

# MODELING AND SENSORY FEEDBACK CONTROL FOR SPACE MANIPULATORS

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## Abstract

This paper treats the positioning control problem of the endtip of space manipulators whose base are uncontrolled. In such a case, the conventional control method for industrial robots based on a local feedback at each joint is not applicable, because a solution of the joint displacements that satisfies a given position and orientation of the endtip is not decided uniquely. We propose a sensory feedback control scheme for space manipulators based on an artificial potential defined in a task-oriented coordinates. Using this scheme, the controller can easily determine the input torque of each joint from the data of an external sensor such as a visual device. Since the external sensor is mounted on the unfixed base, the manipulator must track the moving image of the target in the sensor coordinates. Moreover the dynamics of the base and the manipulator is interactive. However, we can prove that the endtip asymptotically approaches the target stationary in an inertial coordinate frame by the Liapunov's method. Finally results of computer simulation for a 6-link space manipulator model show the effectiveness of the proposed scheme.

## 1 Introduction

For the next generation of the space activities, various missions in satellite orbits are planned. In order to carry out these missions, robots that replace astronauts are indispensable, because the number of astronauts living in the orbit is limited. In particular, many plans of robots for extra vehicle activities have been proposed such as shown Fig.1. They are small spacecrafts(or satellites) equipped with manipulators and visual devices, and are thought to perform a wide variety of tasks while flying freely around the mother ships.

One of the differences of space manipulators from conventional ones on the ground is that their bases, namely spacecrafts, are unfixed. Therefore the degree-of-freedom of motion of the whole system increases by 6. When the endtip is positioned at a target object floating independently of the manipulator system, one may think that the base as well as the joints must be controlled. Of course thrusters and reaction wheels are necessary to translate and rotate bases themselves. Thrusters can output gas jet like on-off and reaction wheels can make moments by changing their angular velocities. In case that they cooperate with joint actuators to control the endtip of the manipulator, their capacities need to be large, or manipulators need to be operated slowly. Hence, it is inadequate to control the base and joints at the same time from the view point of energy, weight, or task efficiency. Therefore it is desirable to be able to control the endtip of manipulator only by the joint actuators, nevertheless the uncontrolled base is moved by the reactions from the manipulator.

When the base is uncontrolled, the base's motion depends on the action of joint. In such a case, the linear and angular momenta of the whole system floating in non-gravitational environment are conserved, because the joint actuators generate no external forces. Considering this constraint, the kinematic degree-of-freedom of the system reduces to the number of the joints. The position and orientation of the endtip, however, depend on the time history of joint displacements. In other words, they cannot be expressed as functions only of the joint displacements at each time. If one applied a conventional local feedback method to this system, one could not define the joint displacements corresponding to a given position and orientation of the endtip. Therefore the positioning control of the space manipulator needs global feedbacks using data of external sensors such as visual devices.

For this problem, Vafa and Dubowsky developed the *virtual manipulator concept*, that is an idealized kinematic chain [1]. When the base's attitude is controlled and its position is uncontrolled, this concept is very useful. Alexander and Cannon showed that the endtip can be controlled by solving inverse dynamics that includes the base's motion [2]. However, much calculation is necessary to get the control inputs by their method. Umetani and Yoshida proposed the *generalized Jacobian matrix*, that relates the endtip velocities to joint velocities by taking the base's motion into consideration [3]. They also showed that a desired endtip velocity can be resolved to joint velocities by this relation. However, they treated only the kinematic problem.

