N90-28630

COMPUTER CODE FOR THE PREDICTION OF NOZZLE ADMITTANCE

Thong V. Nguyen
Aerojet TechSystems Company
Sacramento, California 95813

ACKNOWLEDGEMENTS

This work was supported by the Air Force Astronautics Laboratory, under contract F04611-85-C-0100, Oxygen/Hydrocarbon Injector Characterization. The AFAL program manager is Capt. M. Walker, the ATC program manager is Dr. M. Young, and the ATC project engineer is Mr. J. Pieper.

I. INTRODUCTION

Aerojet TechSystems Company is currently conducting a program (contract F04611-85-C-0100) to formulate a procedure (Ref. 1) which can accurately characterize injector designs for large thrust (0.5 to 1.5 million pounds), high pressure (500 to 3000 psia) LOX/hydrocarbon engines. In this procedure, a rectangular cross-sectional (hereafter will be refered to simply as rectangular) combustion chamber is to be used to simulate the lower tranverse frequency modes of the large scale chamber. The chamber will be sized so that the first width mode of the rectangular chamber corresponds to the first tangential mode of the full-scale chamber. Test data to be obtained from the rectangular chamber will be used to assess the full scale engine stability. This requires the development of combustion stability models for rectangular chambers.

As part of the combustion stability model development, a computer code, NOAD based on existing theory (Ref. 4) has been developed to calculate the nozzle admittances for both rectangular and axisymmetric nozzles.

---

1. Nozzle Admittance

Combustion instability, characterized by organized pressure oscillations in rocket combustion chamber, can cause severe vibrations on various engine system components and payloads. In addition, combustion instabilities may cause excessive mechanical stresses and heat loads on the injector and combustion chamber walls. These may result in extensive damage to both the engine and to the payloads.

Combustion instabilities have been generally classified according to their frequency range: low, intermediate, and high frequency. Significant efforts have been devoted to the understanding of high frequency instability because it is the most common in new engine developments and is the most destructive. Analytical models capable of characterizing this undesirable phenomenon are obviously useful and valuable to engine designers during the development stage. High frequency instability results from the coupling between the combustion process and the acoustic waves in the chamber, which, in general consists of a straight section and a converging section. The converging section is also considered as part of the nozzle, hence the beginning of the converging section is usually called the nozzle entrance. The analysis can be greatly simplified if the acoustic wave equations are solved separately for each of the sections and the solutions are to be matched at the interface of the two sections (at the nozzle entrance). For instance, wave equations are solved for acoustic properties in the converging section; the properties are then evaluated at the nozzle entrance and used as the boundary condition to solve the wave equations in the straight section. The nozzle admittance which is defined as the ratio of the acoustic axial velocity to the local acoustic pressure at the nozzle entrance provides a boundary condition for solving the wave equations in the straight section of the chamber.

References 5 and 7 discussed another valuable aspect of the nozzle admittance, that the relative stabilizing effects of different nozzle designs can be determined from the computed nozzle admittance.

2. Previous works

2.1 Experimental study

A literature search indicates that existing experimental data on nozzle admittances are only for axisymmetric nozzles.

In reference 10, Crocco et al. performed a series of experiments to determine nozzle admittances by directly measuring the velocity and pressure oscillations at the nozzle entrance. A hotwire anemometer and a diaphragm pressure transducer were installed at the nozzle entrance to measure the velocity and the pressure. A technique was devised to make corrections to the real part of the measured nozzle admittance because this component results from the part of the velocity oscillations that are in phase with pressure and temperature oscillations, both of which directly affect the measurements of the velocities. The use of a hotwire anemometer to measure velocity oscillations also limits the accuracy of the data due to the high turbulent noise to signal ratio.

Using the impedance tube method, Zinn et al. (Ref. 3) experimentally determined nozzle admittances for a number of nozzle geometries. This method has the advantage over the direct measurement approach because velocity oscillations do not have to be measured. In this method, the nozzle admittance is shown analytically to be related to a standing wave pattern in the impedance tube and the wave pattern is determined from the pressure amplitudes measured at several axial locations along the tube.

2.2 Theoretical background

The first analytical study of small amplitude wave propagation in axisymmetric nozzles was made by Tsien (Ref. 6). In this work, the perturbations are assumed to be one-dimensional, isothermal; and the mean axial velocity is assumed to vary linearly with the axial distance from the nozzle entrance. Tsien then derives expressions for the transfer function of the rocket nozzle for either very low or very high frequency range. The transfer function of the rocket nozzle is defined as the sum of the ratios of perturbation density and perturbation axial velocity to perturbation pressure at the nozzle entrance. The perturbations are non-dimensionalized appropriately by their local means.

The original study by Tsien was extended by Crocco (Ref. 8) to include non-isothermal case, non-linear velocity distribution and all frequency ranges. The one-dimensional perturbations assumption is however still retained in this study.

