and dissipation rate as

$$\Pi_{ij} - \varepsilon_{ij} \equiv \frac{\epsilon}{\langle q^2\rangle}\Phi_{ij} \tag{3.3}$$

Note, Φ_{ij} is not trace free because it includes the dissipation rate and the pressure diffusion. We model Φ_{ij} by assuming that it is a linear function of $\langle u_i u_j\rangle$, and function of a unit vector

5

normal to the surface, n_i. Under these assumptions, the most general form of Φ_{ij} which satisfies symmetry is

$$\Phi_{ij} = a_{00}n_in_j + a_0\delta_{ij} + a_1\langle u_iu_j\rangle + a_2(\langle u_iu_k\rangle n_jn_k + \langle u_ju_k\rangle n_in_k)$$
$$+ a_3\langle u_ku_l\rangle n_kn_ln_in_j + a_4\langle u_ku_l\rangle n_kn_l\delta_{ij} \tag{3.4}$$

Using Eq. (3.4) and the wall constraint,

$$D_{ij} + \frac{\epsilon}{\langle q^2\rangle}\Phi_{ij} = 0 \qquad \text{as} \quad y \to 0 \tag{3.5}$$

the coefficients can be determined as $a_{00} = a_0 = 0$, $a_1 = 2$, $a_2 = 4$, and $a_3 = 2$, with an undetermined a_4, which is set to be zero according to numerical experiments. The above model, Eq. (3.4), is valid only in the region near the wall. The influence of the wall should vanish at the place far away from the wall. Numerical experiments suggest the use of a damping function of the form $f_w = \exp(-(R_t/C)^2)$, where $R_t = \langle q^2\rangle^2/9\nu\epsilon$, $C = 1.358R_{e\tau}^{0.44}$, $R_{e\tau} = u_\tau\delta/\nu$, u_τ is the friction velocity, and δ is the thickness of the boundary layer or half width of a channel. Therefore, the model in Eq. (3.3) can be written as follows:

$$\Pi_{ij} - \varepsilon_{ij} = -f_w\frac{\epsilon}{\langle q^2\rangle}[2\langle u_iu_j\rangle + 4(\langle u_iu_k\rangle n_jn_k + \langle u_ju_k\rangle n_in_k) + 2\langle u_ku_l\rangle n_kn_ln_in_j] \tag{3.6}$$

Away from the wall, the velocity pressure-gradient correlation is split into a rapid part $\Pi_{ij}^{(1)}$ and a slow part $\Pi_{ij}^{(2)}$. Lumley's high-Reynolds number model[7] for the return to isotropy term $\Pi_{ij}^{(2)} - \varepsilon_{ij}$ has been successfully used in conjunction with wall functions. However, this return to isotropy model does not vanish as $y \to 0$, and should be modified with a damping function if one wants to use it to the wall. We use the same f_w damping function to minimize the arbitrary choice of the damping functions, and set

$$\Pi_{ij}^{(2)} - \varepsilon_{ij} = -(\epsilon\beta b_{ij} + \frac{2}{3}\epsilon\delta_{ij})(1 - f_w), \tag{3.7}$$

where $b_{ij} = \langle u_iu_j\rangle/\langle q^2\rangle$ - $\delta_{ij}/3$. The rapid part of the velocity pressure-gradient, $\Pi_{ij}^{(1)}$, is modeled using the model of Shih and Lumley[2] with a modified coefficient C. Notice that this rapid model decays as $O(y^2)$ as $y \to 0$. Therefore, we use it down to the wall without a damping function. Shih and Lumley[2] set $C = 0.8$ when the model was used in conjunction with wall functions. Here, we modify the coefficient as a function of the turbulent Reynolds number: $C = 0.8[1 - \exp(-(R_t/40)^2)]$; C will approach 0.8 when R_t becomes large.

