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RELIC GRAVITATIONAL WAVES AND EXTENDED INFLATION

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Abstract. In extended inflation, a new version of inflation where the transition from the false-vacuum phase to a radiation-dominated Universe is accomplished by bubble nucleation and percolation, bubble collisions supply a potent—and potentially detectable—source of gravitational waves. The present energy density in relic gravity waves from bubble collisions is expected to be about 10^{-5} of closure density—many orders of magnitude greater than that of the gravity waves produced by quantum fluctuations. Their characteristic wavelength depends upon the reheating temperature T_{RH}: $\lambda \sim 10^4 \, \text{cm} \, (10^{14} \, \text{GeV}/T_{RH})$. If large numbers of black holes are produced—a not implausible outcome—they will evaporate producing comparable amounts of shorter wavelength waves, $\lambda \sim 10^{-6} \, \text{cm} \, (T_{RH}/10^{14} \, \text{GeV})$.

Inflation provides a means of understanding the smoothness and flatness of the Universe, the origin of the primeval density fluctuations necessary to trigger structure formation, and a very elegant solution to the problem of overproduction of magnetic monopoles in unified gauge theories.[1,2] Testing the "inflationary paradigm" is not a simple matter. Inflation makes but three robust predictions: (i) a flat Universe, i.e., $\Omega_{TOT} = 1.0$, where Ω_{TOT} is the ratio of the total energy density to the critical energy density; (ii) the Harrison-Zel'dovich spectrum of adiabatic density perturbations; and (iii) the presence of a spectrum of relic gravitational waves with wavelengths from about 10^5 cm to 10^{28} cm—and the absence of the 0.9 K thermal background of relic gravitational waves that might otherwise be expected. The first two of these predictions can be confronted with a variety of cosmological observations and experiments, including the comparison of the Hubble age with other independent age determinations, the determination of the type (gaussian or nongaussian) and spectrum of anisotropies in the cosmic microwave background radiation (CMBR), the detailed modeling of structure formation and comparison to the observed distribution of galaxies, and the search for exotic dark matter such as axions or neutralinos. (Such dark matter seems to be required if we demand $\Omega_{TOT} = 1.0$ and zero cosmological constant, since the ordinary baryonic contribution to the cosmic density is constrained by primordial nucleosynthesis to be less than about 0.1.[3])

The third test is the most challenging, but also the most decisive. (Indeed, as an historical matter both flatness and the scale-invariant fluctuation spectrum were proposed before inflation.[4]) One source of relic gravitational waves (the same source that leads to the scale-invariant density fluctuation spectrum) is due to quantum fluctuations that arise in all massless fields during inflation: During inflation the transverse, traceless tensor components of the metric (which correspond to the gravitational degrees of freedom) are excited by de Sitter quantum fluctuations. Later, during the post-inflationary epoch, as a given tensor mode re-enters the horizon its rms amplitude is about $h \simeq 2H/\sqrt{\pi}m_{Pl}$, where H is the value of the Hubble parameter during inflation. Once inside the horizon (i.e., physical wavelength λ less than H^{-1}), the mode can be described appropriately as relic gravitons. The spectrum extends from 3×10^{17} cm $(\mathrm{GeV}/T_{RH}^{1/3}M^{2/3})$ to about 10^{28} cm, the scale of the present Hubble volume. (Here, T_{RH} is the reheat temperature and M^4 is the vacuum energy during inflation). The present energy density per octave in relic gravitons is:[5] (i) $\Omega_{GW}h^2 \simeq (4/3\pi)^2(H/m_{Pl})^2$ on the present Hubble scale, $\lambda \simeq H_0^{-1} \simeq 3000h^{-1}\,\mathrm{Mpc} \simeq 10^{28}h^{-1}$ cm; (ii) $\Omega_{GW}h^2$ decreases as λ^2 for scales between the present Hubble scale and about $13h^{-2}$ Mpc; (iii) $\Omega_{GW}h^2 \simeq 10^{-5}(H/m_{Pl})^2$ is constant on scales between $13h^{-2}$ Mpc and about $10^{-7}(\mathrm{GeV}/T_{RH})$ Mpc; and (iv) $\Omega_{GW}h^2$ again de-

creases as λ^2 down to the smallest wavelengths, about $10^{-7} \, (\text{GeV}/T_{RH}^{1/3} M^{2/3}) \, \text{Mpc}$. In these expressions, $\Omega_{GW} \equiv (\lambda d\rho_{GW}/d\lambda)/\rho_{CRIT}$ is the fraction of critical density contributed per octave and the present Hubble parameter $H_0 = 100h \, \text{km sec}^{-1} \, \text{Mpc}^{-1}$. The spectrum of relic gravitons that arises from quantum fluctuations is shown in Fig. 1.