In reference 4, Culick developed a more generalized model in which the perturbations no longer have to be assumed one-dimensional. The mean axial velocity is assumed to vary only in the axial direction and is assumed to follow the equation applicable to one-dimensional, compressible, isentropic flows. Although only the axisymmetric case is discussed in detail in the reference, the theory can be simply extended to rectangular nozzles.

Following Culick's theory, Waugh developed a computer model at Aerojet TechSystems Company to calculate the nozzle admittances of rectangular nozzles.

In reference 5, Crocco and Sirigano derived equations to describe the oscillations in a nozzle. Both axisymmetric and rectangular nozzles are considered. Calculations of nozzle admittance of a specific axisymmetric nozzle geometry are made over a wide range of frequency and entrance Mach numbers, and the results are tabulated. A scaling technique was developed to calculate, using the tabulated results, the nozzle admittances of nozzles having different sizes but similar shapes.

Bell and Zinn (Ref. 2) extended the above theory to include wave-amplitude attenuations. A computer code was developed to determine the nozzle admittances of axisymmetric nozzles. Calculations are made and compared with experimental data. Comparisons show good agreement between the prediction and the measurement.

3. Objective of the present work

While the theories developed in all of the previous works readily apply or can be simply extended to apply to rectangular nozzles, only one computer code has been developed by Waugh to calculate nozzle admittances of rectangular nozzles. Unfortunately, the listing of the computer code is incompleted.

The objective of the present work is to formulate a computer code to calculate the nozzle admittances of not only rectangular but also axisymmetric nozzles. The code was to be formulated such that:

1. It predictes accurately the nozzle admittances over a wide range of frequency and nozzle contours and for any resonance modes.

2. It accounts for the variations of wave amplitudes with time.

3. It generates a file which interfaces directly with the combustion stability computer codes, or it can be easily converted to be used as a subroutine in other combustion stability computer codes.

4. It requires minimum computer time and storage.

II. THEORY

The theory used in the present work follows closely reference 4. First, the continuity and momentum equations are written for an ideal gas. Then, all dependent variables, e.g. pressure, density, velocity, etc., are decomposed into their mean and fluctuating components. The mean components do not vary with time and are assumed to vary only in the axial direction. The fluctuating components, however, vary in all directions and are functions of time. These fluctuating components are assumed to be small so that the products of any two components can be neglected. As a result, equations for the fluctuating components are linear in time, thus their oscillations can be assumed to be sinusoidal. The flow is assumed to be irrotational and the fluctuating velocity components are defined to be the gradients of a velocity potential function. The continuity and momentum equations and an isentropic relation are combined to yield a governing equation for the velocity potential function. The resulting equation is then written for a curvelinear coordinate system which follows the streamline of the mean flow. Using separation of variables technique, the partial differential equation governing the evolution of the potential function is separated into three second-degree ordinary differential equations. For rectangular nozzles, using boundary conditions at the nozzle walls, two of the equations in the tranverse and lateral directions are solved explicitly to give eigenvalues that correspond to tranverse and lateral resonance modes. Similarly for axisymmetric nozzles, boundary conditions at the chamber wall and at the axis of symmetry are used to calculate the eigenvalues that correspond to radial resonance modes. The eigenvalues that correspond to tangential resonance modes are determined by requiring the solutions to the differential equation to be single value functions.

A new integration variable to which nozzle admittance can be related is defined to reduce the differential equation in the axial direction to the first order. This reduction is possible since we are interested in only the ratio of the fluctuating components of the axial velocity and the pressure but not the absolute values of the individual quantities. The resulting first order differential equation, having the same form for both axisymmetric and rectangular cases, is integrated numerically to determine the value of the integration variable at the nozzle entrance. The nozzle admittance is then calculated from an expression that relates the admittance to the value of the integration variable.

No attempts are made to describe the derivations of the equations here since it has been described in full details elsewhere (Ref. 4). A computer code, NOAD was developed using the aforementioned theory.

III. RESULTS AND DISCUSSIONS

Nozzle admittances were calculated using the computer code NOAD over a wide range of frequencies for longitudinal mode and for mixed longitudinal and transverse modes. The calculations were made for rectangular and axisymmetric nozzles having different convergence angles. Calculated results for axisymmetric nozzles are compared with the experimental data of reference 9. In the discussion follows, all calculated results and experimental data are presented in non-dimensional forms. Frequencies are non-dimensionalized by the half height (rectangular nozzles) or by the radius (axisymmetric nozzles) of the nozzles at the entrance, and by the stagnation sound speed. Nozzle admittances are non-dimensionalized by the stagnation sound speed and by the stagnation pressure.

