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INTRODUCTION

Certain fiber composites are often modeled as equivalent homogeneous solids [1]. Continuum models of this sort are useful in analyzing wave propogation in fiber composites when the wavelengths under consideration are long compared to the mean fiber diameter.

The input-output characterization of a homogeneous transversely isotropic plate is investigated by tracing P and SV waves. Following the work in [2], the reflection of a P wave at a stress-free plane boundary in a semi-infinite transversely isotropic medium is considered first. It is reestablished that an incident P wave reflects a similar P wave and an SV wave. It is also reestablished that the angle of reflection is equal to the angle of incidence whenever the plane boundary where the reflection occurs is parallel to the isotropic plane of the transversely isotropic medium. The angle of reflection of the reflected SV wave is found as a function of the angle of incidence of the incident P wave.

The plane of isotropy of the equivalent transversely isotropic continuum plate lies in the midplane of the plate and is parallel to each face of the plate. The P waves experience multiple reflections at each face of the plate. At each reflection a P wave and an SV wave are reflected back into the medium. The SV waves also experience multiple reflections, producing a reflected P wave and a reflected SV wave at each reflection [2]. The reflected SV wave is reflected with an angle of reflection equal to the angle of incidence of the incident SV wave [2] and the reflected P wave is reflected with an angle of reflection equal to the angle of incidence of the incident P wave that produced the incident SV wave. The amplitude coefficients of the P and SV waves reflected by an incident P wave are calculated as functions of the angle of incidence and plotted. Further, the amplitude coefficients of the P and SV waves reflected by an incident SV wave produced by mode splitting are calculated as functions of the angle of incidence of the incident P wave that produced the incident SV wave and plotted.

A path notation is then defined to aid in the tracing of series of reflected P and SV waves through the medium. Paths are determined by path parameters equal to the number of P waves in the path, the number of SV waves in the path and the number of SV wave to SV wave reflections in the path. A path amplitude coefficient is defined in terms of the path parameters and tabulated. It is found that more than one path may have a given set of path parameters and a path multiplicity function is derived to count the number of distinct paths with the same path parameters. A net path amplitude coefficient is defined as the sum of the amplitudes of all paths with the same total combination of P waves and SV waves. The net path amplitude coefficient is tabulated.

Finally, a theoretical output voltage from the receiving transducer is calculated for a tone burst (a periodic input voltage of finite duration) by neglecting the effects of mode splitting.

REFLECTION OF INCIDENT P WAVE AT STRESS-FREE PLANE BOUNDARY IN SEMI-INFINITE TRANSVERSELY ISOTROPIC MEDIUM WITH PLANE OF ISOTROPY PARALLEL TO PLANE BOUNDARY

1. REFLECTED P AND SV WAVES

A plane progressive stress wave may be represented as

$$(u, v, w) = A(P_x, P_y, P_z)\exp\{i\omega(S_x x + S_y y + S_z z - t)\} \tag{1}$$

for example, see [3], where u, v and w are the displacement components of a point in the medium along the x, y and z axes, respectively; A is the amplitude of the particle displacement; P_x, P_y and P_z are the components of the unit vector of particle displacement along the x, y and z axes, respectively; $i = \sqrt{-1}$; ω denotes radian frequency; S_x, S_y and S_z are the components of the slowness vector, which points in the same direction as the normal to the wave front and has magnitude equal to the reciprocal of the phase velocity, see [4], along the x, y and z axes, respectively; and t denotes time.

A plane progressive P wave is incident on the plane boundary of a semi-infinte linearly elastic transversely isotropic continuum with plane of isotropy parallel to the plane boundary. Define a cartesian coordinate system (x, y, z) as follows: the plane boundary of the medium contains the x and y axes, and the z axis is the zonal axis of the medium (see Fig. 1). The generalized Hooke's Law, when written relative to the (x, y, z) coordinate system, is [4]

$$\tau_{xx} = C_{11}u_{,x} + C_{12}v_{,y} + C_{13}w_{,z}$$
$$\tau_{yy} = C_{12}u_{,x} + C_{11}v_{,y} + C_{13}w_{,z}$$
$$\tau_{zz} = C_{13}u_{,x} + C_{13}v_{,y} + C_{33}w_{,z}$$
$$\tau_{xz} = C_{44}(u_{,z} + w_{,x})$$
$$\tau_{yz} = C_{44}(v_{,z} + w_{,y})$$
$$\tau_{xy} = C_{66}(u_{,y} + v_{,x}) \tag{2}$$

where for $i = j$ τ_{ij} is a normal stress and for $i \neq j$ τ_{ij} is a shear stress; "," denotes partial differentiation with respect to the succeeding variable; C_{11}, C_{12}, C_{13}, C_{33} and C_{44} are the five independent elastic constants for a linearly elastic transversely isotropic medium; and C_{66} is equal to $1/2(C_{11} - C_{12})$.

The stresses associated with a plane progressive P wave are evaluated by substituting the expression for the displacement components of a point in the medium, given by eqn. (1), into eqn. (2) to be

$$\tau_{xx} = i\omega A(C_{11}S_xP_x + C_{12}S_yP_y + C_{13}S_zP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{yy} = i\omega A(C_{12}S_xP_x + C_{11}S_yP_y + C_{13}S_zP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{zz} = i\omega A(C_{13}S_xP_x + C_{13}S_yP_y + C_{33}S_zP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{xz} = i\omega A(C_{44}S_zP_x + C_{44}S_xP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{yz} = i\omega A(C_{44}S_zP_y + C_{44}S_yP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{xy} = i\omega A(C_{66}S_yP_x + C_{66}S_xP_y)\exp\{i\omega(S_xx + S_yy + S_zz - t)\} \tag{3}$$

The stress boundary conditions on a stress free plane boundary require [3]

$$\tau_{xz}^{(I)} + \tau_{xz}^{(R)} = 0$$
$$\tau_{yz}^{(I)} + \tau_{yz}^{(R)} = 0$$
$$\tau_{zz}^{(I)} + \tau_{zz}^{(R)} = 0 \tag{4}$$

where the $\tau_{ij}^{(I)}$ represent the stresses associated with the incident wave and the $\tau_{ij}^{(R)}$ represent the stresses associated with any reflected waves. The stresses determined by eqn. (3) satisfy the boundary conditions, eqn. (4), only when the frequency, ω, of any reflected wave is equal to the frequency of the incident wave [3] and

$$S_x^{(I)} = S_x^{(R)}$$
$$S_y^{(I)} = S_y^{(R)} \tag{5}$$

where $S_x^{(I)}$ and $S_y^{(I)}$ are the x and y components of the slowness vector of the incident wave, respectively; and $S_x^{(R)}$ and $S_y^{(R)}$ are the x and y components of the slowness vector of any reflected wave, respectively [2]. Eqn. (5) establishes that the incident and reflected waves lie in the same plane, called the plane of incidence. For computational ease, and without loss of generality, assume the plane of incidence coincides with the y-z plane. Then the x components of the slowness vectors of the incident and the reflected waves vanish, satisfying

$$S_x^{(I)} = S_x^{(R)} = 0 \tag{6}$$

A P wave with slowness surface in a plane containing the zonal axis of a transversely isotropic medium possesses a quasi-longitudinal displacement [5]; therefore, a P wave traveling in the y-z plane in the (x, y, z) coordinate system has unit vector of particle displacement

$$(P_x, P_y, P_z) = (0, P_y^{(P)}, P_z^{(P)}) \tag{7}$$

where $P_y^{(P)}$ and $P_z^{(P)}$ are, respectively, the directionally dependent y and z components of the unit vector of particle displacement of a P wave.

A wave with P_x, P_y and P_z given by eqn. (7) and S_x given by eqn. (6) has stresses determined by eqn. (3) satisfying

$$\tau_{xy} = \tau_{xz} = 0$$

$$\tau_{yz};\, \tau_{xx};\, \tau_{yy};\, \tau_{zz} \neq 0 \tag{8}$$

An SV wave with slowness surface in a plane containing the zonal axis of a transversely isotropic medium is quasi-transverse [5]. Therefore, an SV wave traveling in the plane x=0 in the (x, y, z) coordinate system has P_x, P_y and P_z according to

$$(P_x, P_y, P_z) = (0, P_y^{(SV)}, P_z^{(SV)}) \tag{9}$$

where $P_y^{(SV)}$ and $P_z^{(SV)}$ are, respectively, the directionally dependent y and z components of the unit vector of particle displacement of an SV wave.

A wave with unit vector of particle displacment satisfying eqn. (9) and x component of slowness vector determined by eqn. (6) has stresses calculated from eqn. (3) satisfying

$$\tau_{xy} = \tau_{xz} = 0$$

$$\tau_{yz};\, \tau_{xx};\, \tau_{yy};\, \tau_{zz} \neq 0 \tag{10}$$

An SH wave with slowness surface in a plane containing the zonal axis of a tranversely isotropic medium possesses a transverse displacement [5]. Thus, for an SH wave traveling in the plane x=0 in the (x, y, z) coordinate system the unit vector of particle displacement is given by

$$(P_x, P_y, P_z) = (1, 0, 0) \tag{11}$$

A wave with P_x, P_y and P_z given by eqn. (11) and x component of slowness vector equal to zero according to eqn. (6) has stresses determined from eqn. (3) satisfying

$$
\begin{aligned}
&\tau_{xx} = \tau_{yy} = \tau_{zz} = \tau_{yz} = 0 \\
&\tau_{xy} \neq 0 \\
&\tau_{xz} \neq 0
\end{aligned}
\qquad (12)
$$

A P wave is incident on the plane boundary of a semi-infinite tranversely isotropic medium. The stresses at the point of incidence that are associated with the incident P wave are constrained by eqn. (8) and must satisfy

$$
\begin{aligned}
&\tau_{yz}^{(I)} \neq 0 \\
&\tau_{zz}^{(I)} \neq 0 \\
&\tau_{xz}^{(I)} = 0
\end{aligned}
\qquad (13)
$$

where the superscript $^{(I)}$ is used to denote properties associated with the incident wave.

The stress boundary conditions for any reflected waves are evaluated from eqn. (4) and eqn. (13) and can be expressed as

$$
\begin{aligned}
&\tau_{yz}^{(R)} = -\tau_{yz}^{(I)} \neq 0 \\
&\tau_{zz}^{(R)} = -\tau_{zz}^{(I)} \neq 0 \\
&\tau_{xz}^{(R)} = 0
\end{aligned}
\qquad (14)
$$

where the superscript $^{(R)}$ is used to denote properties associated with any reflected waves. Eqn. (12) indicates that the τ_{xz} component of the stress tensor associated with an SH wave is nonzero. Therefore, no SH wave will be reflected back into the medium by an incident P wave, because such a wave would create a nonzero τ_{xz} stress component and violate the stress boundary conditions, eqn. (14). Eqn. (8) indicates that the components of the stress tensor associated with a P wave satisfy the stress boundary conditions and eqn. (10) indicates that the components of the stress tensor associated with an SV wave also satisfy the stress boundary conditions. Therefore, an incident P wave will result in a reflected P wave and a reflected SV wave [2].

2. SLOWNESS SURFACE OF P WAVE

The equations of motion relative to the (x, y, z) coordinate system are [4]

$$\begin{aligned}
&\tau_{xx},x + \tau_{xy},y + \tau_{xz},z = \rho u,tt \\
&\tau_{xy},x + \tau_{yy},y + \tau_{yz},z = \rho v,tt \\
&\tau_{xz},x + \tau_{yz},y + \tau_{zz},z = \rho w,tt
\end{aligned} \tag{15}$$

where ρ is the density of the medium.

When the components of stress are calculated according to eqn. (3) and the displacement components are calculated according to eqn. (1), the equations of motion can be written as [2]

$$\begin{aligned}
&(C_{11}S_x^2 + C_{66}S_y^2 + C_{44}S_z^2 - \rho)P_x + (C_{11} + C_{66})S_xS_yP_y + (C_{13} + C_{44})S_xS_zP_z = 0 \\
&(C_{12} + C_{66})S_xS_yP_x + (C_{66}S_x^2 + C_{11}S_y^2 + C_{44}S_z^2 - \rho)P_y + (C_{13} + C_{44})S_yS_zP_z = 0 \\
&(C_{13} + C_{44})S_xS_zP_x + (C_{13} + C_{44})S_yS_zP_y + (C_{44}S_x^2 + C_{44}S_y^2 + C_{33}S_z^2 - \rho)P_z = 0
\end{aligned} \tag{16}$$

Eqn. (16) can be put into matrix form as

$$[B]\,(P_x, P_y, P_z)^T = [0] \tag{17}$$

where $[B]$ is a 3x3 matrix with entries

$$b_{11} = (C_{11}S_x^2 + C_{66}S_y^2 + C_{44}S_z^2 - \rho)$$

$$b_{22} = (C_{66}S_x^2 + C_{11}S_y^2 + C_{44}S_z^2 - \rho)$$

$$b_{33} = (C_{44}S_x^2 + C_{44}S_y^2 + C_{33}S_z^2 - \rho)$$

$$b_{12} = b_{21} = (C_{12} + C_{66})S_xS_y$$

$$b_{13} = b_{31} = (C_{13} + C_{44})S_xS_z$$

$$b_{23} = b_{32} = (C_{13} + C_{44})S_yS_z \tag{18}$$

(P_x,P_y,P_z) is the unit vector of particle displacement and [0] is the 3x1 zero matrix. The matrix $[B]$, defined by eqn. (18), is a symmetric matrix; therefore, there exist three real eigenvalues. One eigenvalue corresponds to an SH wave, another eigenvalue corresponds to a P wave, and the third eigenvalue corresponds to an SV wave [2].

The plane wave solution is found by setting the determinant of matrix $[B]$, the matrix of the coefficients of the unit vector of particle displacement, equal to zero [6]. Expanding the determinant of $[B]$ and solving for the three roots yields equations for the three slowness surfaces. The slowness surface for a P wave is [5]

$$\frac{1}{2}\Big\{(C_{11}+C_{44})(S_x^2+S_y^2)+(C_{44}+C_{33})S_z^2+\{[(C_{11}-C_{44})(S_x^2+S_y^2)+(C_{33}-C_{44})S_z^2]^2+ 4(S_x^2+S_y^2)S_z^2[(C_{11}-C_{44})(C_{33}-C_{44})-(C_{13}+C_{44})^2]\}^{1/2}\Big\}=\rho \tag{19}$$

3. ANGLE OF REFLECTION OF REFLECTED P WAVE

According to eqn. (5) the y components of the slowness vectors of the incident P wave, the reflected P wave and the reflected SV wave are equal. Thus eqn. (5) can be written as

$$b = S_y^{(I)} = S_y^{(P)} = S_y^{(SV)} \tag{20}$$

where b is a constant and $S_y^{(I)}$, $S_y^{(P)}$ and $S_y^{(SV)}$ are the y components of the slowness vectors of the incident P wave, the reflected P wave and the reflected SV wave, respectively. The z component of the slowness vector of the reflected P wave, $S_z^{(P)}$, is found from the equation for the slowness surface of a P wave, eqn. (19). The values of the x and y components of the slowness vector of the reflected P wave, $S_x^{(P)}$ and $S_y^{(P)}$, given by eqn. (20) and eqn. (5), respectively, are substituted into eqn. (19) and $S_z^{(P)}$ is found to satisfy

$$S_z^{(P)} = \pm S_z^{(I)} \tag{21}$$

where $S_z^{(I)}$ is the z component of the slowness vector of the incident P wave. The slowness vector of the incident P wave points out of the medium and the slowness vector of the reflected P wave points into the medium (see Fig. 1), therefore the relationship between $S_z^{(P)}$ and $S_z^{(I)}$ is

$$S_z^{(P)} = -S_z^{(I)} \tag{22}$$

The angle of incidence is defined as the angle between the slowness vector of the incident wave and the normal to the boundary at the point of incidence. The angle of incidence of the incident P wave, $\theta^{(I)}$, is then

$$\theta^{(I)} = \arctan\left(\frac{S_y^{(I)}}{-S_z^{(I)}}\right) \tag{23}$$

where the point of incidence is taken to be on the plane boundary of the semi-infinite body (see Fig. 1).

The angle of reflection is defined, in a manner similar to the angle of incidence, as the angle between the slowness vector of the reflected wave and the normal to the boundary at the point of reflection. The angle of reflection of the reflected P wave, $\theta^{(P)}$, is then

$$\theta^{(P)} = \arctan\left(\frac{S_y^{(P)}}{S_z^{(P)}}\right) \tag{24}$$

Evaluating $\theta^{(P)}$ using eqn. (20) and eqn. (22) and comparing the result to eqn. (23) establishes that

$$\theta^{(P)} = \theta^{(I)} \tag{25}$$

Thus, the angle of reflection of the reflected P wave is equal to the angle of incidence of the incident P wave [2].

4. ANGLE OF REFLECTION OF REFLECTED SV WAVE

The equation for the slowness surface of an SV wave is found from the determinant of the matrix $[B]$, defined by eqn. (18), and is [5]

$$\frac{1}{2}\Big\{(C_{11}+C_{44})(S_x^2+S_y^2)+(C_{44}+C_{33})S_z^2-\{[(C_{11}-C_{44})(S_x^2+S_y^2)+(C_{33}-C_{44})S_z^2]^2+ \\ 4(S_x^2+S_y^2)S_z^2[(C_{11}-C_{44})(C_{33}-C_{44})-(C_{13}+C_{44})^2]\}^{1/2}\Big\}=\rho \tag{26}$$

where S_x, S_y and S_z are the x, y and z components of the slowness vector of the SV wave, respectively; ρ is the density of the medium and C_{11}, C_{13}, C_{33} and C_{44} are elastic constants. The z component of the slowness vector of the reflected SV wave, $S_z^{(SV)}$, is found from eqn. (26) by substituting in the values for $S_y^{(SV)}$ and $S_x^{(SV)}$, the y and x components of the slowness vector of the reflected SV wave, given by eqn. (20) and eqn. (5), respectively. The substitution provides two values for $S_z^{(SV)}$, one positive and one negative. The positive value corresponds to a wave traveling into the medium and is labeled $S_z^{(SV)}$. The angle of reflection of the reflected SV wave, $\theta^{(SV)}$, is then

$$\theta^{(SV)} = \arctan\left(\frac{S_y^{(SV)}}{S_z^{(SV)}}\right) \tag{27}$$

The angle of reflection of the reflected SV wave is, in general, not equal to the angle of incidence of the incident P wave [2].

