N90-29797

REDUNDANCY OF SPACE MANIPULATOR ON FREE-FLYING VEHICLE AND ITS NONHOLONOMIC PATH PLANNING

Yoshihiko Nakamura and Ranjan Mukherjee

Mechanical and Environmental Engineering and
Center for Robotic Systems in Microelectronics
University of California, Santa Barbara
California, 93106

ABSTRACT

This paper discusses the nonholonomic mechanical structure of space robots and its path planning. The angular momentum conservation works as a nonholonomic constraint while the linear momentum conservation is a holonomic one. Taking this in to account, a vehicle with a 6 d.o.f. manipulator is described as a 9 variable system with 6 inputs. This fact implies the possibility to control the vehicle orientation as well as the joint variables of the manipulator by actuating the joint variables only if the trajectory is carefully planned, although both of them cannot be controlled independently. It means that assuming feasible-path planning a system that consists of a vehicle and a 6 d.o.f. manipulator can be utilized as 9 d.o.f system. In this paper, first, the nonholonomic mechanical structure of space vehicle/manipulator system is shown. Second, a path planning scheme for nonholonomic systems is proposed using Lyapunov functions.

1. INTRODUCTION

The control of space vehicle/manipulator system possesses inherent issues that have not been considered for on-the-earth robot manipulators, such as the micro gravity, momentum conservation, and preciousness of energy. The kinematics and dynamics of space vehicle/manipulator systems have recently been studied by various researchers.

Alexander and Cannon [1] assumed concurrent use of the thrust force of vehicle and the manipulator joint torque, and proposed a control scheme taking account of the effect of the thrust force in computing the joint torque of manipulator. Dobowsky and Vafa [2] and Vafa [3] proposed a novel concept to simplify the kinematics and dynamics of space vehicle/manipulator system. A *virtual manipulator* is an imaginary manipulator that has similar kinematic and dynamic structure to the real vehicle/manipulator system but fixed at the total center of mass of the system. By solving the motion of the virtual manipulator for the desired motion of endeffector, the motion of vehicle/manipulator system is obtained straightforwardly. On the other hand, Umetani and Yoshida [4] reported a method to continuously control the motion of endeffector without actively controlling the vehicle. The momentum conservations for linear and angular motion are explicitly represented and used as the constraint equations to eliminate dependent variables and obtain the generalized Jacobian matrix that relates the joint motion and the endeffector motion. Longman, Lindberg, and Zadd [5] also discussed the coupling of manipulator motion and vehicle motion. Miyazaki, Masutani, and Arimoto [6] discussed a sensor feedback scheme using the transposed generalized Jacobian matrix.

Both of the linear and angular momentum conservations have been used to eliminate dependent variables[4] [6]. Although both of them are represented by equations of velocities, the linear one can be exhibited by the motion of the center of mass of the total system, which is represented by the equations of positions not of velocities. This implies that the linear momentum conservation is integrable and hence a holonomic constraint. On the other hand, the angular momentum conservation cannot be represented by an integrated form, which means that it is a nonholonomic constraint.

Suppose an n d.o.f. manipulator on a vehicle, the motion of the endeffector is described by n+6 variables, n

of the manipulator and 6 of the vehicle. By eliminating holonomic constraint of linear momentum conservation, the total system is formulated as a nonholonomic system of $n+3$ variables including 3 dependent variables. Although only n variables out of $n+3$ can be independently controlled, with an appropriate path planning scheme it would possible to converge all of $n+3$ variables to a desired values due to the nonholonomic mechanical structure. A similar situation is experienced in our daily life. Although an automobile has two independent variables to control, that is, wheel rotation and steering, it can be parked at an arbitrary place with an arbitrary orientation in two dimensional space. This can be done because it is a nonholonomic system.

To locate the manipulator endeffector at a desired point with a desired orientation, even a vehicle with a 6 d.o.f. manipulator has redundancy because a variety of vehicle orientation can be chosen at the final time. This kind of nonholonomic redundancy would be utilized (1) when the extended Jacobian control results in an infeasible motion due to the physical joint limitation, (2) when the system requires more degrees of freedom to avoid obstacles at the final location of the endeffector, (3) when the vehicle orientation needs to be changed without using propulsion or a momentum gyro, and so on.

