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RADIATIVE DIFFUSIVITY FACTORS IN CIRRUS AND STRATOCUMULUS CLOUDS — APPLICATION TO TWO-STREAM MODELS

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1. Introduction

A diffusion-like description of radiative transfer in clouds and the free atmosphere is often employed. The two stream model is probably the best known example of such a description.[6,12,9] The main idea behind the approach is that only the first few moments of radiance are needed to describe the radiative field correctly. Integration smooths details of the angular distribution of specific intensity and it is assumed that the closure parameters of the theory (diffusivity factors) are only weakly dependent on the distribution. In this paper we investigate the diffusivity factors using the results obtained from both Stratocumulus and Cirrus phases of FIRE experiment. A new theoretical framework is described in which two (upwards and downwards) diffusivity factors are used and a detailed multi-stream model is used to provide further insight about both the diffusivity factors and their dependence on scattering properties of clouds.

2. Diffusivity factors

There are many diffusion-like approximations in radiative transfer theory.[10] The most intuitive being

$$\mathbf{F} = \hat{\mathbf{D}}\frac{d\mathbf{U}}{d\tau} + \mathbf{S}' \tag{2.1}$$

where flux and scalar irradiance vectors

$$\mathbf{F} = \begin{pmatrix} F^+ \\ F^- \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} U^+ \\ U^- \end{pmatrix} \tag{2.2}$$

are defined by the following hemispheric averages

$$F^+ = \int_{-1}^{0} \mu I(\tau,\mu)d\mu \quad F^- = \int_{0}^{1} \mu I(\tau,\mu)d\mu \tag{2.3a}$$

$$U^+ = \int_{-1}^{0} I(\tau,\mu)d\mu \quad U^- = \int_{0}^{1} I(\tau,\mu)d\mu \tag{2.3b}$$

of specific intensity $I(\tau,\mu)$. Vector $\mathbf{S}'$ is related to the source term. The sign convention is as follows: the instrument located on a airplane and facing the Earth's surface will measure the upward flux of radiation, here indicated by the $+$ sign, the corresponding θ values are between $(\pi/2, \pi)$ and $\mu \in (0,-1)$. The instrument facing skywards will measure downward flux, here indicated by the $-$ sign, the corresponding θ values are between $(0, \pi/2)$ and $\mu \in (1, 0)$.

Equation (2.1) is a direct analog to the Fickian diffusion law where the flux quantity is related to the *gradient* of a scalar quantity (scalar irradiance) through the diffusivity matrix $\hat{\mathbf{D}}$. In oceanography[11] and atmospheric science[2] another diffusivity matrix is often employed

$$\mathbf{U} = \mathbf{D}\mathbf{F} \tag{2.4}$$

where $\mathbf{D}$ is the 2×2 matrix

$$\mathbf{D} = \begin{pmatrix} D^+ & 0 \\ 0 & D^- \end{pmatrix} \tag{2.5}$$

and the diffusivities D^- and D^+ are defined as

$$D^- = \frac{U^-(\tau)}{F^-(\tau)} \quad \text{and} \quad D^+ = \frac{U^+(\tau)}{F^+(\tau)}. \tag{2.6}$$

It can be seen that (2.6) involves the first two moments of angular radiance dependence. These quantities are used to close the set of hierarchy of equations for various moments of radiance. The resulting set of equations is called the two-stream approximation.

3. Measured diffusivities from the CSU bugeye

The Marine Stratocumulus Intensive Field Observations (MSIFO) of the First ISCCP(International