A P wave incident, with angle of incidence $\theta^{(I)}$, in the y-z plane in the (x, y, z) coordinate system on the plane boundary of a semi-infinite body has y and z components, $S_y^{(I)}$ and $S_z^{(I)}$, of slowness vector satisfying

$$S_z^{(I)} = \frac{-S_y^{(I)}}{\tan\theta^{(I)}} \tag{28}$$

Substituting eqn. (5) and eqn. (28) into eqn. (19) allows the equation for the slowness surface of a P wave traveling in the plane x=0 to be written as

$$\frac{A}{2}(S_y^{(I)})^2 + \frac{B}{2}(S_y^{(I)})^2\tan^{-2}\theta^{(I)} + \frac{1}{2}\left\{\left[C(S_y^{(I)})^2 + D(S_y^{(I)})^2\tan^{-2}\theta^{(I)}\right]^2 + 4(S_y^{(I)})^4\tan^{-2}\theta^{(I)}(CD - E^2)\right\}^{1/2} = \rho \tag{29}$$

where the conventions:

$$A = C_{11} + C_{44}$$
$$B = C_{44} + C_{33}$$
$$C = C_{11} - C_{44}$$
$$D = C_{33} - C_{44}$$
$$E = C_{13} + C_{44} \tag{30}$$

have been used for computational ease. Eqn. (29) can be solved for $S_y^{(l)}$ yielding

$$S_y^{(l)} = \left\{ \frac{\rho}{\frac{A}{2} + \frac{B}{2}\tan^{-2}\theta^{(l)} + \frac{1}{2}\{[C + D\tan^{-2}\theta^{(l)}]^2 + 4\tan^{-2}\theta^{(l)}[CD - E^2]\}^{1/2}} \right\}^{1/2} \tag{31}$$

An expression for $S_z^{(SV)}$ as a function of $S_y^{(SV)}$ can be derived from the equation for the slowness surface of an SV wave by writing eqn. (26) as

$$-\frac{1}{2}\left\{\left(CS_y^{(SV)2} + DS_z^{(SV)2}\right)^2 + 4S_y^{(SV)2}S_z^{(SV)2}(CD - E^2)\right\}^{1/2} = \rho - \frac{A}{2}S_y^{(SV)2} - \frac{B}{2}S_z^{(SV)2} \tag{32}$$

Squaring both sides of eqn. (32) and simplifying the resulting expression produces a quadratic equation in $(S_z^{(SV)})^2$. Using the quadratic formula to find the roots of this equation produces the expression for $(S_z^{(SV)})^2$

$$S_z^{(SV)2} = \left\{\rho B - \frac{AB}{2}S_y^{(SV)2} + \frac{3CD}{2}S_y^{(SV)2} + E^2 S_y^{(SV)2} + \left\{[3\rho BCD - 2\rho BE^2 - \rho AD^2]S_y^{(SV)2} + \right.\right.$$
$$\left.\left.\left[\frac{-3ABCD}{2} + ABE^2 + 2C^2D^2 - 3CDE^2 + E^4 + \frac{B^2C^2}{4} + \frac{A^2D^2}{4}\right]S_y^{(SV)4} + \rho^2 D^2\right\}^{1/2}\right\} \div$$
$$\left\{\frac{B^2}{2} - \frac{D^2}{2}\right\} \tag{33}$$

The z component of the slowness vector of the reflected SV wave is then computed as a function of $S_y^{(SV)}$ according to

$$S_z^{(SV)} = \left\{ \left\{ \rho B - \frac{AB}{2} S_y^{(SV)2} + \frac{3CD}{2} S_y^{(SV)2} + E^2 S_y^{(SV)2} + \left\{ [3\rho BCD - 2\rho BE^2 - \rho AD^2] S_y^{(SV)2} + \left[\frac{-3ABCD}{2} + ABE^2 + 2C^2D^2 - 3CDE^2 + E^4 + \frac{B^2C^2}{4} + \frac{A^2D^2}{4} \right] S_y^{(SV)4} + \rho^2 D^2 \right\}^{1/2} \right\} \div \left\{ \frac{B^2}{2} - \frac{D^2}{2} \right\} \right\}^{1/2} \tag{34}$$

Substituting eqn. (20) into eqn. (34) allows $S_z^{(SV)}$ to be calculated as a function of the y component of the slowness vector of the incident P wave. The value of $S_z^{(SV)}$ is then given by

$$S_z^{(SV)} = \left\{ \left\{ \rho B - \frac{AB}{2} S_y^{(I)2} + \frac{3CD}{2} S_y^{(I)2} + E^2 S_y^{(SV)2} + \left\{ [3\rho BCD - 2\rho BE^2 - \rho AD^2] S_y^{(I)2} + \left[\frac{-3ABCD}{2} + ABE^2 + 2C^2D^2 - 3CDE^2 + E^4 + \frac{B^2C^2}{4} + \frac{A^2D^2}{4} \right] S_y^{(I)4} + \rho^2 D^2 \right\}^{1/2} \right\} \div \left\{ \frac{B^2}{2} - \frac{D^2}{2} \right\} \right\}^{1/2} \tag{35}$$

Substituting eqn. (20) and eqn. (35) into eqn. (27) allows $\theta^{(SV)}$ the angle of reflection of the reflected SV wave to be computed as a function of $S_y^{(I)}$ according to

$$\theta^{(SV)} = \arctan\left\{ \{S_y^{(I)}\} + \left\{ \left[\rho B - \frac{AB}{2} S_y^{(I)2} + \frac{3CD}{2} S_y^{(I)2} + E^2 S_y^{(SV)2} + \right. \right. \right.$$

$$\left([3\rho BCD - 2\rho BE^2 - \rho AD^2] S_y^{(I)2} + \right.$$

$$\left. \left. \left[\frac{-3ABCD}{2} + ABE^2 + 2C^2D^2 - 3CDE^2 + E^4 + \frac{B^2C^2}{4} + \frac{A^2D^2}{4} \right] S_y^{(I)4} + \rho^2 D^2 \right)^{1/2} \right] \div$$

$$\left. \left. \left[\frac{B^2}{2} - \frac{D^2}{2} \right] \right\}^{1/2} \right\} \tag{36}$$

where $S_y^{(I)}$ is given as a function of $\theta^{(I)}$ by eqn. (31).

The values of the material constants of a typical fiberglass epoxy composite are given in [4] as C_{11}=10.581x10^9 N/m^2, C_{13}=4.679x10^9 N/m^2, C_{33}=40.741x10^9 N/m^2, C_{44}=4.422x10^9 N/m^2, C_{66}=3.243x10^9 N/m^2 and ρ=1850 kg/m^3. The slowness surface of a P wave traveling in the y-z plane in the fiberglass epoxy composite is obtained by substituting the numerical values of the constants into eqn. (19) and setting the x component of the slowness vector equal to zero. The slowness surface of an SV wave traveling in the y-z plane in the fiberglass epoxy composite is found similarly from eqn. (26). The first quadrant of the slowness surfaces thus obtained are shown in Fig. 2.

The angle of reflection of the reflected SV wave is found as a function of the angle of incidence of the incident P wave by substituting the values of the material constants into eqn. (36). The angle of reflection of the reflected SV wave is shown in Fig. 3 as a function of the angle of incidence.

INPUT-OUTPUT CHARACTERIZATION OF FIBER COMPOSITE

1. ANGLES OF INCIDENCE AND REFLECTION OF P AND SV WAVES

Certain fiber composites may be modeled as a linearly elastic tranversely isotropic homogeneous continua [1]. A fiber composite modeled as such a solid in the form of an infinite plate where the plane of isotropy lies in the midplane of the plate is to be studied. A cartesian coordinate system (x, y, z) is chosen such that the x-y plane coincides with the plane of isotropy of the plate and the plate is bounded above by the plane $z=h/2$ and bounded below by the plane $z=-h/2$.

A transmitting transducer and a receiving transducer are assumed to be coupled to the top face of the plate in the y-z plane and separated by a distance L. The input electrical voltage of the transmitting transducer is a known function of time, $V_i(t)$. The output electrical voltage of the receiving transducer is an unknown function of time, $V_o(t)$. The transmitting transducer is assumed to be a point transducer that converts an input electrical voltage into a stress at a point on the boundary of the plate; the stress then travels through the plate as stress waves. The receiving transducer is assumed to be a point transducer that converts the stress at a point on the boundary of the plate into an output voltage.

A representative fiberglass epoxy composite plate will be characterised. The composite plate has an equivalent continuum model with material properties:

ρ=1850 kg/m^3

C_{11}=10.581x10^9 N/m^2

C_{12}=4.098x10^9 N/m^2

C_{13}=4.679x10^9 N/m^2

C_{33}=40.741x10^9 N/m^2

C_{44}=4.422x10^9 N/m^2

C_{66}=3.243x10^9 N/m^2

The transducers are assumed to be separated by a distance L=10 cm and the plate is of thickness h=5 cm.

In the following analysis only those stresses at the receiving transducer associated with P waves generated by the receiving transducer will be considered. The P waves produced by the transmitting transducer experience multiple reflections at each face of the plate before reaching the receiving transducer. Since the isotropic plane of the plate lies in the midplane of the plate and is parallel to the top and bottom faces of the plate each reflection may be treated as the reflection of a plane progressive wave on the plane boundary of a semi-infinite linearly elastic transversely isotropic continuum with plane of isotropy parallel to the boundary. Therefore, at each reflection the incident P wave reflects a P wave and an SV wave; the angle of reflection of the reflected P wave is equal to the angle of incidence of the incident P wave [2] and the angle of the reflection of the reflected SV wave is given by eqn. (36).

Assume a P wave is incident, with angle of incidence $\theta^{(i)}$, on one of the plane boundaries of the plate. An SV wave is reflected back into the medium with angle of reflection $\theta^{(SV)}$. The relationship between the y components, $S_y^{(I)}$ and $S_y^{(SV)}$, of the slowness vectors of the incident P wave and the reflected SV wave, respectively, is given by eqn. (20) as

$$S_y^{(I)} = S_y^{(SV)} \tag{37}$$

The SV wave then travels through the plate and is incident with an angle of incidence equal to $\theta^{(SV)}$ on the opposite face of the plate (see Fig. 4). An SV wave and a P wave are reflected by this new incident SV wave [2]. The reflected SV wave has angle of reflection $\theta^{(RSV)}$ equal to the angle of incidence of the incident SV wave [2],

$$\theta^{(RSV)} = \theta^{(SV)} \tag{38}$$

The reflected P wave has angle of reflection $\theta^{(RP)}$.

The relationship between the y components of the slowness vectors of the incident SV wave, the reflected SV wave and the reflected P wave is [2]

$$S_y^{(SV)} = S_y^{(RSV)} = S_y^{(RP)} \tag{39}$$

where $S_y^{(SV)}$, $S_y^{(RSV)}$ and $S_y^{(RP)}$ are the y components of the slowness vectors of the incident SV wave, the reflected SV wave, and the reflected P wave, respectively. Substituting eqn. (37) into eqn. (39) demonstrates that

$$S_y^{(I)} = S_y^{(RP)} \tag{40}$$

The z component, $S_z^{(RP)}$, of the slowness vector of the P wave reflected by the incident SV wave is found by substituting eqn. (40) into eqn. (11) to satisfy

$$S_z^{(RP)} = \pm S_z^{(I)} \tag{41}$$

where $S_z^{(I)}$ is the z component of the slowness vector of the initial P wave. The P wave reflected by the incident SV wave and the initial P wave are both traveling from the upper face of the plate towards the lower face of the plate (see Fig. 4), so the relationship between $S_z^{(RP)}$ and $S_z^{(I)}$ is

$$S_z^{(RP)} = S_z^{(I)} \tag{42}$$

The angle of incidence $\theta^{(I)}$ of the initial P wave is defined by

$$\theta^{(I)} = \arctan\left(\frac{S_y^{(I)}}{-S_z^{(I)}}\right) \tag{43}$$

and the angle of reflection of the P wave reflected by the incident SV wave, $\theta^{(RP)}$, is defined by

$$\theta^{(RP)} = \arctan\left(\frac{S_y^{(RP)}}{-S_z^{(RP)}}\right) \tag{44}$$

Substituting eqn. (40) and eqn. (42) into eqn. (44) and comparing the result to eqn. (43) establishes that

$$\theta^{(RP)} = \theta^{(I)} \tag{45}$$

Thus, an incident P wave with angle of incidence $\theta^{(I)}$ will cause a series of SV and P waves to be produced by the initial and subsequent reflections. The SV waves will always be reflected with angle of reflection $\theta^{(SV)}$, given by equation (36), and the P waves will always be reflected with angle of reflection equal to $\theta^{(I)}$.

2. AMPLITUDE COEFFICIENTS ASSOCIATED WITH INCIDENT P WAVE

The amplitude coefficients of the P and SV waves reflected by a P wave incident in the y-z plane on the plane boundary of semi-infinite transversely isotropic medium are given by the pair of simultaneous equations [2]

$$S_z^{(I)}P_y^{(I)} + S_y^{(I)}P_z^{(I)} + A^{(P-P)}(S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)}) + \\ A^{(P-SV)}(S_z^{(SV)}P_y^{(SV)} + S_y^{(SV)}P_z^{(SV)}) = 0 \tag{46}$$

and

$$C_{13}S_y^{(I)}P_y^{(I)} + C_{33}S_z^{(I)}P_z^{(I)} + A^{(P-P)}(C_{13}S_y^{(P)}P_y^{(P)} + C_{33}S_z^{(P)}P_z^{(P)}) + \\ A^{(P-SV)}(C_{13}S_y^{(SV)}P_y^{(SV)} + C_{33}S_z^{(SV)}P_z^{(SV)}) = 0 \tag{47}$$

where $A^{(P-P)}$ is the amplitude coefficient of the reflected P wave and $A^{(P-SV)}$ is the amplitude coefficient of the reflected SV wave; $S_y^{(I)}$, $S_y^{(P)}$ and $S_y^{(SV)}$ are the y components of the slowness vectors of the incident P wave, reflected P wave and reflected SV wave, respectively; $S_z^{(I)}$, $S_z^{(P)}$ and $S_z^{(SV)}$ are the z components of the slowness vectors of the incident P wave, reflected P wave and reflected SV wave, respectively; $P_y^{(I)}$, $P_y^{(P)}$ and $P_y^{(SV)}$ are the y components of the unit vectors of particle displacement of the incident P wave, reflected P wave and reflected SV wave, respectively; $P_z^{(I)}$, $P_z^{(P)}$ and $P_z^{(SV)}$ are the z components of the unit vectors of particle displacement of the incident P wave, reflected P wave and reflected SV wave, respectively; and C_{13} and C_{33} are elastic constants. Eqn. (46) can be solved to provide the explicit expression for $A^{(P-P)}$,

$$A^{(P-P)} = -\frac{S_z^{(I)}P_y^{(I)} + S_y^{(I)}P_z^{(I)} + A^{(P-SV)}(S_z^{(SV)}P_y^{(SV)} + S_y^{(SV)}P_z^{(SV)})}{S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)}} \tag{48}$$

Substituting eqn. (48) into eqn. (47) yields

$$C_{13}S_y^{(I)}P_y^{(I)} + C_{33}S_z^{(I)}P_z^{(I)} -$$
$$\frac{S_z^{(I)}P_y^{(I)} + S_y^{(I)}P_z^{(I)} + A^{(P-SV)}(S_z^{(SV)}P_y^{(SV)} + S_y^{(SV)}P_z^{(SV)})}{S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)}} \times$$
$$[C_{13}S_y^{(P)}P_y^{(P)} + C_{33}S_z^{(P)}P_z^{(P)}] + A^{(P-SV)}[C_{13}S_y^{(SV)}P_y^{(SV)} + C_{33}S_z^{(SV)}P_z^{(SV)}] = 0 \qquad (49)$$

Eqn. (49) is an implicit function for $A^{(P\text{-}SV)}$ dependent only on the material properties and the slowness vectors and the particle displacement vectors of the incident and reflected waves. This equation can be solved to provide the explicit function for $A^{(P\text{-}SV)}$

$$\begin{aligned}A^{(P-SV)} = \{&-C_{13}S_y^{(I)}P_y^{(I)} - C_{33}S_z^{(I)}P_z^{(I)} + [(S_z^{(I)}P_y^{(I)} + S_y^{(I)}P_z^{(I)}) \times \\ &(C_{13}S_y^{(P)}P_y^{(P)} + C_{33}S_z^{(P)}P_z^{(P)})] \div [S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)}]\} \\ \{&-[(S_z^{(SV)}P_y^{(SV)} + S_y^{(SV)}P_z^{(SV)})(C_{13}S_y^{(P)}P_y^{(P)} + C_{33}S_z^{(P)}P_z^{(P)})] + \\ &[S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)}] + C_{13}S_y^{(SV)}P_y^{(SV)} + C_{33}S_z^{(SV)}P_z^{(SV)}\}^{-1}\end{aligned} \qquad (50)$$

The expression for $A^{(P\text{-}SV)}$ given by eqn. (50) can be substituted into eqn. (48) to produce an expression for $A^{(P\text{-}P)}$ independent of $A^{(P\text{-}SV)}$. The amplitude coefficient of the reflected P wave is then

$$\begin{aligned}A^{(P-P)} = &-(S_z^{(I)}P_y^{(I)} + S_y^{(I)}P_z^{(I)})(S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)})^{-1} - \\ &(S_z^{(SV)}P_y^{(SV)} + S_y^{(SV)}P_z^{(SV)})(S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)})^{-1} \times \\ &\{-C_{13}S_y^{(I)}P_y^{(I)} - C_{33}S_z^{(I)}P_z^{(I)} + (S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)}) \times \\ &(S_z^{(I)}P_y^{(I)} + S_y^{(I)}P_z^{(I)}) \div (C_{13}S_y^{(P)}P_y^{(P)} + C_{33}S_z^{(P)}P_z^{(P)})\} \times \\ &\{-(S_z^{(SV)}P_y^{(SV)} + S_y^{(SV)}P_z^{(SV)})(C_{13}S_y^{(P)}P_y^{(P)} + C_{33}S_z^{(P)}P_z^{(P)}) + \\ &(S_z^{(P)}P_y^{(P)} + S_y^{(P)}P_z^{(P)}) + C_{13}S_y^{(SV)}P_y^{(SV)} + C_{33}S_z^{(SV)}P_z^{(SV)}\}^{-1}\end{aligned} \qquad (51)$$

The value of $S_y^{(I)}$ is found as a function of the angle of incidence $\theta^{(I)}$ from eqn. (31) to be

$$S_y^{(I)} = \left\{ \frac{\rho}{\frac{A}{2} + \frac{B}{2}\tan^{-2}\theta^{(I)} + \frac{1}{2}\{[C + D\tan^{-2}\theta^{(I)}]^2 + 4\tan^{-2}\theta^{(I)}[CD - E^2]\}^{1/2}} \right\}^{1/2} \tag{52}$$

where ρ is the density of the medium and A, B, C, D and E are functions of the material properties defined by eqn. (30). Eqn. (20) establishes the equality of $S_y^{(I)}$, $S_y^{(P)}$ and $S_y^{(SV)}$ and allows $S_y^{(P)}$ and $S_y^{(SV)}$ also to be found as functions of $\theta^{(I)}$. Substituting eqn. (52) into eqn. (28) establishes the value of $S_z^{(I)}$ as

$$S_z^{(I)} = -\frac{1}{\tan\theta^{(I)}} \times$$

$$\left\{ \frac{\rho}{\frac{A}{2} + \frac{B}{2}\tan^{-2}\theta^{(I)} + \frac{1}{2}\{[C + D\tan^{-2}\theta^{(I)}]^2 + 4\tan^{-2}\theta^{(I)}[CD - E^2]\}^{1/2}} \right\}^{1/2} \tag{53}$$

Substituting eqn. (52) into eqn. (20) to find $S_y^{(SV)}$ and substituting that value into eqn. (32) provides the expression for $S_z^{(SV)}$ in terms of $\theta^{(I)}$, as

$$S_z^{(SV)} = \frac{1}{\tan\theta^{(SV)}} \times$$

$$\left\{ \frac{\rho}{\frac{A}{2} + \frac{B}{2}\tan^{-2}\theta^{(I)} + \frac{1}{2}\{[C + D\tan^{-2}\theta^{(I)}]^2 + 4\tan^{-2}\theta^{(I)}[CD - E^2]\}^{1/2}} \right\}^{1/2} \tag{54}$$

where $\theta^{(SV)}$ is the angle of reflection of the reflected SV wave and is determined as a function of $\theta^{(I)}$ by eqn. (36). The relationship between $S_z^{(I)}$ and $S_z^{(P)}$ is determined by eqn. (22) so the z component of the slowness vector of the reflected P wave can also be found as a function of $\theta^{(I)}$.

Thus, the components of the slowness vectors of the incident and reflected waves are determined by eqns. (20), (22), (52), (53) and (54) as functions of only the material properties and the angle of incidence.