In this paper, we propose a path planning scheme to control both of the vehicle orientation and the manipulator joints by actuating manipulator joints only. First, the nonholonomic mechanical structure of space vehicle/manipulator system is shown. Second, a path planning scheme for nonholonomic systems is proposed using Lyapunov functions. Since the planning scheme is given in a general form, it can be applied to other many nonholonomic planning problems, such as the path planning of 2 d.o.f. vehicles for 3 d.o.f. motion in a plane, planning of contact point motion of multifingered hands with spherical rolling contacts, and so on.

2. ANGULAR MOMENTUM CONSERVATION AS NONHOLONOMIC CONSTRAINT

2.1 Nomenclature

frame I	Inertia frame.
frame V	Vehicle frame.
frame B	Manipulator base frame
frame E	Manipulator endeffector frame
frame K	k-th body frame. k-th link frame of manipulator for $k = 1, \cdots, n$. n-th link frame is identical to the manipulator endeffector frame. Vehicle frame for $k = 0$.
m_k	Mass of the k-th body (kg). 0-th body is the vehicle. k-th body ($k \geq 1$) is the k-th link of the manipulator.
${}^I\boldsymbol{r}_k \in R^3$	Position vector from the origin of the inertia frame to the center of mass of k-th body represented in theinertia frame. (m)
${}^B\boldsymbol{r}_k \in R^3$	Position vector from the origin of the manipulator base frame to the center of mass of k-th body represented in the manipulator base frame.(m)
${}^I\boldsymbol{\omega}_k \in R^3$	Angular velocity of the k-th body in the inertia frame.(rad/s)
${}^k\boldsymbol{I}_k \in R^{3\times3}$	Inertia matrix of the k-th body about its center of mass in the k-th body frame. (kgm^2)
${}^I\boldsymbol{I}_k \in R^{3\times3}$	Inertia matrix of the k-th body about its center of mass in the inertia frame. (kgm^2)
$\dot{\boldsymbol{\theta}}_1 \in R^6$	Linear velocity of the center of mass and angular velocity of the vehicle in inertia frame. (m/s, rad/s)
$\boldsymbol{\theta}_2 \in R^n$	Joint variables ($q_1, \cdots, q_n$) of the manipulator. (rad)
${}^I\boldsymbol{A}_B \in R^{3\times3}$	Rotation matrix from the inertia frame to the manipulator base frame.
${}^I\boldsymbol{A}_k \in R^{3\times3}$	Rotation matrix from the inertia frame to thek-th body frame (vehicle frame for $k = 0$, k-th link frame of the manipulator for $k = 1, \cdots, n$).
$\boldsymbol{J}_2^k \in R^{3\times n}$	Jacobian matrix of the position of the center of mass of k-th body ($k = 1, \cdots, n$) in the manipulator base frame. (m)
$\boldsymbol{E}_i \in R^{i\times i}$	$i \times i$ identity matrix.
α, β, γ	z-y-x Euler angles.

2.2 Kinematics of Space Vehicle/Manipulator System

The basic equations of kinematics of space vehicle/manipulator system is developed in this subsection. Fig. 1 shows a model of space vehicle/manipulator system. Five kinds of frames, the inertia frame, the vehicle frame, the manipulator base frame, the k-th link frames, and the manipulator endeffector frame, are represented by I, V, B, K, and E respectively. The link frames of the manipulator are defined by Denavit-Hartenberg convention [7]. The vehicle frame is assumed to be fixed at the center of mass of the vehicle.

Supposing zero linear and angular momentum at initial time, the linear and angular momentum conservations are represented by

$$\sum_{k=0}^{n} m_k {}^I\dot{\boldsymbol{r}}_k = 0, \tag{1}$$

$$\sum_{k=0}^{n} ({}^I\boldsymbol{I}_k {}^I\boldsymbol{\omega}_k + m_k {}^I\boldsymbol{r}_k \times {}^I\dot{\boldsymbol{r}}_k) = 0, \tag{2}$$

The vehicle and manipulator motions are described by the following $\dot{\boldsymbol{\theta}}_1$ and $\boldsymbol{\theta}_2$.

$$\dot{\boldsymbol{\theta}}_1 = \begin{pmatrix} {}^I\dot{\boldsymbol{r}}_0 \\ {}^I\boldsymbol{\omega}_0 \end{pmatrix} \tag{3}$$

$$\boldsymbol{\theta}_2 = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix} \tag{4}$$