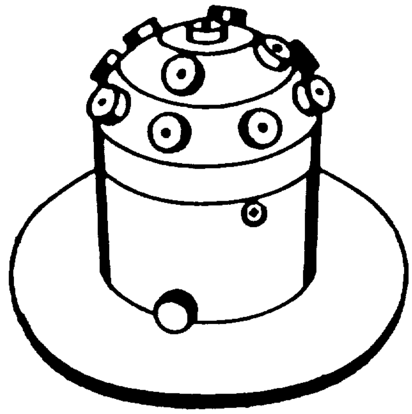

Figure 1: CSU bugeye instrument

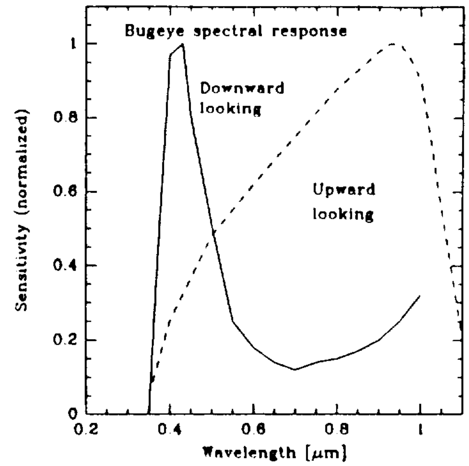

Figure 2: Spectral response of CSU Bugeyes

Satellite Cloud Climate Program) Regional Experiment (FIRE) was conducted off the coast of California at and in the vicinity of San Nicolas Island in July 1987. On San Nicolas Island, a tethered balloon with radiometric instrumentation from Colorado State University (CSU) probed the marine stratocumlus.[7] The shortwave radiometric instrumentation measured both the downwelling and upwelling irradiances, and consisted of four Epply pyranometers (measuring wavelengths from 0.3 to 2.8 μm and from 0.7 to 2.8 μm), and two CSU bugeyes. The bugeye measurements will be used to derive the radiative diffusivities.

The Bugeye, the CSU Multidirectional Photodiode Radiometer[4] measures the angular distribution of the radiance field. It consists of a hemispherical array of thirteen silicon photodiodes and associated electrical circuitry mounted on an aluminum housing (Fig. 1). The upward looking bugeye had diodes with a 10° field of view. Each diode of the downward looking bugeye had a 50° field of view. The spectral range of both bugeyes is from 0.36 to 1.10 μm but the diodes of the downward looking bugeye were covered with a blue tinted Schott glass filters. Figure 2 shows the spectral response of the downward looking bugeye (solid line) and the upward looking bugeye (dashed line). The peak sensitivity of the downward looking bugeye is at 0.40 μm while the upward looking bugeye has a peak sensitivity at about 0.93 μm. The bugeye voltages are actually irradiances seen within each bugeye diode's field of view. These voltages were normalized to the sensitivity of the first diode after being corrected for a zero offset. A field of view correction was then applied to each diode voltage to provide the proper steradian weighting on the hemisphere. A correction for the pitch and roll of the platform was also accounted for. From the 13 diode measurements, the diffusivity factor was deduced as

$$D = \frac{\sum_{i=1}^{13} V_i A_i(\text{hemi})/A(\text{diode})}{\sum_{i=1}^{13} V_i A_i(\text{hemi})/A(\text{diode}) \cos\theta_i} \tag{3.1}$$

where V_i is the voltage of diode i, A(diode) is the steradian field of view of each diode, A_i(hemi) is the steradian area that diode i voltage represents, and θ_i is the zenith/nadir angle (accounting for the pitch and roll of the platform) of diode i. Comparison of (3.1) to (2.6) and (2.3) emphasizes the approximations used to integrate over angle. To test the approximation, the irradiances obtained from summation of the diode measurements (the denominator of (3.1)), were compared to the shortwave irradiances obtained directly from the Epply pyranometer. An excellent relationship (not shown) between these measurements was obtained despite the different spectral characteristics of the bugeyes and pyranometers. The diffusivities for the downwelling radiation (D^-) and for the upwelling radiation (D^+) are shown in Figure 3 for a Sc cloud smapled on the morning of Julian Day 189 (8 July 1987). The diffusivity values average about 1.62 with D^+ being slightly larger than D^-. Cloud base is at about 970 mb and cloud top is at about 930 mb. As the bugeye entered the base of the cloud the value of D^+ rapidly increases until D^+ assumes a in-cloud profile and then decreases through out the cloud. D^- increases with height through the cloud to cloud top where it has has the same value as D^+.

We also performed some preliminary calculations of diffusivities for the downward looking (and only) aircraft bugeye from FIRE Cirrus, Oct. 28, 1986. The results for several passes through two cirrus samples give results of $D^+ \approx 1.7$.

