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Robust Eigensystem Assignment for State Estimators Using Second-Order Models

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Abstract

A novel design of a state estimator is presented using second-order dynamic equations of mechanical systems. The eigenvalues and eigenvectors of the state estimator are assigned by solving the second-order eigenvalue problem of the structural system. Three design methods for the state estimator are given in this paper. The first design method uses collocated sensors to measure the desired signals and their derivatives. The second design method uses prefilters to shift signal phases to obtain estimates of the signal derivatives. These two methods are used to build a second-order state estimator model. The third design method is the conventional one which converts a typical second-order dynamic model to a first-order model, and then builds a state estimator based on the first-order model. It is shown that all the three designs for state estimation are similar. A numerical example representing a large space structure is given for illustration of the design methods presented in this paper.

Introduction

Structural dynamic systems are generally described by second-order differential equations with symmetric and sparse structural matrices. Structural engineers and analysts perform dynamic analyses by taking full advantage of the symmetry and sparsity of the structural matrices to minimize the computational burden and keep physical insight intact. For example, it is obviously easier to solve the eigenvalues of a symmetric and sparse matrix than a general matrix. On the other hand, control theory including estimation theory are established using first-order dynamic equations. Existing control software tools today are thus written in first-order forms. In applications to structural dynamic systems, a composite state vector is used to transform the second-order dynamic equations to a larger dimensional first-order form. Transformation to the first-order form not only increases the dimension by a factor of two but also destroys the symmetry of the structural matrices. As a result, significant model reduction is generally required before any controller or state estimator design can be accomplished, because of the numerical difficulty associated with the solution of high dimensional equations such as Riccati equations. A number of researchers[1-5] have investigated model reduction to circumvent the dimension problem. An alternative approach to model reduction is to preserve the second-order dynamic equations in designing the controller or state estimator. Recently, several researchers[6-10] have addressed the computational advantages of designing controllers and state estimators directly using the second-order structural models.

The state estimator plays a major role in controller designs using state feedback under the constraint that the number of sensors is less than the number of states. Second-order state estimator models have just recently received attention in the literature. An optimal state estimator known as Kalman Filter has been used in Ref. 6 for discretized second-order structural models. Robust computational procedures for solving Kalman Filter estimation error covariance matrices have been developed for second-order models in Ref. 7. A dissipative state estimator in second-order form was introduced in Ref. 8. This state estimator was analogous to a dissipative controller[9] with collocated sensors and actuators whereby positive definite feedback gains were designed to insure stability. The computational advantages of second-order state estimator models are discussed in Refs. 10 and 11.

The objective of this paper is to develop a robust state estimator for use with robust controllers. This research was stimulated by the work in Refs. 10 and 11 where sub-optimal second-order observers were developed using optimal control theory. The approach of this paper is to extend the technique presented in Ref. 12 for robust eigensystem assignment of second-order controllers. As in Ref. 12, the technique takes advantage of a second-order form of the system equations (instead of transforming to a first-order form) which results in considerable computational efficiency. The technique can handle any forms of feedbacks, i.e., displacement, velocity, and acceleration. It is known that the controller and state estimator in the first-order form are dual in a mathematical

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sense, which implies that the design freedoms are identical. The question arises whether the same statement is true for the second-order form. It will be shown that the design freedoms for the second-order state estimators are only half of those for second-order controllers (see Ref. 12). Therefore, this paper presents methods of obtaining the additional freedoms which are needed to complete the design process.

In this paper, three methods for the design of state estimators having second-order models are presented using eigensystem assignment techniques. The first design method uses collocated sensors to measure signals and their corresponding derivatives to gain full freedom to build a second-order state estimator model. The technique used in this design assigns the eigenvalues such that the resulting closed-loop system is robust with respect to system parameter uncertainties. This is accomplished by requiring the closed-loop eigenvectors to be as close as possible to the column space of a well-conditioned matrix. The second design method uses a second-order prefilter design in place of collocated measurements of the signals and their derivatives as required in the first design method. Here, the prefilter is designed to shift the phase of the signals, thus replicating the effects of the signal derivatives in the first design. The third design method is the traditional state estimator design in which the estimator is constructed based on a first-order model of the system. However, the gain matrix is computed through a second-order model. It is demonstrated that a second-order state estimator together with a prefilter design has the same design freedoms as the conventional first-order state estimator. A numerical example is given to demonstrate the proposed method.

State Estimators with Measurement Signals and their Derivatives

In vibration control of flexible structures, two set of second-order linear, constant coefficient, ordinary differential equations are frequently used. These equations, in matrix form, are

$$M\ddot{x} + D\dot{x} + Kx = Bu \tag{1}$$

$$y = H_v\dot{x} + H_d x. \tag{2}$$

Equation (1) is the system dynamic equation having x as the state vector of dimension n, and M, D, and K as the mass, damping and stiffness matrices, respectively, which generally are symmetric and sparse. The $n \times p$ influence matrix B describes the control force distribution for the $p \times 1$ control force vector u. Equation (2) is the measurement equation having y as the measurement vector of length m, H_v the $m \times n$ velocity influence matrix and H_d the $m \times n$ deflection influence matrix.

If the measurement vector y in Eq. (2) is used directly for a feedback control design, an output feedback controller is obtained. The output feedback control is generally attractive because it is simple and easy for real time implementation. However, a stable and robust output feedback controller may require either too many measurements which are not practical, or some measurement devices which are not yet available and need to be developed. On the other hand, the state feedback control law assumes that all states are measurable. In many practical control designs for flexible structures, it is physically or economically impractical to install the sensors that would be necessary to measure all the states. For such cases, a state estimator is needed to estimate the states from the measurement outputs, and provide enough freedom for a stable and robust feedback controller design.

The basic approach of estimating the states is to simulate the state and output measurement equations of the system on a computer with an assumed initial state vector. In other words, Eqs. (1) and (2) are simulated on a computer with the same input u as applied to the actual physical system. For noise-free and uncertainty-free cases, the states of the simulated system, i.e. the estimated states, will then be identical to the states of the actual systems if initial states are the same. However, the actual system may be subjected to unmeasurable disturbances which can not be used in the simulation but affect the output measurements. In order to make sure that the estimated state does not deviate too much from the actual state values, the difference between the actual output and the estimated output should be used as one of the driving inputs in the estimation equation.

Let the state estimation and the output equations be

$$M\ddot{\hat{x}} + D\dot{\hat{x}} + K\hat{x} = Bu + L_d(y - \hat{y}) + L_v(\dot{y} - \dot{\hat{y}}) \tag{3}$$

$$\hat{y} = H_v\dot{\hat{x}} + H_d\hat{x} \tag{4}$$

where $\hat{x}$ is the estimated state vector of length n, $\hat{y}$ the estimated output of length m, and L_d and L_v the $n \times m$ state estimator gain matrices. Here, in contrast to the conventional approach, an additional term, $L_v(\dot{y} - \dot{\hat{y}})$, is added to Eq. (3) to penalize the difference between the actual output derivative and estimated output derivative. Why the additional term is added in Eq. (3) will be explained in detail later. Note that it is not wise to differentiate measurement signals to generate this additional term for real time implementation. Additional sensors are recommended for use in measuring the derivatives of measurement signals.