${}^I\dot{\boldsymbol{r}}_k$ is computed by

$$\begin{aligned} {}^I\dot{\boldsymbol{r}}_k &= {}^I\dot{\boldsymbol{r}}_0 + {}^I\boldsymbol{\omega}_0 \times ({}^I\boldsymbol{r}_k - {}^I\boldsymbol{r}_0) + {}^I\boldsymbol{A}_B \boldsymbol{J}_2^k \dot{\boldsymbol{\theta}}_2 \\ &= (\boldsymbol{E}_3 \quad -{}^I\boldsymbol{R}_{0k})\,\dot{\boldsymbol{\theta}}_1 + {}^I\boldsymbol{A}_B\,\boldsymbol{J}_2^k\,\dot{\boldsymbol{\theta}}_2 \end{aligned} \tag{5}$$

where ${}^I\boldsymbol{R}_{0k}$ and ${}^I\boldsymbol{r}_{0k}$ are defined by

$${}^I\boldsymbol{R}_{0k} = \begin{pmatrix} 0 & -{}^I r_{0k\,z} & {}^I r_{0k\,y} \\ {}^I r_{0k\,z} & 0 & -{}^I r_{0k\,x} \\ -{}^I r_{0k\,y} & {}^I r_{0k\,x} & 0 \end{pmatrix} \tag{6}$$

$${}^I\boldsymbol{r}_k - {}^I\boldsymbol{r}_0 = \begin{pmatrix} {}^I r_{0k\,x} \\ {}^I r_{0k\,y} \\ {}^I r_{0k\,z} \end{pmatrix} \tag{7}$$

On the other hand, ${}^I\boldsymbol{I}_k {}^I\boldsymbol{\omega}_k$ is given by

$${}^I\boldsymbol{I}_k {}^I\boldsymbol{\omega}_k = {}^I\boldsymbol{A}_k {}^k\boldsymbol{I}_k {}^I\boldsymbol{A}_k^{T}\, {}^I\boldsymbol{\omega}_k \tag{8}$$

$${}^I\boldsymbol{\omega}_k = \begin{cases} (0 \quad \boldsymbol{E})\,\dot{\boldsymbol{\theta}}_1 & \text{for } k = 0 \\ {}^I\boldsymbol{\omega}_0 + \sum_{j=1}^{k} {}^I\boldsymbol{A}_j \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \dot{q}_j & \text{for } k = 1, \cdots, n \end{cases} \tag{9}$$

By substituting eqs. (5) and (8) into eqs. (1) and (2) and summarizing them in a matrix form, the linear and angular momentum conservations are represented by the following equation.

$$\boldsymbol{H}_1\dot{\boldsymbol{\theta}}_1 + \boldsymbol{H}_2\dot{\boldsymbol{\theta}}_2 = 0 \tag{10}$$

$$H_1 = \begin{pmatrix} \sum_{k=0}^{n} m_k E & -\sum_{k=0}^{n} m_k\, {}^I R_{0k} \\ \sum_{k=0}^{n} m_k\, {}^I R_k & \sum_{k=0}^{n} {}^I A_k\, {}^k I_k\, {}^I A_k^{\,T} - \sum_{k=0}^{n} m_k\, {}^I R_k\, {}^I R_{0k} \end{pmatrix} \tag{11}$$

$$H_2 = \begin{pmatrix} \sum_{k=0}^{n} m_k\, {}^I A_B\, J_2^k \\ \sum_{k=0}^{n} m_k\, {}^I R_k\, {}^I A_B\, J_2^k + P \end{pmatrix} \tag{12}$$

where

$${}^I R_k = \begin{pmatrix} 0 & -{}^I r_{k\,z} & {}^I r_{k\,y} \\ {}^I r_{k\,z} & 0 & -{}^I r_{k\,x} \\ -{}^I r_{k\,y} & {}^I r_{k\,x} & 0 \end{pmatrix} \tag{13}$$

$$P = (\, P_1 \quad P_2 \quad \cdots \quad P_n\,) \tag{14}$$

$$P_i = \left(\sum_{k=i}^{n} {}^I A_i\, {}^i I_i\, {}^I A_i^{\,T} \right) {}^I A_i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

In eq. (13), ${}^I r_{k\,x}$, ${}^I r_{k\,y}$ and ${}^I r_{k\,z}$ are x, y and z components of ${}^I \boldsymbol{r}_k$ respectively.