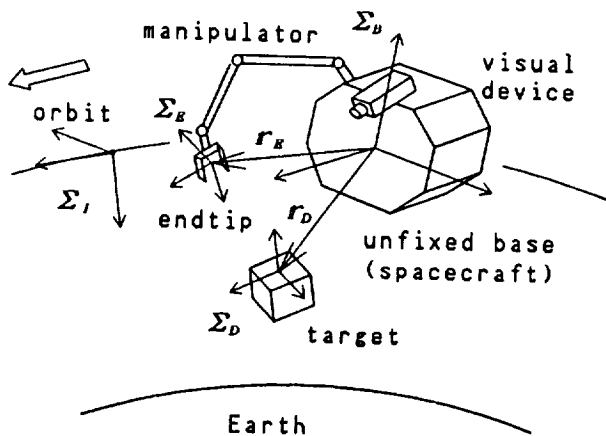


Fig.1: Space robot

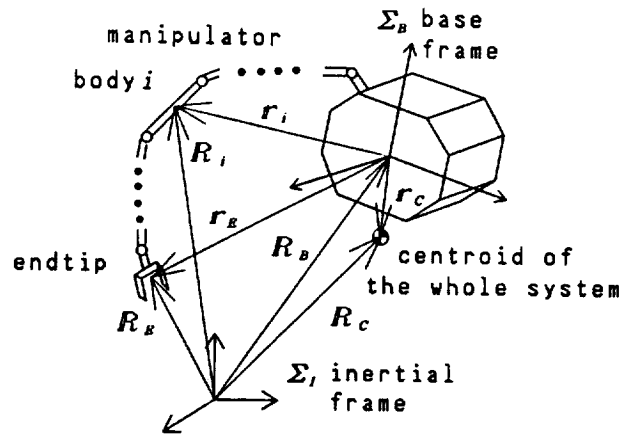


Fig.2: Model of the space manipulators

We propose a sensory feedback control scheme that position the endtip at the target specified in an inertial space without using inverse dynamics. This is based on the concept of an artificial potential defined in a task-oriented coordinate system [4,5]. Using this scheme, the deviations of the endtip measured with external sensor on the base is fed back to the each joint multiplied by the transpose of the Jacobian. Since the external sensor is mounted on the free-motion base, the sensor signal indicates as if the target were moving. Thus the manipulator must track the moving target, though this target is stationary in the inertial coordinate frame in fact. In general, it is difficult to show the asymptotic stability of the position control system tracking a moving point. Fortunately it is possible to prove it by using the Liapunov's method and the dynamical relationship of the motion between the base and joints.

In this paper, first we discuss the kinematics of space manipulators, and an algorithm to derive the generalized Jacobian from the conventional Jacobian is shown. Next, considering the dynamics of space manipulators, we represent the new control scheme, and discuss its stability. Finally we show the effectiveness of the proposed scheme through the computer simulation.

## 2 Kinematics

In this section, we consider kinematic relation between space manipulators and their bases by using the conservation laws of linear and angular momenta of the whole system.

### 2.1 Modeling

A space manipulator system in the earth's satellite orbit can be approximately considered to be floating in the non-gravitational environment. We treat this system as a set of  $n + 1$  rigid bodies connected with  $n$  joints that form a tree configuration. Each joint is numbered in series of 1 to  $n$ , and these displacements are represented by the joint vector  $q = [q_1, q_2, \dots, q_n]^T$ . On the other hand, each body is numbered in series of 0 to  $n$ . In particular, we assign  $B$  to the number of the base body. The mass and inertia tensor of  $i$ -th body are  $m_i$  and  $I_i$  respectively.

Let us define two coordinate frames. One is the inertial coordinate frame,  $\Sigma_I$ , that in the orbit where the system exists; another is the base coordinate frame,  $\Sigma_B$ , that is attached on the base, whose origin is located at the centroid of the base. In this paper, we express all vectors in terms of frame  $\Sigma_B$ . The reasons why we introduce the base coordinate frame are to contrast the properties of space manipulators with that ground-fixed manipulators and to use the data measured in this frame for the control scheme later. In order to represent the orientation of frame  $\Sigma_B$  relative to frame  $\Sigma_I$ , we use three appropriate parameters such as roll, pitch, and yaw and their vector form representation  $\phi \in R^3$ .

As shown in Fig.2, let  $\mathbf{R}_i$  and  $\mathbf{r}_i$  be position vectors pointing the centroid of the  $i$ -th body with reference to frames  $\Sigma_I$  and  $\Sigma_B$  respectively. The relation between them can be written by

$$\mathbf{R}_i = \mathbf{R}_B + \mathbf{r}_i. \quad (1)$$

where  $\mathbf{R}_B$  is the position vector pointing the centroid of the base with reference to  $\Sigma_I$ .