For the third moments, we use the model of Hanjalic and Launder[4], but with a different model constant:

$$\langle u_iu_ju_k\rangle = -0.07\frac{\langle q^2\rangle}{\epsilon}[\langle u_ku_p\rangle\langle u_iu_j\rangle_{,p} + \langle u_ju_p\rangle\langle u_iu_k\rangle_{,p} + \langle u_iu_p\rangle\langle u_ju_k\rangle_{,p}] \tag{3.8}$$

3.2 Dissipation Rate Equation

We write the dissipation rate equation as

$$\epsilon_{,t} + U_i\epsilon_{,i} - (\nu\epsilon_{,i} - \langle\epsilon u_i\rangle)_{,i} = -\frac{\epsilon\tilde{\epsilon}}{\langle q^2\rangle}\Psi \tag{3.9}$$

6

At the level of the second order closure, we assume that Ψ is a function of $\langle u_i u_j \rangle$, ϵ, ν, $U_{i,j}$ and $U_{i,jk}$. Since Ψ is an invariant, it should be at least a function of II, III, R_t, $\frac{\langle u_i u_i \rangle U_{i,i}}{\epsilon}$, $U_{i,jk} U_{i,jl} \langle u_k u_l \rangle / \frac{\epsilon}{\langle q^2 \rangle} \frac{\bar{\epsilon}}{\langle q^2 \rangle} \frac{\epsilon}{\nu}$, $U_{i,jk} U_{j,il} \langle u_k u_l \rangle / \frac{\epsilon}{\langle q^2 \rangle} \frac{\bar{\epsilon}}{\langle q^2 \rangle} \frac{\epsilon}{\nu}$, and $U_{i,jk} U_{l,ij} \langle u_k u_l \rangle / \frac{\epsilon}{\langle q^2 \rangle} \frac{\bar{\epsilon}}{\langle q^2 \rangle} \frac{\epsilon}{\nu}$, where II and III are the invariants of b_{ij}. We then expand Ψ in Taylor series about the invariants and take only first order linear terms. We obtain

$$
\begin{aligned}
\Psi = \psi_0 &+ \psi_1 \frac{\langle u_i u_j \rangle U_{i,j}}{\epsilon} \\
&+ \psi_2 \frac{U_{i,jk} U_{i,jl} \langle u_k u_l \rangle}{\frac{\epsilon}{\langle q^2 \rangle} \frac{\bar{\epsilon}}{\langle q^2 \rangle} \frac{\epsilon}{\nu}} + \psi_3 \frac{U_{i,jk} U_{j,il} \langle u_k u_l \rangle}{\frac{\epsilon}{\langle q^2 \rangle} \frac{\bar{\epsilon}}{\langle q^2 \rangle} \frac{\epsilon}{\nu}} + \psi_4 \frac{U_{i,jk} U_{l,ij} \langle u_k u_l \rangle}{\frac{\epsilon}{\langle q^2 \rangle} \frac{\bar{\epsilon}}{\langle q^2 \rangle} \frac{\epsilon}{\nu}}
\end{aligned}
\tag{3.10}
$$

where the coefficients ψ_0, ψ_1, ψ_2, ψ_3 and ψ_4 are, in general, functions of II, III and R_t. Now if we consider the terms in (3.10) containing the second derivative of the mean velocity as a model of the term $-2\nu \langle u_j u_{i,k} \rangle U_{i,kj}$ (the term in the exact equation of ϵ containing the second derivative of the mean velocity) and use continuity, $\langle u_j u_{k,k} \rangle = 0$, we obtain $\psi_3 = 0$, $\psi_4 = -\psi_2$. In addition, at high Reynolds number or when the small scales are isotropic, the dissipation rate should not respond instantaneously to a change in the mean velocity derivatives. Under this constraint, we require ψ_2 to vanish as the turbulence becomes isotropic. The simplest form that satisfies this condition is $\psi_2 = -0.15(1 - F)$, where Lumley's parameter[7] $F = 1 + 9 b_{ij} b_{jk} b_{ki} - 4.5 b_{ij} b_{ij}$ approaches one as the turbulence becomes isotropic. Finally, the modeled dissipation rate equation can be written as