The relationship between the endeffector, $\dot{\theta}_1$ and $\dot{\theta}_2$ is described in the following form.

$$\dot{h} = J_1 \dot{\theta}_1 + J_2 \dot{\theta}_2 \tag{15}$$

where

$$\dot{h} = \begin{pmatrix} {}^I \dot{r}_E \\ {}^I \omega_E \end{pmatrix}$$

J_1 and J_2 are the pure geometrical Jacobian matrices which do not take account of the momentum conservations. In eq. (10), $H_1 \in R^{6\times 6}$ is always nonsingular. Therefore, eq. (10) is identical to

$$\dot{\theta}_1 = -H_1^{-1} H_2 \dot{\theta}_2 \tag{16}$$

Substituting eq. (16) into eq. (15) offers

$$\dot{h} = \left(-J_1 H_1^{-1} H_2 + J_2 \right) \dot{\theta}_2 \tag{17}$$

Umetani and Yoshida [4] named the coefficient matrix of the above equation *the generalized Jacobian matrix*. In this derivation, the momentum conservations of eq. (10) are used as constraints equations and eliminated in the final equation.

2.3 Holonomic and Nonholonomic Constraints

Eq. (1) can be analytically integrated as follows:

$$\begin{aligned} \int_0^t \sum_{k=0}^{n} m_k\, {}^I \dot{r}_k \, dt &= \sum_{k=0}^{n} m_k\, {}^I r_k(t) - \sum_{k=0}^{n} m_k\, {}^I r_k(0) \\ &= 0 \end{aligned} \tag{18}$$

The above equation physically means that the total center of mass of the system does not move. ${}^I \boldsymbol{r}_k$ is computed by

$$ {}^{I}\boldsymbol{r}_k = {}^{I}\boldsymbol{A}_B \, {}^{B}\boldsymbol{r}_k + {}^{I}\boldsymbol{r}_0 \tag{19} $$

where ${}^{I}\boldsymbol{A}_B$ is a function of the vehicle orientation only. ${}^{B}\boldsymbol{r}_k$ is a function of the joint variables of the manipulator only. Knowing the vehicle orientation, the joint variables, and the initial position of the total center of mass, the vehicle position ${}^{I}\boldsymbol{r}_k$ can be obtained by substituting eq. (19) into eq. (18). Therefore, the linear momentum conservation is considered a holonomic constraint because it is integrable.

Although eqs. (1) and (2) are both represented by velocities, eq. (2) can not be analytically integrated and, therefore, it is a nonholonomic constraint. The physical characteristic of nonholonomic constraint is exhibited by the fact that even if the manipulator joints return to the initial joint variables after a sequence of motion, the vehicle orientation may not be the same as its initial value. The vehicle orientation can be eliminated as a dependent variable as we did in deriving eq. (17). In next section, we propose to control both of the independent and dependent variables by controlling the independent ones only.

The basic system equation is obtained by taking the vehicle orientation and $\boldsymbol{\theta}_2$ as the state variable and the $\dot{\boldsymbol{\theta}}_2$ as the input variable. First, the coefficient matrix of eq. (16) is divided into a top $3 \times n$ matrix and a bottom $3 \times n$ matrix as follows:

$$ \boldsymbol{H} = \begin{pmatrix} \boldsymbol{H}_r \\ \boldsymbol{H}_\omega \end{pmatrix} = -\boldsymbol{H}_1^{-1}\boldsymbol{H}_2 \tag{20} $$

The state variable $\boldsymbol{x}$ and the input variable $\boldsymbol{u}$ are defined by

$$ \boldsymbol{x} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \\ \boldsymbol{\theta}_2 \end{pmatrix} \in R^{n+3} \tag{21} $$

$$ \boldsymbol{u} = \dot{\boldsymbol{\theta}}_2 \in R^n \tag{22} $$

α,β, and γ are the z-y-x Euler angles of the vehicle with respect to the inertia frame. The relationship between the Euler angles and ${}^{I}\boldsymbol{\omega}_0$ is given by

$$ {}^{I}\boldsymbol{\omega}_0 = \boldsymbol{N} \begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{pmatrix} \tag{23} $$

where

$$ \boldsymbol{N} = \begin{pmatrix} 0 & -sin\alpha & cos\alpha\, cos\beta \\ 0 & cos\alpha & sin\alpha\, \cos\beta \\ 1 & 0 & -sin\beta \end{pmatrix} $$