Moreover, let  $\mathbf{V}_i$  and  $\mathbf{\Omega}_i$  be linear and angular velocities with reference to frame  $\Sigma_I$ ,  $\mathbf{v}_i$  and  $\boldsymbol{\omega}_i$  be linear and angular velocities with reference to frame  $\Sigma_B$ . They have relations as follows;

$$\mathbf{V}_i = \mathbf{v}_i + \mathbf{V}_B + \mathbf{\Omega}_B \times \mathbf{r}_i, \quad (2)$$

$$\mathbf{\Omega}_i = \boldsymbol{\omega}_i + \mathbf{\Omega}_B, \quad (3)$$

where  $\mathbf{V}_B$  and  $\mathbf{\Omega}_B$  are linear and angular velocities of the centroid of the base with reference to frame  $\Sigma_I$ , and operator 'x' represents outer product of vectors. The Jacobian matrices corresponding to the linear and angular velocities of the centroid of each body can be obtained as functions of the joint vector  $\mathbf{q}$ ;

$$\mathbf{v}_i = J_{L_i}(\mathbf{q}) \dot{\mathbf{q}}, \quad (4)$$

$$\boldsymbol{\omega}_i = J_{A_i}(\mathbf{q}) \dot{\mathbf{q}}. \quad (5)$$

## 2.2 Linear and Angular Momenta of the System

Since we treat in this paper the case in which no gravitational forces act and no actuators generating external forces are employed, the linear and angular momenta of the whole system are conserved. We assume here that the system is stationary in initial state. Thus these momenta are always zero. Using the relations given in the previous sub-section, we express these momenta as a linear combinations of  $\mathbf{V}_B$ ,  $\mathbf{\Omega}_B$ , and  $\dot{\mathbf{q}}$ .

The linear momentum,  $\mathbf{P}$ , and angular momentum,  $\mathbf{L}$ , of the whole system are defined by;

$$\mathbf{P} \stackrel{\text{def}}{=} \sum_{i=0}^n m_i \mathbf{V}_i, \quad (6)$$

$$\mathbf{L} \stackrel{\text{def}}{=} \sum_{i=0}^n \{ {}^B I_i \mathbf{\Omega}_i + m_i \mathbf{R}_i \times \mathbf{V}_i \}, \quad (7)$$

where  ${}^B I_i$  means the inertia tensor in term of the base frame,  $\Sigma_B$ . Let us define the followings for the centroid of the whole system;

$$m_C \stackrel{\text{def}}{=} \sum_{i=0}^n m_i, \quad (8)$$

$$\mathbf{r}_C(\mathbf{q}) \stackrel{\text{def}}{=} \sum_{i=0, i \neq B}^n m_i \mathbf{r}_i(\mathbf{q}) / m_C, \quad (9)$$

$$J_C(\mathbf{q}) \stackrel{\text{def}}{=} \sum_{i=0, i \neq B}^n m_i J_{L_i}(\mathbf{q}) / m_C \in R^{3 \times n}. \quad (10)$$

Substituting equations(1),(2), (3), (4),and (5) into equations(6) and (7) yields

$$\begin{bmatrix} \mathbf{P} \\ \mathbf{L} \end{bmatrix} = \begin{pmatrix} H_V & H_{V\Omega} \\ H_{V\Omega}^T & H_\Omega \end{pmatrix} \begin{bmatrix} \mathbf{V}_B \\ \mathbf{\Omega}_B \end{bmatrix} + \begin{pmatrix} H_{Vq} \\ H_{\Omega q} \end{pmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{O} \\ \mathbf{R}_B \times \mathbf{P} \end{bmatrix}. \quad (11)$$

Each block is defined by

$$H_V \stackrel{\text{def}}{=} m_C U_3 \in R^{3 \times 3}, \quad (12)$$

$$H_{V\Omega} \stackrel{\text{def}}{=} -m_C [\mathbf{r}_C \times] \in R^{3 \times 3}, \quad (13)$$

$$H_{Vq} \stackrel{\text{def}}{=} m_C J_C \in R^{3 \times n}, \quad (14)$$

$$H_{\Omega} \stackrel{\text{def}}{=} \sum_{i=0, i \neq B}^n \{ {}^B I_i + m_i D(\mathbf{r}_i) \} + I_B \in R^{3 \times 3}, \quad (15)$$

$$H_{\Omega q} \stackrel{\text{def}}{=} \sum_{i=0, i \neq B}^n \{ {}^B I_i J_{A_i} + m_i [\mathbf{r}_i \times] J_{L_i} \} \in R^{3 \times n}, \quad (16)$$

where  $U_3$  is  $3 \times 3$  unit matrix, and two matrix functions  $[\mathbf{r} \times]$  and  $D(\mathbf{r})$  for a vector argument  $\mathbf{r} = [r_x, r_y, r_z]^T$  are defined as follows;

$$[\mathbf{r} \times] \stackrel{\text{def}}{=} \begin{pmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{pmatrix} \in R^{3 \times 3}, \quad (17)$$

$$D(\mathbf{r}) \stackrel{\text{def}}{=} [\mathbf{r} \times]^T [\mathbf{r} \times] \in R^{3 \times 3}. \quad (18)$$