The gravitational waves just entering the horizon today ($\lambda \sim 10^{28} \, \text{cm}$) lead to a quadrupole anisotropy in the temperature of the CMBR of magnitude comparable to their dimensionless amplitude: $\delta T/T \sim H/m_{Pl}$. The observed isotropy of the CMBR on large angles,[6] $\delta T/T \lesssim 3 \times 10^{-5}$, constrains H/m_{Pl} to be less than about 3×10^{-5}. In turn, this constrains the entire spectrum of relic gravitational waves. In particular, the long plateau region is constrained to contribute at most 10^{-14} of the critical density. The maximal spectrum of gravitational waves is shown in Fig. 1. The intrinsically small amplitude of the spectrum of gravitational waves—which traces to the CMBR isotropy constraint— makes prospects for their detection bleak. Note in this connection that the dimensionless *rms* amplitude h_λ and the energy density per octave are related by, $h_\lambda \simeq \lambda\sqrt{G\rho_\lambda}$, so that short-wavelength fluctuations correspond to smaller absolute metric distortions, for a fixed energy density.

Our main purpose in this *Letter* is to point out that in models of extended inflation[7] there is an additional and probably much more important source of gravitational waves, whose fractional contribution to critical density is generically about 10^{-5}. The origin of these gravitational waves traces to the fundamental difference between slow-rollover inflation and extended inflation, the different mechanism for terminating the transition between inflationary and normal evolution. Whereas in slow-rollover inflation the transition is basically smooth, proceeding through the decay of the inflaton field,[8] in extended inflation the transition occurs through bubble nucleation and percolation. Bubble collisions result in significant production of gravitational waves. The characteristic wavelength of these gravity waves depends upon the reheat temperature: $\lambda \sim 10^4 \, \text{cm} \, (10^{14} \, \text{GeV}/T_{RH})$. It is also possible—and even highly plausible—that mini black holes are produced at the collision sites. As we shall see, these holes are expected to be so small that they evaporate rapidly ($\lesssim 10^{-3} \, \text{sec}$) through the Hawking process. One consequence of such black hole evaporations will be the production of comparable amounts of gravitational radiation, at shorter wavelengths. (The production of primordial black holes and their other consequences have been considered in Ref. 9.)

The reason that reheating by bubble nucleation works in extended inflation is the fact that the nucleation rate per Hubble volume per Hubble time ($\varepsilon \equiv \Gamma/H^4$) varies during inflation: At early times the Universe is hung up in the false vacuum (as in old inflation)

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because ε is much less than unity, while at late times ε becomes greater than unity and bubbles nucleate rapidly, and percolation occurs returning the Universe to a radiation-dominated phase. The rate at which ε changes from being less than unity to being greater than unity determines the spectrum of bubble sizes.[10] In order that there not be too many large bubbles, which ultimately result in large temperature anisotropies in the CMBR, the transition must happen relatively fast;[7] to wit, we will assume that there is a characteristic bubble size $\bar{\lambda}$. Precisely how ε evolves is very model dependent; in the simplest model of extended inflation the nucleation rate (per volume per time) Γ is constant, while the expansion rate H varies because the gravitational constant varies. In other models the variation of the nucleation rate is more important in determining the variation of ε. Here, we will simply assume that there is a characteristic bubble size at the epoch of reheating. Given a specific model, it is a straightforward matter to take into account the spectrum of bubble sizes.