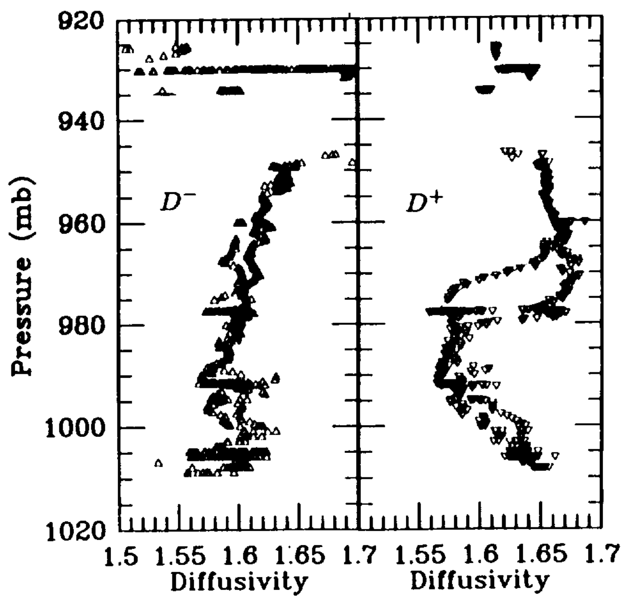

Figure 3: Diffusivity coefficient. Marine stratocumulus. FIRE 8 July 1987

4. Model results

A. Diffusivity factors from the detailed radiative transfer code

To demonstrate further the structure and dependency of diffusivity factors under various conditions, we have employed a comprehensive radiation model[13] which includes scattering and absorption by molecules, as well as by particles. The midlatitude summer[8] atmospheric profile is employed in which a cloud layer of 1 km thick is located with a base 1 km above the surface. The cloud layer is assumed to be homogeneous with a mean liquid water content of $0.2\mathrm{gm}^{-3}$ and a mean effective radius of 10μm. Three spectral bands of $0.52-0.57\mu$m, $1.28-1.53\mu$m, and $2.38-2.91\mu$m are selected, respectively to represent the cases of no, moderate and strong absorption by water droplets. Three solar zenith angles of 0°, 30° and 75° are also chosen to investigate the angular dependence of diffusivity factor. The results of these calculations are shown in Fig. 4. These calculations clearly demonstrate the dependence of D^+ and D^- on absorption strength, solar zenith angle, and optical depth. The diffusion domain in which $D^+(\tau,\mu_0) = D^+$ and $D^-(\tau,\mu_0) = D^-$ is also apparent for large τ. The results also indicate that the diffusivity is higly variable for clouds of $\tau < 1$.

B. Two-stream approximation

The azimuthally and hemisphericaly averaged monochromatic radiative transfer equation for diffuse radiation, $I(\tau,\mu)$, in a plane-parallel, horizontally homogeneous medium which scatters, emits and absorbs can be written as

$$\frac{dF^+(\tau)}{d\tau} = U^+(\tau) - \tilde{\omega}_0 \int_0^1 \beta(\mu) I(\tau,\mu) d\mu - \tilde{\omega}_0 \int_0^1 \varphi(\mu) I(\tau,-\mu) d\mu - S^+(\tau) \quad (4.1)$$

$$\frac{dF^-(\tau)}{d\tau} = -U^-(\tau) + \tilde{\omega}_0 \int_0^1 \beta(\mu) I(\tau,-\mu) d\mu + \tilde{\omega}_0 \int_0^1 \varphi(\mu) I(\tau,\mu) d\mu + S^-(\tau) \quad (4.2)$$

where the backscatter fraction is

$$\beta(\mu) = \frac{1}{2}\int_0^1 P(\mu,-\mu')d\mu' \quad (4.3)$$

and the forward scattering fraction

$$\varphi(\mu) \equiv 1 - \beta(\mu) = \frac{1}{2}\int_0^1 P(\mu',\mu)d\mu' \quad (4.4)$$