$$
\begin{aligned}
\epsilon_{,t} + U_i \epsilon_{,i} = &(\nu \epsilon_{,i} - \langle \epsilon u_i \rangle)_{,i} - \psi_0 \frac{\epsilon \bar{\epsilon}}{\langle q^2 \rangle} \\
&- \psi_1 \frac{\bar{\epsilon}}{\langle q^2 \rangle} \langle u_i u_j \rangle U_{i,j} - \psi_2 \frac{\nu \langle q^2 \rangle}{\epsilon} \langle u_k u_l \rangle (U_{i,jl} - U_{l,ij}) U_{i,jk}
\end{aligned}
\tag{3.11}
$$

where ψ_0, and ψ_1 are Lumley's coefficients.[7] To close the above equation, we use Hanjalic and Launder's diffusion model with a different model constant:

$$
\langle \epsilon u_k \rangle = -0.07 \frac{\langle q^2 \rangle}{2\epsilon} \langle u_k u_p \rangle \epsilon_{,p}
\tag{3.12}
$$

4. FULLY DEVELOPED TURBULENT CHANNEL FLOW

To test the models developed in this paper, we chose a fully developed channel flow. The Reynolds number based on the friction velocity and the half width of the channel Re_τ is 180, for which direct numerical simulation[9] and experimental data[10] are available for comparison. The Reynolds-stress equations for this flow are one-dimensional and steady, therefore the model testing will be accurate.

The main results of the k-ϵ models are shown in figures (1) to (4). These figures present predictions for the mean velocity, turbulent kinetic energy and its dissipation rate and eddy viscosity profiles, using several k-ϵ models[13],[14],[15],[16] including the present one. These results show clearly that the present k-ϵ model gives significant improvements over previous k-ϵ models according to the comparisons with the direct numerical simulations. The main results of the present second order closure model are shown in the figures (5) to (8). All the predictions are in a good agreement with the direct numerical simulation data. Figures (5) and (6)

7

present the comparisons of predicted mean velocity and turbulent kinetic energy using different models (both k-ϵ models and second order closure models) with the numerical simulation data. The comparison with data of the normal components of the stresses are shown in figure (7). Predictions for all the components are quantitatively accurate. The near wall behavior of the three normal components is also reflected by the present models. None of the previous models have succeeded in predicting these trends.[5],[6]

It should be noted, in our comparison with the data, the channel flow was computed at fairly low Reynolds number. At high Reynolds numbers, the models still predict the log law profile for the mean velocity. Comparison with new channel data[18] (see figure (8)) at $Re_\tau = 393$ shows that the normal stresses near the wall are still well predicted. However, in the core region the model slightly overpredicts the level of the turbulent kinetic energy.

ACKNOWLEGEMENT

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TABLE 1. Constants and functions for various $K - \epsilon$ models

Model	ϵ_w-BC	C_μ	$C_{\epsilon 1}$	$C_{\epsilon 2}$	σ_k	σ_ϵ
JL	0	.09	1.45	2.0	1.0	1.3
Reynolds	$\nu\frac{\partial^2 K}{\partial y^2}$	.084	1.0	1.83	1.69	1.3
LB	$\nu\frac{\partial^2 K}{\partial y^2}$	.09	1.44	1.92	1.0	1.3
Chien	0	.09	1.35	1.8	1.0	1.3
NH	0	.09	1.45	1.9	1.0	1.3
Present	$\nu\frac{\partial^2 K}{\partial y^2}$	.09	1.45	2.0	1.3	1.3

Model	f_μ	f_1	f_2
JL	$\exp(\frac{-2.5}{1+R_t/50})$	1.0	$1 - .3\exp(-R_t^2)$
Reynolds	$1 - \exp(-.0198R_k)$	1.0	$[1 - .3\exp(-R_t^2/9)]f_\mu$
LB	$[1 - \exp(-.0165R_k)]^2 \times (1 + \frac{20.5}{R_t})$	$1 + (\frac{.05}{f_\mu})^3$	$1 - \exp(-R_t^2)$
Chien	$1 - \exp(-.0115y^+)$	1.0	$1 - .22\exp(-R_t^2/36)$
NH	$[1 - \exp(-y^+/26.5)]^2$	1.0	$1 - .3\exp(-R_t^2)$
Present	Eq. (2.9)	1.0	$1 - .22\exp(-R_t^2/36)$