Calculations were made for six different rectangular and axisymmetric nozzles having convergence angles of 15, 30 and 45 degrees. The nozzle contours of the rectangular nozzles and the axisymmetric nozzles having the same convergence angles are similar. The radii (or the half height) at the entrance and at the throat of the nozzles are 5.69 and 2.11 inches, respectively. Using the specific heat ratio of 1.4 for air, the entrance Mach numbers were calculated to be approximately 0.08 for the axisymmetric nozzles, and they were calculated to be approximately 0.221 for the rectangular nozzles. The radii of curvature of the nozzle contours at the entrance and at the throat are 2.5 inches.

The calculated real and imaginary parts of the admittances for longitudinal modes are plotted against the frequency as shown in Figures 1, 2, and 3. The experimental data obtained from reference 9 for axisymmetric nozzles are also shown on the figures. The figures show good agreement between the calculated results and the experimental data. It should be noted that the data were obtained only for axisymmetric nozzles. It can be seen that the dependence of the nozzle admittances on the frequency for rectangular nozzles are trendwise similar to that for axisymmetric nozzles. The peaks and valleys appear more frequent over the frequency range for the rectangular nozzles than for the axisymmetric nozzles.

Calculations are then made for mixed longitudinal and first tangential modes and the results are shown in Figures

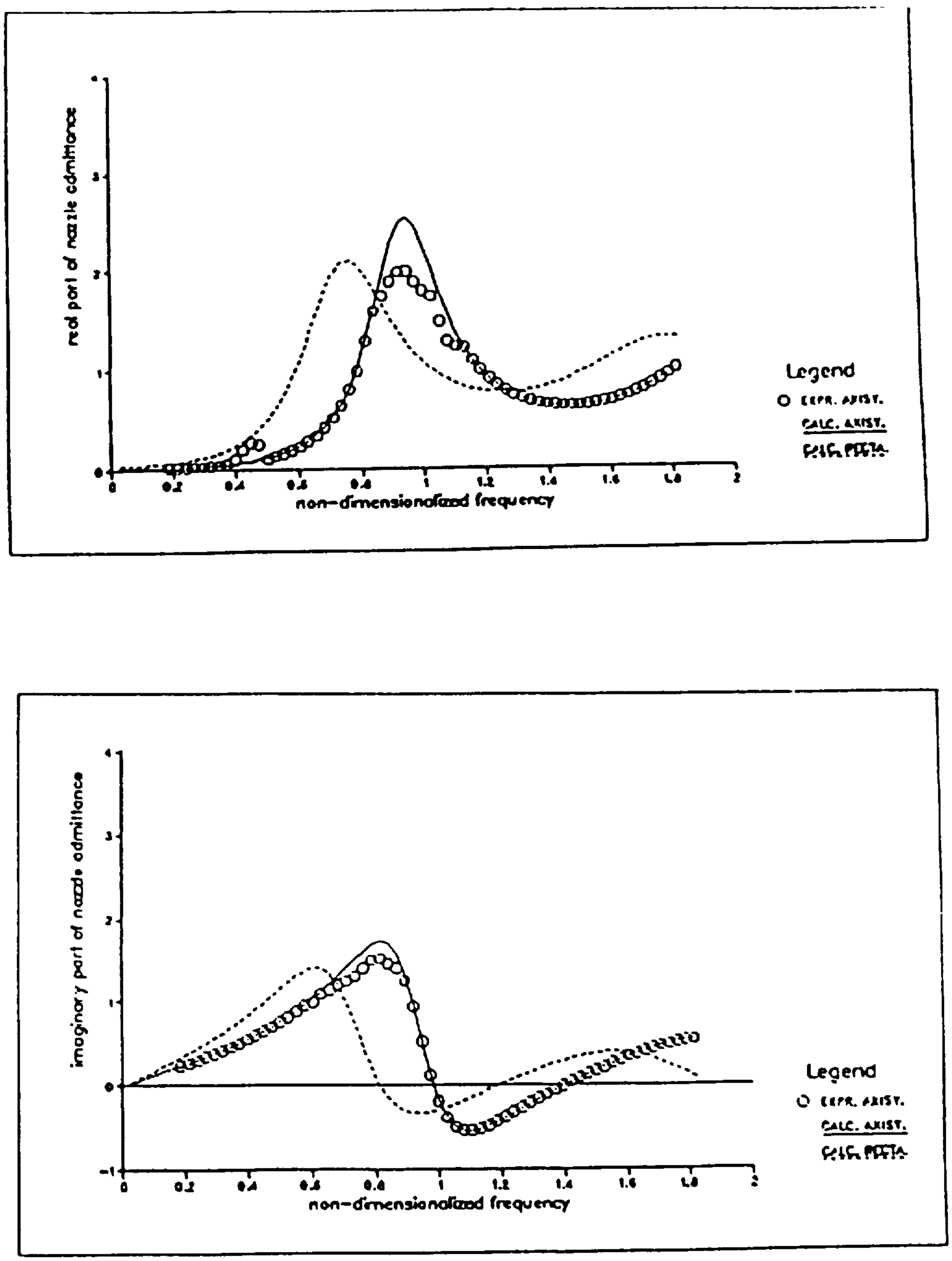
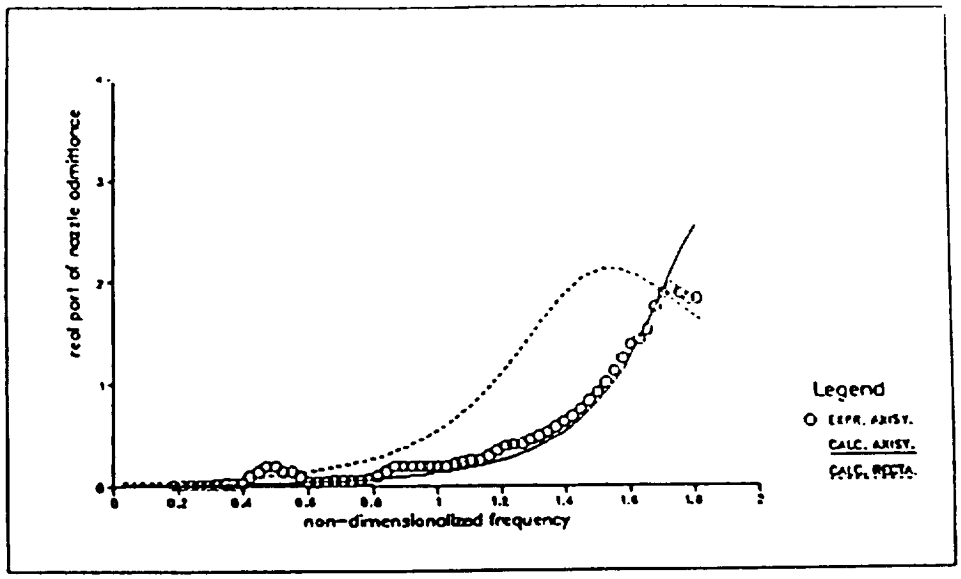