The y component of the unit vector of particle displacement of a P wave traveling in a transversely isotropic continuum is calculated in [5] and can be expressed for a wave traveling in the y-z plane as [2]

$$P_y = \frac{H_p - DS_z^2}{[(H_p - DS_z^2)^2 + E^2 S_y^2 S_z^2]^{1/2}} \tag{55}$$

where the conventions of eqn. (30) have been used. Similarly, the z component of the unit vector of particle displacement of a P wave traveling in the y-z plane can be expressed as

$$P_z = \frac{ES_y S_z}{[(H_p - DS_z^2)^2 + E^2 S_y^2 S_z^2]^{1/2}} \tag{56}$$

The coefficient H_p in eqn. (55) and eqn. (56) is defined by [2]

$$H_p = \frac{1}{2}\left\{CS_y^2 + DS_z^2 + \left[(CS_y^2 + DS_z^2)^2 - 4S_y^2 S_z^2 (CD - E^2)\right]^{1/2}\right\} \tag{57}$$

Substituting eqn. (28) into eqn. (57) allows the coefficient H_p to be written as a function of only $S_y^{(I)}$ and the material properties. The H_p coefficient associated with the incident P wave, $H_p^{(I)}$, is then

$$H_p^{(I)} = \frac{1}{2}\left\{C(S_y^{(I)})^2 + D(S_y^{(I)})^2 \tan^{-2}\theta^{(I)} + \left[\left(C(S_y^{(I)})^2 + D(S_y^{(I)})^2 \tan^{-2}\theta^{(I)}\right)^2 - \right.\right.$$

$$\left.\left. 4(S_y^{(I)})^4 \tan^{-2}\theta^{(I)}(CD - E^2)\right]^{1/2}\right\} \tag{58}$$

Eqn. (58) can be factored as

$$H_p^{(I)} = (S_y^{(I)})^2 H_p' \tag{59}$$

where

$$H_p' = \frac{1}{2}\left\{C + D\tan^{-2}\theta^{(I)} + [(C + D\tan^{-2}\theta^{(I)})^2 - 4\tan^{-2}\theta^{(I)}(CD - E^2)]^{1/2}\right\} \tag{60}$$

Using eqns. (20), (22) and (28), eqn. (57) can be evaluated to provide an expression for $H_p^{(P)}$, the H_p coefficient associated with the reflected P wave, as

$$H_p^{(P)} = \frac{1}{2}\left\{C(S_y^{(I)})^2 + D(S_y^{(I)})^2 \tan^{-2}\theta^{(I)} + \left[\left(C(S_y^{(I)})^2 + D(S_y^{(I)})^2 \tan^{-2}\theta^{(I)}\right)^2 - \right.\right.$$

$$\left.\left. 4(S_y^{(I)})^4 \tan^{-2}\theta^{(I)}(CD - E^2)\right]^{1/2}\right\} \tag{61}$$

Comparing eqn. (61) and eqn. (58) establishes that

$$H_p^{(I)} = H_p^{(P)} \tag{62}$$

The y component of the unit vector of particle displacement of the incident P wave is computed by substituting eqn. (28) and eqn. (59) into eqn. (55) to be

$$P_y^{(I)} = \left[(S_y^{(I)})^2 H_p' - D(S_y^{(I)})^2 \tan^{-2}\theta^{(I)} \right] \div \\ \left[\left((S_y^{(I)})^2 H_p' - D(S_y^{(I)})^2 \tan^{-2}\theta^{(I)} \right)^2 + E^2 (S_y^{(I)})^4 \tan^{-2}\theta^{(I)} \right]^{-1/2} \tag{63}$$

Canceling a factor of $(S_y^{(I)})^2/(S_y^{(I)})^2$ from the right-hand side of eqn. (63) produces an expression for $P_y^{(I)}$ dependent on $\theta^{(I)}$ and the material properties. The y component of the unit vector of particle displacement of the incident P wave can then be written as

$$P_y^{(I)} = \frac{H_p' - D\tan^{-2}\theta^{(I)}}{[(H_p' - D\tan^{-2}\theta^{(I)})^2 + E^2 \tan^{-2}\theta^{(I)}]^{1/2}} \tag{64}$$

Similar substitutions allow the z component of the unit vector of particle displacment of the incident P wave to be calculated as a function of the angle of incidence and the material properties. Eqn. (28) and eqn. (59) are substituted into eqn. (56); a factor of $(S_y^{(I)})^2/(S_y^{(I)})^2$ is canceled from the resulting expression and $P_z^{(I)}$ is then determined by

$$P_z^{(I)} = -\frac{E}{\tan\theta^{(I)}} \left[(H_p' - D\tan^{-2}\theta^{(I)})^2 + E^2 \tan^{-2}\theta^{(I)} \right]^{-1/2} \tag{65}$$

The components of the unit vector of particle displacement of the reflected P wave are found by similar methods. Eqns. (20), (22), (28), (59) and (62) are substituted into eqn. (55) to produce an expression for $P_y^{(P)}$ dependent on only the material properties and $\theta^{(I)}$. Comparing the equation for $P_y^{(P)}$ resulting from the above substitutions to eqn. (65) establishes the relationship between $P_y^{(P)}$ and $P_y^{(I)}$ as

$$P_y^{(P)} = P_y^{(I)} \tag{66}$$

Substituting eqns. (20), (22), (28), (59) and (62) into eqn. (56) provides an expression for $P_z^{(P)}$ dependent only on $\theta^{(I)}$ and the material properties. The value of $P_z^{(P)}$ is given by

$$P_z^{(P)} = \frac{E}{\tan\theta^{(I)}}\left[(H_p' - D\tan^{-2}\theta^{(I)})^2 + E^2\tan^{-2}\theta^{(I)}\right]^{-1/2} \tag{67}$$

Comparing eqn. (67) and eqn. (64) establishes

$$P_z^{(P)} = -P_z^{(I)} \tag{68}$$

The components of the unit vector of particle displacement of an SV wave traveling in a transversely isotropic continuum are given in [5] and can be written for the present case of a wave traveling in the y-z plane in the (x, y, z) coordinate system as [2]

$$P_y = \frac{H_{sv} - DS_z^2}{[(H_{sv} - DS_z^2)^2 + E^2S_y^2S_z^2]^{1/2}} \tag{69}$$

and

$$P_z = \frac{ES_yS_z}{[(H_{sv} - DS_z^2)^2 + E^2S_y^2S_z^2]^{1/2}} \tag{70}$$

where

$$H_{sv} = \frac{1}{2}\left\{CS_y^2 + DS_y^2 - \left[(CS_y^2 + DS_z^2)^2 - 4S_y^2S_z^2(CD - E^2)\right]^{1/2}\right\} \tag{71}$$

and the conventions defined by eqn. (30) are used.

Substituting the value of $S_z^{(SV)}$ given by eqn. (32) into eqn. (71) allows the H_{sv} coefficient associated with the reflected SV wave, $H_{sv}^{(SV)}$, to be written as

$$H_{sv}^{(SV)} = \frac{1}{2}\left\{C(S_y^{(SV)})^2 + D(S_y^{(SV)})^2 \tan^{-2}\theta^{(SV)} - \left[\left(C(S_y^{(SV)})^2 + D(S_y^{(SV)})^2 \tan^{-2}\theta^{(SV)}\right)^2 - \right.\right.$$

$$\left.\left. 4(S_y^{(SV)})^4 \tan^{-2}\theta^{(SV)}(CD - E^2)\right]^{1/2}\right\} \tag{72}$$

where $\theta^{(SV)}$ is the angle of reflection of the reflected SV wave and is given as a function of the angle of incidence $\theta^{(I)}$ of the incident P wave by eqn. (36). The right-hand side of eqn. (72) can be factored as

$$H_{sv}^{(SV)} = (S_y^{(SV)})^2 H_{sv}' \tag{73}$$

where

$$H_{sv}' = \frac{1}{2}\left\{C + D\tan^{-2}\theta^{(SV)} - [(C + D\tan^{-2}\theta^{(SV)})^2 - 4\tan^{-2}\theta^{(SV)}(CD - E^2)]^{1/2}\right\} \tag{74}$$

The y component of the unit vector of particle displacement of the reflected SV wave can be calculated by substituting eqn. (32) and eqn. (73) into eqn. (69), then, canceling a factor of $(S_y^{(SV)})^2/(S_y^{(SV)})^2$ from the resulting expression to be

$$P_y^{(SV)} = \frac{H_{sv}' - D\tan^{-2}\theta^{(SV)}}{[(H_{sv}' - D\tan^{-2}\theta^{(SV)})^2 + E^2\tan^{-2}\theta^{(SV)}]^{1/2}} \tag{75}$$

Similar substitutions allow the z component of unit vector of particle displacement of the reflected SV wave to be written as a function of $\theta^{(I)}$ and the material properties. Eqn. (32) and eqn. (73) are substituted into eqn. (70); a factor of $(S_y^{(SV)})^2/(S_y^{(SV)})^2$ is canceled from the right-hand side of the ensuing expression and $P_z^{(SV)}$ is then given by

$$P_z^{(SV)} = \frac{E}{\tan\theta^{(SV)}}\left[(H_{sv}' - D\tan^{-2}\theta^{(SV)})^2 + D\tan^{-2}\theta^{(SV)}\right]^{-1/2} \tag{76}$$

The amplitude coefficients of the P wave and SV wave reflected by an incident P wave can be determined as functions of only the angle of incidence $\theta^{(I)}$ of the incident P wave and the material properties. The value of $S_y^{(I)}$, the y component of the slowness vector of the incident P wave, is found as a function of $\theta^{(I)}$ from eqn. (52). The y components, $S_y^{(P)}$ and $S_y^{(SV)}$, of the slowness vectors of the reflected P wave and reflected SV wave, respectively, are set equal to $S_y^{(I)}$ according to eqn. (20). The value of $S_z^{(I)}$, the z component of the slowness vector of the incident P wave, is determined by eqn. (53) and the z component, $S_z^{(P)}$, of the slowness vector of the reflected P wave is set equal to the negative of $S_z^{(I)}$ according to eqn. (22). The value of $S_z^{(SV)}$, the z component of the slowness vector of the reflected SV wave, is found from eqn. (54) where $\theta^{(SV)}$ is the angle of reflection of the reflected SV wave and is defined in terms of $\theta^{(I)}$ by eqn. (36). The values of $P_y^{(I)}$ and $P_z^{(I)}$, the y and z components, respectively, of the unit vector of particle displacement of the incident P wave, are determined by eqn. (64) and eqn. (65) where the coefficient H_p' is determined by eqn. (60). The y component of the unit vector of particle displacement of the reflected P wave, $P_y^{(P)}$, is set equal to $P_y^{(I)}$ according to eqn. (66); and $P_z^{(P)}$, the z component of the unit vector of particle displacement of the reflected P wave, is set equal to the negative of $P_z^{(I)}$ according to eqn. (68). The values of $P_y^{(SV)}$ and $P_z^{(SV)}$, the y and z components of the unit vector of particle displacement of the reflected SV wave, are found from eqn. (75) and eqn. (76) where the coefficient H_{sv}' is determined by eqn. (74).

The values of the y and z components of the slowness vectors of the incident P wave and the reflected P and SV waves and the y and z components of the unit vectors of particle displacement of the incident P wave and the reflected P and SV waves, determined as functions of $\theta^{(I)}$, can then be substituted into eqn. (50) and eqn. (51) to obtain an expression for $A^{(P\text{-}SV)}$, the amplitude coefficient of the reflected SV wave, and an expression for $A^{(P\text{-}P)}$, the amplitude coefficient of the reflected P wave, as functions of $\theta^{(I)}$. Thus, the amplitude coefficients of the reflected waves are determined by only the angle of incidence of the incident wave and the material properties.

The amplitude coefficient of the P wave reflected by an incident P wave is calculated using the material properties of the representative fiberglass epoxy composite and is shown as a function of the angle of incidence of the incident P wave in Fig. 5. The amplitude coefficient of the SV wave reflected by an incident P wave is calculated similarly and is shown as a function of angle of incidence in Fig. 6.

3. AMPLITUDE COEFFICIENTS ASSOCIATED WITH INCIDENT SV WAVE PRODUCED BY MODE SPLITTING

The amplitude coefficients $A^{(SV\text{-}SV)}$ and $A^{(SV\text{-}P)}$ of the SV wave and the P wave, respectively, reflected by an incident SV wave are given by the pair of simultaneous equations [2]

$$\begin{aligned} &S_z^{(ISV)}P_y^{(ISV)} + S_y^{(ISV)}P_z^{(ISV)} + A^{(SV-P)}(S_z^{(RP)}P_y^{(RP)} + S_y^{(RP)}P_z^{(RP)}) + \\ &\quad A^{(SV-SV)}(S_z^{(RSV)}P_y^{(RSV)} + S_y^{(RSV)}P_z^{(RSV)}) = 0 \end{aligned} \tag{77}$$

and

$$C_{13}S_y^{(ISV)}P_y^{(ISV)} + C_{33}S_z^{(ISV)}P_z^{(ISV)} + A^{(SV-P)}(C_{13}S_y^{(RP)}P_y^{(RP)} + C_{33}S_z^{(RP)}P_z^{(RP)}) +$$
$$A^{(SV-SV)}(C_{13}S_y^{(RSV)}P_y^{(RSV)} + C_{33}S_z^{(RSV)}P_z^{(RSV)}) = 0 \qquad (78)$$

where $S_y^{(ISV)}$ and $S_z^{(ISV)}$ are the y and z components of the slowness vector of the incident SV wave, respectively; $S_y^{(RSV)}$ and $S_z^{(RSV)}$ are the y and z components of the slowness vector of the reflected SV wave, respectively; $S_y^{(RP)}$ and $S_z^{(RP)}$ are the y and z components of the slowness vector of the reflected P wave, respectively; $P_y^{(ISV)}$ and $P_z^{(ISV)}$ are the y and z components of the unit vector of particle displacement of the incident SV wave, respectively; $P_y^{(RSV)}$ and $P_z^{(RSV)}$ are the y and z components of the unit vector of particle displacement of the reflected SV wave, respectively; and $P_y^{(RP)}$ and $P_z^{(RP)}$ are the y and z components of the unit vector of particle displacement of the reflected P wave, respectively.

Eqn. (77) can be solved for $A^{(SV-P)}$ in terms of $A^{(SV-SV)}$ as

$$A^{(SV-P)} = -\{S_z^{(ISV)}P_y^{(ISV)}P_z^{(ISV)} + S_y^{(ISV)} + A^{(SV-SV)}(S_z^{(RSV)}P_y^{(RSV)} + S_y^{(RSV)}P_z^{(RSV)})\} \div$$
$$\{S_z^{(RP)}P_y^{(RP)} + S_y^{(RP)}P_z^{(RP)}\} \qquad (79)$$

Substituting eqn. (79) into eqn. (78) provides an expression for $A^{(SV-SV)}$ independent of $A^{(SV-P)}$. Simplifying the resulting equation produces the explicit function for $A^{(SV-SV)}$

$$A^{(SV-SV)} = \{-C_{13}S_y^{(ISV)}P_y^{(ISV)} - C_{33}S_z^{(ISV)}P_z^{(ISV)} + [(S_z^{(ISV)}P_y^{(ISV)} + S_y^{(ISV)}P_z^{(ISV)}) \times$$
$$(C_{13}S_y^{(RP)}P_y^{(RP)} + C_{33}S_z^{(RP)}P_z^{(RP)})] \div [S_z^{(RP)}P_y^{(RP)} + S_y^{(RP)}S_z^{(RP)}]\}$$
$$\{[-(S_z^{(RSV)}P_y^{(RSV)} + S_y^{(RSV)}P_z^{(RSV)})(C_{13}S_y^{(RP)}P_y^{(RP)} + C_{33}S_z^{(RP)}P_z^{(RP)})] \div$$
$$[S_z^{(RP)}P_y^{(RP)} + S_y^{(RP)}P_z^{(RP)}] + [C_{13}S_y^{(RSV)}P_y^{(RSV)} + C_{33}S_z^{(RSV)}P_z^{(RSV)}]\}^{-1} \qquad (80)$$

Substituting eqn. (80) into eqn. (79) yields an expression for $A^{(SV-P)}$ independent of $A^{(SV-SV)}$. The amplitude coefficient of the reflected P wave is

$$
\begin{aligned}
A^{(SV-P)} = &-\{S_z^{(ISV)}P_y^{(ISV)} + S_y^{(ISV)}P_z^{(ISV)} + \{-C_{13}S_y^{(ISV)}P_y^{(ISV)} + C_{33}S_z^{(ISV)}P_z^{(ISV)} + \\
&[(S_z^{(ISV)}P_y^{(ISV)} + S_y^{(ISV)}P_z^{(ISV)})(C_{13}S_y^{(RP)}P_y^{(RP)} + C_{33}S_z^{(RP)}P_z^{(RP)})][S_z^{(RP)}P_y^{(RP)}S_y^{(RP)}P_z^{(RP)}]^{-1}\} \times \\
&\{[-(S_z^{(RSV)}P_y^{(RSV)} + S_y^{(RSV)}P_z^{(RSV)})(C_{13}S_y^{(RP)}P_y^{(RP)} + C_{33}S_z^{(RP)}P_z^{(RP)})] \div \\
&[S_z^{(RP)}P_y^{(RP)} + S_y^{(RP)}P_z^{(RP)}] + [C_{13}S_y^{(RSV)}P_y^{(RSV)} + C_{33}S_z^{(RSV)}P_z^{(RSV)}]\}^{-1} \times \\
&\{S_z^{(RSV)}P_y^{(RSV)} + S_y^{(RSV)}P_z^{(RSV)}\}\}\{S_z^{(RP)}P_y^{(RP)} + S_y^{(RP)}P_z^{(RP)}\}^{-1}
\end{aligned}
\tag{81}
$$

Assume the incident SV wave was produced by the reflection of a P wave incident with angle of incidence $\theta^{(I)}$. Then, the y and z components of the slowness vector satisfy

$$S_y^{(ISV)} = S_y^{(SV)} \tag{82}$$

and

$$S_z^{(ISV)} = S_z^{(SV)} \tag{83}$$

where $S_y^{(SV)}$ and $S_z^{(SV)}$ are the y and z components, respectively, of the slowness vector of the SV wave reflected by the initial P wave. The y component of the slowness vector of the SV wave reflected by the initial P wave is determined from eqn. (20) and eqn. (52), and $S_z^{(SV)}$ is determined from eqn. (54). The components of the slowness vector of the incident SV wave are thus determined as functions of the angle of incidence of the initial P wave, $\theta^{(I)}$.

The relationship between the y components of the slowness vectors of the incident SV wave and the reflected SV and P waves is given by eqn. (39) as [2]

$$S_y^{(ISV)} = S_y^{(RSV)} = S_y^{(RP)} \tag{84}$$

The z component of the slowness vector of the reflected SV wave is found by substituting the expression for $S_y^{(RSV)}$ determined by eqn. (84) into the equation for the slowness surface of an SV wave, eqn. (26), to satisfy

$$S_z^{(RSV)} = \pm S_z^{(ISV)} \tag{85}$$

If the slowness vector of the incident SV wave is pointing in the positive z direction then the slowness vector of the reflected SV wave is pointing in the negative z direction and vice versa (see Fig. 4); therefore, the relationship between $S_z^{(RSV)}$ and $S_z^{(ISV)}$ is

$$S_z^{(RSV)} = -S_z^{(ISV)} \tag{86}$$

where $S_z^{(RSV)}$ and $S_z^{(ISV)}$ are the z components of the slowness vectors of the reflected SV wave and the incident SV wave, respectively. Substituting eqn. (84) into eqn. (82) and then using eqn. (20) and eqn. (52) provides an expression for $S_y^{(RSV)}$ dependent on only the material properties and $\theta^{(I)}$. Substituting eqn. (86) into eqn. (83) and substituting the resulting expression into eqn. (54) provides an expression for $S_z^{(RSV)}$ as a function of the material properties and $\theta^{(I)}$.

The relationship between $S_y^{(RP)}$ and the y component, $S_y^{(I)}$, of the slowness vector of the initial incident P wave is given by eqn. (40) as

$$S_y^{(RP)} = S_y^{(I)} \tag{87}$$

The relationship between $S_z^{(RP)}$ and the z component, $S_z^{(I)}$, of the slowness vector of the initial incident P wave is given by eqn. (42) as

$$S_z^{(RP)} = S_z^{(I)} \tag{88}$$

Substituting eqn. (87) into eqn. (52) produces an expression for $S_y^{(RP)}$ dependent only on the material properties and $\theta^{(I)}$, and substituting eqn. (88) into eqn. (52) produces an expression for $S_z^{(RP)}$ as a function of $\theta^{(I)}$.

The components of the unit vector of particle displacement of an SV wave traveling in a transversely isotropic continuum are given in [5] and are reproduced for the present case of a wave traveling in the y-z plane in the (x, y, z) coordinate system in eqn. (69) and eqn. (70) following [2]. The incident SV wave is the same wave as the SV wave reflected by the initial incident P wave, therefore, the y and z components of the unit vector of particle displacement of the incident SV wave are equal to the y and z components, $P_y^{(SV)}$ and $P_z^{(SV)}$, of the SV wave reflected by the incident P wave;

$$P_y^{(ISV)} = P_y^{(SV)} \tag{89}$$

and

$$P_z^{(ISV)} = P_z^{(SV)} \tag{90}$$

where $P_y^{(SV)}$ and $P_z^{(SV)}$ are given by eqn. (75) and eqn. (76), respectively.