Equation (4.1-4.2) doesn't involve any approximations. Unfortunately it is a system of 2 equations and 6 unknowns: F^+, U^+, F^-, U^- and 2 independent integrals involving scattering fraction. To overcome this difficulty all two-stream models introduce some kind of closure hypothesis.[9,1] The particular choice below follows the discussion given in Preisendorfer[11] and Buglia.[1] To close the system of equation we need the relationships between F, U (eq. 2.1 or 2.1, for instance), and the integrals involving backscatter. For this purpose we introduce diffusivities as defined by (2.6) and assume that the back and forward scatter fractions are independent of angle. It follows that

$$\frac{d\mathbf{F}}{d\tau} = \mathbf{AF} + \mathbf{S} \quad (4.5)$$

where $\mathbf{A}$ is the 2×2 matrix

$$\mathbf{A} = \begin{pmatrix} \gamma_1^+ & -\gamma_2^- \\ \gamma_2^+ & -\gamma_1^- \end{pmatrix} \quad (4.6)$$

and

$$\gamma_1^+ = D^+(1-\tilde{\omega}_0\varphi), \quad \gamma_1^- = D^-(1-\tilde{\omega}_0\varphi) \quad (4.7a)$$
$$\gamma_2^+ = D^+\tilde{\omega}_0\beta, \quad \gamma_2^- = D^-\tilde{\omega}_0\beta \quad (4.7b)$$

and

$$\mathbf{F} = \begin{pmatrix} F^+ \\ F^- \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} -S^+ \\ S^- \end{pmatrix} \quad (4.8)$$