Reheating through bubble collisions is an inherently violent and nonspherical process, and so one expects copious production of gravitational waves. To estimate this production, we characterize the size of the bubbles when they collide as $\bar{\lambda} \equiv fH^{-1}$, where H is the Hubble parameter at the end of inflation and f is expected to be of order unity. Further, since the growth of bubbles is inherently relativistic we assume that the time scale associated with bubble collisions is also $\bar{\lambda}$. The emission of gravitational waves during the collision of a few bubbles is characterized by a luminosity given by $\mathcal{L}_{GW} \sim G(d^3Q/dt^3)^2$, where Q is the quadrupole moment of the energy distribution and G is the gravitational constant. (Since the bubble collision process is relativistic higher multipoles will also be very important; however, the quadrupole formula will serve to give the correct scaling.) It follows that the energy liberated in gravitational waves during the collision process is

$$\Delta E_{GW} \sim \bar{\lambda}\mathcal{L}_{GW} \sim G\frac{M_B^2}{\bar{\lambda}}, \tag{1}$$

where $M_B \simeq \bar{\lambda}^3 M^4$ is the mass-energy of a typical bubble. (As before, M^4 is the false vacuum energy.) From Eq. (1) we estimate that the fraction of the false-vacuum energy that goes into gravitational waves is $\epsilon \sim \Delta E_{GW}/M_B \sim f^2$, and since $f \sim \mathcal{O}(1)$, there is every reason to expect that after reheating a significant fraction of the energy density in the Universe is present in the form of gravitational waves of wavelength $\bar{\lambda}$.

The total energy density in radiation released by bubble collisions is

$$\rho_R \equiv \frac{g_*\pi^2}{30}T_{RH}^4 \simeq M^4, \tag{2}$$

where $T_{RH} \simeq \sqrt{30/\pi^2 g_*}M$ is the reheat temperature and g_* counts the total number of ultrarelativistic degrees of freedom (1 for each internal bosonic degree of freedom and 7/8 for

each fermionic). Although an accurate calculation seems out of reach at present, it is possible to estimate roughly the amplitude h_λ of the gravitational waves. Using the fact that $\epsilon \rho_R = \rho_{GW} \sim G^{-1}(h_\lambda/\bar\lambda)^2$, it follows that $h_\lambda \sim \epsilon$. Assuming for the moment that the gravitational constant does indeed remain constant, then as the Universe expands h_λ evolves as R^{-1} and $\bar\lambda$ increases as R (R is the cosmic scale factor). Further, if we assume that the expansion is adiabatic after reheating, then the entropy per comoving volume, which is proportional to $g_*(T)R^3 T^3$, remains constant. It is a simple matter to relate the value of the scale factor today to that at reheating: $R_0/R_{RH} = [g_*(T_{RH})/g_*(3\,\mathrm{K})]^{1/3}(T_{RH}/3\,\mathrm{K})$. From this it follows that the present amplitude h_λ and wavelength λ of the bubble-produced gravitational waves is:

$$h_\lambda \sim 10^{-26}\left(\frac{10^{14}\mathrm{GeV}}{T_{RH}}\right)\epsilon; \tag{3a}$$

$$\lambda \sim 10^4\,\mathrm{cm}\left(\frac{T_{RH}}{10^{14}\mathrm{GeV}}\right)\epsilon^{1/2}; \tag{3b}$$

where we have taken $g_*(T_{RH})$ to be 300 and $g_*(3\,\mathrm{K}) \simeq 3.4$.

In a similar manner one can use the constancy of the entropy per comoving volume and the fact that ρ_{GW} evolves as R^{-4} to find the ratio of the energy density in gravitational waves to that in photons at any epoch:

$$\frac{\rho_{GW}}{\rho_\gamma} = \epsilon\left(\frac{g_*(T_{RH})}{2}\right)\left(\frac{g_*(T)}{g_*(T_{RH})}\right)^{4/3}. \tag{4}$$

Relic gravitational waves contribute energy density just like any relativistic species; based upon primordial nucleosynthesis we know that any additional relativistic species can contribute no more to the energy density than photons.[3] At the critical epoch of nucleosynthesis, when the neutron-to-proton ratio freezes out ($T \sim \mathrm{MeV}$), $g_* \simeq 10.75$ (for three, light neutrino species), so that we have $\rho_{GW}/\rho_\gamma \simeq 1.8\epsilon \lesssim 1$, which implies that ϵ must be less than about 0.5 (again we use $g_*(T_{RH}) = 300$). Using the fact that the fraction of critical density contributed by photons today is $\Omega_\gamma h^2 \simeq 2.6 \times 10^{-5}$, we can compute that contributed by gravitational waves: $\Omega_{GW}h^2 \simeq 10^{-5}\epsilon$.