The coefficient H_{sv}, defined by eqn. (70), can be evaluated for the reflected SV wave by substituting eqn. (86) into eqn. (28) to establish

$$S_z^{(RSV)} = \frac{-S_y^{(SV)}}{\tan\theta^{(SV)}} \tag{91}$$

then, substituting eqn. (82) and eqn. (91) into eqn. (71) to evaluate the H_{sv} coefficient associated with the reflected SV wave, $H_{sv}^{(RSV)}$, as

$$H_{sv}^{(RSV)} = \frac{1}{2}\left\{C(S_y^{(SV)})^2 + D(S_y^{(SV)})^2\tan^{-2}\theta^{(SV)} - \left[\left(C(S_y^{(SV)})^2 + D(S_y^{(SV)})^2\tan^{-2}\theta^{(SV)}\right)^2 - 4(S_y^{(SV)})^4\tan^{-2}\theta^{(SV)}(CD - E^2)\right]^{1/2}\right\} \tag{92}$$

Comparing eqn. (92) and eqn. (74) establishes that

$$H_{sv}^{(RSV)} = (S_y^{(SV)})^2 H_{sv}' \tag{93}$$

The y component of the unit vector of particle displacement is found by substituting eqns. (82), (91) and (93) into eqn. (69) to be

$$P_y^{(RSV)} = \left[(S_y^{(SV)})^2 H_{sv}' - D(S_y^{(SV)})^2 \tan^{-2}\theta^{(SV)} \right] \div \left[\left((S_y^{(SV)})^2 H_{sv}' - D(S_y^{(SV)})^2 \tan^{-2}\theta^{(SV)} \right)^2 + 4E^2 (S_y^{(SV)})^4 \tan^{-2}\theta^{(SV)} \right]^{1/2} \tag{94}$$

Canceling a factor of $(S_y^{(SV)})^2/(S_y^{(SV)})^2$ from the right-hand side of eqn. (94) and comparing the resulting expression to eqn. (75) establishes that

$$P_y^{(RSV)} = P_y^{(SV)} \tag{95}$$

The z component of the unit vector of particle displacement of the reflected SV wave is found by similar substitutions. Eqns. (84), (89) and (93) are substituted into eqn. (70). A factor of $(S_y^{(SV)})^2/(S_y^{(SV)})^2$ is canceled from the right-hand side of the subsequent equation to produce an expression for $P_z^{(RSV)}$ dependent on only the material properties and $\theta^{(SV)}$, the angle of incidence of the incident SV wave. Comparing the expression for $P_z^{(RSV)}$ thus generated to eqn. (76) produces the relationship

$$P_z^{(RSV)} = -P_z^{(SV)} \tag{96}$$

The components of the unit vector of particle displacement of a P wave traveling in a transversely isotropic contiuum are given in [5] and are reproduced for the present case of a wave traveling in the y-z plane in eqn. (55) and eqn. (56) following [2]. The coefficient H_p, defined by eqn. (57), can be evaluated for the reflected P wave by substituting eqn. (88) into eqn. (28) producing the expression

$$S_z^{(RP)} = \frac{-S_y^{(I)}}{\tan\theta^{(I)}} \tag{97}$$

then, substituting eqn. (87) and eqn. (97) into eqn. (57) to evaluate the H_p coefficient associated with the reflected P wave, $H_p^{(RP)}$, as

$$H_p^{(RP)} = \frac{1}{2}\left\{C(S_y^{(I)})^2 + D(S_y^{(I)})^2\tan^{-2}\theta^{(I)} + \left[\left(C(S_y^{(I)})^2 + D(S_y^{(I)})^2\tan^{-2}\theta^{(I)}\right)^2 - \right.\right.$$

$$\left.\left. 4(S_y^{(I)})^4\tan^{-2}\theta^{(I)}(CD - E^2)\right]^{1/2}\right\} \tag{98}$$

Comparing eqn. (88) and eqn. (60) establishes that

$$H_p^{(RP)} = (S_y^{(I)})^2 H_p{}' \tag{99}$$

The y component of the unit vector of particle displacement of the reflected P wave is found by substituting eqns. (88), (97) and (99) into eqn. (55) to be

$$P_y^{(RP)} = \left[(S_y^{(I)})^2 H_p{}' - D(S_y^{(I)})^2\tan^{-2}\theta^{(I)}\right] \div$$
$$\left[\left((S_y^{(I)})^2 H_p{}' - D(S_y^{(I)})^2\tan^{-2}\theta^{(I)}\right)^2 + E^2(S_y^{(I)})^2\tan^{-2}\theta^{(I)}\right]^{1/2} \tag{100}$$

Canceling a factor of $(S_y^{(SV)})^2/(S_y^{(SV)})^2$ from the right-hand side of eqn. (100) and comparing the resulting expression to eqn. (64) produces the relationship

$$P_y^{(RP)} = P_y^{(I)} \tag{101}$$

The z component of the unit vector of particle displacement of the reflected P wave is found by similar substitutions. Eqns. (88), (97) and (99) are substituted into eqn. (56). A factor of $(S_y^{(SV)})^2/(S_y^{(SV)})^2$ is canceled from the right-hand side of the subsequent expression for $P_z^{(P)}$ and the resulting expression is compared to eqn. (65) to establish that

$$P_z^{(RP)} = P_z^{(I)} \tag{102}$$

The amplitude coefficients of the SV wave and the P wave reflected by the incident SV wave produced by mode splitting can then be determined as functions of only the angle of incidence $\theta^{(I)}$ of the incident P wave, that produced the incident SV wave, and the material properties. The value of $S_y^{(ISV)}$, the y component of the slowness vector of the incident SV wave, is found as a function of $\theta^{(I)}$ by substituting eqn. (82) into eqn. (52). The y components, $S_y^{(RSV)}$ and $S_y^{(RP)}$, of the slowness vectors of the reflected SV wave and reflected P wave, respectively, are set equal to $S_y^{(RSV)}$ according to eqn. (84). The value of $S_z^{(ISV)}$, the z component of the slowness vector of the incident SV wave is found by substituting eqn. (83) into eqn. (54), where $\theta^{(SV)}$ is defined as a function of $\theta^{(I)}$ by eqn. (36). The z component, $S_z^{(RSV)}$, of the slowness vector of the reflected SV wave is set equal to the negative of $S_z^{(ISV)}$ according to eqn. (86). The value of $S_z^{(RP)}$, the z component of the slowness vector of the reflected P wave, is found by substituting eqn. (88) into eqn. (53). The values of $P_y^{(ISV)}$ and $P_z^{(ISV)}$, the y and z components of the unit vector of particle displacement of the incident SV wave, are found by substituting eqn. (89) into eqn. (75) and substituting eqn. (90) into eqn. (76), respectively. The coefficient H_w' is determined as a function of $\theta^{(I)}$ by eqn. (74). The y component, $P_y^{(RSV)}$, of the unit vector of particle displacement of the reflected SV wave is found by substituting eqn. (95) into eqn. (75), and $P_z^{(RSV)}$, the z component of the unit vector of particle displacement of the reflected SV wave, is found by substituting eqn. (95) into eqn. (76). The y and z components, $P_y^{(RP)}$ and $P_z^{(RP)}$, of the unit vector of particle displacement of the reflected P wave are found by substituting eqn. (101) into eqn. (64) and substituting eqn. (102) into eqn. (65), respectively.

The values of the y and z components of the slowness vectors of the incident SV wave and the reflected SV and P waves and the y and z components of the unit vectors of particle displacement of the incident SV wave and the reflected SV and P waves, determined as functions of $\theta^{(I)}$, can then be substituted into eqn. (80) and eqn. (81) to determine $A^{(SV\text{-}SV)}$, the amplitude coefficient of the reflected SV wave, and $A^{(SV\text{-}P)}$, the amplitude coefficient of the reflected P wave, as functions of $\theta^{(I)}$. Thus, the amplitude coefficients of the waves reflected by an SV wave produced by mode splitting are determined in terms of only the angle of incidence of the P wave that produced the incident SV wave and the material properties.

The amplitude coefficient of the P wave reflected by an incident SV wave produced by mode splitting is computed using the material properties of the representative fiberglass epoxy composite. The amplitude coefficient of the reflected P wave is shown as a function of the angle of incidence of the initial P wave in Fig. 7. The amplitude coefficient of the SV wave reflected by an incident SV wave produced by mode splitting is calculated similarly. The amplitude coefficient of the reflected SV wave is shown as a function of the angle of incidence of the initial P wave in Fig. 8.

4. WAVE PATHS

It is assumed that the receiving transducer does not produce an output voltage unless there exists a series of reflected waves beginning (in this case) with a P wave produced by the transmitting transducer, and ending with a P wave incident at the receiving transducer. Let such a series of reflected waves be called a path from the transmitting transducer to the receiving transducer. Both P waves and SV waves experience mode splitting when incident on the stress-free plane boundary of a transversely isotropic continuum [2]; therefore, a path may include both P and SV waves.

A simple path from the transmitting transducer to the receiving transducer may be represented as

$$M = P \cdot P \cdot SV \cdot P \tag{103}$$

Eqn. (103) should be read as follows: M is the path from the transmitting transducer to the receiving transducer that begins with a P wave produced by the transmitting transducer that is incident on the lower face of the plate, producing a second P wave that is, in turn, incident on the upper face of the plate, producing an SV wave that then is incident on the lower face of the plate, producing a final P wave that is incident at the receiving transducer. The complexity of this description demonstrates the usefulness of the path notation.

A more complicated path of demonstrative value is

$$N = P \cdot P \cdot SV \cdot P \cdot SV \cdot SV \cdot P \cdot P \tag{104}$$

Path N is interesting because it includes all four possible types of reflection: P wave producing P wave, P wave producing SV wave, SV wave producing P wave, and SV wave producing SV wave. These four types of reflection have amplitude coefficients $A^{(P\text{-}P)}$, $A^{(P\text{-}SV)}$, $A^{(SV\text{-}P)}$ and $A^{(SV\text{-}SV)}$.

5. NUMBER OF P AND SV WAVES IN PATH: WAVE INDICES

When the initial P wave of a path from the transmitting tranducer to the receiving transducer is incident with a known angle of incidence, $\theta^{(I)}$, on the lower face of the plate the angles of incidence and reflection for all subsequent P waves and SV waves in the path are also known. The angle of reflection and the angle of incidence of any P wave in the path are both equal to $\theta^{(I)}$, and the angle of reflection and the angle of incidence of any SV wave in the path are both equal to $\theta^{(SV)}$, which is defined as a function of $\theta^{(I)}$ by eqn. (36).

A path from the transmitting transducer to the receiving transducer is therefore characterized by the angle $\theta^{(I)}$ with which the initial P wave of the path is incident on the lower face of the plate. Because the plate is of thickness h, the total distance r_p traveled between reflections by any P wave in the path is (see Fig. 9)

$$r_p = \frac{h}{\cos\theta^{(I)}} \tag{105}$$

The distance b_p traveled in the y direction between reflections by the P wave is

$$b_p = r_p \sin\theta^{(I)} \tag{106}$$

Using eqn. (105) to evaluate r_p in eqn. (106), the distance b_p can be written as

$$b_p = h \tan\theta^{(l)} \tag{107}$$

The distance r_{sv} traveled between reflections by any SV wave in the path is (see Fig. 9)

$$r_{sv} = \frac{h}{\cos\theta^{(SV)}} \tag{108}$$

where $\theta^{(sv)}$ is defined as a function of $\theta^{(l)}$ by eqn. (36). The distance b_{sv} traveled in the y direction between reflections by the SV wave is

$$b_{sv} = r_{sv} \sin\theta^{(SV)} \tag{109}$$

Evaluating r_{sv} in eqn. (109) from eqn. (108) allows the distance b_{sv} to be written as

$$b_{sv} = h \tan\theta^{(SV)} \tag{110}$$

The transmitting transducer and the receiving transducer are assumed to lie in the plane x=0 and be separated by a distance L in the y direction; therefore, the total distance traveled in the path in the y direction must be L. Thus, if there are r P waves in the path and s SV waves in the path, then the wave indices r and s must satisfy the relation

$$r b_p + s b_{sv} = L \tag{111}$$

Substituting eqn. (107) and eqn. (110) into eqn. (111) and dividing both sides of the resulting expression by h allows the relationship between r and s to be written as

$$r \tan\theta^{(l)} + s \tan\theta^{(SV)} = \frac{L}{h} \tag{112}$$

Since the first wave in the path is produced by the transmitting transducer and every other wave in the path is produced by a reflection in the path the total number of reflections in the path must be one less than the total number of waves in the path, $r + s$.

Furthermore, the transmitting transducer and the receiving transducer are assumed to be coupled to the same face of the plate; therefore, the total number of reflections in any path from the transmitting transducer to the receiving transducer must be an odd number. Thus, the total number of waves in the path must be an even number, mathematically expressed as

$$\frac{r+s}{2} \in \{1,2,3,\ldots\} \tag{113}$$

where $\in$ denotes "is a member of".

In order for a path to be of interest, the characteristic angle of the path, $\theta^{(i)}$, the number of P waves in the path, r, and the number of SV waves in the path, s, must satisfy eqn. (112) and eqn. (113).

6. PATH AMPLITUDE COEFFICIENT AND REFLECTION INDEX

When the characteristic angle of a path, $\theta^{(i)}$, is known, the four amplitude coefficients associated with the path, $A^{(P\text{-}SV)}$, $A^{(P\text{-}P)}$, $A^{(SV\text{-}SV)}$ and $A^{(SV\text{-}P)}$, can be calculated from eqns. (50), (51), (80) and (81), respectively. A path amplitude coefficient, A, can then be defined as the ratio between the amplitudes of the final P wave and the initial P wave of the path, for a nondispersive and nonattenuating medium. This path amplitude coefficient can be calculated as a function of $\theta^{(i)}$ according to

$$\mathrm{A} = (A^{(P-SV)})^{l}\,(A^{(P-P)})^{m}\,(A^{(SV-SV)})^{n}\,(A^{(SV-P)})^{p} \tag{114}$$

where l is the number of P wave to SV wave reflections in the path, m is the number of P wave to P wave reflections in the path, n is the number of SV wave to SV wave reflections in the path, and p is the number of SV wave to P wave reflections in the path.

When the path is written as a list of P's and SV's following the notation of eqn. (103), l is the number of times the character string $\mathrm{P \cdot SV}$ occurs in the list, m is the number of times the character string $\mathrm{P \cdot P}$ occurs in the list, n is the number of times the character string $\mathrm{SV \cdot SV}$ occurs in the list, and p is the number of times the character string $\mathrm{SV \cdot P}$ occurs in the list. The path amplitude coefficient for the path defined as N, A(N), is then found from eqn. (104) and eqn. (114) to be

$$\mathrm{A(N)} = (A^{(P-SV)})^2 (A^{(P-P)})^2 (A^{(SV-SV)})^1 (A^{(SV-P)})^2 \tag{115}$$

The characteristic angle, $\theta^{(l)}$, together with the wave indices r and s do not uniquely determine the path amplitude coefficient. More than one path may have the same $\theta^{(l)}$, r and s; furthermore, paths with the same $\theta^{(l)}$, r and s may or may not have the same amplitude coefficients. The three paths $\mathrm{P_1}$, $\mathrm{P_2}$ and $\mathrm{P_3}$ defined by

$$\begin{aligned}
\mathrm{P_1} &= \mathrm{P \cdot SV \cdot P \cdot SV \cdot P \cdot P} \\
\mathrm{P_2} &= \mathrm{P \cdot SV \cdot SV \cdot P \cdot P \cdot P} \\
\mathrm{P_3} &= \mathrm{P \cdot P \cdot P \cdot SV \cdot SV \cdot P}
\end{aligned} \tag{116}$$

have the same wave indices, r=4 and s=2, and therefore, the same characteristic angle, calculated from eqn. (112), but are distinct paths. The path amplitude coefficients, $\mathrm{A(P_2)}$ and $\mathrm{A(P_3)}$, of paths $\mathrm{P_2}$ and $\mathrm{P_3}$, respectively, are equal and computed as

$$\mathrm{A(P_2)} = \mathrm{A(P_3)} = (A^{(P-SV)})^1 (A^{(P-P)})^2 (A^{(SV-SV)})^1 (A^{(SV-P)})^1 \tag{117}$$

but path $\mathrm{P_1}$ has a different path amplitude coefficient, $\mathrm{A(P_1)}$, computed as

$$\mathrm{A(P_1)} = (A^{(P-SV)})^2 (A^{(P-P)})^1 (A^{(SV-P)})^2 \tag{118}$$

Therefore, another index, a reflection index n, must be found to uniquely determine the path amplitude coefficient.

The path amplitude coefficient of any path is determined by the characteristic angle of the path and the number of each type of reflection in the path. The last wave in any path of interest is a P wave; therefore, every SV wave in the path must reflect in the path. The number of SV wave to SV wave reflections in the path summed with the number of SV wave to P wave reflections in the path must therefore be equal to the total number of SV waves in the path, s. If there are n SV wave to SV wave reflections in the path, then there must be s-n SV wave to P wave reflections in the path. Since the first wave in the path is a P wave and the last wave in the path is a P wave, the number of P wave to SV wave reflections in the path must be equal to the number of SV wave to P wave reflections in the path, s-n. The total number of reflections in the path is equal to r+s-1, one less than the total number of waves in the path. Since the number of SV wave reflections in the path is equal to the number of SV waves in the path, s, the number of P wave reflections in the path must be equal to r-1, one less than the number of P waves in the path. Therefore, the number of P wave to P wave reflections in the path must be equal to (r-1)-(s-n). The path amplitude coefficient $A_{r,s,n}$ can then be calculated as a function of the characteristic angle $\theta^{(i)}$, the wave indices r and s, equal to the number of P and SV waves in the path, respectively, and the reflection index n, equal to the number of SV wave to SV wave reflections in the path, according to

$$A_{r,s,n} = (A^{(P-SV)})^{s-n} (A^{(P-P)})^{(r-1)-(s-n)} (A^{(SV-SV)})^{n} (A^{(SV-P)})^{s-n} \tag{119}$$

7. MULTIPLICITY FUNCTION

The number of distinct paths from the transmitting transducer to the receiving transducer that have characteristic angle $\theta^{(i)}$, wave indices r and s, and reflection index n, is found by applying elementary counting theory to the lists of the characters P and SV that represent the paths. The lists are partitioned into character strings, where a character string is an ordered subset of a list, of the form $\mathrm{P \cdot SV \ldots SV}$ and character strings of the single character P. The character strings of the form $\mathrm{P \cdot SV \ldots SV}$ are chosen such that the final SV in the character string is followed by a P in the list. The single character character strings are chosen such that the P is followed by a second P in the list. Thus, each character string in a partition begins with a P and ends with the character preceding the next P in the list. Therefore, if a P is followed by a P, in the list, the first P becomes a single character character string in the partition; otherwise, a character string consisting of the first P followed by as many SV's as occur in the list, until the next P is encountered, is included in the partition (see Table 3 for examples).

Let the number of character strings, in a list, of the form $\mathrm{P \cdot SV \ldots SV}$, containing exactly q+1 SV's, be represented by k_q. Thus, k_q is the number of character strings in the path representing one P wave to SV wave reflection and q SV wave to SV wave reflections. The number of character strings in a list of the form $\mathrm{P \cdot SV}$ is then given by k_0, since in this case q=0. Let the number of character strings of the single character P be denoted by k_p. The number of SV wave to SV wave reflections is exactly n, so k_q must be zero for all q greater than n. Furthermore, the total number of SV wave to SV wave reflections in the path is equal to

$$\sum_{q=1}^{n} k_q q$$

so the multiplicity coefficients k_q must satisfy the relation

$$\sum_{q=1}^{n} k_q q = n \tag{120}$$

The number of SV waves in the path is equal to

$$\sum_{q=0}^{n} k_q(q+1)$$

and must be exactly s. Therefore, the k_q's must also satisfy

$$\sum_{q=0}^{n} k_q(q+1) = s \tag{121}$$

In order for the receiving transducer to be excited, the last wave in the path must be a P wave. Therefore, k_p must be at least one; mathematically, this can be represented by

$$k_p \geq 1 \tag{122}$$

The number of P waves in the path is equal to

$$k_p + \sum_{q=0}^{n} k_q$$

and is exactly r. Therefore, the multiplicity coefficients, k_q and k_p, satisfy

$$k_p + \sum_{q=0}^{n} k_q = r \tag{123}$$

The number of distinct paths from the transmitting transducer to the receiving transducer ending in a P wave, that have r P waves, s SV waves, n SV wave to SV wave reflections, and multiplicity coefficients k_n, k_{n-1}, ..., k_0 and k_p, satisfying eqns. (120), (121), (122) and (123), is equal the number of distinct lists with k_n objects of type 1, k_{n-1} objects of type 2, ..., k_0 objects of type n+1 and k_p-1 objects of type n+2. The value of k_p is reduced by one to remove one object of type n+2 from random placement in the list in order to place it at the end of the list; guaranteeing the last wave in the path is a P wave. The number ν of the distinct lists with k_i objects of type i for each of n+2 types of objects is given by [7]

$$\nu = \binom{\sum_{i=1}^{n+2} k_j}{k_1, k_2, \ldots, k_{n+1}, k_{n+2}} \tag{124}$$

where

$$\binom{m}{n_1, n_2, \ldots, n_{\alpha-1}, n_\alpha}$$

is the multinomial coefficient defined by

$$\binom{m}{n_1, n_2, \ldots, n_{\alpha-1}, n_\alpha} = \begin{cases} 0 & \text{if} \quad n_i < 0 \quad \text{for any} \quad i \\ \dfrac{m!}{n_1!n_2!\ldots n_{\alpha-1}!n_\alpha!} & \text{if} \quad n_i \geq 0 \quad \text{for all} \quad i \end{cases} \tag{125}$$

where x! represents the factorial expansion of x, (x)(x-1)(x-2)...(2)(1).

The number N of distinct paths is then

$$N = \binom{(k_p - 1) + \sum_{q=0}^{n} k_q}{k_p - 1, k_0, \ldots, k_{n-1}, k_n} \tag{126}$$

Substituting eqn. (120) and eqn. (123) into eqn. (126) allows the expression for N to be simplified to

$$N = \binom{r-1}{k_p - 1, k_0, \ldots, k_{n-1}, k_n} \tag{127}$$

The total number of paths, with r P waves, s SV waves and n SV wave to SV wave reflections, is found by summing eqn. (127) over all possible values of the multiplicity coefficients. Using the constraints of eqns. (120), (121), (122) and (123) allows the total number of distinct paths $N_{r,s,n}$ to be calculated as

$$N_{r,s,n} = \sum_{k_n=0}^{1} \sum_{k_{n-1}=0}^{\alpha_{n-1}} \cdots \sum_{k_2=0}^{\alpha_2} \binom{r-1}{k_p - 1, k_0, \ldots, k_{n-1}, k_n} \tag{128}$$

where

$$\alpha_q = \left\lfloor \frac{n - \sum_{i=q+1}^{n} k_i i}{q} \right\rfloor \tag{129}$$

where $\lfloor x \rfloor$ denotes the greatest integer less than x,

$$k_1 = n - \sum_{i=2}^{n} k_i i \tag{130}$$

$$k_0 = s - \sum_{i=1}^{n} k_i (i+1) \tag{131}$$

and

$$k_p = r - \sum_{i=0}^{n} k_i \tag{132}$$

The characteristic angle, $\theta^{(t)}$, the number of P waves, r, the number of SV waves, s, and the number of SV wave to SV wave reflections, n, have been held constant throughout the counting process; therefore, all paths counted by eqn. (128) have the same path amplitude coefficient.