The system equation becomes

$$ \dot{\boldsymbol{x}} = \boldsymbol{K}\,\boldsymbol{u} \tag{24} $$

where

$$ \boldsymbol{K} = \begin{pmatrix} \boldsymbol{N}^{-1}\boldsymbol{H}_\omega \\ \boldsymbol{E}_n \end{pmatrix} \in R^{(n+3)\times n} \tag{25} $$

2.4 Nonholonomic Redundancy

The system represented by eq. (25) has a unique feature in the fact that the input variable may not be found even if a smooth desired trajectory of $\boldsymbol{x}$ is provided because it has less number of input variable. It is impossible to plan a feasible trajectory without taking full account of the dynamics of eq. (25). This is a general feature of nonholonomic mechanical systems. An automobile can move around in two dimensional space and orient itself if we drive it properly, although it has only two variables to control, that is, wheel rotation and steering. In this case, the state variables are three and the inputs are two.

By appropriately planning the trajectory, the desired final values of the vehicle orientation and the manipulator joint variables could be reached. To locate the manipulator endeffector at a desired point with a desired orientation, even a vehicle with a 6 d.o.f. manipulator has redundancy because a variety of vehicle orientation can be chosen at the final time. The choice of the final vehicle orientation can be done based on the conventional control or planning schemes of kinematically redundant manipulators [8, 9, 10]. It is a problem to find an appropriate configuration among the configurations attained by 3 d.o.f. selfmotion.

The nonholonomic redundancy would be utilized (1) when the extended Jacobian control results in an infeasible motion due to the physical joint limitation, (2) when the system requires more degrees of freedom to avoid obstacles at the final location of the endeffector, (3) when the vehicle orientation needs to be changed without using propulsion or a momentum gyro, and so on.

3. PATH PLANNING USING LYAPUNOV FUNCTIONS

3.1 First Lyapunov function

In this section, the input variable $\boldsymbol{u}$ is synthesized based on the Lyapunov's direct method [11] so that the vehicle orientation and the joint variables should converge to their desired values.

The following function is chosen as a candidate of the Lyapunov function.

$$v_1 = \frac{1}{2}\Delta\boldsymbol{x}^T \boldsymbol{A}\Delta\boldsymbol{x} \tag{26}$$

$$\Delta\boldsymbol{x} = \boldsymbol{x}_d - \boldsymbol{x} \tag{27}$$

where $\boldsymbol{A}$ is a positive definite constant matrix. $v_1 = 0$ is attained only when $\boldsymbol{x}_d = \boldsymbol{x}$. The time derivative of v_1 is computed as follows:

$$\dot{v}_1 = -\Delta\boldsymbol{x}^T\boldsymbol{A}\dot{\boldsymbol{x}} = -\Delta\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{K}\boldsymbol{u} \tag{28}$$

where eq. (24) was substituted. Now, choosing the input variable as

$$\boldsymbol{u}_1 = (\boldsymbol{A}\boldsymbol{K})^T\Delta\boldsymbol{x}, \tag{29}$$

the rate of change of the Lyapunov function becomes

$$\dot{v}_1 = -\boldsymbol{u}_1^T\boldsymbol{u}_1 \leq 0 \tag{30}$$

If the equality of eq. (30) holds only when $\boldsymbol{x}_d = \boldsymbol{x}$, Lyapunov's theorem [11] can conclude its global stability. However, eq. (30) is not the case. $\dot{v}_1$ becomes zero when $\Delta\boldsymbol{x}$ is in the null space of $(\boldsymbol{A}\boldsymbol{K})^T$, which is a three dimensional space.

3.2 Avoiding Null Space of $(\boldsymbol{A}\boldsymbol{K})^T$

The LaSalle's theorem [12] says that the state variable $\boldsymbol{x}$ converges to $\boldsymbol{x}_d$ if $\boldsymbol{x} = \boldsymbol{x}_d$ is the unique entry of the maximum invariant set. When $\Delta\boldsymbol{x}$ is at the null space of $(\boldsymbol{A}\boldsymbol{K})^T$ and it stays within the null space thereafter, all the points on this trajectory are the entries of the maximum invariant set. In this subsection, the unit vector is chosen such that $\Delta\boldsymbol{x}$ should avoid the null space as much as possible and get out of the null space if it is there.