Since the metric perturbations at the locus of bubble collisions are of order unity, we may expect the production of large numbers of black holes. It would be difficult to be very quantitative about this hard dynamical problem even if the model parameters were precisely known, which of course they are not. However, a few qualitative and semi-quantitative remarks can be made. Since the bubble walls have energy of order $M_B \sim M^4/H^3 \simeq m_{Pl}^3/M^2$ when they collide, and the problem is basically geometrical, we would expect mini black holes of this mass to be formed.

These black holes will have a Hawking temperature $T_H \sim m_{Pl}^2/M_B \sim M^2/m_{Pl}$, and will evaporate in a time $\tau \sim M_B^3/m_{Pl}^4 \sim m_{Pl}^5/M^6$. If the fraction of the false vacuum energy that is converted into small black holes is greater than about $(M/m_{Pl})^2$, the energy density of small black holes will come to dominate the energy density of the Universe before they evaporate. In this case, the radiation black holes release when they evaporate will overwhelm the radiation released during reheating: The entropy of the Universe and necessarily the baryon number is produced by black hole evaporation. The temperature of the Universe after the mini black holes evaporate and the particles radiated thermalize should be about $T_a \sim M^3/m_{Pl}^2$. (Of course, to ensure that the Universe is radiation-dominated during nucleosynthesis T_a must be greater than about 1 MeV, which restricts M to be greater than about 10^{11} GeV. At somewhat higher values of M there may be effects on the electroweak and quark/hadron phase transitions, which we have not yet investigated.)

The amount of gravitational waves produced in the evaporation process will be comparable to that in photons, from which it follows that the ratio of energy density in gravity waves to that in photons evolves as

$$\frac{\rho_{GW}}{\rho_\gamma} = \left(\frac{g_*(T)}{g_*(T_a)}\right)^{4/3}; \tag{5a}$$

which implies that today $\Omega_{GW}h^2 \simeq 10^{-6}$, comparable to that produced by bubble collisions. (In this case the gravity waves produced by bubble collisions will be greatly diluted by the entropy produced by mini black hole evaporations.) Unlike the other particles radiated as the mini holes evaporate, the gravitons radiated will not thermalize and will have a distribution characterized by the temperature T_H. (However, once radiated into the Universe, they do not correspond to a black body distribution of gravitons at this temperature, because their number density is too small by a factor of $T_a^3/T_H^3 \sim M^3/m_{Pl}^3$.) The present wavelength of these gravity waves is very different than those produced by bubble collisions:

$$\lambda \sim T_H^{-1}\left(\frac{T_a}{3\,K}\right) \simeq 10^{-6}\,\text{cm}\left(\frac{T_{RH}}{10^{14}\,\text{GeV}}\right). \tag{5b}$$

In the case that the mini black holes contribute only a small fraction of the energy density of the Universe when they evaporate (fraction of false vacuum energy converted into black holes less than about $(M/m_{Pl})^2$), the gravitational waves radiated are subdominant to those produced by bubble collisions.

Many of the present models of extended inflation are based upon Jordan-Brans-Dicke-like theories of gravity.[11] In such theories, the gravitational constant is not constant; rather

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its value is set by the value of some scalar field that evolves with time. If the value of the gravitational constant today is different than that at the epoch of reheating, we must re-examine our previous estimates for gravitational-wave production.

To begin, write the gravitational part of the action as:

$$S = - \int d^4x \frac{\mathcal{R}}{16\pi G};$$ (6)

where $\mathcal{R}$ is the curvature scalar and G is the effective gravitational constant. When $\mathcal{R}$ is linearized to extract the graviton degrees of freedom, and the metric is specialized to the Robertson-Walker form, the graviton part of the action becomes

$$S = \int d\eta \ d^3x \frac{R^2}{16\pi G}[(\partial_\eta h)^2 - (\partial_i h)^2];$$ (7)

where η, defined by $d\eta = dt/R$, is conformal time, and for simplicity the indices on the metric perturbation $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ have been suppressed. The graviton wave equation is

$$\frac{\partial}{\partial \eta}\left[\frac{R^2}{G}\frac{\partial}{\partial \eta}h\right] = \frac{\partial}{\partial x}\left[\frac{R^2}{G}\frac{\partial}{\partial x}h\right].$$ (8)