The total number of distinct paths with wave indices r and s and reflection index n, $N_{r,s,n}$, and the associated path amplitude coefficient, $A_{r,s,n}$, are tabulated in Table 1 for all paths with ten or fewer transits of the plate for the fiberglass epoxy composite. The characteristic angle is found from eqn. (112) by fixing r and s and substituting in the values of L and h, L=10 cm and h=5 cm. The values of $A_{r,s,n}$ are found from eqn. (119) and are observed from Table 1 to be either greater than or less than zero for different values of r, s and n. This is due to the fact that $A^{(SV\text{-}SV)}$, the ratio of the amplitude of particle displacement of the SV wave reflected by an incident SV wave to the amplitude of particle displacement of the incident SV wave, is greater than zero and $A^{(P\text{-}SV)}$, $A^{(P\text{-}P)}$ and $A^{(SV\text{-}P)}$ are all less than zero (see Figs. 5,6,7 and 8). A negative amplitude coefficient indicates that the direction of the particle displacement due to the reflected wave, at the point of reflection, is in the opposite sense of the direction of the particle displacement due to the incident wave. Whereas, a positive amplitude coefficient indicates that the direction of the particle displacement due to the reflected wave, at the point of reflection, is in the same sense as the direction of the particle displacement due to the incident wave.

8. PHASE VELOCITY AND TIME DELAY

The distance r_p traveled between reflections by any P wave in a path with characteristic angle $\theta^{(t)}$ is given eqn. in (105). If there are exactly r P waves in the path then the total distance R_p traveled by P waves in the path is

$$R_p = r\frac{h}{\cos\theta^{(I)}} \tag{133}$$

where h is the thickness of the plate.

The distance r_{sv} traveled between reflections by any SV wave in a path with characteristic angle $\theta^{(I)}$ is given by eqn. (108). If there are exactly s SV waves in the path then the total distance R_{sv} traveled by SV waves in the path is

$$R_{sv} = s\frac{h}{\cos\theta^{(SV)}} \tag{134}$$

where $\theta^{(sv)}$ is defined as a function of $\theta^{(I)}$ by eqn. (36).

The directionally dependent phase velocity of a P wave traveling in a transversely isotropic continuum, $C_p(\theta)$, is, for example, see [4]

$$C_P(\theta) = \left(\frac{C_{44} + C_{11}\sin^2\theta + C_{33}\cos^2\theta + \sqrt{G}}{2\rho}\right)^{1/2} \tag{135}$$

where

$$G = [(C_{11} - C_{44})\sin^2\theta + (C_{44} - C_{33})\cos^2\theta]^2 + 4(C_{13} + C_{44})^2\sin^2\theta\cos^2\theta \tag{136}$$

The directionally dependent phase velocity of an SV wave traveling in a transversely isotropic contiuum, $C_{sv}(\theta)$, is, for example, see [4]

$$C_{SV}(\theta) = \left(\frac{C_{44} + C_{11}\sin^2\theta + C_{33}\cos^2\theta - \sqrt{G}}{2\rho}\right)^{1/2} \tag{137}$$

where G is defined by eqn. (136).

The time delay $t_{r,s}$ is defined as the time taken for the final P wave of a path with r P waves and s SV waves to reach the receiving transducer after the path is initiated by the transmitting transducer. The time delay is then

$$t_{r,s} = \frac{R_P}{C_P(\theta^{(I)})} + \frac{R_{SV}}{C_{SV}(\theta^{(SV)})} \tag{138}$$

Thus, the time delay is a function of only the geometry of the transducer arrangement, the characteristic angle, $\theta^{(I)}$, the wave indices, r and s, and the material properties of the plate.

9. NET PATH AMPLITUDE COEFFICIENT

Eqn. (138) establishes that all paths from the transmitting transducer to the receiving transducer containing r P waves and s SV waves experience the same time delay in reaching the receiving transducer. Furthermore, all paths with r P waves and s SV waves will be in phase at the receiving transducer. Therfore, the amplitudes of all paths with wave indices r and s may be added algebraically at the receiving transducer. A net path amplitude coefficient $A_{r,s}$ may be defined as the ratio between sum of the amplitudes of the final P waves of all paths with r P waves and s SV waves and the amplitude of the initial P wave of any path with wave indices r and s, when the medium through which the waves are traveling is nondispersive and nonattenuating.

The net path amplitude coefficient is found by summing the product $N_{r,s,n}\, A_{r,s,n}$ over all possible values of the reflection index n, where $N_{r,s,n}$ is the total number of paths with r P waves, s SV waves and n SV wave to SV wave reflections and is given by eqn. (128); and $A_{r,s,n}$ is the path amplitude coefficient of any path with wave indices r and s and reflection index n and is given by eqn. (119). The bounds on n are found by considering the exponents of the amplitude coefficients in eqn. (119). Clearly, if s is equal to zero there can be no SV wave to SV wave reflections, and for all values of s greater than zero there must be at least one P wave to SV wave reflection. Therefore, n must satisfy the relationship

$$n \in \begin{cases} \{0\} & \text{if} \quad s = 0 \\ \{0,1,\ldots,s-1\} & \text{if} \quad s > 0 \end{cases} \tag{139}$$

where $\in$ denotes "is a member of". The lower bound on n is computed by noting that the exponents of $A^{(P\text{-}P)}$ and $A^{(SV\text{-}SV)}$ must be nonnegative for all values of n. The lower bound on n is then

$$n \geq \max(s+1-r,0) \tag{140}$$

The net path amplitude coefficient is then

$$A_{r,s} = \begin{cases} (A_r^{(P-P)})^{r-1} & \text{if} \quad s = 0 \\ \sum\limits_{n=\max(s+1-r,0)}^{s-1} N_{r,s,n} A_{r,s,n} & \text{if} \quad s > 0 \end{cases} \tag{141}$$

The net path amplitude coefficient $A_{r,0} = (A_r^{(P\text{-}P)})^{r\text{-}1}$ corresponds to the path P·P·P...P containing (r-1) P wave to P wave reflections and no SV waves.

The net path amplitude coefficient $A_{r,s}$ is shown in Table 2 for all paths with ten or fewer transits of the plate for the fiberglass epoxy composite. The value of $A_{r,s}$ is found by summing the product $A_{r,s,n}N_{r,s,n}$ over all values of n. The maximum and minimum values of n are found from eqn (139) and eqn. (140) and the values of $A_{r,s,n}$ and $N_{r,s,n}$ are taken from Table 1.

10. ASSUMPTIONS ON TRANSDUCERS

The transmitting transducer and the receiving transducer are assumed to be longitudinal transducers that transform an electrical voltage into a uniform longitudinal stress or a uniform longitudinal stress into an electrical voltage. The following approach parallels that of [8].

If an input voltage V_i of amplitude V and frequency ω is applied according to

$$V_i = V \exp\{-i\omega t\} \tag{142}$$

where $i = \sqrt{-1}$ and t denotes time, the stress τ_{ij} that is introduced into the medium at the transducer-medium interface by the transmitting transducer is

$$\tau_{ij}(t) = F_1(\omega) V \exp\{-i(\omega t + \phi_1)\} \tag{143}$$

where $F_1(\omega)$ is the frequency dependent transduction ratio for the transmitting transducer in transforming a voltage into a stress and ϕ_1 is a phase angle. In eqn. (142) and eqn. (143) the harmonic character of the signals is expressed in complex notation, but only the real parts of these and subsequent equations should be considered. The amplitude T of the applied stress is then

$$T = F_1(\omega) V \tag{144}$$

Similarly, if a stress wave producing a stress component τ_{ij}' of amplitude T' and frequency ω that impinges on the receiving transducer is defined as

$$\tau_{ij}'(t) = T' \exp\{-i\omega t\} \tag{145}$$

then the output voltage V_o from the receiving transducer is

$$V_o(t) = F_2(\omega) T' \exp\{-i(\omega t + \phi_2)\} \tag{146}$$

where $F_2(\omega)$ is the frequency dependent transduction ratio for the receiving transducer in transforming a shear stress to a voltage, and ϕ_2 is a phase angle. Thus, the amplitude V' of the output electrical voltage is

$$V' = F_2(\omega)T' \tag{147}$$

The characteristics of $F_1(\omega)$ and $F_2(\omega)$ are unknown except that the product $F_1(\omega)F_2(\omega)$ is dimensionless.

11. DIRECTIVITY FUNCTIONS

The directivity functions of the stresses associated with P and SV waves traveling in semi-infinite transversely isotropic continua are evaluated from the far-field asymptotic solutions of the displacement components produced by a harmonic point load imbedded in an infinite body in [2]. The directivity functions $D^{K}{}_{ij}^{(P)}$ of the τ_{ij} stress associated with P waves whose slowness vectors are confined to the y-z plane and are produced by a point load located on the plane boundary of a semi-infinite body and acting in the K direction in the (x, y, z) coordinate system are reproduced.

Define a coefficient λ_n according to

$$\lambda_n = \frac{H{,}S_x^2 + H{,}S_y^2 + H{,}S_z^2}{|K_n|} \tag{148}$$

where $|\ |$ denotes the "magnitude of"

$$\begin{aligned} K_n = \Sigma[&H{,}S_z^2(H{,}S_xS_x \cdot H{,}S_yS_y - H{,}S_xS_x^2) + \\ &(2H{,}S_x \cdot H{,}S_y(H{,}S_xS_z \cdot H{,}S_yS_z - H{,}S_xS_y \cdot H{,}S_zS_z)] \end{aligned} \tag{149}$$

and

$$H = \left[\frac{C_{44}}{\rho}S_z^2 + \frac{C_{11}}{\rho}(S_x^2+S_y^2) - 1\right]\left[\frac{C_{44}}{\rho}(S_x^2+S_y^2) + \frac{C_{33}}{\rho}S_z^2 - 1\right] -$$

$$\left(\frac{C_{44}+C_{13}}{\rho}\right)^2 S_z^2(S_x^2+S_y^2) \tag{150}$$

Σ denotes the sum with respect to cyclic permutation of S_x, S_y and S_z; "," denotes differentiation with respect to the succeeding variable carried out assuming S_x, S_y and S_z are independent variables; C_{11}, C_{13}, C_{33} and C_{44} are elastic constants of the material; and ρ is the density of the material.

The directivity function $D^{Y}{}_{yz}{}^{(P)}$ of the τ_{yz} shear stress associated with a P wave whose slowness vector is confined to the plane x=0 and that is produced by a harmonic point load located on the plane boundary of an infinite half-space and acting in the y direction in the (x, y, z) coordinate system is

$$D_{yz}^{Y(P)} = \frac{\lambda_n(0,S_y^*,S_z^*)\omega C_{44}S_z^*}{2\pi}\left(\frac{-C_{13}}{\rho}S_y^{*2} + \frac{C_{33}}{\rho}S_z^{*2} - 1\right) \tag{151}$$

where $(0,S_y^*,S_z^*)$ is the point on the slowness surface of a P wave where the normal to the slowness surface is parallel to the line connecting the point load to the point at which the directivity function is to be evaluated, ω denotes the radian frequency of the harmonic point load and λ_n is evaluated at $(0,S_y^*,S_z^*)$. The directivity function $D^{Y}{}_{zz}{}^{(P)}$ of the τ_{zz} normal stress is

$$D_{zz}^{Y(P)} = -\frac{\lambda_n(0,S_y^*,S_z^*)\omega S_y^*}{2\pi}\left(\frac{C_{13}C_{44}}{\rho}S_y^{*2} - \frac{C_{33}C_{44}}{\rho}S_z^{*2} - C_{13}\right) \tag{152}$$

The directivity function $D^{z\,(P)}_{yz}$ of the τ_{yz} shear stress associated with a P wave produced by a harmonic point load acting in the z direction in the (x, y, z) coordinate system is

$$D^{z(P)}_{yz} = \frac{\lambda_n(0,S_y^*,S_z^*)\omega C_{44}S_y^*}{2\pi}\left(\frac{C_{11}}{\rho}S_y^{*2} + \frac{C_{33}-C_{44}-C_{13}}{\rho}S_z^{*2} - 1\right) \tag{153}$$

Similarly, the directivity function $D^{z\,(P)}_{zz}$ of the τ_{zz} normal stress is

$$D^{z(P)}_{zz} = \frac{\lambda_n(0,S_y^*,S_z^*)\omega S_z^*}{2\pi}\left(\frac{C_{11}C_{13}-C_{13}C_{44}-C_{13}^2}{\rho}S_y^{*2} + \frac{C_{33}^2}{\rho}S_z^{*2} - C_{33}\right) \tag{154}$$

The directivity functions $D^{K\,(P)}_{ij}$ of the stresses associated with a P wave whose slowness surface is confined to the plane x=0 and that is produced by a harmonic point load can be evaluated at a point M in a semi-infinite body by finding the point $(0,S_y^*,S_z^*)$ on the slowness surface of a P wave where the normal to the slowness surface is parallel to the line $\overline{OM}$ connecting the point load to the point M (see Fig. 10). The slope m of the line $\overline{OM}$ is found from Fig. 10 to be

$$m = \frac{\Delta y}{\Delta z} = -\frac{1}{\tan\theta^{(l)}} \tag{155}$$

The normal to the slowness surface of a P wave is found at any point in the y-z plane by setting the x component of the slowness vector, S_x, equal to zero and differentiating the equation for the slowness surface, eqn. (19), with respect to S_z, the z component of the slowness vector. Solving the subsequent equation for the slope, $-dS_y/dS_z$, of the normal yields

$$\frac{dS_y}{dS_z} = \left\{ BS_z + [(3CD - 2E^2)S_y^2 S_z + D^2 S_z^3] \times \right.$$

$$\left. [C^2 S_y^4 + (6CD - 4E^2)S_y^2 S_z^2 + D^2 S_z^4]^{-1/2} \right\} \div$$

$$\left\{ AS_y + [C^2 S_y^3 + (3CD - 2E^2)S_y S_z^2] \times \right.$$

$$\left. [C^2 S_y^4 + (6CD - 4E^2)S_y^2 S_z^2 + D^2 S_z^4]^{-1/2} \right\} \tag{156}$$

where the conventions of eqn. (30) have been used. Setting eqn. (155) equal to eqn. (156) establishes the constraint on S_y^* and S_z^*, the point on the slowness surface where the normal is parallel to the line connecting the transmitting transducer to the point M,

$$\frac{1}{\tan\theta^{(I)}} = \left\{ BS_z^* + \left[(3CD - 2E^2)S_y^{*2} S_z^* + D^2 S_z^{*3} \right] \times \right.$$

$$\left. \left[C^2 S_y^{*4} + (6CD - 4E^2)S_y^{*2} S_z^{*2} + D^2 S_z^{*4} \right]^{-1/2} \right\} \div$$

$$\left\{ AS_y^* + \left[C^2 S_y^{*3} + (3CD - 2E^2)S_y^* S_z^{*2} \right] \times \right.$$

$$\left. \left[C^2 S_y^{*4} + (6CD - 4E^2)S_y^{*2} S_z^{*2} + D^2 S_z^{*4} \right]^{-1/2} \right\} \tag{157}$$

The point $(0, S_y^*, S_z^*)$ is then found by setting S_x equal to zero in eqn. (19) and solving eqn. (19) and eqn. (157) simultaneously.

12. STRESS FIELD RADIATED BY TRANSMITTING TRANSDUCER

If there were no bottom boundary to the plate and the stress waves were propogating in an infinite half-space, the path from the transmitting transducer to the receiving transducer with characteristic angle $\theta_{2m,0}^{(I)}$ consisting of $2m$ P waves and no SV waves would travel to a point $M_{2m,0}$, in a time $t_{2m,0}$, traveling a distance R_{p2m} (see Fig. 11). The amplitude of the hypothetical stress τ_{ij} at the point $M_{2m,0}$ is $T_{2m,0}^{(M)}$ and is defined as [2]

$$T_{2m,0}^{(M)} = T \frac{D_{ij\ 2m}^{K(P)}}{R_{p2m}} \exp\left\{-\alpha R_{p2m}\right\} \tag{158}$$

where T is the magnitude of the longitudinal stress generated by the transmitting transducer in the z direction, $D_{ij\ 2m}^{K(P)}$ is the value of the appropriate directivity function evaluated for the point $M_{2m,0}$ and α is the P wave attenuation constant of the medium. The P wave path, however, does not propagate in an infinite half-space, but instead experiences $2m$-1 reflections. The amplitude $T_{o2m,0}$ of the stress at the receiving transducer is thus obtained by modifying eqn. (158) as

$$T_{o2m,0} = \frac{T D_{ij\ 2m}^{K(P)} (A_{2m,0}^{(P-P)})^{2m-1}}{R_{p2m}} \exp\left\{-\alpha R_{p2m}\right\} \tag{159}$$

where $A_{2m,0}^{(P-P)}$ is the P wave to P wave reflection coefficient, given by eqn. (51), evaluated for a characteristic angle of $\theta_{2m,0}^{(I)}$.

The amplitude $V_{o2m,0}$ of the output voltage from the receiving transducer is obtained from eqn. (147) and eqn. (159) and is

$$V_{o2m,0} = \frac{F_2(\omega) T D_{ij\ 2m}^{K(P)} (A_{2m,0}^{(P-P)})^{2m-1}}{R_{p2m}} \exp\left\{-\alpha R_{p2m}\right\} \tag{160}$$

Substituting eqn. (144) into eqn. (160) yields

$$V_{o_{2m,0}} = \frac{F_1(\omega)F_2(\omega)VD_{ij\,2m}^{K(P)}\left(A_{2m,0}^{(P-P)}\right)^{2m-1}}{R_{p2m}}\exp\left\{-\alpha R_{p_{2m}}\right\} \tag{161}$$

where V is the amplitude of the input voltage. Introducing a possible electrical signal amplification factor, K, eqn. (161) can be written as

$$V_{o_{2m,0}} = \frac{KF_1(\omega)F_2(\omega)VD_{ij\,2m}^{K(P)}\left(A_{2m,0}^{(P-P)}\right)^{2m-1}}{R_{p2m}}\exp\left\{-\alpha R_{p_{2m}}\right\} \tag{162}$$

The amplitude $T_{o_{r,s}}$ of the stress at the receiving transducer associated with a path with characteristic angle $\theta_{r,s}^{(t)}$ consisting of r P waves and s SV waves can be understood qualitatively by visualizing the waves as traveling in a hypothetical multi-layered half-space [8] (see Fig. 12). It is assumed that each layer is identical to the original plate and that the layers are bonded together in a manner such that an incident wave in one layer produces no reflected waves, but produces transmitted P and SV waves in the next layer and that the transmission coefficients are the same as the reflection coefficients in the original plate. The amplitude of the stress is then set equal to $T_{r,s}^{(M)}$ the value of the stress amplitude at point $M_{r,s}$ in the hypothetical half-space. The value of $T_{r,s}^{(M)}$ is then

$$T_{r,s}^{(M)} = \frac{T\bar{D}_{ij\,r,s}^{K}A_{r,s}}{R_{p_r}+R_{sv_s}}\exp\left\{-\left(\alpha R_{p_r}+\beta R_{sv_s}\right)\right\} \tag{163}$$

where T is the magnitude of the stress generated by the transmitting transducer in the z direction, $\bar{D}_{ij\,r,s}^{K}$ is an equivalent directivity function for a path with wave indices r and s, $A_{r,s}$ is the path amplitude coefficient, given by eqn. (119), R_{p_r} is the total distance traveled by P waves in the path, R_{sv_s} is the total distance traveled by SV waves in the path, α is the P wave attenuation constant of the medium and β is the SV wave attenuation constant of the medium.

If $(A_{2_m}^{(P\text{-}P)})^{2m-1}$ is much greater than $A_{r,s}$ for all $s \neq 0$ and the assumptions

$$\overline{D}_{ij}^{K} \sim D_{ij}^{K(P)} \tag{164}$$

and

$$\alpha \sim \beta \tag{165}$$

are then made, it is apparent that the stresses at the receiving transducer associated with paths containing P and SV waves are much smaller than the stresses at the receiving transducer associated with paths containing only P waves. Therefore, in the input-output characterization of fiber composites which may be modeled as transversely isotropic contimuum plates, the effects of mode splitting may be neglected. Only those paths from the transmitting transducer to the receiving transducer consisting of $2m$ P waves and no SV waves will produce significant stresses at the receiving transducer when eqn. (164) and eqn. (165) are satisfied.