Model	Π	D	E	ν_T
JL	0	$-2\nu\big(\frac{\partial\sqrt{K}}{\partial y}\big)^2$	$2\nu\nu_T\big(\frac{\partial^2 U}{\partial y^2}\big)^2$	$O(y^3)$
Reynolds	0	0	0	$O(y^5)$
LB	0	0	0	$O(y^4)$
Chien	0	$-\frac{2\nu K}{y^2}$	$-\frac{2\nu\epsilon}{y^2}\exp(-.5y^+)$	$O(y^3)$
NH	0	$-2\nu\big(\frac{\partial\sqrt{K}}{\partial y}\big)^2$	$\nu\nu_T(1 - f_\mu)\big(\frac{\partial^2 U}{\partial y^2}\big)^2 O(y^4)$	
Present	Eq. (2.10)	0	$\nu\nu_T\big(\frac{\partial^2 U}{\partial y^2}\big)^2$	$O(y^3)$

$R_t = K^2/\nu\epsilon$, $R_k = \sqrt{K}y/\nu$, $y^+ = u_\tau y/\nu$.
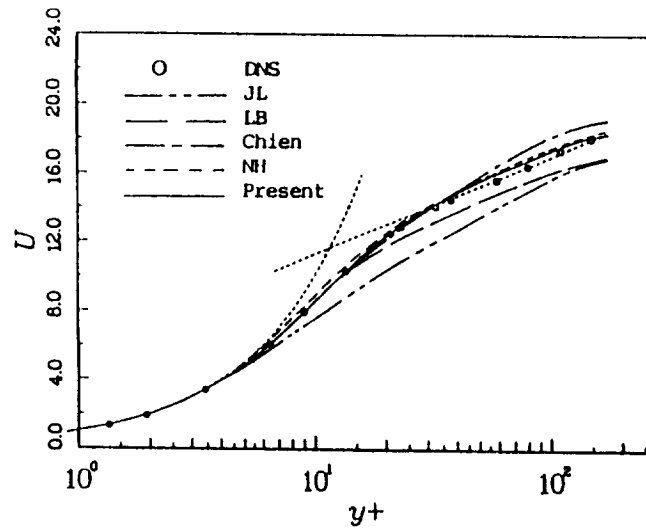
Figure 1. Comparisons between k-ϵ models and
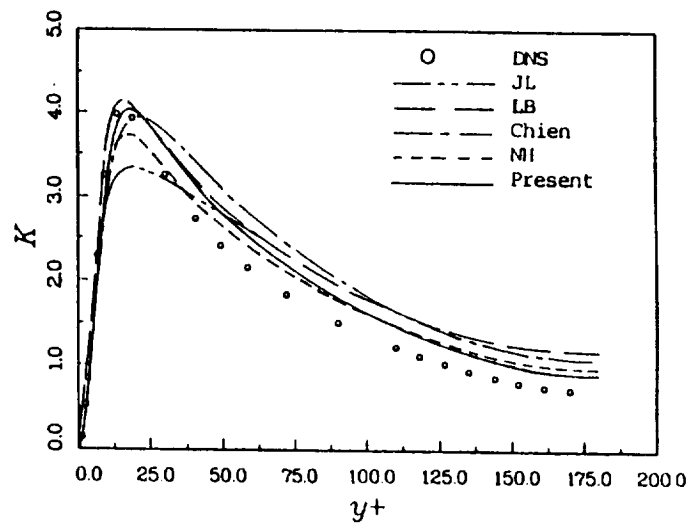DNS[9]-data for mean velocity profiles in the 2D channel.