To determine the matrices L_v and L_d, define the estimation error as

$$e = x - \hat{x}. \tag{5}$$

Subtracting Eq. (3) from Eq. (1) and employing the relationship given in Eqs. (2) and (4), the error equation becomes

$$[M + L_vH_v]\ddot{e} + [D + L_vH_d + L_dH_v]\dot{e} + [K + L_dH_d]e = 0. \tag{6}$$

A question arises as to if an appropriate choice of the gain matrices L_v and L_d will move the eigenvalues of Eq. (6) to the left-hand plane so that the steady-state value of e(t) for any initial condition is zero, i.e. $\lim_{t\to\infty} e(t) = 0$. The following paragraphs present a novel way of synthesizing the matrices L_v and L_d.

Assume that the system, Eqs. (1) and (2), is observable. The left eigenvectors $\overline{\phi}_k^*$ and eigenvalues λ_k for the system in Eq. (6) are related by the equation

$$\overline{\phi}_k^T \left\{[M + L_v H_v]\lambda_k^2 + [D + L_v H_d + L_d H_v]\lambda_k + [K + L_d H_d]\right\} = 0;$$

$$k = 1, \ldots, n. \tag{7}$$

The subscript k refers to the mode number. Rearranging Eq. (7) in a compact matrix form yields

$$\begin{bmatrix} \overline{\phi}_k^T & \overline{\phi}_k^T(\lambda_k L_v + L_d) \end{bmatrix} \begin{bmatrix} (\lambda_k^2 M + \lambda_k D + K) \\ (\lambda_k H_v + H_d) \end{bmatrix} = 0;$$

$$k = 1, ..., n. \tag{8}$$

The transpose of this equation gives

$$\begin{bmatrix} (\lambda_k^2 M + \lambda_k D + K) & (\lambda_k H_v^T + H_d^T) \end{bmatrix} \begin{bmatrix} \overline{\phi}_k \\ (\lambda_k L_v^T + L_d^T)\overline{\phi}_k \end{bmatrix}$$

$$\equiv \Gamma_k \phi_k = 0; \;\; k = 1, ..., n. \tag{9}$$

If the closed-loop eigenvalues λ_k $(k = 1, \ldots, n)$ and their complex conjugates are assigned, Eq. (9) can be used to determine the gain matrices L_v and L_d. Because the vector ϕ_k is in the null space of the matrix Γ_k, it is necessary to compute the null spaces of the matrices Γ_k $(k = 1, \ldots, n)$ corresponding to the desired eigenvalues λ_k $(k = 1, \ldots, n)$. To obtain the nontrivial solution space of the homogeneous equation (9), the singular value decomposition (SVD) is applied to the matrix Γ_k yielding

$$\Gamma_k = U_k \Sigma_k V_k^* = U_k \begin{bmatrix} \sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{\sigma k}^* \\ V_{ok}^* \end{bmatrix}. \tag{10}$$

Because λ_k in Γ_k is a complex value, all the quantities are complex except the diagonal matrix σ_k which contains the nonzero and positive singular values. Here the superscript $*$ means transpose and complex conjugate. It follows that the matrix V_{ok} represents a set of orthogonal basis vectors spanning the null space of the matrix Γ_k so that

$$\Gamma_k \phi_k = \Gamma_k V_{ok} c_k = 0 \tag{11}$$

where c_k is an arbitrary column vector with an appropriate length. Note that if Γ_k is well-conditioned (i.e. not close to a matrix of lesser rank which is easily found from the singular values; hence the advantage of using SVD), the above basis for null space V_{ok} can be computed more efficiently by taking the QR decomposition of Γ_k. If the matrix $[\lambda_k^2 M + \lambda_k D + K]$ is invertible, the vector $[\overline{\phi}_k^* \;\; \hat{\phi}_k^*]^*$, where $\hat{\phi}_k$ is an arbitrary vector of length p and $\overline{\phi}_k = -[\lambda_k^2 M + \lambda_k D + K]^{-1}[\lambda_k H_v^T + H_d^T]\hat{\phi}_k$, is in the null space of the matrix Γ_k.

To obtain an expression for gain matrices L_v and L_d, choose a particular set of vectors, $\phi_k = V_{0k} c_k$ $(k = 1, \ldots, n)$ satisfying Eq. (9), corresponding to some choice c_k, and partition the vector ϕ_k into two components such that

$$\Gamma_k \phi_k \equiv \Gamma_k \begin{bmatrix} \overline{\phi}_k \\ \hat{\phi}_k \end{bmatrix} \equiv \Gamma_k \begin{bmatrix} \overline{V}_{ok} \\ \hat{V}_{ok} \end{bmatrix} c_k = 0. \tag{12}$$

Comparison of Eqs. (9) and (12) yields

$$L_v^T \overline{\Phi} \Lambda + L_d^T \overline{\Phi} = \hat{\Phi} \tag{13}$$

where the $n \times n$ matrix $\overline{\Phi} = [\overline{\phi}_1, \overline{\phi}_2, \ldots, \overline{\phi}_n]$, the $m \times n$ matrix $\hat{\Phi} = [\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_n]$ and the $n \times n$ matrix $\Lambda = diag[\lambda_1, \lambda_2, \ldots, \lambda_n]$. To solve for the gain matrices L_v and L_d. (Both matrices are real), decompose Eq. (13) into real and imaginary parts to yield

$$L_v^T[\overline{\Phi}_r \Lambda_r - \overline{\Phi}_i \Lambda_i] + L_d^T \overline{\Phi}_r = \hat{\Phi}_r \tag{14a}$$

$$L_v^T[\overline{\Phi}_r \Lambda_i + \overline{\Phi}_i \Lambda_r] + L_d^T \overline{\Phi}_i = \hat{\Phi}_i \tag{14b}$$

or in matrix form

$$L^T \overline{\Psi} \equiv [\, L_v^T \quad L_d^T \,] \begin{bmatrix} (\overline{\Phi}_r \Lambda_i + \overline{\Phi}_i \Lambda_r) & (\overline{\Phi}_r \Lambda_r - \overline{\Phi}_i \Lambda_i) \\ \overline{\Phi}_i & \overline{\Phi}_r \end{bmatrix}$$
$$= [\, \hat{\Phi}_i \quad \hat{\Phi}_r \,] \equiv \hat{\Psi}. \tag{15}$$

Here, the subscripts r and i respectively refer to the real and imaginary parts of the associated quantities. The gain matrices L_v and L_d can be obtained from Eq. (15)

$$L^T \equiv [\, L_v^T \quad L_d^T \,] = \hat{\Psi} \overline{\Psi}^{-1}$$
$$= [\, \hat{\Phi}_i \quad \hat{\Phi}_r \,] \begin{bmatrix} (\overline{\Phi}_r \Lambda_i + \overline{\Phi}_i \Lambda_r) & (\overline{\Phi}_r \Lambda_r - \overline{\Phi}_i \Lambda_i) \\ \overline{\Phi}_i & \overline{\Phi}_r \end{bmatrix}^{-1}. \tag{16}$$