13. OUTPUT VOLTAGE DUE TO TONE BURST

Assume the input voltage V_i to the transmitting transducer is a periodic function of time, of center frequency ω and duration t_i. Mathematically the input voltage can be expressed as the sum of two periodic functions of frequency ω that are 180° out of phase, one beginning at time t=0 and one beginning at time t=t_i. The input voltage is then given by

$$V_i = V\exp\{i\omega t\}\,\mathrm{U}(t) - V\exp\{i\omega\}\,\mathrm{U}(t - t_i) \tag{166}$$

where V is the amplitude of the input voltage, $i = \sqrt{-1}$, and U(x) is the unit step function defined as

$$\mathrm{U}(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad x \geq 0 \end{cases} \tag{167}$$

The periodic nature of the signal in eqn. (166) and subsequent equations is expressed in complex notation, but only the real part of the signal is to be considered.

The voltage output V_o at the receiving transducer is found by considering the two terms on the right-hand side of eqn. (166) independently and superposing the voltage output associated with each input signal. Consider the input voltage V_i' given by the first term on the right-hand side of eqn. (166)

$$V_i' = V\exp\{i\omega t\}\,\mathrm{U}(t) \tag{168}$$

The stress waves produced by a transmitting transducer with input voltage V_i' will have a periodic character with respect to time. Therefore, the amplitude of the output voltage associated with each wave path will also be periodic in time and the total output voltage V_i' must be found by the superposition of the contributions of each wave path.

Only those paths with $2m$ P waves and no SV waves will be considered. Eqn. (112) can then be rewritten as

$$2m\tan\theta_{2m}^{(l)} = \frac{L}{h} \tag{169}$$

where $\theta_{2m}^{(l)}$ is the characteristic angle of the path, L is the separation of the transducers and h is the thickness of the plate. Eqn. (169) indicates that the characteristic angle of the plate must satisfy

$$\theta_{2m}^{(l)} = \arctan\left(\frac{L}{2mh}\right) \tag{170}$$

All the trignometric functions of $\theta_{2m}^{(l)}$ can be evaluated from eqn. (170) (see Fig. 13).

A P wave path with $2m$ P waves will be in phase at the receiving transducer with a P wave traveling in a semi-infinite half-space to a point M_m (see Fig. 14). The τ_{ij} stress associated with a plane progressive P wave is given by eqn. (3) and can be calculated at point M_m as

$$\tau_{ij_M} = T_{M_m} \exp\left\{i\left[\omega\left(S_{y_m} L - S_{z_m} 2mh - t\right)\right]\right\} \tag{171}$$

where T_{M_m} is the amplitude of the stress and S_{y_m} and S_{z_m} are the y and z components of the slowness vector of the P wave, respectively. The stress τ_{ij_o} at the receiving transducer is then

$$\tau_{ij_o} = T_{o_m} \exp\left\{i\left[\omega\left(S_{y_m} L - S_{z_m} 2mh - t\right)\right]\right\} \tag{172}$$

where T_{o_m} is the amplitude of the stress and is given by eqn. (159). The output voltage V_{o_m}' at the receiving transducer is found by substituting eqn. (172) into eqn. (146) to be

$$V_{o_m}'(t) = F_2(\omega) T_{o_m} \exp\left\{i\left[\omega\left(S_{y_m} L - S_{z_m} 2mh - t\right) + \phi\right]\right\} \tag{173}$$

where ϕ is a phase angle. The output voltage is then calculated using eqn. (144) and eqn. (159) and including a possible electrical signal amplification factor K as

$$V_{o_m}' = \frac{KF_1(\omega)F_2(\omega)VD^{K(P)}_{ij\,2m}(A^{(P-P)}_{2m})^{2m-1}}{R_{p2m}} \exp\left\{-\alpha R_{p_{2m}}\right\} \times$$

$$\exp\left\{i\left[\omega\left(S_{y_m}L - S_{z_m}2nh - t\right) + \phi\right]\right\} \tag{174}$$

where V is the amplitude of the input voltage, $D^{K}{}_{ij}{}^{(P)}{}_{2m}$ is the appropriate directivity function, $A_{2m}{}^{(P\text{-}P)}$ is the P wave to P wave reflection coefficient and $R_{p_{2m}}$ is the total distance traveled by the wave path. The total output voltage V_o' is found by summing eqn. (174) over all m and retarding each contribution by the delay time t_{2m} to be

$$V_o' = \sum_{m=1}^{\infty} \frac{KF_1(\omega)F_2(\omega)VD^{K(P)}_{ij\,2m}(A^{(P-P)}_{2m})^{2m-1}}{R_{p2m}} \exp\left\{-\alpha R_{p_{2m}}\right\} \times$$

$$\exp\left\{i\left[\omega\left(S_{y_m}L - S_{z_m}2mh - t\right) + \phi\right]\right\} \mathrm{U}(t - t_{2m}) \tag{175}$$

where t_{2m} is given by eqn. (138) with s=0.

The output voltage V_o'' associated with the input voltage characterized by the second term on the right-hand side of eqn. (166) is found similarly to be

$$V_o'' = \sum_{m=1}^{\infty} \frac{-KF_1(\omega)F_2(\omega)VD^{K(P)}_{ij\,2m}(A^{(P-P)}_{2m})^{2m-1}}{R_{p2m}} \exp\left\{-\alpha R_{p_{2m}}\right\} \times$$

$$\exp\left\{i\left[\omega\left(S_{y_m}L - S_{z_m}2mh - t\right) + \phi\right]\right\} \mathrm{U}(t - (t_i + t_{2m})) \tag{176}$$

where t_i is the duration of the input signal.

The output voltage V_o associated with the input voltage V_i determined by eqn. (166) is found by adding eqn. (175) and eqn. (176) to be

$$V_o = KF_1(\omega)F_2(\omega)V \sum_{m=1}^{\infty} \frac{D_{ij\,2m}^{K(P)}(A_{2m}^{(P-P)})^{2m-1}}{R_{p2m}} \exp\left\{-\alpha R_{p2m}\right\} \times$$

$$\exp\left\{i\left[\omega\left(S_{y_m}L - S_{z_m}2mh - t\right) + \phi\right]\right\}\{U(t - t_{2m}) - U(t - (t_i + t_{2m}))\} \qquad (177)$$

where $D^{K}{}_{ij}{}^{(P)}{}_{2m}$ is given by eqn. (151), (152), (153) or (154), $A_{2m}{}^{(P-P)}$ is given by eqn. (51), R_{p2m} is given by eqn. (133), S_{y_m} is given by eqn. (31), S_{z_m} is given by eqn. (28), t_{2m} is given by eqn. (138), t_i is the duration of the input signal and the characteristic angle $\theta_{2m}{}^{(I)}$ is given by eqn. (170). The sum in eqn. (177) does not include the effects of mode splitting, therefore the predicted output voltage V_o will not exactly equal the true output voltage. However, the effects of mode splitting have been shown to be negligible for the initial wave paths. Therefore, the initial predicted output voltage should correspond closely to experimental data.

14. SUMMARY OF IMPORTANT RESULTS

The angle of reflection of an SV wave reflected by an incident P wave is determined as a function of the angle of incidence of the incident P wave by eqn. (36).

The angle of reflection of a P wave reflected by an SV wave produced by mode splitting is found to be equal to the angle of incidence of the incident P wave that produced the incident SV wave, eqn. (45).

The amplitude coefficients of the P and SV waves reflected by an incident P wave, $A^{(P-P)}$ and $A^{(P-SV)}$, are found as functions of the angle of incidence of the incident P wave by eqn. (50) and eqn. (51), respectively.

The amplitude coefficients of the P and SV waves reflected by an incident SV wave produced by mode splitting, $A^{(SV-P)}$ and $A^{(SV-SV)}$, are found as functions of the angle of incidence of the incident P wave that produced the incident SV wave by eqn. (81) and eqn. (80), respectively.

A path amplitude coefficient, $A_{r,s,n}$, is defined for any path from the transmitting transducer to the receiving transducer in terms of wave indices r and s, defined as the number of P waves in the path and the number of SV waves in the path, respectively, and a reflection index n, defined as the number of SV wave to SV wave reflections in the path, eqn. (114).

The number of paths having the same wave and reflection indices, $N_{r,s,n}$, are counted in accordance with eqn. (127).

A net path amplitude coefficient, $A_{r,s}$, equal to the product $A_{r,s,n}N_{r,s,n}$ summed over all values of the reflection index for a given pair of wave indices is computed in eqn. (141). The product $A_{r,s,n}N_{r,s,n}$ can be physically interpreted as the sum of the amplitudes of all paths having wave indices r and s and reflection index n.

The magnitude of the longitudinal stress at a point in a hypothetical multilayered half-space due to all paths having the same wave indices is determined by eqn. (163) for all time after the arrival of the first path having those wave indices.

An output voltage from the receiving transducer is determined as a function of time by eqn. (177).

CONCLUSION AND DISCUSSION

The input-output characterization of certain fiber composite plates can be studied using a transversely isotropic continuum model when the wavelengths under consideration are long compared to the mean fiber diameter. For wavelengths close to, or less than, the mean fiber diameter, a continuum plate model is no longer appropriate and the inhomogenaities of the composite must be considered.

In the analysis of a transversely isotropic continuum plate an incident P wave is found to reflect a P wave and an SV wave during each reflection at the top or bottom face of the plate. The angle of reflection of the reflected P wave is equal to the angle of incidence of the incident P wave and the angle of reflection of the reflected SV wave is uniquely determined by the angle of incidence of the incident P wave. When the reflected SV wave is in turn incident on the opposite face of the plate, a P wave and an SV wave are again reflected. This second reflected SV wave is reflected with an angle of reflection equal to the angle of incidence of the incident SV wave and this second reflected P wave is reflected with an angle of reflection equal to the angle of incidence of the original incident P wave. Thus, tracing a series of reflected P and SV waves through the medium is simplified because all the reflected P waves are reflected with angle of reflection equal to the angle of incidence of the initial P wave and all reflected SV waves are reflected with angle of reflection equal to the angle of reflection of the initial reflected SV wave. This simple relationship exists because the isotropic plane of the plate is parallel to the top and bottom faces of the plate. If this special geometry is not present a more complicated analysis considering each reflection individually is needed to trace a series of reflected waves through the plate.

A path from the transmitting transducer to the receiving transducer may contain both P and SV waves (no SH waves are produced during the reflections). The path amplitude coefficient is found from the product of the reflection coefficients encountered at each reflection. Because more than one path may have the same number of P and SV waves, a net path amplitude coefficient is found by summing the path amplitude coefficients of all paths having the same total combination of P waves and SV waves. Table 2 shows the magnitudes of the net path amplitude coefficients associated with paths containing no SV waves to be an order of magnitude or more larger than the magnitudes of the net path amplitude coefficients associated with paths containing SV waves.

Examining Figs. 5, 6, 7 and 8 reveals that the magnitudes of the reflection coefficients for P wave to P wave and SV wave to SV wave reflections are generally close to one, especially for the small angles of incidence common for paths from the transmitting transducer to the receiving transducer. On the other hand, the magnitudes of the reflection coefficients for P wave to SV wave and SV wave to P wave reflections are much smaller than one for small angles of incidence. Therefore, paths containing both P waves and SV waves, which thus must contain at least one P wave to SV wave reflection and one SV wave to P wave reflection in order to start and end with P waves, should be expected to have path amplitude coefficients with magnitudes much less than the magnitudes of the path amplitude coefficients associated with paths containing no SV waves. Table 1 reveals that for those values of the wave indices and reflection index for which the mutiplicity function is large the magnitude of the path amplitude coefficients tend to be much much less than one. Therefore, the increased multiplicity of paths containing both P waves and SV waves does not cause a significant increase in the magnitude of the net path amplitude coefficient.

The magnitude of the net path amplitude coefficients associated with paths containing P and SV waves can be expected to be much smaller than the magnitude of the path amplitude coefficients associated with paths containing only P waves. Therefore, the input-output characterization of a transversely isotropic continuum plate by P waves can be carried out neglecting the effects of mode splitting and tracing only those paths containing no SV waves.

This study enhances the theoretical understanding of the nondestructive evaluation (NDE) of transversely isotropic media such as certain fiber composites. It also provides impetus for further study on the input-output characterization of transversely isotropic media by P waves by simplifying the analysis.

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Table 1 Characteristic angle $\theta^{(I)}$, number of P waves r, number of SV waves s, reflection index n, number of distinct paths $N_{r,s,n}$ and path amplitude coefficient $A_{r,s,n}$ are shown for all paths with 10 or fewer transits of plate.

$\theta^{(I)}$ (degrees)	r	s	n	$N_{r,s,n}$	$A_{r,s,n}$
45.0	2	0	0	1	-0.907
37.5	2	2	1	1	0.106
31.5	2	4	3	1	0.069
27.5	2	6	5	1	0.049
24.0	2	8	7	1	0.037
30.0	3	1	0	2	-0.066
27.5	3	3	2	2	-0.050
	3	3	1	2	0.003
23.5	3	5	4	2	-0.036
	3	5	3	4	0.001
20.5	3	7	6	2	-0.027
	3	7	5	6	0.001
26.5	4	0	0	1	-0.924
23.5	4	2	1	3	0.038
	4	2	0	3	-0.001
21.5	4	4	3	3	0.031
	4	4	2	9	-0.001
	4	4	1	3	0.000
18.5	4	6	5	3	0.022
	4	6	4	15	-0.001
	4	6	3	10	0.000
20.5	5	1	0	1	-0.023
18.5	5	3	2	4	-0.023
	5	3	1	2	0.001
	5	3	0	4	0.000
17.0	5	5	4	4	-0.019
	5	5	3	24	0.001
	5	5	2	24	0.000
	5	5	1	4	0.000

(Table 1 Continued)

Table 1 (Continued)

$\theta^{(I)}$ (degrees)	r	s	n	$N_{r,s,n}$	$A_{r,s,n}$
18.5	6	0	0	1	-0.941
17.0	6	2	1	5	0.019
	6	2	0	10	0.000
15.5	6	4	3	5	0.016
	6	4	2	30	0.000
	6	4	1	30	0.000
	6	4	0	5	0.000
15.5	7	1	0	6	-0.016
14.5	7	3	2	6	-0.013
	7	3	1	30	0.000
	7	3	0	20	0.000
14.0	8	0	0	1	-0.952
13.0	8	2	1	7	0.011
	8	2	0	21	0.000
12.0	9	1	0	8	-0.009
11.5	10	0	0	1	-0.956

Table 2 Net path amplitude coefficient $A_{r,s}$ as function of number of P waves r and number of SV waves s for all paths with 10 or fewer transits of plate.

r \ s:	0	1	2	3	4	5	6	7	8
r	$A_{r,s}$								
2	-0.907		0.106		0.069		0.049		0.037
3		-0.132		-0.094		-0.068		-0.048	
4	-0.924		0.111		0.084		0.051		
5		-0.023		-0.090		-0.052			
6	-0.941		0.095		0.080				
7		-0.096		-0.078					
8	-0.952		0.077						
9		-0.072							
10	-0.956								

Table 3 Partitions, wave indices, reflection index and multiplicity coefficients for sample wave paths.

Wave Path	Partition	r	s	n	k_p	k_0	k_1	k_2
P P SV SV P SV P P	P P SV SV P SV P P	5	3	1	3	1	1	0
P SV SV SV P P P P	P SV SV SV P P P P	5	3	2	4	0	0	1
P P SV P SV P SV P	P P SV P SV P SV P	5	3	0	2	3	0	0
P SV P P SV SV P P	P SV P P SV SV P P	5	3	1	3	1	1	0

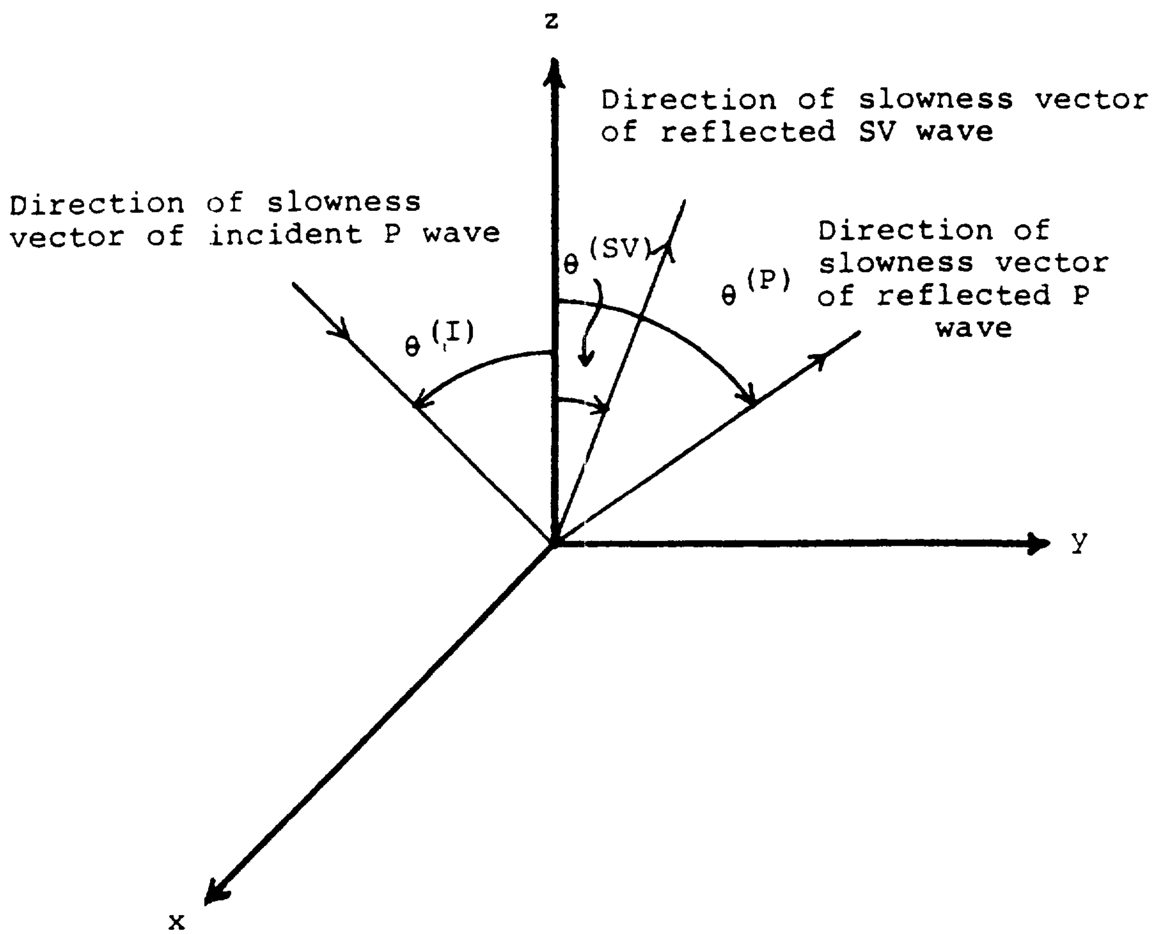

Fig. 1 Angle of incidence of incident P wave $\theta^{(I)}$, angle of reflection of reflected P wave $\theta^{(P)}$ and angle of reflection of reflected SV wave $\theta^{(SV)}$ shown in y-z plane of (x,y,z) coordinate system.

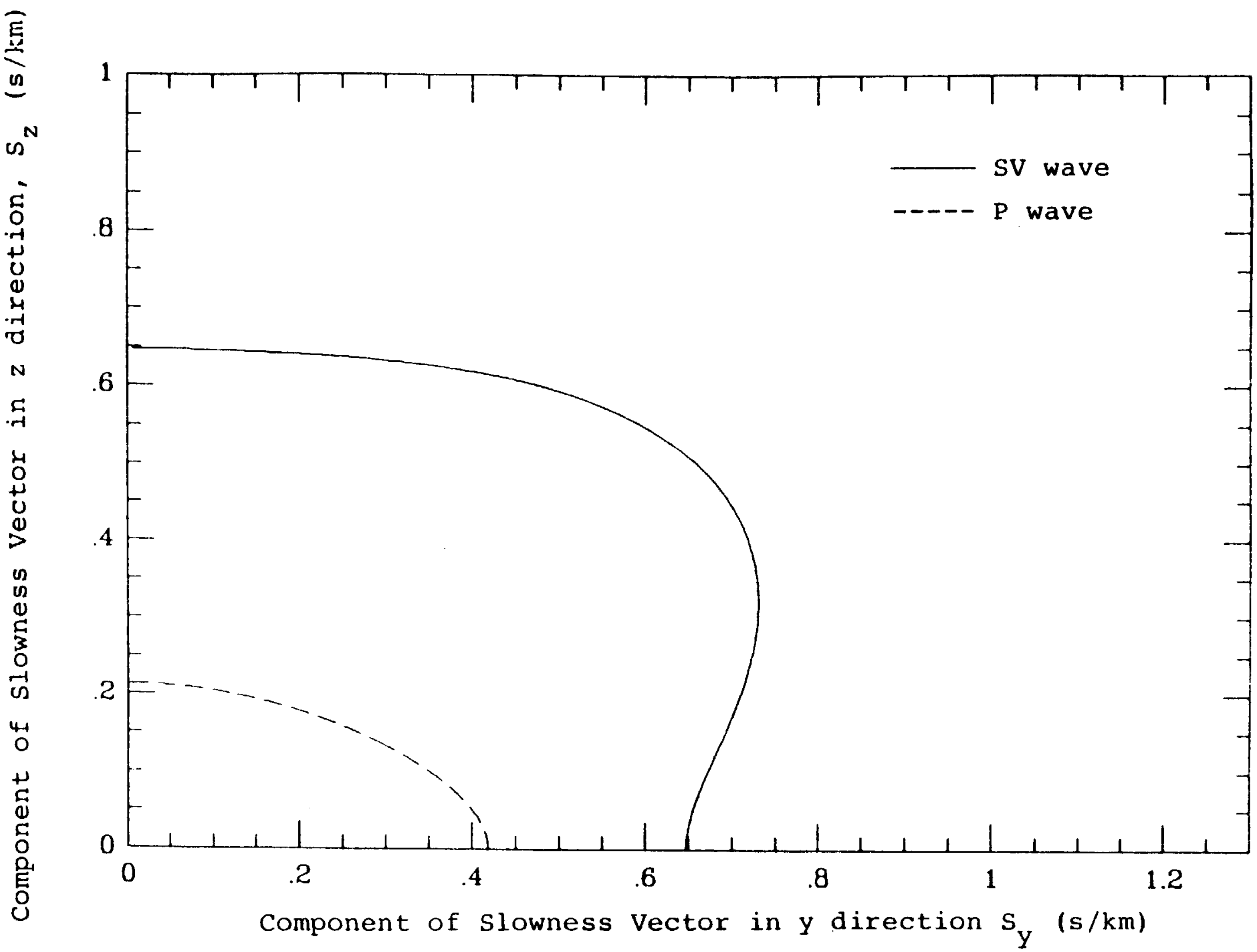

Fig. 2 First quadrant of slowness surfaces of P wave and SV wave in fiberglass epoxy composite in y-z plane.