Figure 1: Nozzle admittances for axisymmetric and rectangular nozzles having 15 convergence half angle. Longitudinal modes.



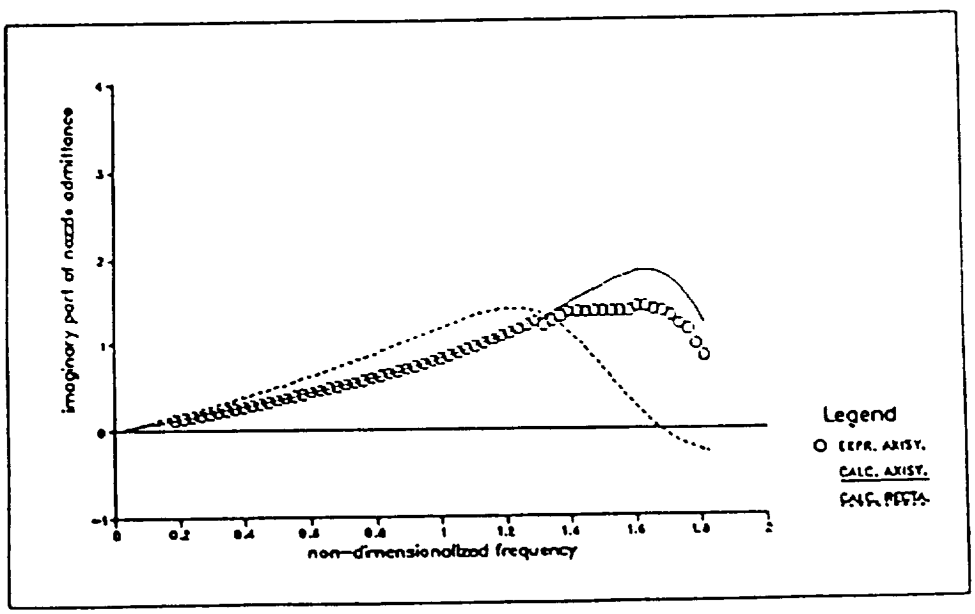
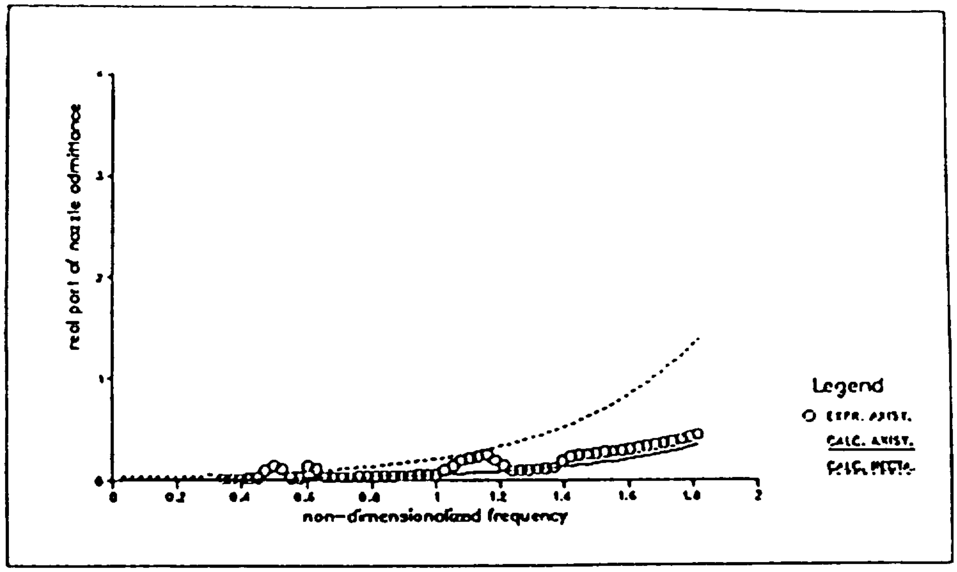