A matrix inversion is required in the computation of the gain matrices L_v and L_d. However, if the number of eigenvalues to be assigned is less than the number assignable, n, Eq. (11) becomes underdetermined which leads naturally to a minimum gain solution. To assure that the above matrix is well-conditioned for inversion, the condition number of the matrix $\overline{\Psi}$ should be the smallest possible. Interestingly, the above numerical requirement for the well-conditioning of the matrix inversion problem corresponds exactly to the eigenvalue conditioning problem since $\overline{\Psi}$ consists of eigenvalues and eigenvectors. For sufficiently small damping D and small real part of the closed eigenvalue Λ_r, $\overline{\Phi}_i$ in Eq. (15) approaches zero, because all the null spaces corresponding to Γ_k $(k = 1, \ldots, n)$ are nearly in the real domain. The matrix $\overline{\Psi}$ in Eq. (15) can be approximated by $\overline{\Psi} = diag[\overline{\Phi}_r \Lambda_i \quad \overline{\Phi}_r]$. In structural dynamics terminology, it indicates that for small damping and small real part of the eigenvalues assigned for the closed-loop model, the real part of the closed-loop eigenvector matrix $\overline{\Phi}_r$ dominates the conditioning of the matrix $\overline{\Psi}$. The closed-loop eigenvectors are chosen and discussed in the following section.

Eigenvector Assignment for Robustness

Define an $n \times n$ well-conditioned matrix H_0, with vectors $h_{01}, h_{02}, \ldots, h_{0n}$ as its columns. Then, the closed-loop eigenvectors are chosen to be as close as possible to the range space of the columns of matrix H_0 to achieve a robust closed-loop design. If the open-loop conditioning is good to begin with, then the columns of matrix H_0 may be chosen to correspond to the open-loop eigenvectors, i.e.,

$$h_{0k} = \psi_{ok} \quad ; \quad k = 1, 2, \ldots, n. \tag{17}$$

In general, the choice of the open-loop eigenvector ψ_{ok} is arbitrary as long as the resulting closed-loop eigenvectors are linearly independent. If the control system is used only to provide active damping, and the closed-loop damped frequencies are quite close to their open-loop values, then it is best to use the open-loop eigenvector corresponding to an eigenvalue with the same (or similar) frequency for the vector ψ_{ok} in Eq. (17).

Alternatively, the matrix H_0 may be chosen to be an arbitrary unitary matrix (with perfect conditioning), or the closest unitary matrix to the open-loop eigenvectors. In the latter case, the matrix H_0 is then the solution of the constrained least square problem minimizing

$|\Psi_o - H_0|$

subject to $H_0^* H_0 = I$

which leads to

$$H_0 = UW^* \tag{18}$$

where U and W are, respectively, the left and right singular vectors of the open-loop eigenvector matrix Ψ_o, and I is an $n \times n$ identity matrix. It is noted that by choosing the closed-loop eigenvectors to be as close as possible to the column space of matrix H_0 the closed-loop conditioning of the second-order system represented by Eq. (9) will be enhanced. However, an additional requirement is needed to ensure the well-conditioning of the actual system of Eq. (6), and that is to require that the estimator gains be as small as possible.

Having defined H_0, the closed-loop eigenvectors $\overline{\phi}_k$, $\overline{\phi}_k = \overline{V}_{ok} c_k$, $k = 1, 2, \ldots, n$, and the corresponding coefficient vectors c_k are computed through the following sequential three steps.

Step (1): Obtain the vector in the attainable closed-loop eigenvector space, $\overline{V}_{o1}$ (see Eq. (12)), corresponding to the first closed-loop eigenvalue, which is as close as possible to the range space of the columns of matrix $Q_0 \equiv \begin{bmatrix} H_0 \\ N_0 \end{bmatrix}$, where N_0 is a $p \times n$ null matrix. The vector can be obtained from the algorithm described by Golub and Van Loan[13] for the computation of principal angles and vectors of a subspace pair. Expand both matrices $\overline{V}_{o1}$ and Q_0 in terms of their QR decomposition, i.e.,

$$\overline{V}_{o1} = Q_{\overline{V}_{o1}} R_{\overline{V}_{o1}} \quad ; \quad Q_0 = Q_{Q_0} R_{Q_0}. \tag{19}$$

Here $Q_{\overline{V}_{o1}}$ and Q_{Q_0} are orthonormal matrices of dimensions $(n+p) \times p$ and $(n+p) \times n$, respectively, and $R_{\overline{V}_{o1}}$ and R_{Q_0} are $p \times p$ and $n \times n$ upper triangular matrices, respectively. Project the vectors $Q_{V_{o1}}$ unto Q_{Q_0} to obtain

$$H = Q_{V_{o1}}^* Q_{Q_0}. \tag{20}$$

The singular values of matrix H are cosines of the principal angles of the subspace pair $\{\Re(Q_{\overline{V}_{o1}}), \Re(Q_{Q_0})\}$.[13] Where $\Re(\ )$ denotes the range space of ( ). Taking the singular value decomposition of H, gives

$$H = YSZ^* \tag{21}$$

in which Y and Z are unitary matrices, and the matrix S contains the singular values of H, i.e.,

$$S = \begin{bmatrix} \sigma_1 & \cdots & \cdots & 0 \\ \vdots & \sigma_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \sigma_p \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \equiv \begin{bmatrix} \cos\theta_1 & \cdots & \cdots & 0 \\ \vdots & \cos\theta_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \cos\theta_p \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}. \tag{22}$$

Note that the singular values $\sigma_1, \ldots, \sigma_p$ are all positive and less than or equal to 1 because the matrix H is formed by orthonormal matrices as shown in Eq. (20). The angles $\theta_1, \ldots, \theta_p$ are the principal angles of the subspace pair. Having in mind that $1 \geq \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_p$, the vector $\overline{\phi}_1$ in the orthonormal columns of $Q_{\overline{V}_{o1}}$ which is closest to the column space of Q_{Q_0} is then

$$\overline{\phi}_1 = Q_{\overline{V}_{o1}} y = \overline{V}_{o1} R_{\overline{V}_{o1}}^{-1} y \tag{23}$$

in which y represents the first column of matrix Y. Choosing the first closed-loop eigenvector to be $\overline{\phi}_1$, the coefficients c_1 of Eq. (12) become

$$c_1 = R_{\overline{V}_{o1}}^{-1} y. \tag{24}$$

Note that $R_{\overline{V}_{o1}}$ defined in Eq. (19) is a nonsingular matrix.

Step (2): Reduce the column space of matrix Q_0 by the vector $\overline{\phi}_1$, in order to ensure the linear independence of the closed-loop eigenvectors. If $\overline{\phi}_1$ is not in the column space of Q_0, then reduce the column space by the closest vector in the space to the vector $\overline{\phi}_1$. The new subspace is spanned by the columns of matrix Q_1 defined as

$$Q_1 = [Q_0 - Q_{Q_0} z z^* Q^*_{Q_0} Q_0] \tag{25}$$

where z is the first column of matrix Z. Note that the columns of matrix Q_1 are orthogonal to the vector $Q_{Q_0} z$, i.e., $z^* Q^*_{Q_0} Q_1 = 0$. Equation (25) may not be computationally efficient. It is shown here just for simplicity and clarity, and other computational procedures to compute Q_1 may be used instead.