To take account of the null space of $(\boldsymbol{A}\boldsymbol{K})^T$ we introduce the second Lyapunov function v_2 such that

$$v_2 = \frac{\Delta\boldsymbol{x}^T\left(\boldsymbol{I} - (\boldsymbol{A}\boldsymbol{K})(\boldsymbol{A}\boldsymbol{K})^{\#}\right)\Delta\boldsymbol{x}}{\Delta\boldsymbol{x}^T\Delta\boldsymbol{x} + \epsilon_1} \tag{31}$$

where ϵ_1 is a positive small constant. v_2 becomes equal to zero when $\Delta\boldsymbol{x} = 0$.

Since

$$\begin{aligned}
&\Delta x^T \left(I - (AK)(AK)^{\#}\right) \Delta x \\
=&\Delta x^T \left(I - (AK)(AK)^{\#}\right)^T \left(I - (AK)(AK)^{\#}\right) \Delta x \\
=& \| \left(I - (AK)(AK)^{\#}\right) \Delta x \|^2,
\end{aligned} \tag{32}$$

the numerator of eq. (31) implies the squared Euclidean norm of the orthogonal projection of Δx on the null space of $(AK)^T$. If we define ϕ such that

$$\cos\phi = \frac{\| \left(I - (AK)(AK)^{\#}\right) \Delta x \|}{\| \Delta x \|}, \qquad 0 \le \phi \le \frac{\pi}{2} \tag{33}$$

ϕ means an angle between Δx and the hyperplane of the null space of $(AK)^T$, and can be considered as a distance of Δx from the null space as shown in Fig. 2. For $\epsilon_1 = 0$ the second Lyapunov function becomes

$$v_2 = \cos^2\phi \qquad 0 \le \phi \le \frac{\pi}{2}. \tag{34}$$

In eq. (31), ϵ_1 allows for v_2 not to take extreme values and to be defined at Δx. In eq. (34) v_2 is monotonously reduced as ϕ grows, and takes zero at $\phi = \pi/2$, which means the farthest point from the null space.

Taking the derivative of v_2 with respect to time, we have

$$\dot{v}_2 = \frac{\partial v_2}{\partial x}\dot{x} = \frac{\partial v_2}{\partial x} K u. \tag{35}$$

If we choose u_2 such as

$$u_2 = -K^T \left(\frac{\partial v_2}{\partial x}\right)^T, \tag{36}$$

and use it as u, then $\dot{v}_2 \le 0$, and u_2 works to avoid the null space by driving toward $\phi = \pi/2$.

We integrate u_1 and u_2 in a hierarchical manner such that

$$u = k_1 u_1 + k_2 \left(I - u_1 u_1^{\#}\right) u_2 \tag{37}$$

where $u_1^{\#}$ is the pseudoinverse of u_1, k_1 and k_2 are positive constants. Since $\left(I - u_1 u_1^{\#}\right) u_2$ is the orthogonal projection of u_2 onto the hyperplane that is perpendicular to u_1, the first and second terms are mutually perpendicular. This results in following properties of eq. (37).

The second term of eq.(37) has no effect on the convergence speed of v_1 † because substituting eq. (37) into eq. (28) we have

$$\dot{v}_1 = -u_1^T\{u_1 + k_1 \left(I - u_1 u^{\#}\right) u_2\} = -u_1^T u_1 \tag{38}$$

where $u_1^T \left(I - u_1 u_1^{\#}\right) = \left(u_1 - u_1 u_1^{\#} u_1\right)^T = 0$ is used.

Let's consider the effect of the second term of v_2. Substituting the second term of eq. (37) into eq. (35) along with eq. (36), we obtain

$$\begin{aligned}
\dot{v}_2 &= -\frac{\partial v_2}{\partial x} K \left(I - u_1 u_1^{\#}\right) K^T \left(\frac{\partial v_2}{\partial x}\right)^T \\
&= -\frac{\partial v_2}{\partial x} K \left(I - u_1 u_1^{\#}\right)^T \left(I - u_1 u_1^{\#}\right) K^T \left(\frac{\partial v_2}{\partial x}\right)^T \le 0
\end{aligned} \tag{39}$$

† The convergence speed $\dot{v}_1$ is the same for both u_1 and u of eq. (37) only in local sense. Since the global trajectory of x varies depending on the choice of the input, the global convergence speed would be different.