Figure 2: Nozzle admittances for axisymmetric and rectangular nozzles having 30 convergence half angle. Longitudinal modes.



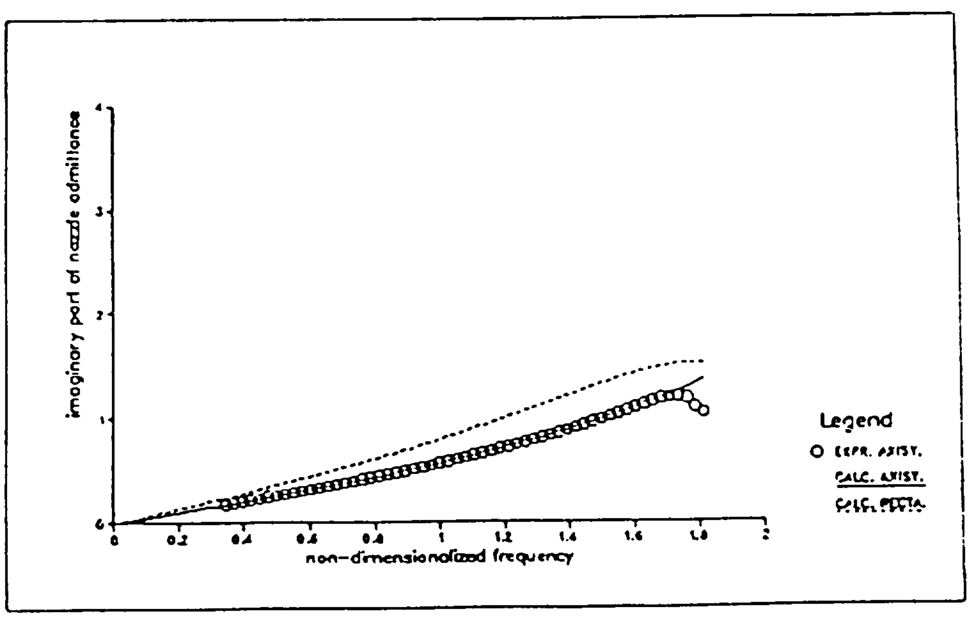


Figure 3: Nozzle admittances for axisymmetric and rectangular nozzles having 45 convergence half angle. Longitudinal modes.

4, 5 and 6. Again, the experimental data obtained from the reference 9 for the axisymmetric nozzles are shown in the figures for comparisons. The agreement between the predictions and the measurements is also good.

IV. SUMMARY AND CONCLUSIONS

A computer code, NOAD was developed to calculate the nozzle admittances of rectangular and axisymmetric nozzles. Several statements can be made with regards to the computer code and the calculated results:

1. The computer code is capable to calculate the nozzle admittances of rectangular nozzles and axisymmetric nozzles.

2. Correlations of calculated results and experimental data with regard to the nozzle admittances of axisymmetric nozzles are quite good. Although there is no experimental data to verify the predictions for rectangular nozzles, it is believed that the model also predicts accurately the nozzle admittances for rectangular nozzles.

3. The dependencies of nozzle admittances on frequency for rectangular nozzles and for axisymmetric nozzles having the same nozzle contours are trendwise similar. The variation trends of the nozzle admittances for rectangular and axisymmetric nozzles are also similar when the acoustic resonance goes from one mode to another.