Provided that the variations in R^2/G are slow, one may find approximate solutions by the method of geometrical optics; they take the form $h \sim \alpha(\eta)\exp[i(kx - \omega t)]$, where in the zeroth approximation k and ω are constant and at next order $\alpha^2 \propto G/R^2$. The comoving energy density ($E \propto R^3 \rho_{GW}$),

$$E = \frac{\delta(\sqrt{g}\mathcal{L})}{\delta(\partial h/\partial t)}\frac{\partial h}{\partial t} - \mathcal{L},$$

is now easily evaluated (note that $\mathcal{L} = 0$ at lowest order). One finds that $E \propto 1/R$, as for ordinary radiation, with no G dependence. In spite of the time variation of G, the energy density in gravity waves still decreases as R^{-4}. Moreover, our original estimate of the fraction of the false-vacuum energy that goes into gravitational waves did not depend upon G. Therefore, our previous results for Ω_{GW} and h_λ are unaffected. (The effect of the variation of G on the amplitude of the long wavelength gravitational waves produced as quantum fluctuations is nontrivial and is discussed in Ref. 12.)

Finally, we comment briefly on the detectability of such relic gravitational waves. They are by their nature a stochastic background. While the fraction of critical density they contribute is expected generically to be of order 10^{-5}, their characteristic wavelength depends upon the reheat temperature and whether they were produced by bubble collisions or black hole evaporations: $\lambda \sim 10^4 \, \text{cm}(10^{14} \, \text{GeV}/T_{RH})$ in the first case and

$\lambda \sim 10^{-6}\,\text{cm}(T_{RH}/10^{14}\,\text{GeV})$ in the latter case. The most promising means for detecting such gravitational waves appears to be either the proposed laser interferometric gravitational wave observatory (LIGO) or a beam in space (for a recent review of the means and prospects for detection of gravitational waves see Ref. 13). While a first generation LIGO does not appear to have the necessary sensitivity, an advanced, second generation LIGO or a beam in space look more promising (see Fig. 1).

Our conclusions, necessarily somewhat tentative in the absence of a detailed dynamical simulation, are as follows. In models of extended inflation one expects an additional source of gravitational waves of a characteristic wavelength determined by the reheating temperature and whether the dominant source is bubble collisions or black hole evaporations, that contribute about 10^{-5} of critical density. The prospects for their detection depend crucially upon their wavelength and therefore the reheat temperature. Moreover, if detected, their characteristic wavelength would provide a measure of the reheat temperature and a means of arguing that they were not produced during another cosmological phase transition that proceeded via bubble nucleation, e.g., the electroweak or QCD phase transitions.[14] By way of contrast, the gravitational waves that arise as de Sitter space quantum fluctuations have a spectrum that extends from about $10^5\,\text{cm}$ to about $10^{28}\,\text{cm}$ and a much smaller amplitude. While the wavelengths of these gravitational waves span a very wide range, their small amplitude makes their detection seem remote at present. For some parameters and scenarios, there is danger of producing a large enough density in gravitational waves to spoil the success of cosmic nucleosynthesis calculations or to interfere with the electroweak or quark/hadron transitions. Evidently consideration of possible gravitational radiation from the "popping" of vacuum bubbles at the end of extended inflation gives new theoretical and possibly even observational handles on this spectacular moment in the history of the Universe.

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FIGURE CAPTION

Figure 1: Fraction of critical density in gravitational waves per octave $\Omega_{GW}h^2$ vs. wavelength λ. Shown are the 0.9 K background of gravity waves expected in the standard cosmology, the stochastic background produced by bubble collisions in extended inflation (for $T_{RH} = 3 \times 10^{10}$ GeV), the maximal spectrum of gravity waves that arise as quantum fluctuations in inflation ($M \simeq 10^{16}$ GeV and $T_{RH} \sim 3 \times 10^{10}$ GeV), the limit provided by the large-angle isotropy of the CMBR, and the projected capabilities of some proposed detectors (from Ref. 13).