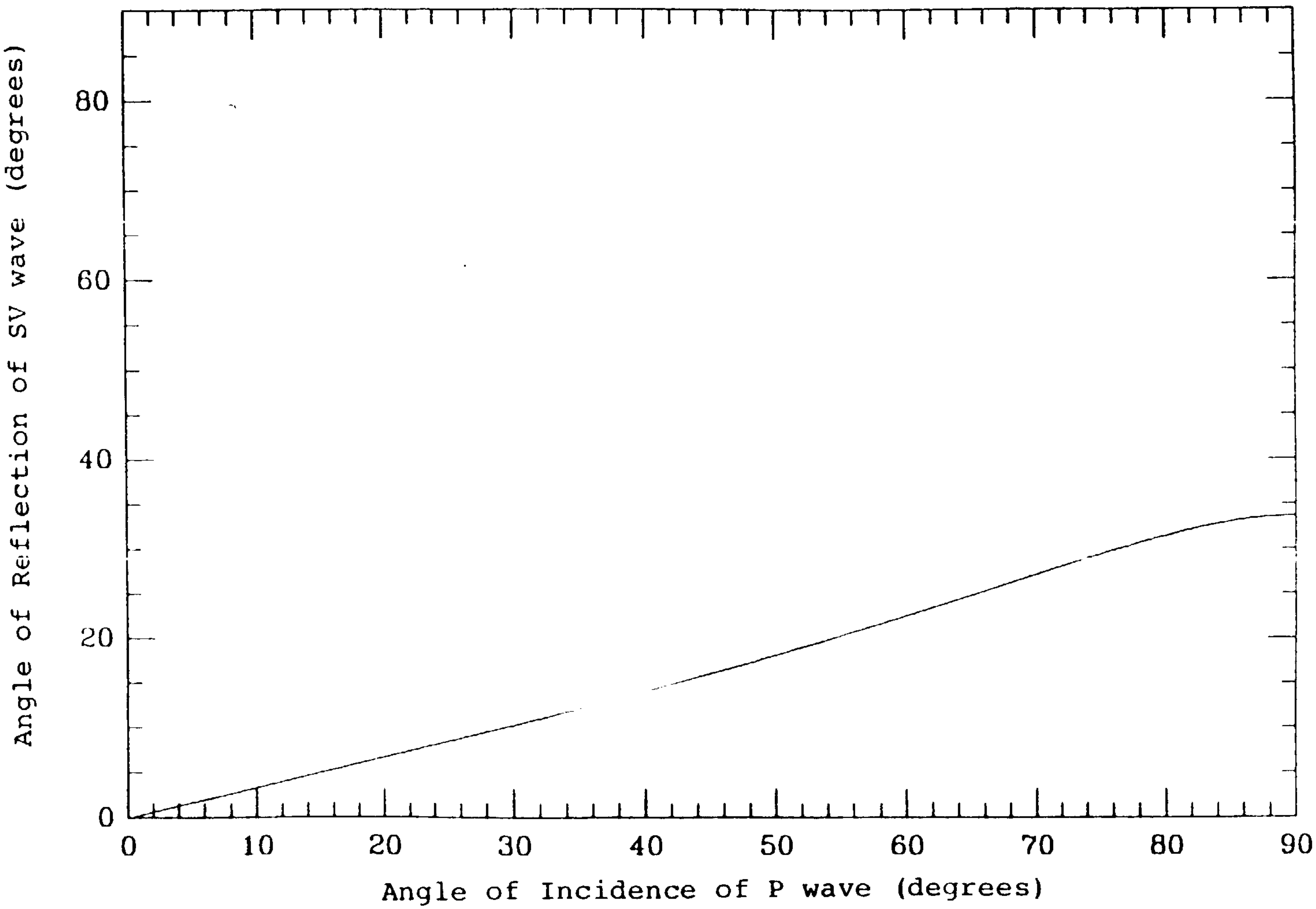

Fig. 3 Angle of reflection of reflected SV wave as function of angle of incidence of incident P wave for P wave incident on stress-free plane boundary of continuum plate.

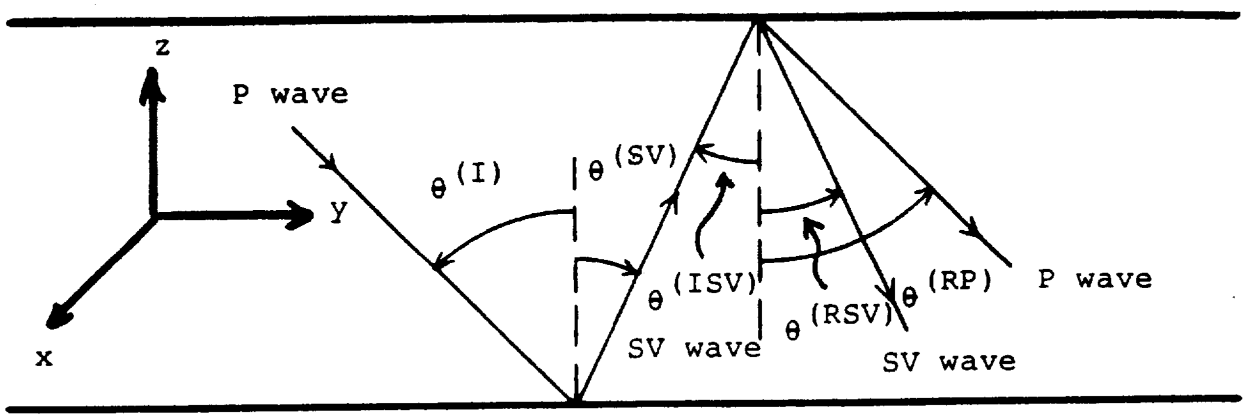

Fig. 4 Incident P wave produces SV wave which experiences mode splitting at opposite face of plate.

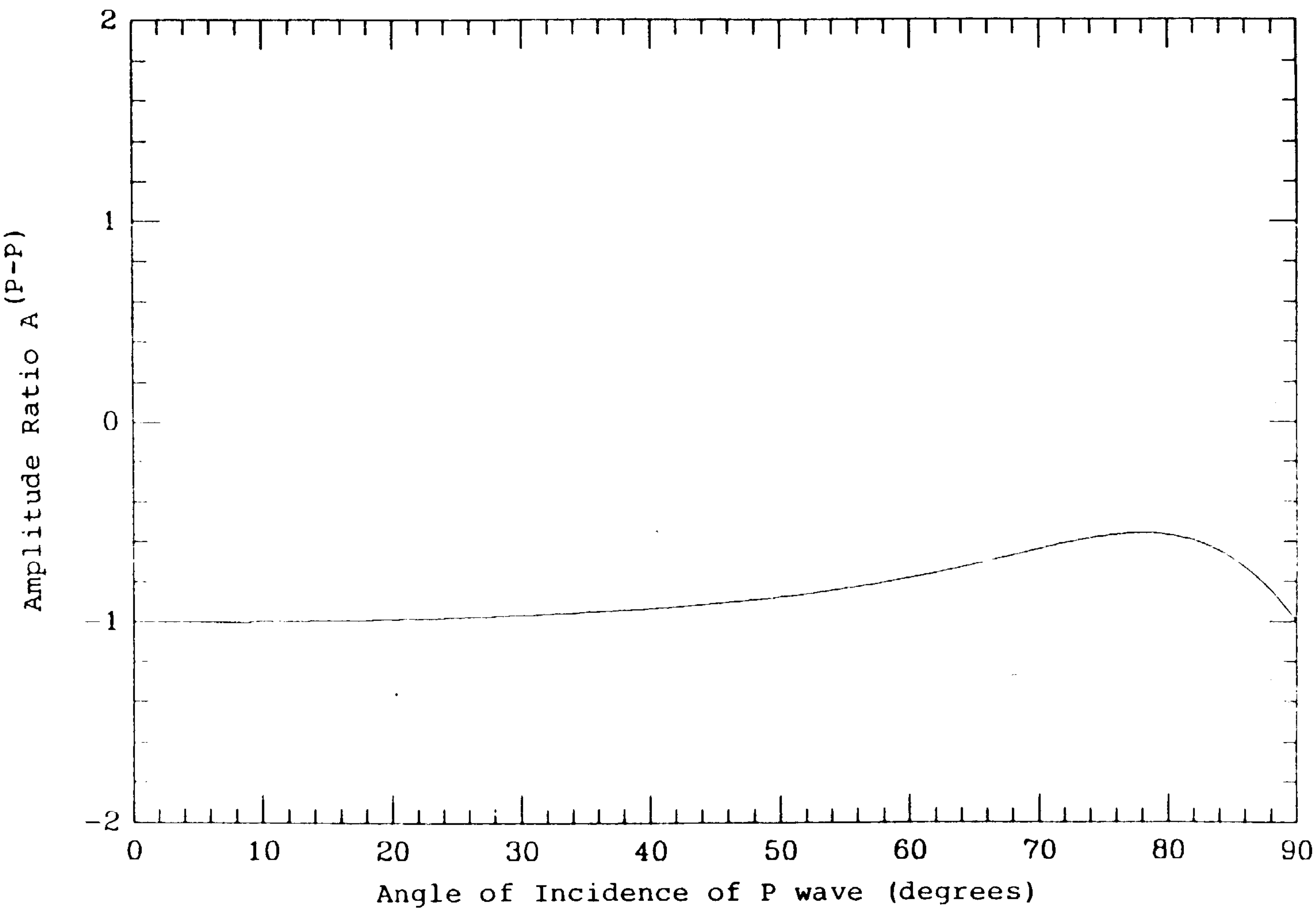

Fig. 5 Amplitude coefficient of P wave reflected by incident P wave as function of angle of incidence of incident P wave.

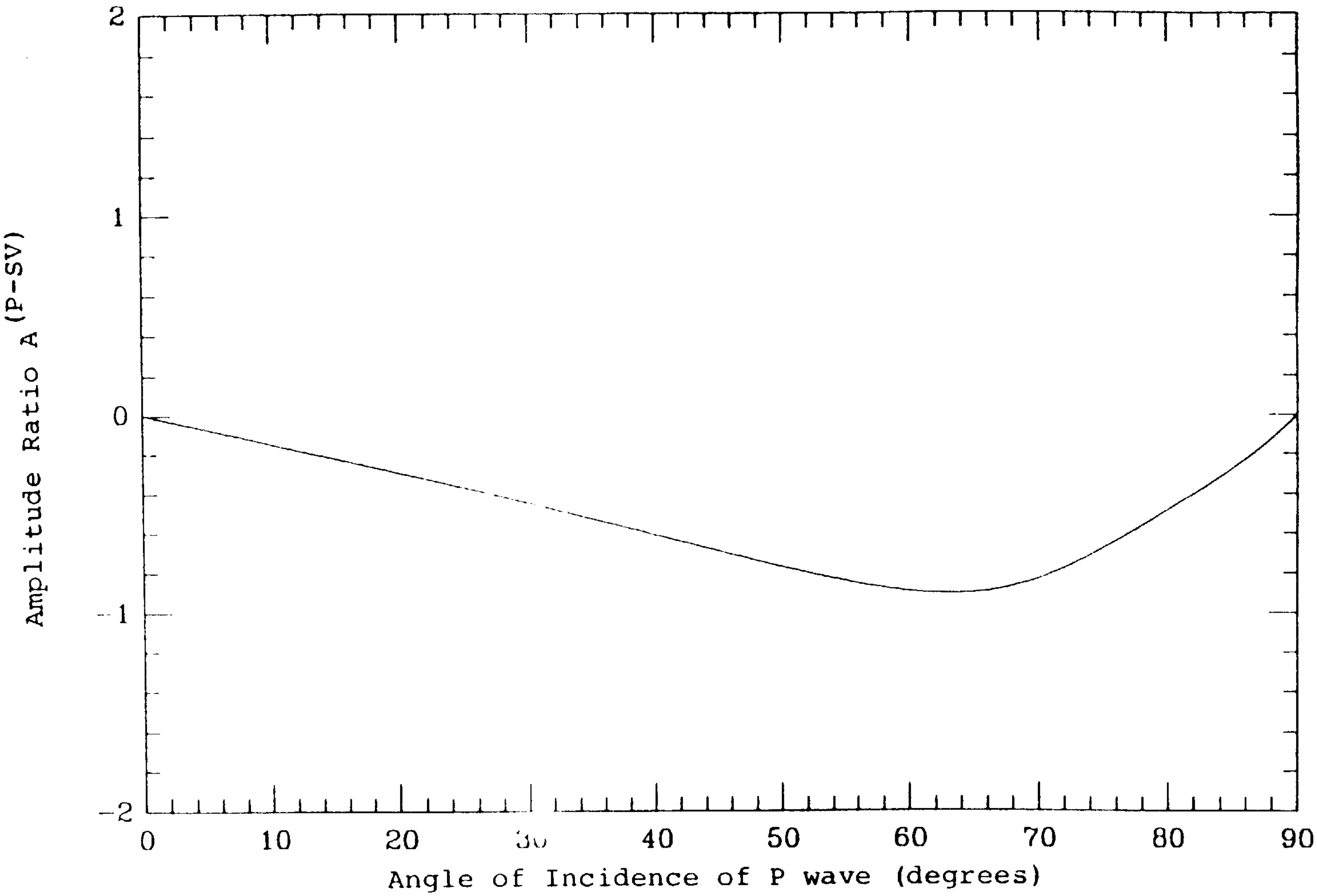

Fig. 6 Amplitude coefficient of SV wave reflected by incident P wave as function of angle of incidence of incident P wave.

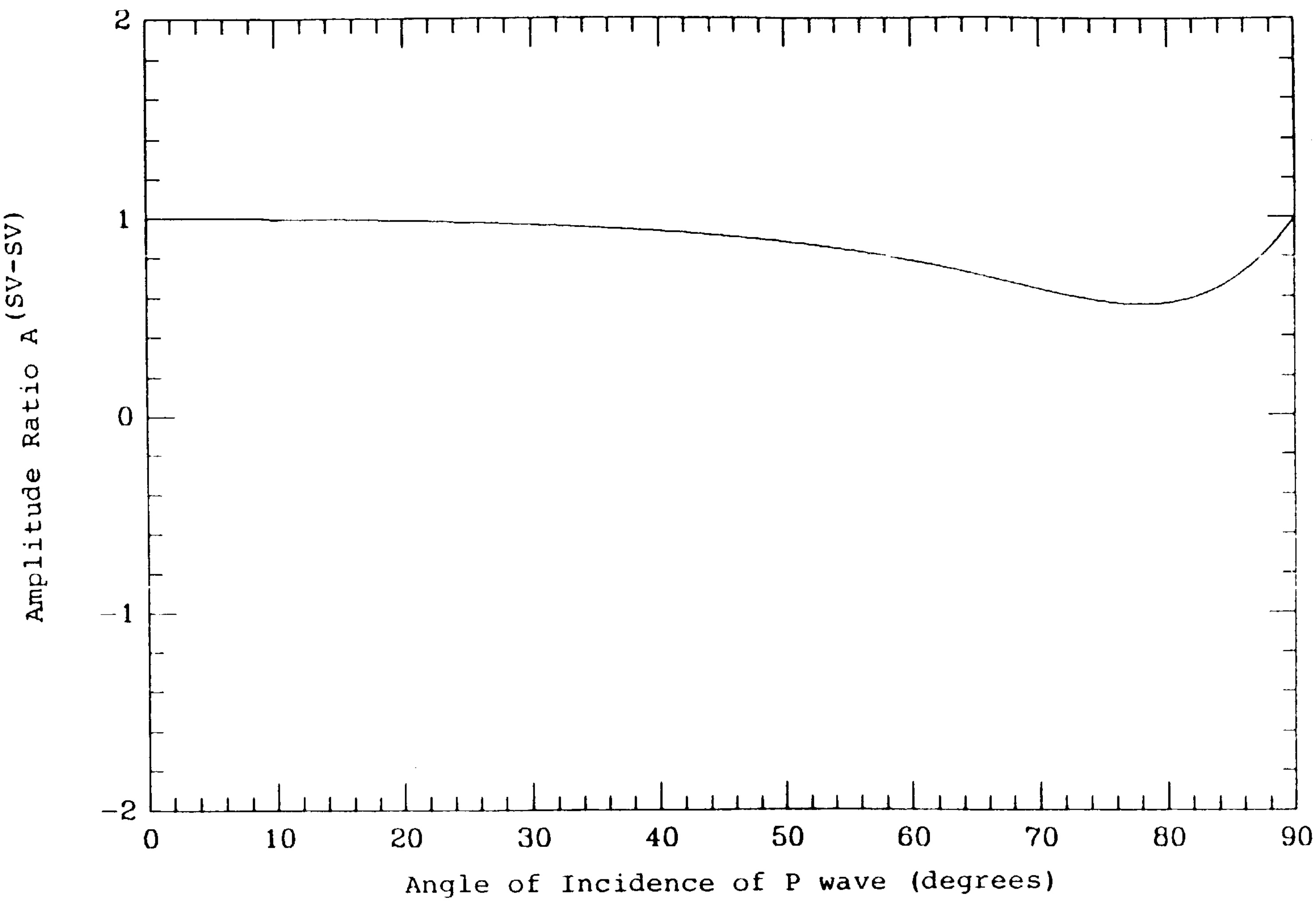

Fig. 7 Amplitude coefficient of SV wave reflected by incident SV wave produced during mode splitting as function of angle of incidence of initial P wave.

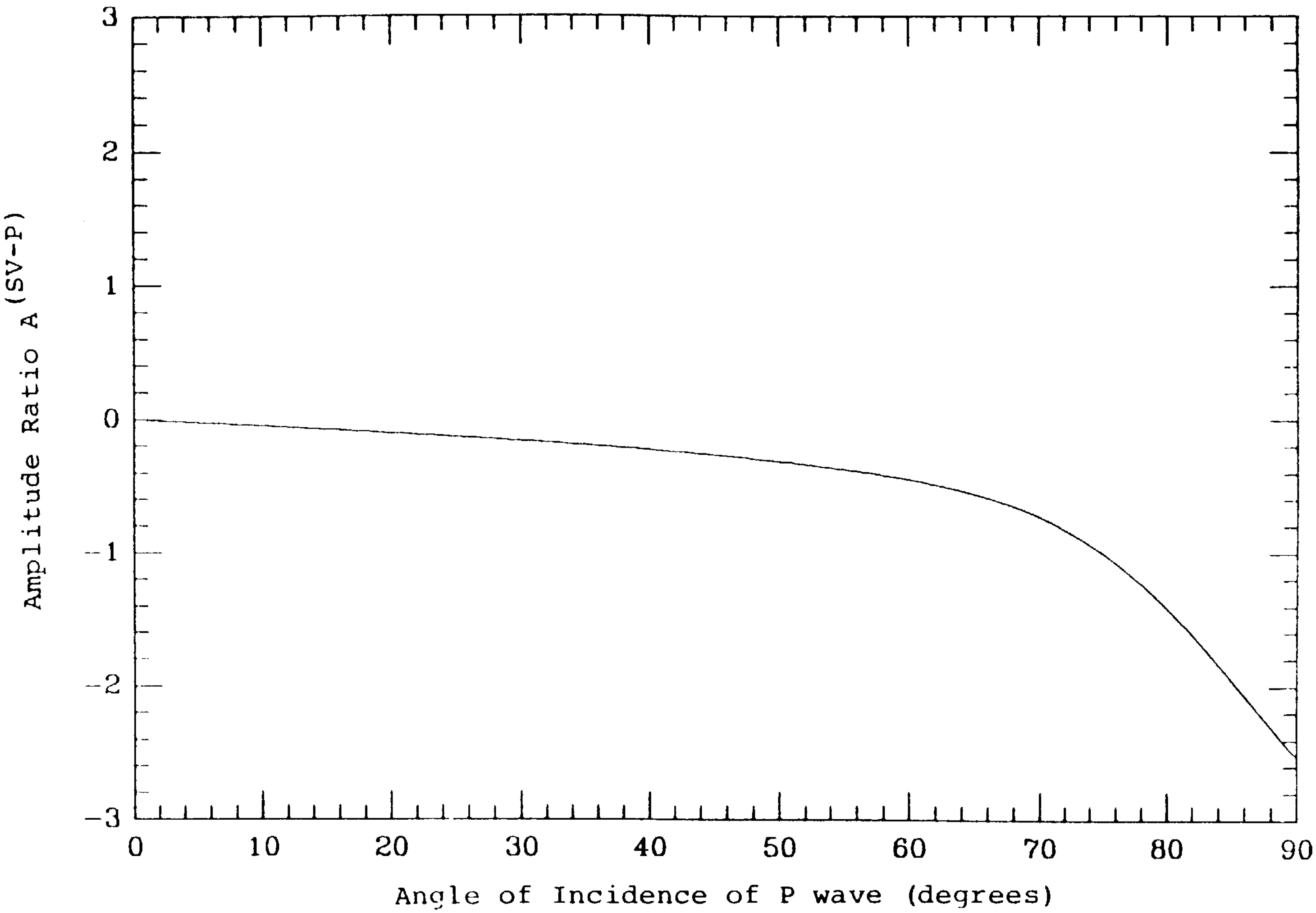

Fig. 8 Amplitude coefficient of P wave reflected by incident SV wave produced during mode splitting as function of angle of incidence of initial P wave.