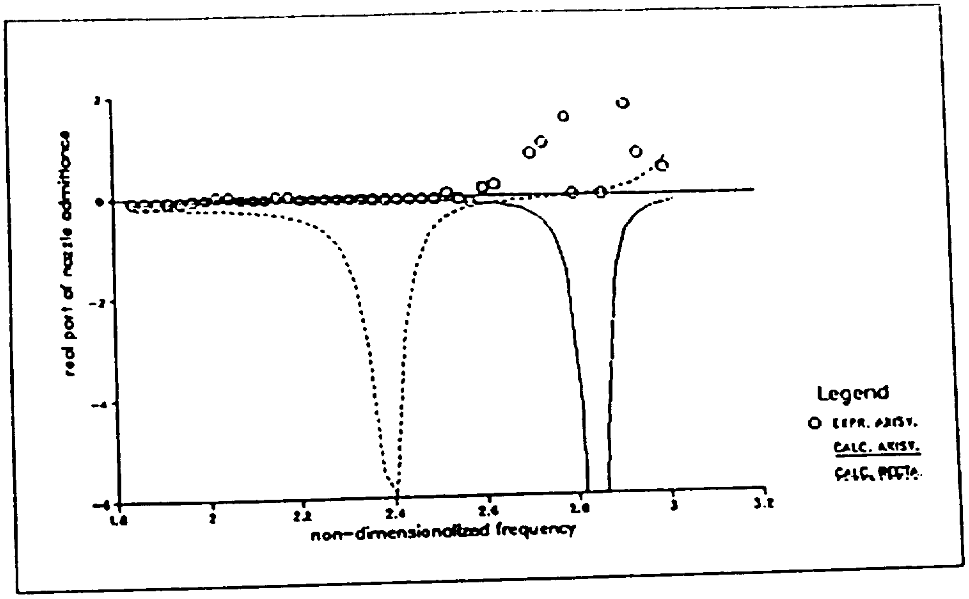
4. A typical run of NOAD for 30 frequency values requires approximately 16 CPU seconds on the micro-VAX at ATC.

V. REFERENCES

1. Oxygen/Hydrocarbon Injector Characterization. Phase I industry briefing by Aerojet TechSystems Comapny under contract F04611-85-C-0100, Air Force Rocket Propulsion Laboratory, February 1986.

2. Bell, W. A., and Zinn, B. T., "The Prediction of Three Dimensional Liquid-Propellant Rocket Nozzle Admittances". NASA CR-121129, February, 1983.

3. Zinn, B. T.. Bell, W. A., Daniel, B. R., and Smith Jr., A. J., "Experimental Determination of Three-Dimensional Liquid Rocket Nozzle Admittances". AIAA Journal, volume 11, March, 1973.



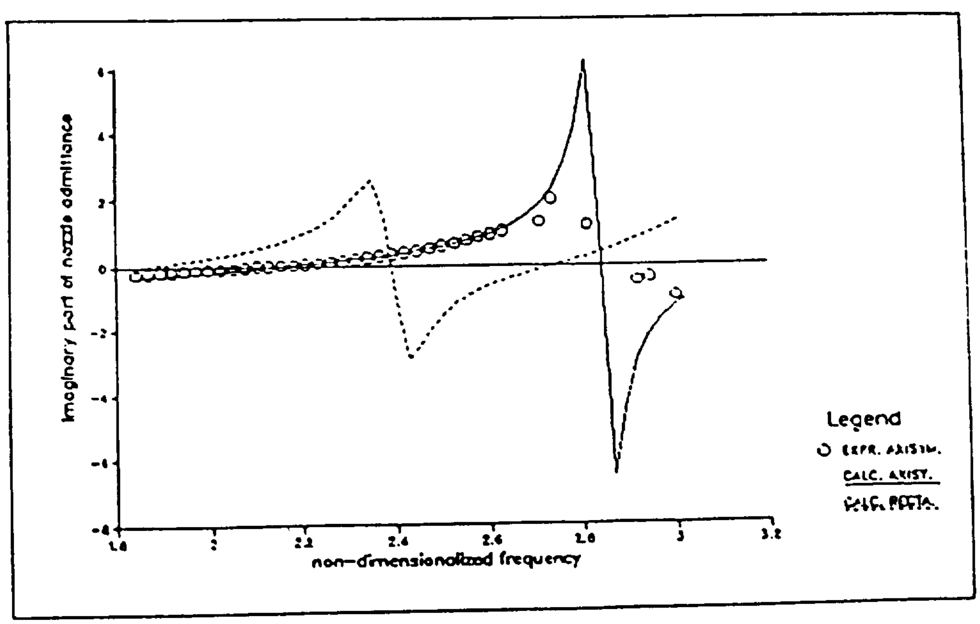


Figure 4: Nozzle admittances for axisymmetric and rectangular nozzles having 15 convergence half angle. Mixed 1T and longitudinal modes.