Figure 2. Comparisons between k-ϵ models and
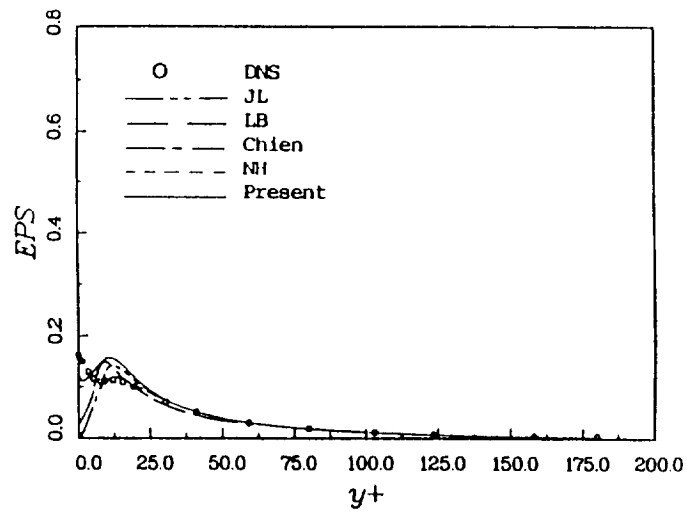DNS[9]-data for turbulent kinetic energy in the 2D channel.

10

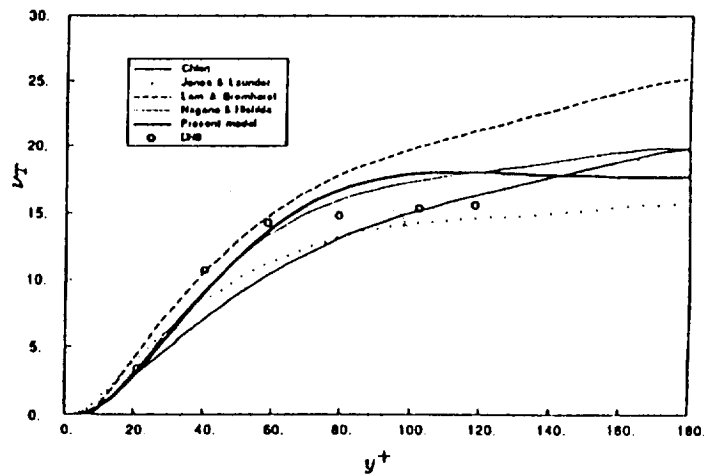Figure 3. Comparisons between k-ϵ models and DNS[9]-data for the dissipation rate in the 2D channel.

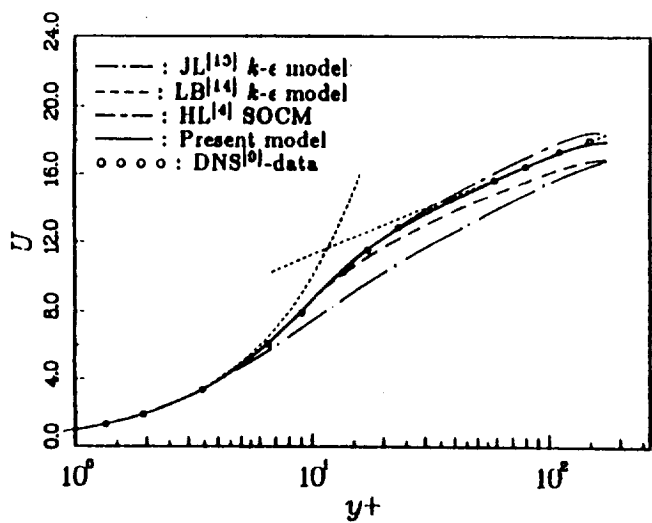Figure 4. Comparisons between k-ϵ models and DNS[9]-data for eddy viscosity in the 2D channel.