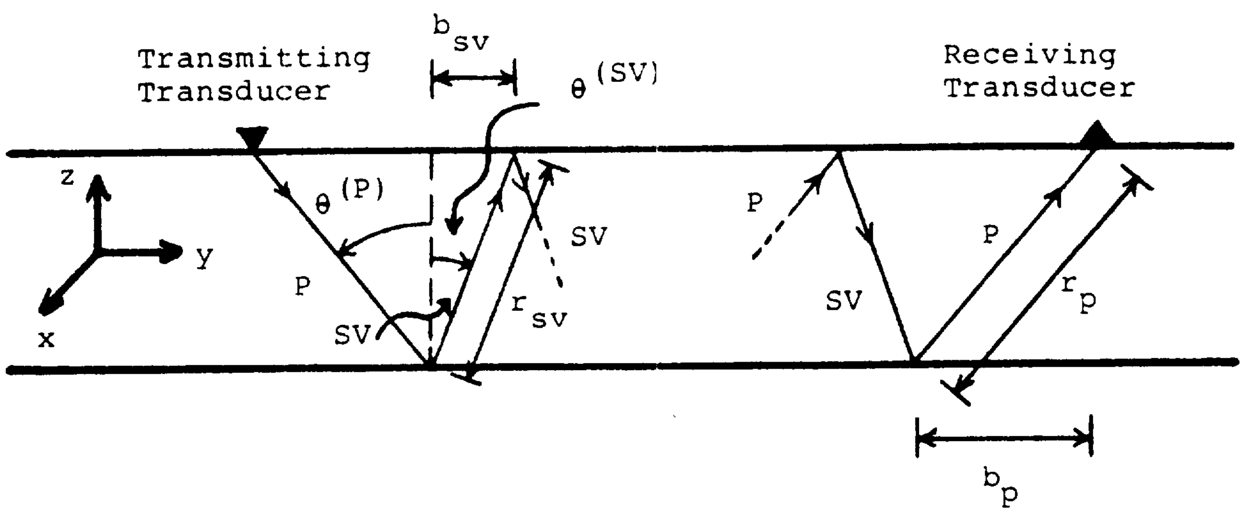

Fig. 9 Wave path P·SV·SV...P·SV·P from transmitting transducer to receiving transducer containing both P and SV waves.

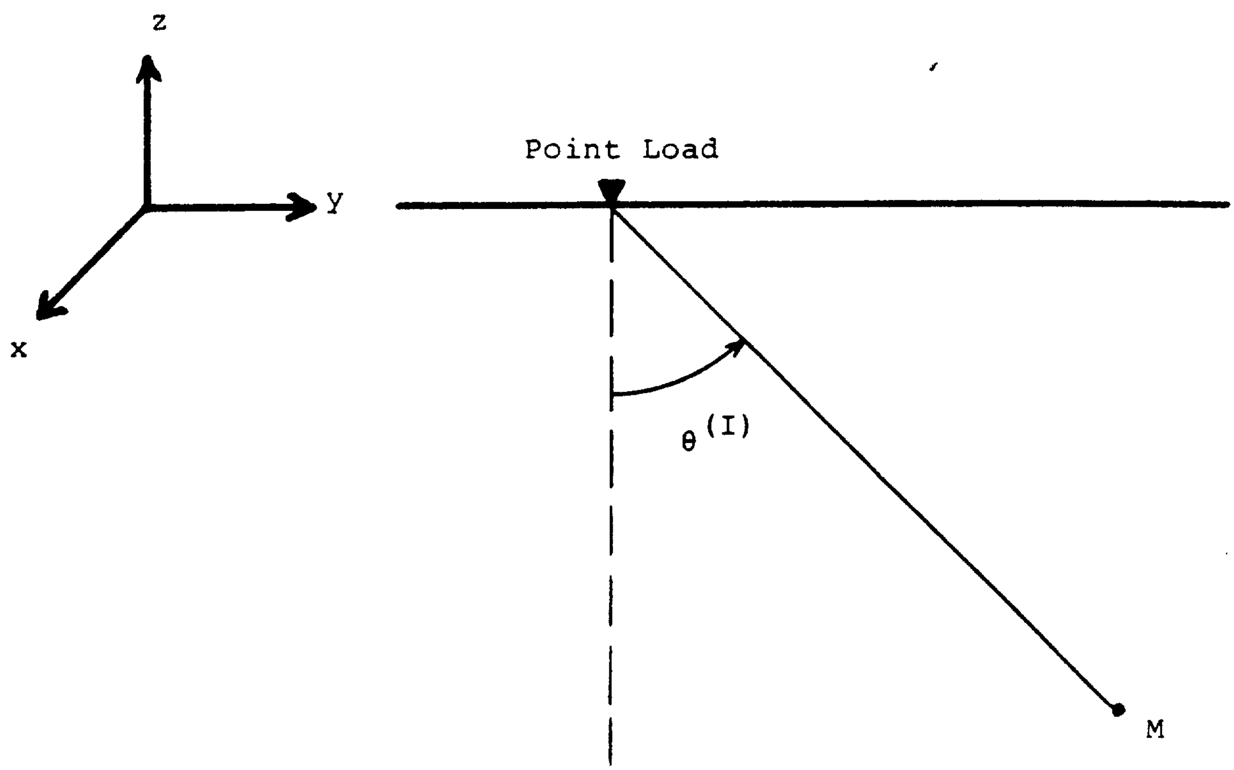

Fig. 10 P wave traveling to point M in semi-infinite body.

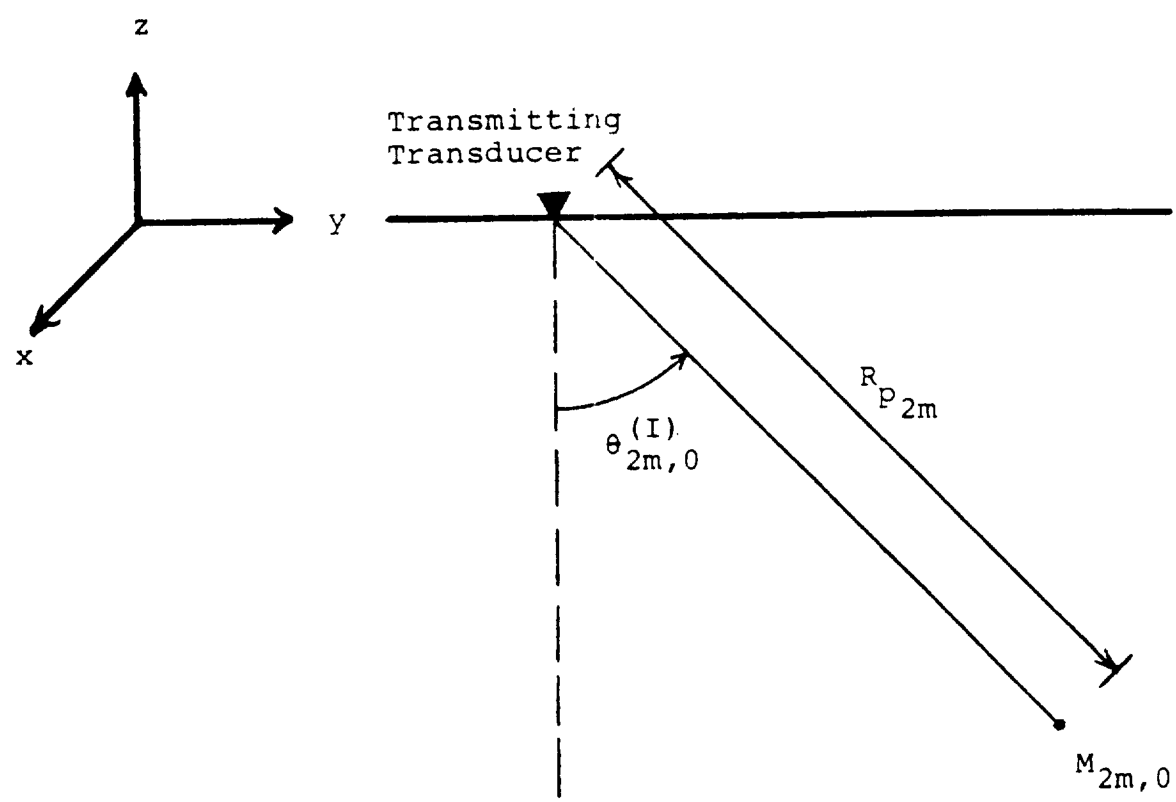

Fig. 11 P wave traveling in hypothetical half-space travels distance $R_{P_{2m}}$ to point $M_{2m,0}$.

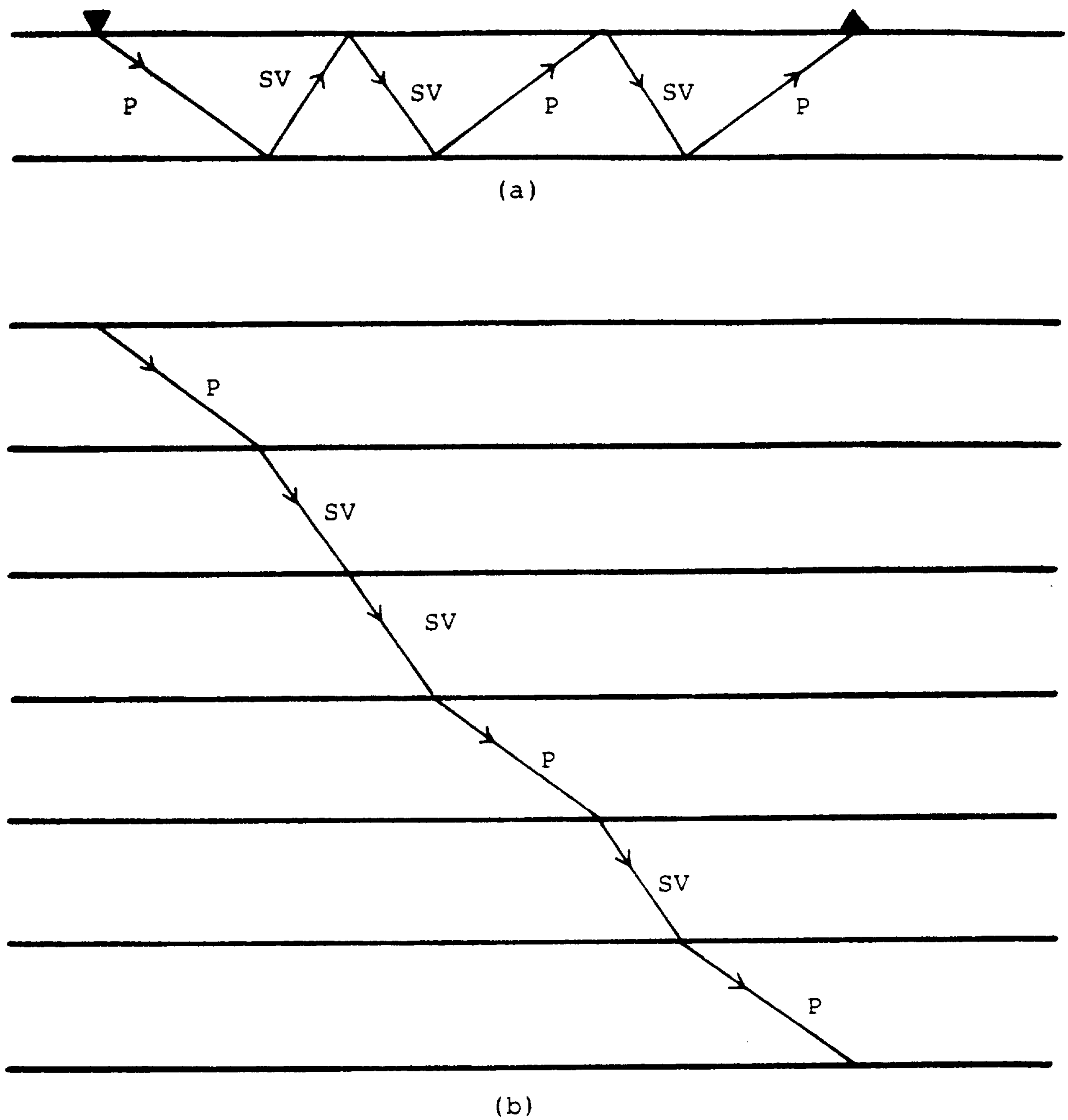

Fig. 12 Schematics of (a) wave path P·SV·SV·P·SV·P in continuum plate and (b) wave path P·SV·SV·P·SV·P visualized in hypothetical multilayered half-space.

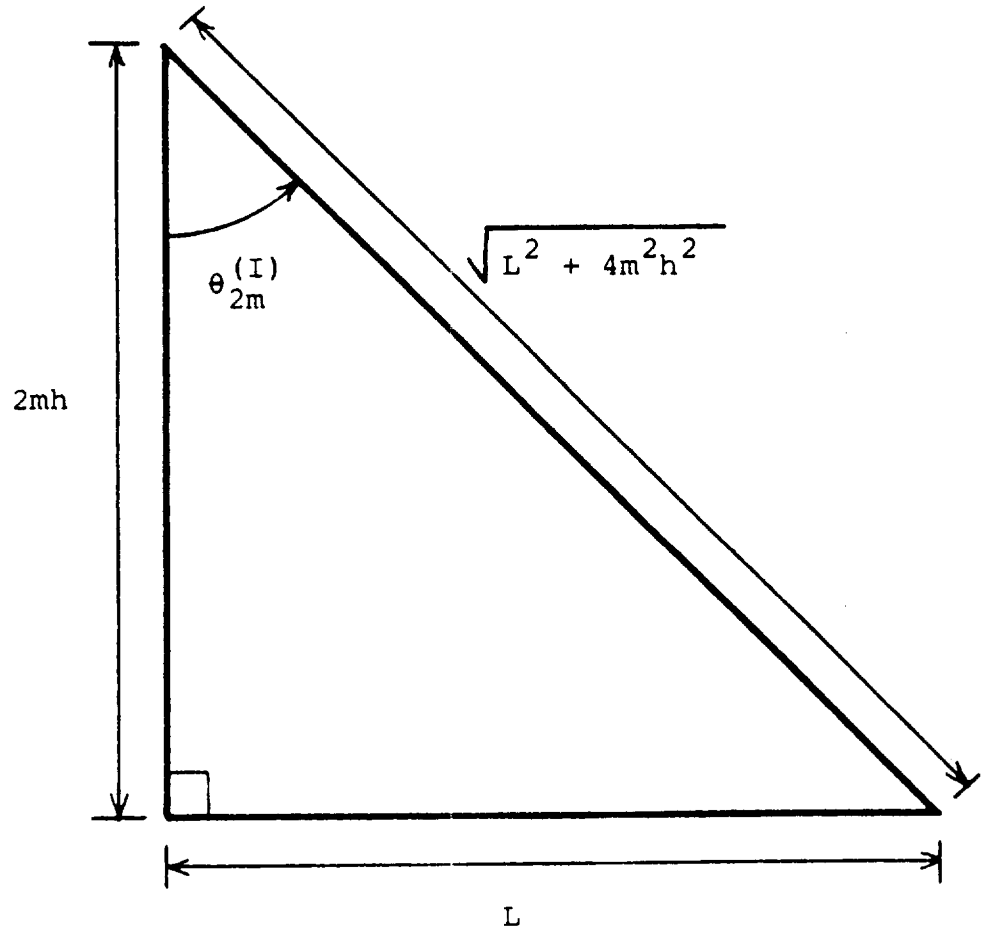

Fig. 13 Right triangle from which trigonometric functions of $\theta_{2m}^{(I)}$ are derived.

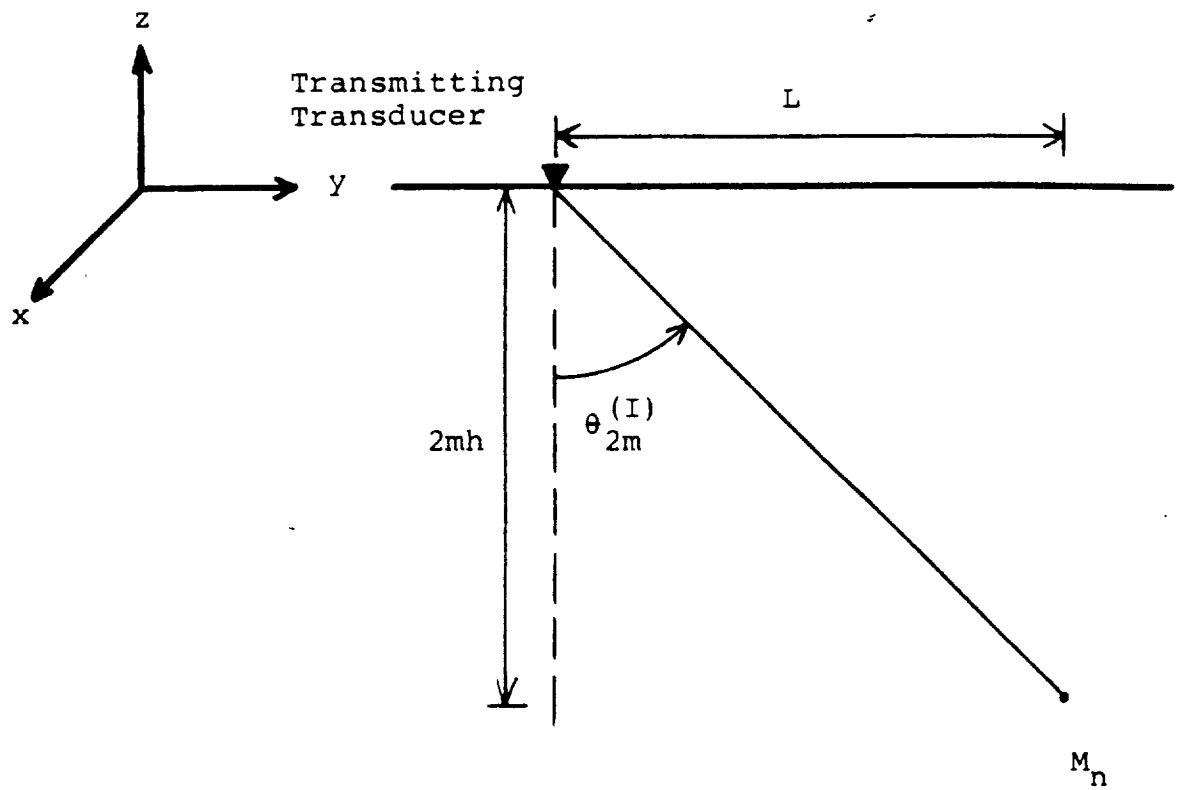

Fig. 14 P wave traveling in hypothetical half-space to point M_n, making angle $\theta_{2m}^{(I)}$ with respect to z direction.

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16. Abstract

Input-output characterization of fiber composites is studied theoretically by tracing P waves in the media. A new path notation to aid in the tracing of P and the reflection-generated SV wave paths in the continuum plate is developed. A theoretical output voltage from the receiving transducer is calculated for a tone burst. The study enhances the quantitative and qualitative understanding of the nondestructive evaluation (NDE) of fiber composites which can be modeled as transversely isotropic media.

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