Step (3): Repeat steps (1) and (2) for the remaining $n-1$ eigenvalues. For the i^{th} eigenvalue the column space of matrix Q_{i-1} is reduced by the vector $Q_{Q_{i-1}} z$ resulting in a new matrix Q_i,

$$Q_i = [Q_{i-1} - Q_{Q_{i-1}} z z^* Q^*_{Q_{i-1}} Q_{i-1}]. \tag{26}$$

In summary, following the developments outlined in steps (1)-(3), the closed-loop eigenvectors are chosen to be as close as possible to the column space of a well-conditioned matrix H_0 and the estimator gain matrix elements are designed to be as small as possible, thereby resulting in a robust closed-loop design.

Note that the above formulations are nearly identical to the eigensystem assignment with full state feedback[12] except for slight differences in the matrix Γ_k defined in Eq. (9), i.e. the matrix B in the case of full state feedback is replaced by matrix $[\lambda_k H_v^T + H_d^T]$ for the state estimator. Computationally, both state feedback and state estimator designs are identical.

Let us come back to discuss the term $L_v(\dot{y} - \dot{\hat{y}})$ which was added in Eq. (3) to gain more freedoms and make the estimator problem dual to the state feedback problem. Examination of Eq. (13) reveals that when $L_v^T = 0$, $L_d^T = \hat{\Phi}\overline{\Phi}^{-1}$ which is, in general, a complex matrix. It thus contradicts the requirement that the gain matrix L_d^T must be real. Note that $\hat{\Phi}\overline{\Phi}^{-1}$ is function of the closed-loop eigenvalues of the state estimator. Consequently, it is immediately concluded that, with the absence of L_v^T, the solution of the gain matrix L in real domain, in general, does not exist. However, there may exist certain eigenvalues for the state estimator such that $\hat{\Phi}\overline{\Phi}^{-1}$ is real. For example, given a set of desired estimator eigenvalues, the gain matrices, L_v and L_d, are computed from Eq. (16). Using the approach shown in Ref. 10, L_v is omitted from Eq. (6), and the eigenvalues of the system in Eq. (6) may be still in the left-half plane. However, the eigenvalues thus obtained are, in general, different from the desired ones which are originally used to compute the matrices L_v and L_d. However, if the eigenvalues satisfy the performance requirements, it can be used for real time implementation. To this end, it is concluded that, using the second-order dynamic model for design of state estimators, requires an additional expense in the sense that collocated sensors may be needed for measuring the signals and their derivatives.

State Estimators with Measurement Signals and Prefilters

In some cases, collocated sensors for measuring signals and their derivatives are not available. An alternate way is needed to gain the freedom for designing the state estimators using second-order dynamic models. The purpose of adding sensors, as discussed previously, is to measure signal derivatives to obtain signals with different phases so as to gain enough freedom for the state estimator design. There are many other ways to shift signal phases without additional sensors. An alternative is the use of prefilter to approximately estimate the signal derivatives.

Let the state estimation and the output equations be

$$M\ddot{\hat{x}} + D\dot{\hat{x}} + K\hat{x} = Bu + L_d(y - \hat{y}) + L_v(z - \hat{z}) \tag{27}$$

$$\ddot{z} + P\dot{z} + Qz = y \quad (i.e.\ \ H_v\dot{x} + H_d x) \tag{28}$$

and

$$\ddot{\hat{z}} + P\dot{\hat{z}} + Q\hat{z} = \hat{y} \quad (i.e.\ \ H_v\dot{\hat{x}} + H_d\hat{x}) \tag{29}$$

where the vector z of length m contains the filtered signals, $\hat{z}$ the estimated output vector of length m, and L_v the $n \times m$ gain matrix associated with the error between z and $\hat{z}$. Here, the additional term, $L_v(z-\hat{z})$ provides the freedom necessary to design the state estimators for second-order models. The phase differences between the measured signals y and the shifted signals z are determined by the $m \times m$ square matrices P and Q. For simplicity in real time implementation, the matrices P and Q may be chosen to be diagonal such that signals are not coupled in the prefiltering processes. Indeed, in this case, a scalar second-order equation is obtained for each signal which can be easily implemented by an analog computer. Subtracting Eq. (27) from Eq. (1) with the aid of Eqs. (28) and (29) yields the error equation

$$M\ddot{e} + [D + L_d H_v]\dot{e} + [K + L_d H_d]e + L_v e_z = 0 \tag{30}$$

and

$$\ddot{e}_z + P\dot{e}_z + Qe_z - H_v\dot{e} - H_d e = 0 \tag{31}$$

where $e_z = z - \hat{z}$ is the error between the filtered-signals and the estimated filtered-signals. Equations (30) and (31) can be rewritten in the following matrix form

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{pmatrix} \ddot{e} \\ \ddot{e}_z \end{pmatrix} + \begin{bmatrix} D + L_d H_v & 0 \\ -H_v & P \end{bmatrix} \begin{pmatrix} \dot{e} \\ \dot{e}_z \end{pmatrix} + \begin{bmatrix} K + L_d H_d & L_v \\ -H_d & Q \end{bmatrix} \begin{pmatrix} e \\ e_z \end{pmatrix} = 0. \tag{32}$$

The left eigenvectors $[\overline{\phi}_k^T \ \ \tilde{\phi}_k^T]$ $(k = 1, \ldots, n+m)$ and the corresponding eigenvalues λ_k for the system of Eq. (32) are related through the following equation

$$[\bar{\phi}_k^T \;\; \tilde{\phi}_k^T]\left\{\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\lambda_k^2 + \begin{bmatrix} D+L_dH_v & 0 \\ -H_v & P \end{bmatrix}\lambda_k + \begin{bmatrix} K+L_dH_d & L_v \\ -H_d & Q \end{bmatrix}\right\} = 0 \tag{33}$$

where subscript k refers to the eigenvalue number. The transpose of this equation is

$$\left\{\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\lambda_k^2 + \begin{bmatrix} D+H_v^TL_d^T & -H_v^T \\ 0 & P^T \end{bmatrix}\lambda_k + \begin{bmatrix} K+H_d^TL_d^T & -H_d^T \\ L_v^T & Q^T \end{bmatrix}\right\}\begin{bmatrix} \bar{\phi}_k \\ \tilde{\phi}_k \end{bmatrix} = 0. \tag{34}$$

This equation can be decomposed into two parts which are

$$\left\{\lambda_k^2 M + \lambda_k[D + H_v^T L_d^T] + [K + H_d^T L_d^T]\right\}\bar{\phi}_k - [\lambda_k H_v^T + H_d^T]\tilde{\phi}_k = 0 \tag{35a}$$

and

$$L_v^T\bar{\phi}_k + [\lambda_k^2 + \lambda_k P^T + Q^T]\tilde{\phi}_k = 0. \tag{35b}$$

Solving for $\tilde{\phi}_k$ from Eq. (35b) and substituting it into Eq. (35a) yields

$$[(\lambda_k^2 M + \lambda_k D + K^T) \;\; (\lambda_k H_v^T + H_d^T)] \cdot \begin{bmatrix} \bar{\phi}_k \\ \{L_d^T + [\lambda_k^2 + \lambda_k P^T + Q^T]^{-1}L_v^T\}\bar{\phi}_k \end{bmatrix} \equiv \Gamma_k\phi_k = 0. \tag{36}$$

This equation is nearly identical to Eq. (9) in the sense that Γ_k's in both equations are identical, and therefore have the same null space. This simply means that the computational procedure developed previously can also be used in computing the gain matrices L_v and L_d. However, the way to compute L_v and L_d in Eq. (36) is somewhat different from that of Eq. (9).