$\dot{v}_2$ becomes zero only when $(\boldsymbol{I} - \boldsymbol{u}_1\boldsymbol{u}_1{}^{\#})\,\boldsymbol{K}^T(\partial v_2/\partial \boldsymbol{x})^T = 0$. Otherwise $\dot{v}_2$ is always negative. This means that the second term of eq. (37) tries to reduce v_2 although the total $\boldsymbol{u}$ of eq. (37) does not be guarantee the negativeness of $\dot{v}_2$ because of the effect of the first term.

To summarize eqs. (28),(29),(35),and (36), the proposed hierarchical Lyapunov function approach can be represented as follows

$$\boldsymbol{u} = k_1\,\boldsymbol{u}_1 + k_2\,(\boldsymbol{I} - \boldsymbol{u}_1\boldsymbol{u}_1{}^{\#})\,\boldsymbol{u}_2 \tag{40}$$

$$\boldsymbol{u}_i = -\boldsymbol{K}^T\left(\frac{\partial v_i}{\partial \boldsymbol{x}}\right)^T, \qquad \text{for } i = 1,2 \tag{41}$$

It should be noted that if we consider v_i as the ith manipulation variable, $(\partial v_i/\partial \boldsymbol{x})\,\boldsymbol{K}$ as its Jacobian matrix with respect to the input variable $\boldsymbol{u}$, then eq. (40) is identical to the *task-priority approach* developed for kinematically redundant manipulators[10], having

$$\dot{v}_i = -\frac{\partial v_i}{\partial \boldsymbol{x}}\boldsymbol{K}\boldsymbol{K}^T\left(\frac{\partial v_i}{\partial \boldsymbol{x}}\right)^T, \qquad \text{for } i = 1,2 \tag{42}$$

as the desired trajectories of the manipulation variables. This approach cannot guarantee that $\boldsymbol{x} = \boldsymbol{x}_d$ is the unique entry of the maximum invariant set [12] and, therefore, the trajectory may halt at some point in the null space of $(\boldsymbol{A}\boldsymbol{K})^T$. However, if the second Lyapunov function can successfully avoid the null space of $(\boldsymbol{A}\boldsymbol{K})^T$, $\boldsymbol{x}$ converges to $\boldsymbol{x}_d$.

4. CONCLUSION

A new insight of the mechanical structure of space vehicle/ manipulator systems was given. By utilizing the nonholonomic structure, not only the manipulator joints, but also vehicle orientation can be controlled only by actuating the joint variables, although both of the vehicle motion and the manipulator joints cannot be controlled independently. Therefore, it is essential to plan a feasible trajectory. A nonlinear control scheme was synthesized using Lyapunov's direct method. This scheme can be used not only for real-time control, but for planning of a feasible motions of vehicle and manipulator. To verify the effectiveness of the proposed approach, numerical simulation is currently being undertaken at the Center for Robotic Systems in Microelectronics, University of California, Santa Barbara.

ACKNOWLEDGEMENT

This material is supported by the National Science Foundation under Contract number 8421415. Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the author and do not necessarily reflect the views of the Foundation.