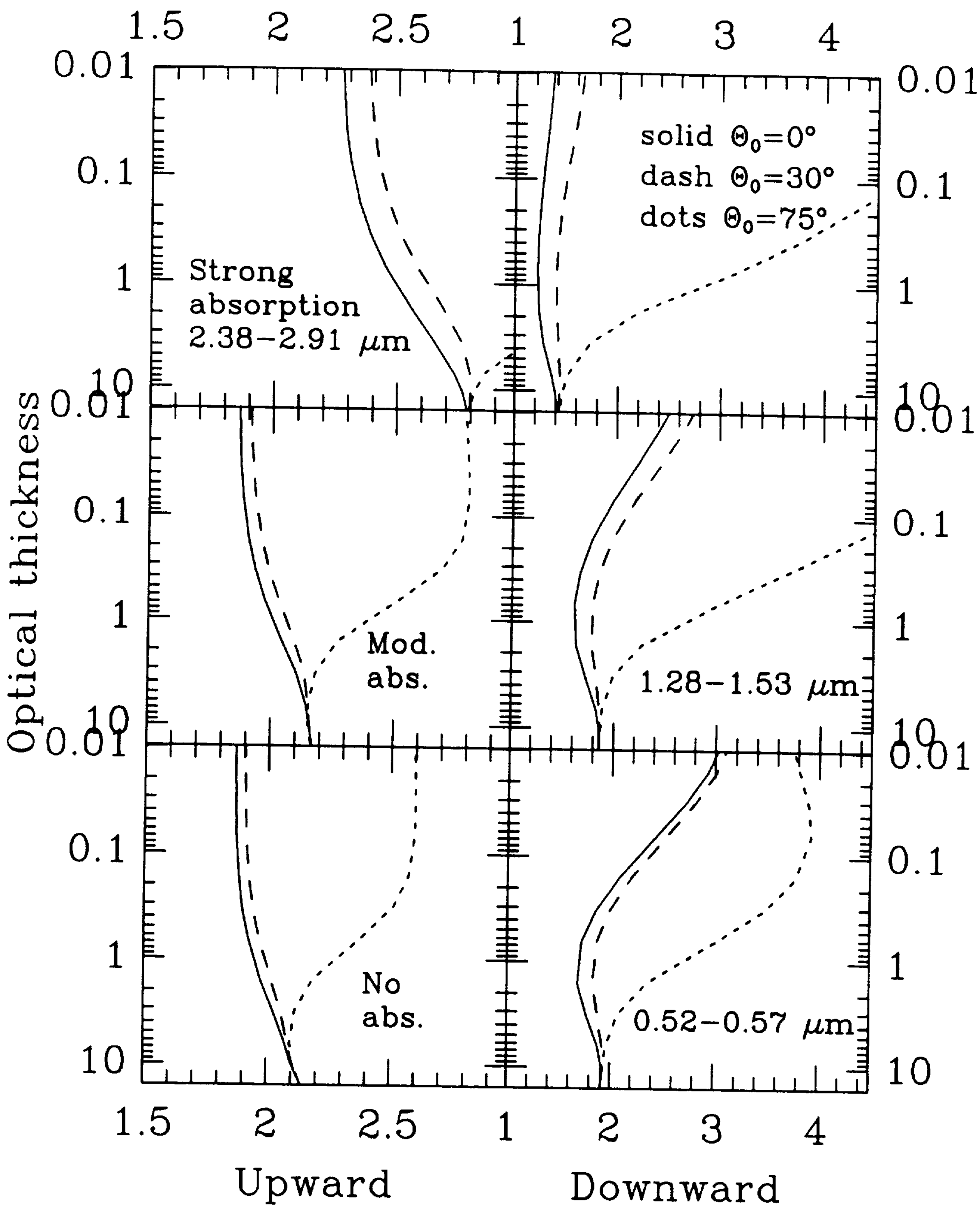

Figure 4: Diffusivity factors from detailed radiative transfer model

Matrix $\mathbf{A}$ depends on local optical properties of the atmosphere (such as the single scattering albedo $\tilde{\omega}_o$ and the asymmetry parameter g). With the typical assumption that $D^+ = D^- = D$, i.e. that the properties of the homogeneous layer are radiatively isotropic, we write scalars γ_1 and γ_2 as

$$\gamma_1 = D\,(1 - \tilde{\omega}_0\varphi) \tag{4.9a}$$

$$\gamma_2 = D\tilde{\omega}_0\beta \tag{4.9b}$$

For the case $D^+ = D^-$, the matrix $\mathbf{A}$ exhibits specific structure and is defined by two scalars. It is this fact which allows us to introduce only two physical parameters defining the homogeneous and isotropic medium: transmittance t and reflectance r. For the anisotropic (but homogeneous) layer, for which $D^+ \neq D^-$, the matrix structure (4.6) is defined by four scalar entries and two transmittances and two reflectances are needed to define the system. The apparent[11] anisotropy of the medium is forced by differences in the distributions of the intensity incident at cloud top and base. The solution of (4.5) may be written in the form[5]

$$\mathbf{F}\,(\tau) = \mathbf{PF}\,(\tau_t) \tag{4.10}$$

where $\mathbf{P}$ is a 2×2 fundamental solution matrix. The propagator for a homogeneous atmosphere, i.e. for constant $\mathbf{A}$, is

$$\mathbf{F}\,(\tau) = e^{\mathbf{A}(\tau-\tau_t)}\mathbf{F}\,(\tau_t) \tag{4.11}$$

for a layer of thickness $\tau - \tau_t$. For the case of a two-stream model and we can represent the propagator as

$$\mathbf{P}_{\text{exp}} = \frac{1}{\lambda_1 - \lambda_2}\left[e^{\lambda_1\tau}\,(\mathbf{A} - \lambda_2\mathbf{1}) - e^{\lambda_2\tau}\,(\mathbf{A} - \lambda_1\mathbf{1})\right] \tag{4.12}$$

where $\mathbf{1}$ is the 2×2 identity matrix. The eigenvalues λ_1 and λ_2 are roots of the characteristic equation

$$\lambda^2 - p\lambda + q = 0 \tag{4.13}$$

and

$$p = \text{tr}\mathbf{A} = \gamma_1^+ - \gamma_1^- \tag{4.14a}$$

$$q = \det\mathbf{A} = \gamma_2^+\gamma_2^- - \gamma_1^+\gamma_1^- \tag{4.14a}$$

It can be shown that the reflectances and transmittances are directly related to the propagator

$$\begin{pmatrix} r^+ & t^+ \\ t^- & r^- \end{pmatrix} = \frac{1}{p_{11}}\begin{pmatrix} -p_{12} & 1 \\ p_{11}p_{22} - p_{21}p_{12} & p_{21} \end{pmatrix} \tag{4.15}$$

where

$$p_{11} = \frac{1}{l}\left[e^{\lambda_1\tau}(\gamma_1^+ - \lambda_2) - e^{\lambda_2\tau}(\gamma_1^+ - \lambda_1)\right] \tag{4.16a}$$