If the closed-loop eigenvalues λ_k ($k = 1, \ldots, n+m$) and their complex conjugates are assigned, the gain matrices can be determined as follows. As shown in Eq. (12), choose a particular set of vectors ϕ_k ($k = 1, \ldots, n+m$) satisfying Eq. (36) and partition vector ϕ_k into two components, then Eq. (36) implies

$$\{L_d^T + [\lambda_k^2 + \lambda_k P^T + Q^T]^{-1}L_v^T\}\bar{\phi}_k = \hat{\phi}_k \quad ; \quad k = 1, \ldots, n+m. \tag{37}$$

This equation can be decomposed into real part and imaginary parts to solve for the gain matrices L_v and L_d (Both matrices are real), containing $2m \times n$ unknown elements. However, the computational procedure is not as straightforward as the previous case, Eq. (13). Because P and Q are design parameters for the prefilter equations, they may be chosen to be diagonal for simplicity in real time implementation as well as computation of L_v and L_d in Eq. (37).

First-Order State Estimator Models

There are two reasons why first-order state estimators for flexible structures are presented in this section. First, a computational procedure which is different from conventional ones is developed here. Second, a comparison between the first-order and the second-order state estimators is given for better understanding of the characteristics and merits of both approaches.

Equation (1) can be rewritten in a first-order form,

$$\begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{bmatrix} 0 & I \\ -K & -D \end{bmatrix}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix}u. \tag{38}$$

Correspondingly, the state estimation equation becomes

$$\begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}\begin{pmatrix} \dot{\hat{x}} \\ \ddot{\hat{x}} \end{pmatrix} = \begin{bmatrix} 0 & I \\ -K & -D \end{bmatrix}\begin{pmatrix} \hat{x} \\ \dot{\hat{x}} \end{pmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix}u + \begin{bmatrix} L_v \\ L_d \end{bmatrix}[H_d \;\; H_v]\begin{pmatrix} x - \hat{x} \\ \dot{x} - \dot{\hat{x}} \end{pmatrix}. \tag{39}$$

Note that the gain matrices L_v and L_d used here are somewhat different from those shown in the above section (Eq. (3)), even though they look similar. To determine the matrices L_v and L_d, the state estimation error is defined by

$$e^T = [(x - \hat{x})^T \;\; (\dot{x} - \dot{\hat{x}})^T]^T. \tag{40}$$

Observe that the estimation error defined in Eq. (5) for a second-order model is different from the one defined above, Eq. (40). There is no estimation error for velocity terms involved in Eq. (5). Instead, the estimated velocity errors are incorporated in the state estimation equation, Eq. (3). Subtracting Eq. (39) from Eq. (38) yields the error equation

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\dot{e} + \begin{bmatrix} 0 & -I \\ K & D \end{bmatrix}e + \begin{bmatrix} L_v \\ L_d \end{bmatrix}[H_d \;\; H_v]\,e = 0. \tag{41}$$

The left eigenvectors $\{\phi_{kv}^T \;\; \phi_{kd}^T\}$ ($k = 1, \ldots, 2n$) and the corresponding eigenvalues λ_k for the system given by Eq. (41) are written as

$$[\phi_{kv}^T \;\; \phi_{kd}^T]\left\{\begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}\lambda_k + \begin{bmatrix} 0 & -I \\ K & D \end{bmatrix} + \begin{bmatrix} L_v \\ L_d \end{bmatrix}[H_d \;\; H_v]\right\} = 0. \tag{42}$$

where subscript k refers to the eigenvalue number. The transpose of this equation is

$$\left\{\begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}\lambda_k + \begin{bmatrix} 0 & K \\ -I & D \end{bmatrix} + \begin{bmatrix} H_d^T \\ H_v^T \end{bmatrix}\begin{bmatrix} L_v^T & L_d^T \end{bmatrix}\right\}\begin{bmatrix} \phi_{kv} \\ \phi_{kd} \end{bmatrix} = 0. \tag{43}$$

This equation can be decomposed into two parts which are

$$\lambda_k\phi_{kv} + K\phi_{kd} + H_d^T\left[L_v^T\phi_{kv} + L_d^T\phi_{kd}\right] = 0 \tag{44a}$$

and

$$\lambda_k M\phi_{kd} - \phi_{kv} + D\phi_{kd} + H_v^T\left[L_v^T\phi_{kv} + L_d^T\phi_{kd}\right] = 0. \tag{44b}$$

Premultiplying Eq. (44b) by λ_k and adding the resulting equation to Eq. (44a) yields

$$\left[\lambda_k^2 M + \lambda_k D + K\right]\phi_{kd} + \left[\lambda_k H_v^T + H_d^T\right]\left[L_v^T\phi_{kv} + L_d^T\phi_{kd}\right] = 0 \tag{45}$$

or in a compact matrix form

$$\left[(\lambda_k^2 M + \lambda_k D + K) \quad (\lambda_k H_v^T + H_d^T)\right] \cdot \begin{bmatrix} \phi_{kd} \\ (L_v^T\phi_{kv} + L_d^T\phi_{kd}) \end{bmatrix} \equiv \Gamma_k\phi_k = 0. \tag{46}$$

This equation is nearly identical to Eq. (9) in the sense that Γ_k's in both equations are identical, and therefore have the same null space. This is a significant result which simply indicates that the computational procedure developed above for the second-order model can also be used in computing the gain matrices L_v and L_d for the first-order model. The matrices L_v and L_d can be determined when all the null spaces of the matrix Γ_k corresponding to the eigenvalues λ_k are computed. However, the way to compute L_v and L_d in Eq. (46) is somewhat different from those in Eq. (9).