11

Figure 5. Mean velocity profiles, $Re_\tau = 180$.
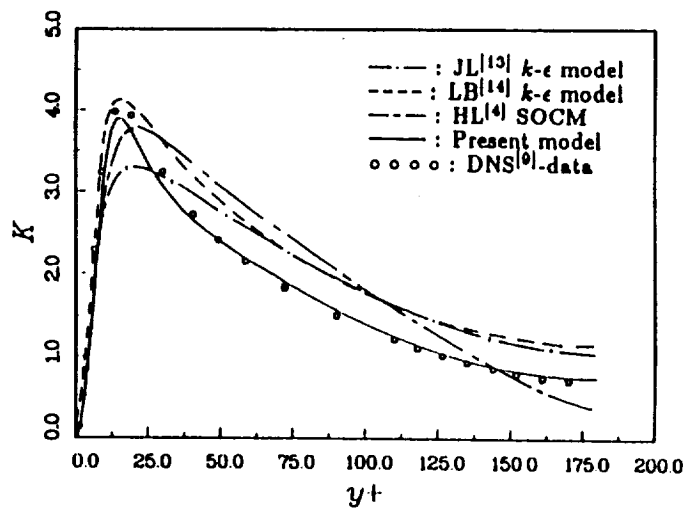
Figure 6. Turbulent kinetic energy profiles, $Re_\tau = 180$.

12
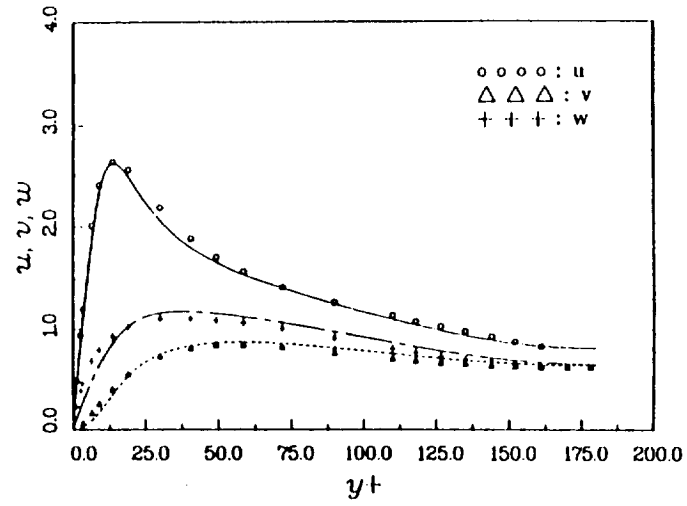
Figure 7. RMS of fluctuating velocities, $Re_r = 180$.
Lines represent present model; Symbols are DNS[9]-data.
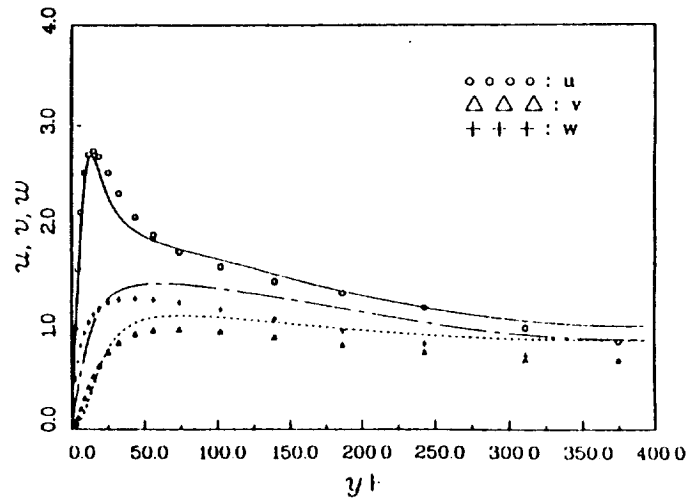
Figure 8. RMS of fluctuating velocities, $Re_r = 393$.
Lines represent present model; Symbols are DNS[15]-data.

13

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16. Abstract

This paper presents an improved k-ϵ model and a second order closure model for low Reynolds number turbulence near a wall. For the k-ϵ model, a modified form of the eddy viscosity having correct asymptotic near-wall behavior is suggested, and a model for the pressure diffusion term in the turbulent kinetic energy equation is proposed. For the second order closure model, we modify the existing models for the Reynolds-stress equations to have proper near-wall behavior. A dissipation rate equation for the turbulent kinetic energy is also reformulated. The proposed models satisfy realizability and will not produce unphysical behavior. Fully developed channel flows are used for model testing. The calculations are compared with direct numerical simulations. It is shown that the present models, both the k-ϵ model and the second order closure model, perform well in predicting the behavior of the near wall turbulence. Significant improvements over previous models are obtained.

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