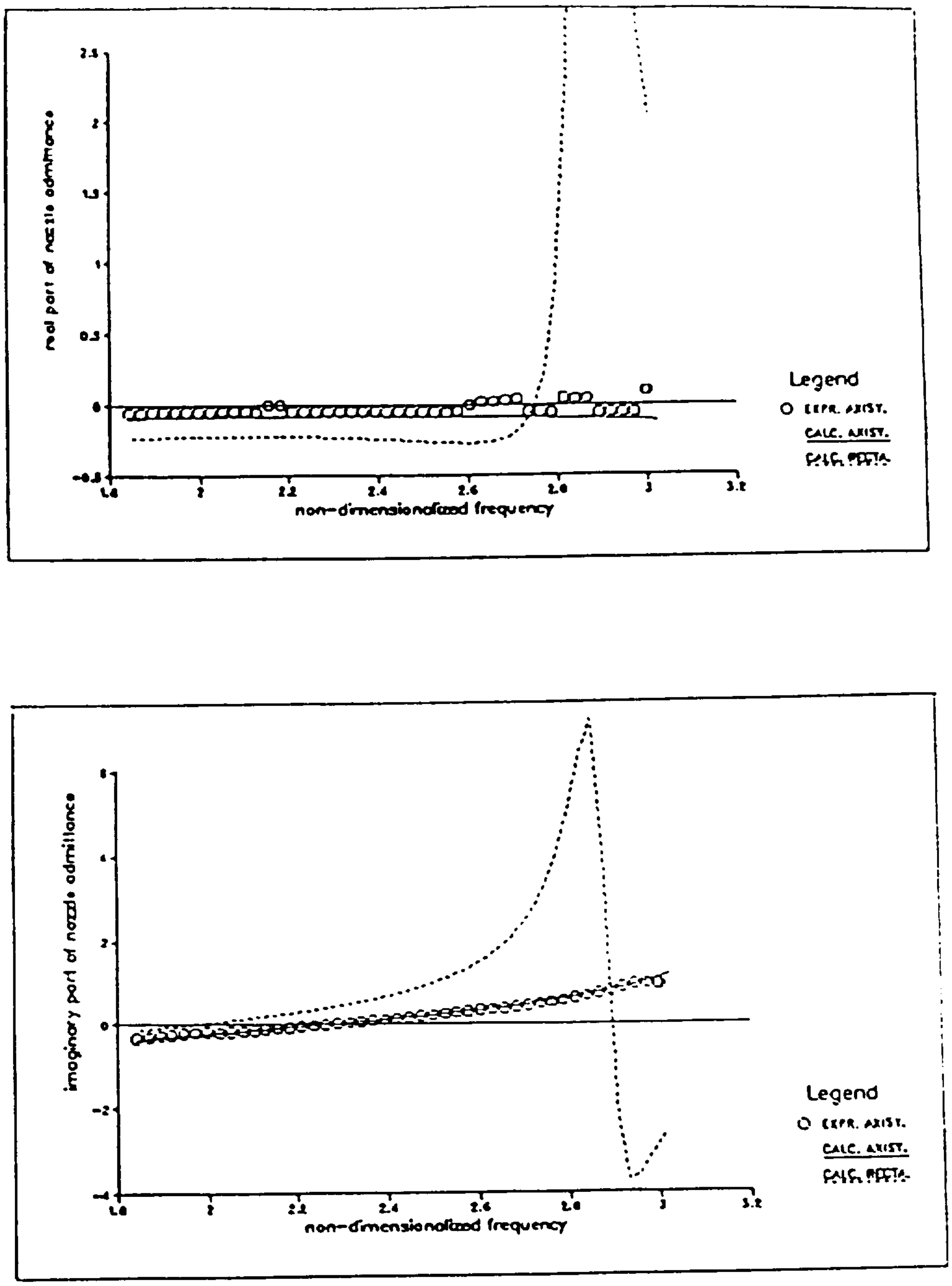


Figure 5: Nozzle admittances for axisymmetric and rectangular nozzles having 30 convergence half angle. Mixed 1T and longitudinal modes.