If the closed-loop eigenvalues λ_k $(k = 1, \ldots, n)$ are assigned including their complex conjugates, the gain matrices L_v and L_d can be determined as follows. Following Eq. (12), choose a particular set of vectors ϕ_k $(k = 1, \ldots, n)$ satisfying Eq. (46) and partition the vector ϕ_k into two components, $\phi_k^T = \left[\phi_{kd}^T \;\; \hat{\phi}_k^T\right]$. Equation (46) implies

$$L_v^T\phi_{kv} + L_d^T\phi_{kd} = \hat{\phi}_k \quad ; \quad k = 1, \ldots, n \tag{47}$$

where ϕ_{kv} can be solved from Eq. (44)

$$\phi_{kv} = -(K\phi_{kd} + H_d^T\hat{\phi}_k)/\lambda_k \quad ; \quad k = 1, \ldots, n. \tag{48}$$

Both Eqs. (47) and (48) are $n \times n$ equations compared to a $2n \times 2n$ equations solved for a typical first-order model such as Eq. (43). In other words, there are $2n \times n$ less equations to solve for the matrices L_v and L_d for each eigenvalue using Eqs.(47) and (48). Therefore, a total of $2n \times n \times n$ equations are computationally saved for n assigned eigenvalues. Now let $\overline{\Phi} = [\phi_{1d}, \ldots, \phi_{nd}]$, $\tilde{\Phi} = [\phi_{1v}, \ldots, \phi_{nv}]$ and $\hat{\Phi} = [\hat{\phi}_1, \ldots, \hat{\phi}_n]$. Equation (47) gives

$$L_v^T\tilde{\Phi}_r + L_d^T\overline{\Phi}_r = \hat{\Phi}_r \tag{49a}$$

$$L_v^T\tilde{\Phi}_i + L_d^T\overline{\Phi}_i = \hat{\Phi}_i \tag{49b}$$

or in matrix form

$$\begin{bmatrix} L_v^T & L_d^T \end{bmatrix}\begin{bmatrix} \tilde{\Phi}_i & \tilde{\Phi}_r \\ \overline{\Phi}_i & \overline{\Phi}_r \end{bmatrix} = \begin{bmatrix} \hat{\Phi}_i & \hat{\Phi}_r \end{bmatrix} \tag{50}$$

where the subscript r and i respectively refer to the real and imaginary parts of the associated quantities. The gain matrices L_v and L_d can then be solved by using

$$\begin{bmatrix} L_v^T & L_d^T \end{bmatrix} = \begin{bmatrix} \hat{\Phi}_i & \hat{\Phi}_r \end{bmatrix}\begin{bmatrix} \tilde{\Phi}_i & \tilde{\Phi}_r \\ \overline{\Phi}_i & \overline{\Phi}_r \end{bmatrix}^{-1}. \tag{51}$$

Again, a matrix inversion is required in the computation of the gain matrices L_v and L_d.

Numerical Example

The second-order state estimator is used to control the vibrational motion of the flexible truss structure shown in Fig. 1. The structure generically represents a test article for the NASA's Controls-Structures-Interaction program. It is composed of a L-shaped bus, a reflector, a laser feed, and two suspension cables used to simulate on-orbit conditions. The original finite element model is composed of 350 grid points with six degrees of freedom per grid point, resulting in a 2100 degree-of-freedom. However, for preliminary control designs a reduced order model comprised of the first nine modes of the structure, covering a bandwidth of $0-5Hz$, is used. These modes include four pendulum modes (modes due to suspension effects) and five flexural as well as torsional modes. The actuation of the control forces is provided through six proportional gas thrusters located at various points on the body as illustrated in Figure 1. Six inertial measurement units and six accelerometers provide twelve measurements of linear velocity and accelerations at six locations along the body (almost collocated with the actuators) as indicated in Fig. 2.

The open-loop eigenvalues and the desired closed-loop eigenvalues are summarized in Table 1.

Table 1.

Open-Loop Eigenvalues	Closed-Loop Eigenvalues
$0.0000 + 0.7746i$	$-0.0501 + 1.0000i$
$0.0000 + 0.8019i$	$-0.0601 + 1.2000i$
$0.0000 + 0.8043i$	$-0.0751 + 1.5000i$
$0.0000 + 3.9237i$	$-0.1964 + 3.9237i$
$0.0000 + 13.4800i$	$-0.6748 + 13.4800i$
$0.0000 + 15.2194i$	$-0.7619 + 15.2194i$
$0.0000 + 18.9833i$	$-0.9504 + 18.9833i$
$0.0000 + 19.4861i$	$-0.9755 + 19.4861i$
$0.0000 + 23.4345i$	$-1.1732 + 23.4345i$

The closed-loop eigenvalues are chosen such that the first four pendulum modes are provided with 30% damping and the remaining modes with 5% damping. The closed-loop damped frequencies (imaginary part of the eigenvalues) are larger than the open-loop values for the first three modes, but the same for the remaining modes. The closed-loop eigenvalues are assigned via a constant full state feedback, using the robust second-order assignment technique described in Ref. 12 in conjunction with the first design method described in the section of State Estimators with Measurement Signals and their Derivatives. Here, however, the estimated state is used instead of the actual state in the feedback loop.

Using the closest unitary matrix to the open-loop eigenvector matrix as the choice for matrix H_0, the eigensystem assignment technique results in a well-conditioned closed-loop system with a modal matrix condition number, $c(\Psi)$, of 26.76 which compares quite well with open-loop condition number of 23.43. The forbenius norm of the gain matrix is quite small at 6.89. These results indicate that proposed eigensystem assignment technique is quite effective and can lead to viable closed-loop designs.

The estimator gains are also obtained using the eigensystem assignment technique described in the first design method. The closed-loop eigenvalues of the estimator system are chosen equal to the closed-loop eigenvalues of the actual system given in Table 1 except that the real part of all the eigenvalues are chosen at -0.5 to achieve an acceptable performance for the overall system. The resulting state estimator has a good conditioning of 67.06 and a low norm of the gain matrix at 1.10 which further illustrates the effectiveness of the proposed design procedure to obtain well-conditioned closed-loop systems with small control effort.

To verify the feasibility of the designed state estimator, a numerical simulation was carried out wherein the dynamic behavior of the closed-loop system for an initial disturbance is investigated. The time history for the first pendulum mode and its associated estimation error are presented in Figs. 2(a) and 2(b), respectively, for an intial velocity of 1.0 in all coordinates. Similarly, the time histories for the first flexible mode, and its corresponding estimation error are given in Figs. 2(c) and 2(d). The time histories of the control forces are respectively illustrated in Figs. 3(a)-3(f). The results indicate that the initial disturbance is practically damped out in $10 - 12$ seconds.

Concluding Remarks

Three design methods for a state estimator were presented in this paper. The first two methods were used to build a second-order state estimator model. The third design method was the traditional state estimator design using a first-order model, but the gain matrix was computed through a second-order model. Careful examination of the the third method reveals that the first-order state estimation equation does include a filter equation. Indeed, when a second-order model is converted into a first-order model, an additional first-order equation is generated which is then implicitely used to build a filter equation. Consequently, it can be concluded that as long as a prefilter design is added in the second-order dynamic model, full freedom to design a state estimator is obtained. From the computational point of view, the second-order models are more attractive for use in designing the state estimators, because the dimension of the mathematical models remain unchanged, rather than an increase by a factor of two for the first-order models. Furthermore, the fundamental structure of the mathematical models such as the symmetry and sparsity of the mass, damping and stiffness matrices is maintained. The disadvantages of the state estimation using second-order models include the requirements of additional sensors or prefilters for real time implementation.