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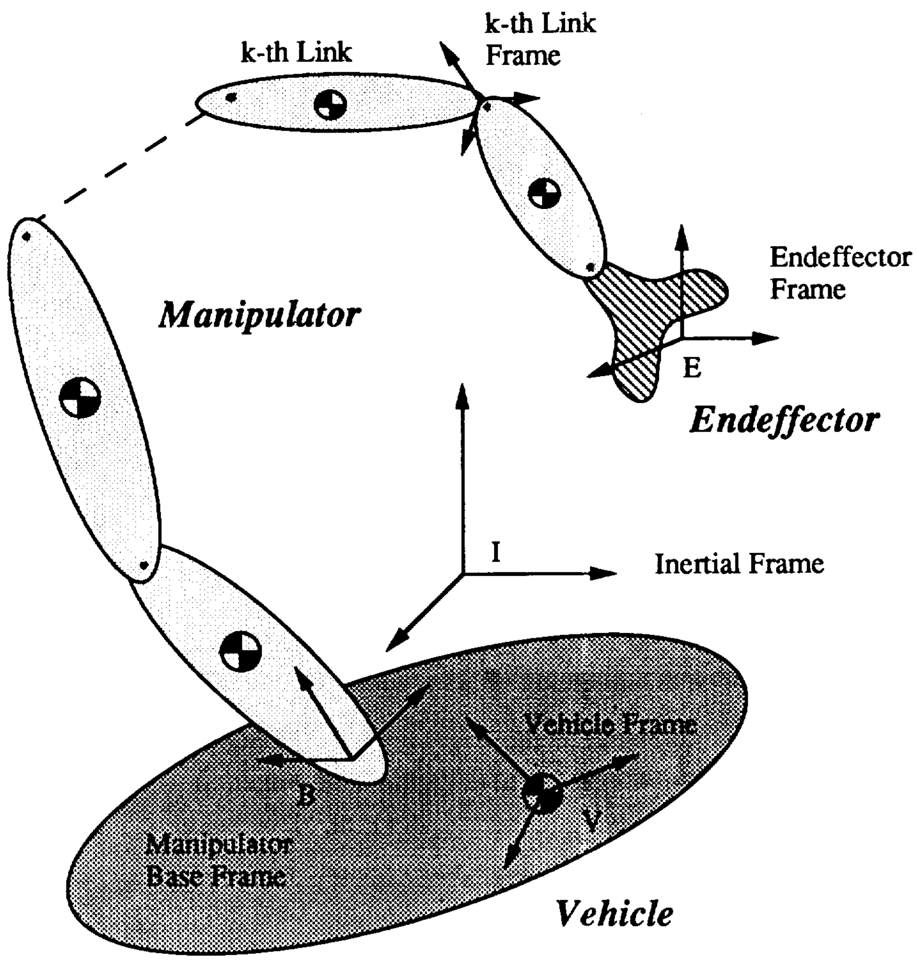
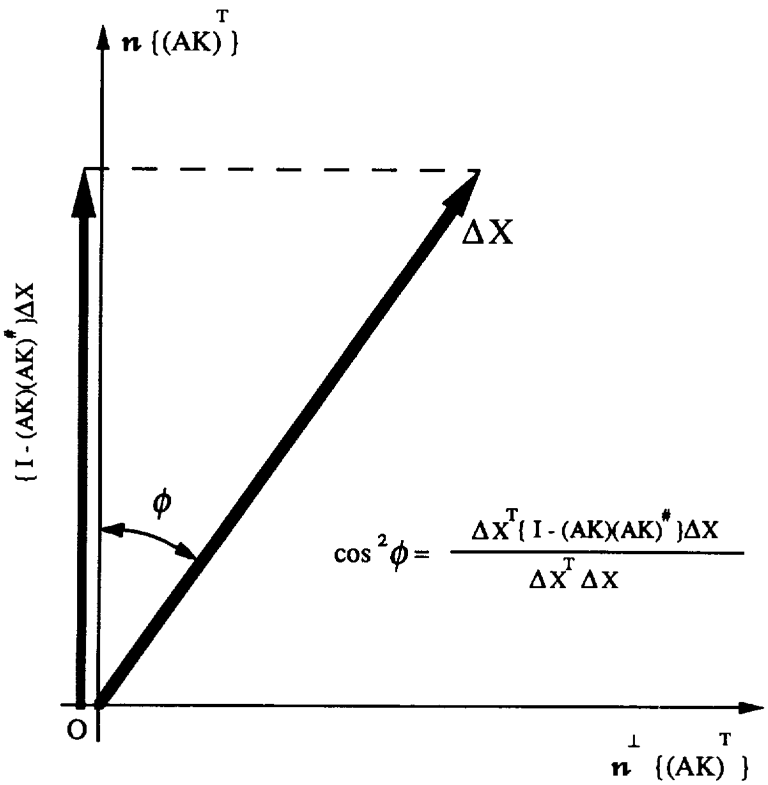


Fig 1. Five Coordinate Frames for the Space Vehicle / Manipulator System.



$n\{(AK)^T\}$: the null space of $(AK)^T$

$n^{\perp}\{(AK)^T\}$: the orthogonal complement space of $n\{(AK)^T\}$

Fig 2. Physical Meaning of the 2nd Lyapunov Function and Definition of Angle ϕ .