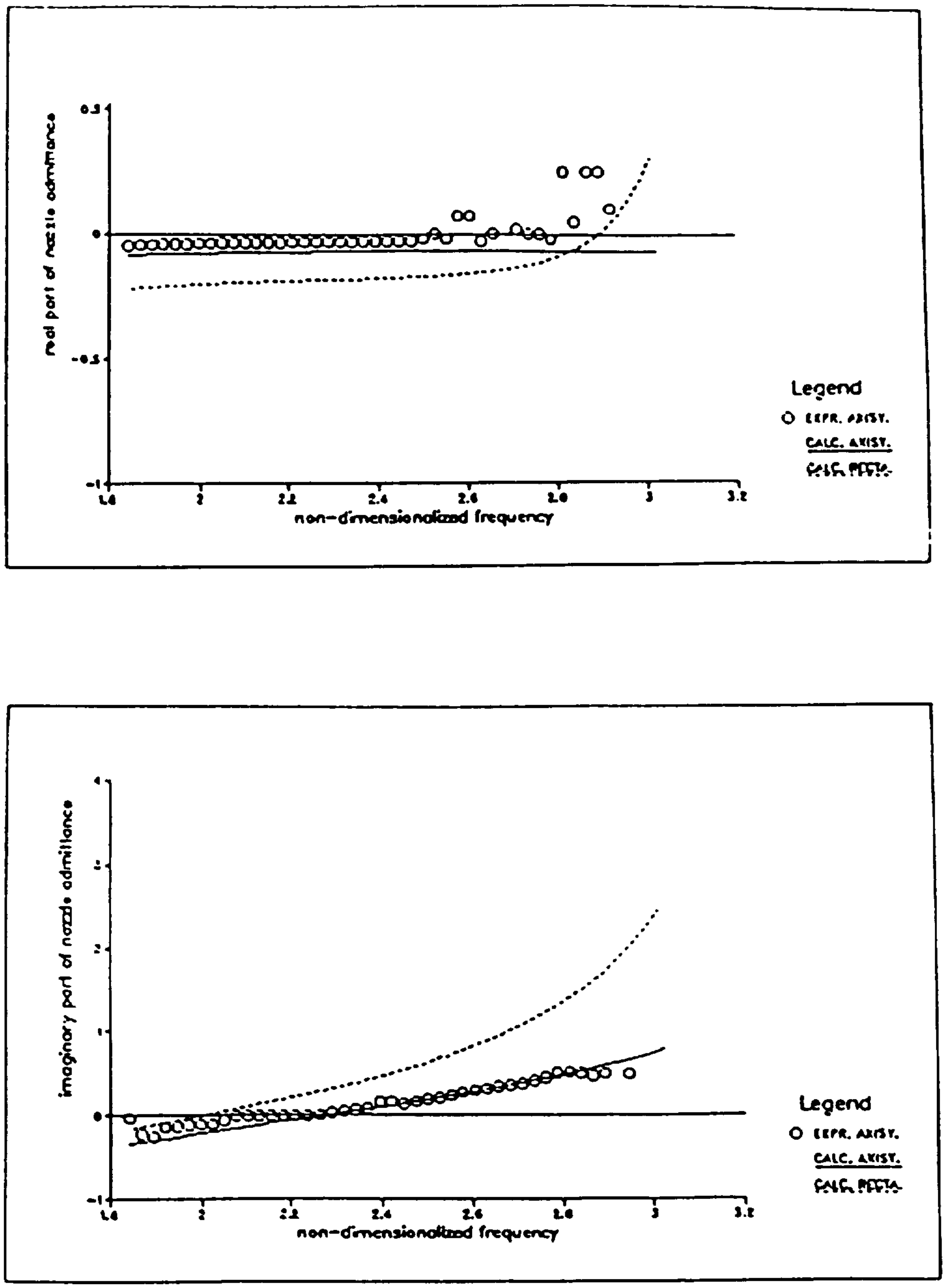


Figure 6: Nozzle admittances for axisymmetric and rectangular nozzles having 45 convergence half angle. Mixed 1T and longitudinal modes.

4. Culick, F. E. C., "Stability of High Frequency Pressure Oscillations in Gas and Liquid Rocket Combustion Chambers". Ph.D thesis, Massachusetts Institute of Technology, June 1961.

5. Crocco, L., and Sirignano, W. A., "Behavior of Supercritical Nozzles under Three Dimensional Oscillatory Conditions". Technical report number 790. Princeton University, Department of Aerospace and Mechanical Sciences, April, 1967.

6. Tsien, H. S., "The Transfer Functions of Rocket Nozzles". ARS journal, volume 22, May-June 1952.

7. Sigman, R. K., and Zinn, B. T., "A Finite Element Approach for Predicting Nozzle Admittances". Journal of Sound and Vibration, 1983.

8. Crocco, L., and Cheng, S., "Theory of Combustion Stability in Liquid Propellant Rocket Motors". Published for The Advisory Group for Aeronautical Research and Development, North Atlantic Treaty Organization by Butterworths Scientific Publications, 1956.

9. Bell, W. A., "Experimental Determination of Three-Dimensional Liquid Rocket Nozzle Admittances", Ph.D Thesis, Georgia Institute of Technology, July 1972.

10. Crocco, L., Monti, R., and Grey, J., "Verification of Nozzle Admittance Theory by Direct Measurement of the Admittance Parameter". ARS Journal, June 1961.