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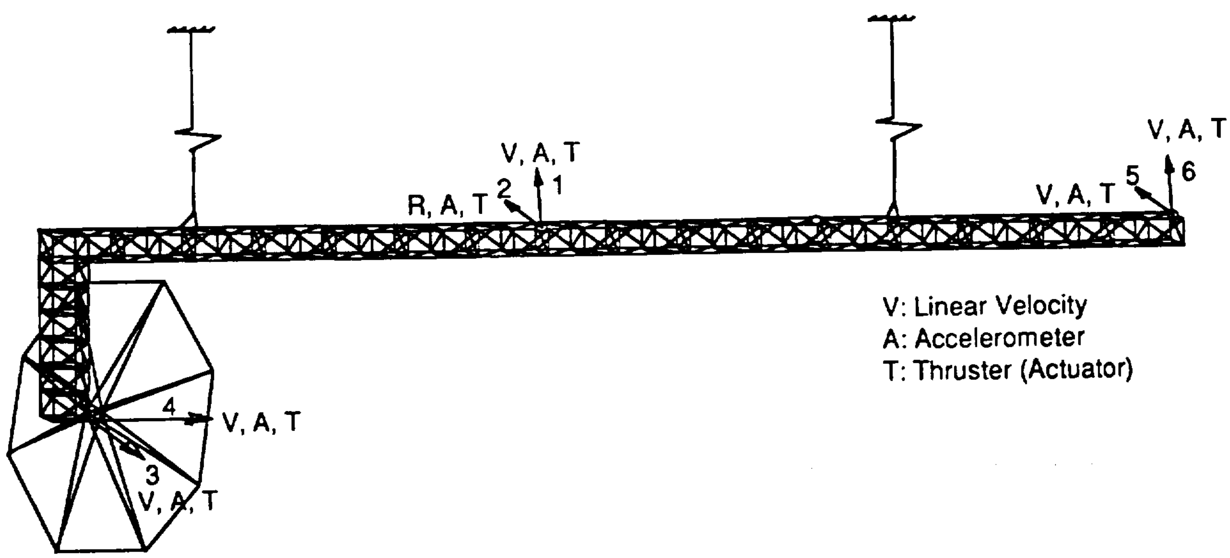
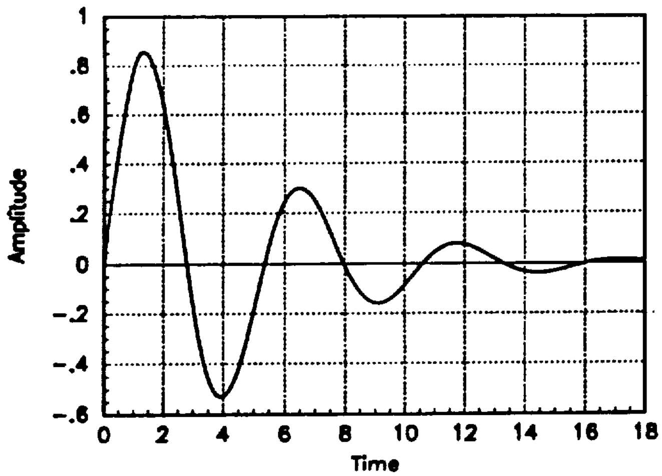
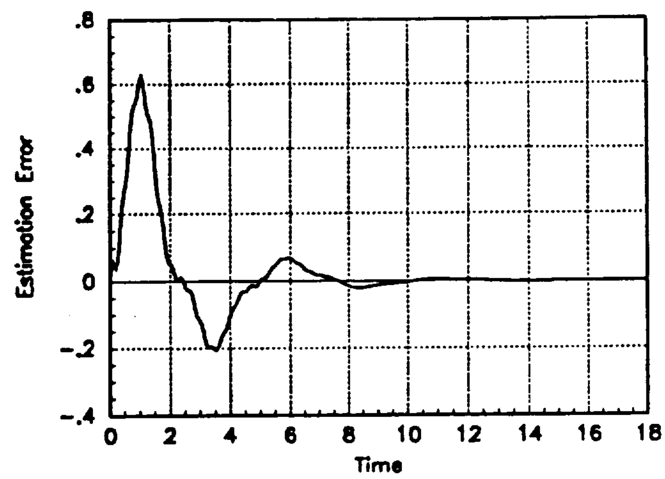


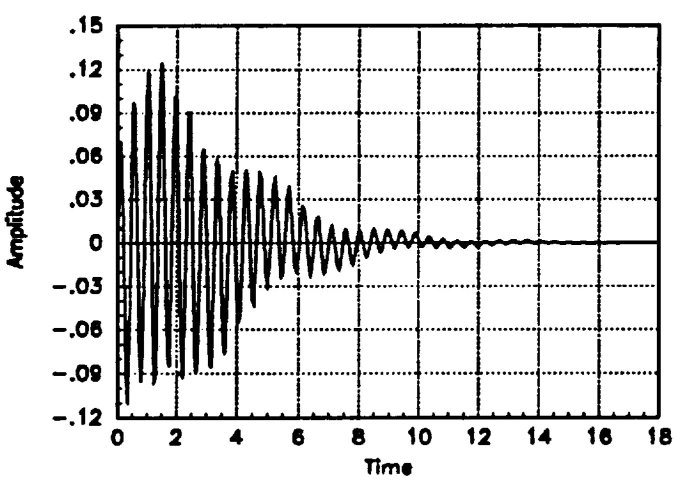
Fig. 1 NASA Controls-Structures Interaction Test Article