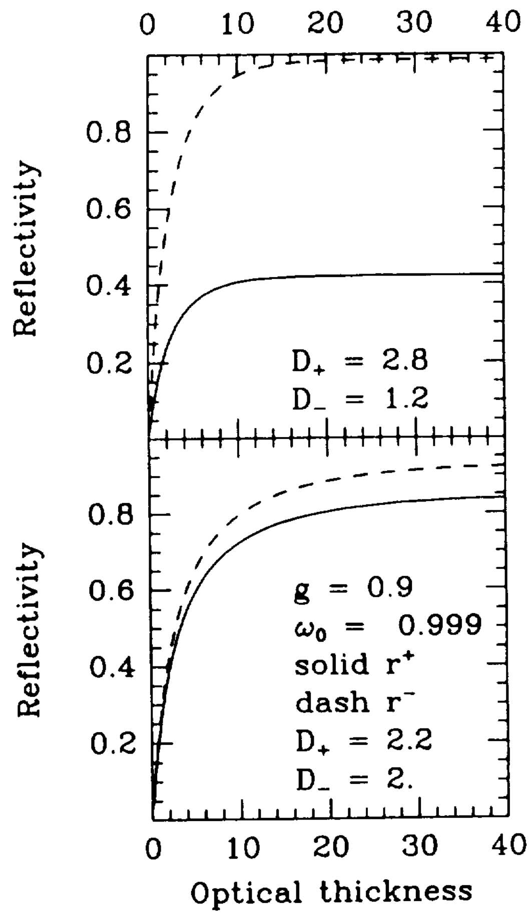

Figure 5: Reflectivities from the four parameter two-stream model

$$p_{12} = -\frac{1}{l}\gamma_2^-\left(e^{\lambda_1\tau} - e^{\lambda_2\tau}\right) \tag{4.16b}$$

$$p_{21} = \frac{1}{l}\gamma_2^+\left(e^{\lambda_1\tau} - e^{\lambda_2\tau}\right) \tag{4.16c}$$

$$p_{22} = -\frac{1}{l}\left[e^{\lambda_1\tau}(\gamma_1^- + \lambda_2) - e^{\lambda_2\tau}(\gamma_1^- + \lambda_1)\right] \tag{4.16d}$$

and $l = \lambda_1 - \lambda_2$. Figure 5 presents results for two sets of diffusivities. First $D^+ = 2.2$, $D^- = 2$ which corresponds to the no absorption case (compare Fig. 4). The second set $D^+ = 2.8$, $D^- = 1.2$ is more typical of the absorption case. The results indicate that the medium becomes anisotropic ($r^+ \neq r^-$) and that reflectances and transmittances of the cloud are sensitive to changes in the diffusivity matrix.

5. Summary

The diffusivity factor has been studied in the context of two-stream approximation of the radiative transfer equation using data obtained from both Stratocumlus and Cirrus phases of FIRE project. Detailed radiative transfer calculations have been employed. The theoretical framework of the two-stream model for homogeneous but anisotropic (anisotropy being forced by the boundary conditions) atmospheric layer is described. It is shown that the total extinction matrix is defined by four components. Preliminary results indicate that the diffusion coefficients depend on the specific spectral region under consideration, absorption strength, optical thickness and solar zenith angles. In the strong absorption case, upward and downward diffusivities separate and the two-stream model predicts large differences in upward and downward reflectances and transmittances for such a case. Model results indicate the existence of a diffuse region in which the diffusion matrix doesn't vary with respect to solar zenith angle and/or increased optical thickness. Experimental data indicate the variability with the sun's zenith angle of the downward diffusivity in the free air overlying the cloud. In the cloud layer the upward diffusivity is larger than the downward diffusivity which is in agreement with the strong absorption case obtained from the numerical model. Downward diffusivity is fairly constant throughout the cloud and below the cloud layer. The upward diffusivity adjusts rapidly to the existence of the cloud layer. Model results indicate that a 'skin' layer in which rapid changes in diffusivity occur is of the order of $\tau = 1$. Thus constant-D models may not be suitable for thin cirrus clouds.

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