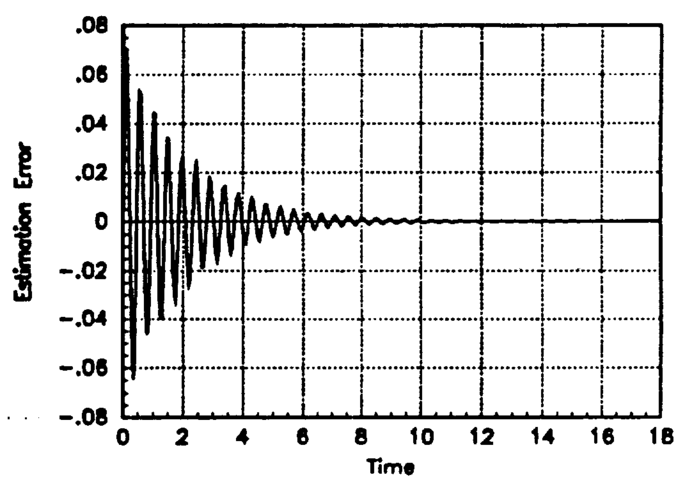
a) Actual Amplitude of the first mode



b) Estimation error of the first mode amplitude

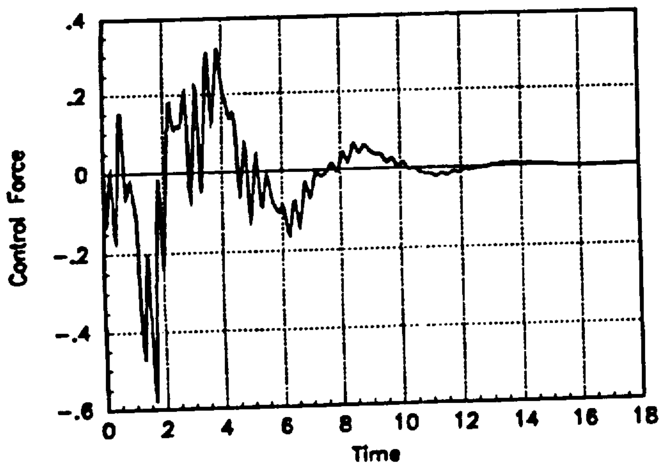


c) Actual Amplitude of the fifth mode

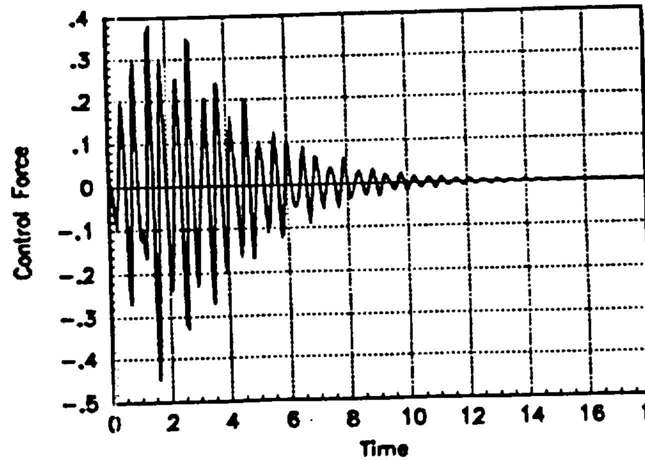


d) Estimation error of the fifth mode amplitude

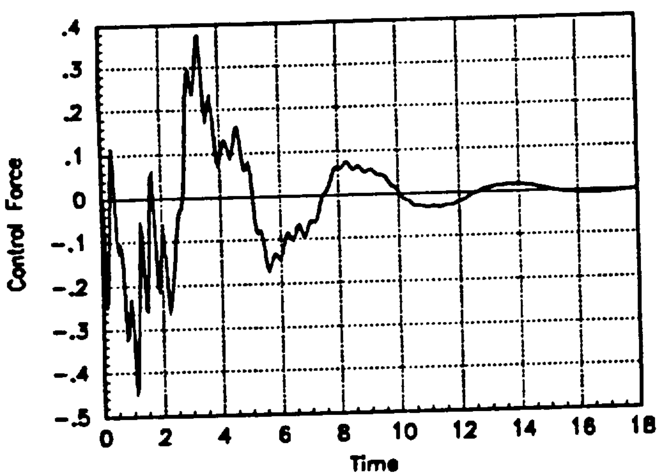
Figure 2 Time histories of modal amplitudes and estimation errors



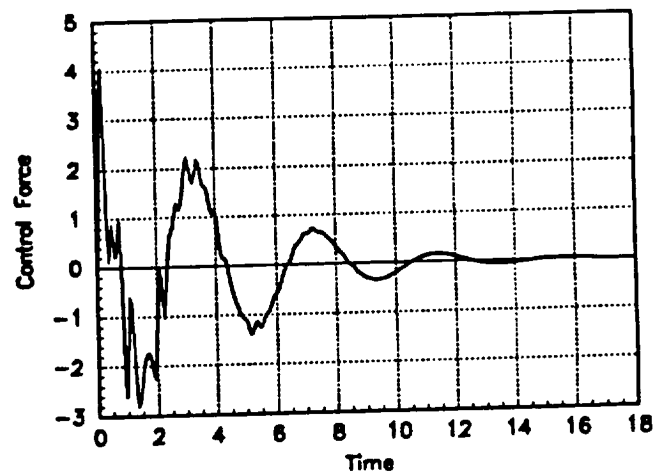
a) Thruster #1



b) Thruster #2



c) Thruster #3



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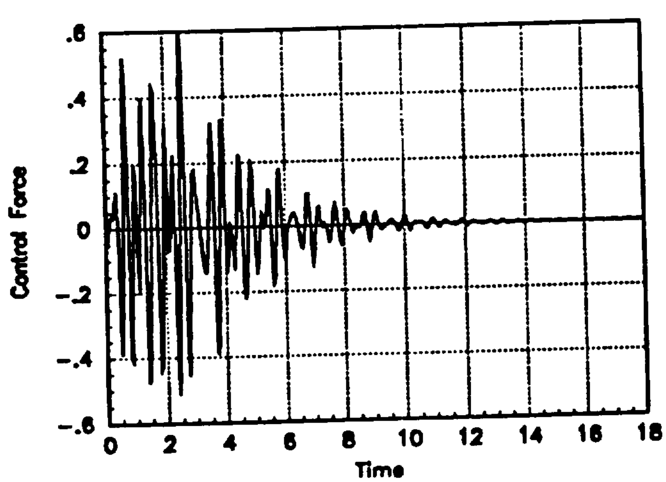
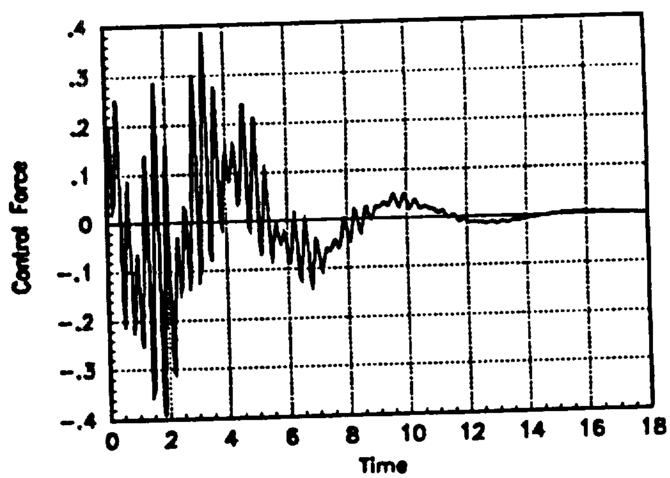


e) Thruster #5



f) Thruster #6

Figure 3 Time histories of control forces

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15. Supplementary Notes

16. Abstract

A novel design of a state estimator is presented using the second-order dynamic equations. The eigenvalues and eigenvectors or the state estimator are assigned, via the second-order eigenvalue problem of the structural system. These design methods for the state estimator are given in this paper. The first design method uses collocated sensors to measure the desired signals and their derivatives. The second design methods used prefilters to shift signal phases to obtain similar effects as the collocated sensors does for signal derivatives. The above two methods build a second-order state estimator model. For comparison, the third design method is the conventional one which converts a typical second-order dynamic model to a first-order model, and then builds a state estimator based on the first-order model. It is shown that all the three designs for state estimation are similar. A numerical example representing a large space structure is given for illustration of the design methods presented in this paper.

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