Since the zero linear and angular momenta are assumed, substituting  $\mathbf{P} \equiv \mathbf{O}$  and  $\mathbf{L} \equiv \mathbf{O}$  into equation(11) and solving it for  $\mathbf{V}_B$  and  $\boldsymbol{\Omega}_B$ , we obtain

$$\begin{bmatrix} \mathbf{V}_B \\ \boldsymbol{\Omega}_B \end{bmatrix} = \begin{pmatrix} -J_C(\mathbf{q}) - [\mathbf{r}_C(\mathbf{q}) \times] H_B^{-1}(\mathbf{q}) H_M(\mathbf{q}) \\ -H_B^{-1}(\mathbf{q}) H_M(\mathbf{q}) \end{pmatrix} \dot{\mathbf{q}}, \quad (19)$$

where

$$H_B \stackrel{\text{def}}{=} H_{\Omega} - m_C D(\mathbf{r}_C) \in R^{3 \times 3}, \quad (20)$$

$$H_M \stackrel{\text{def}}{=} H_{\Omega q} - m_C [\mathbf{r}_C \times] J_C \in R^{3 \times n}. \quad (21)$$

Matrix  $H_{\Omega}$  and  $H_B$  are the inertia tensors of the whole system with respect to rotation about the centroid of *the base*,  $\mathbf{R}_B$ , and the centroid of *the whole system*,  $\mathbf{R}_C$ . The inverse of inertia tensor  $H_B$  always exists, because it is positive definite. Equation(19) is the significant and basic relation for space manipulators that represents how the base is translated and rotated ( $\mathbf{V}_B, \boldsymbol{\Omega}_B$ ) by the joint action( $\dot{\mathbf{q}}$ ).

Between angular velocity of the base,  $\boldsymbol{\Omega}_B$ , and time derivative of the orientation parameters of the base,  $\dot{\boldsymbol{\phi}}$ , there is a relation

$$\boldsymbol{\Omega}_B = N(\boldsymbol{\phi}) \dot{\boldsymbol{\phi}}, \quad N(\boldsymbol{\phi}) \in R^{3 \times 3}. \quad (22)$$

From equation(19)and(22), we can derive

$$N(\boldsymbol{\phi}) \dot{\boldsymbol{\phi}} = -H_B^{-1}(\mathbf{q}) H_M(\mathbf{q}) \dot{\mathbf{q}}. \quad (23)$$

Since this equation is not integrated, the orientation of the base depends on the time history of the joint variables. Therefore the position and orientation of the endtip cannot be expressed as functions only of the joint variables,  $\mathbf{q}$ . Conversely even though the position and orientation of the endtip are given, it is impossible to obtain the joint variables to satisfy the given conditions in the way industrial robots always do. This is the reason why the global control feeding back information of the endtip is required.

### 2.3 Generalized Jacobian Matrix

Applying equation (19), the linear and angular velocities,  $\mathbf{V}_E$  and  $\boldsymbol{\Omega}_E$ , can be expressed as a linear combination only of the joint velocities. The equations with respect to the endtip motion such as equations(2), (3), (4), and (5) are hold, thus substituting equations(19) into them yield

$$\mathbf{V}_E = \hat{J}_L(\mathbf{q}) \dot{\mathbf{q}}, \quad (24)$$

$$\boldsymbol{\Omega}_E = \hat{J}_A(\mathbf{q}) \dot{\mathbf{q}}, \quad (25)$$

where

$$\hat{J}_L \stackrel{\text{def}}{=} J_L - J_C + [(\mathbf{r}_E - \mathbf{r}_C) \times] H_B^{-1} H_M \in R^{3 \times n}, \quad (26)$$

$$\hat{J}_A \stackrel{\text{def}}{=} J_A - H_B^{-1} H_M \in R^{3 \times n}. \quad (27)$$

In equations(26), vector  $\mathbf{r}_E$  is the position of the endtip with respect to the base. Matrices  $J_L$  and  $J_A$  are respectively *the conventional Jacobian matrices* that correspond to the linear and angular velocities relative to the base. On the other hand, matrices  $\hat{J}_L$  and  $\hat{J}_A$  are called *the generalized Jacobian matrices*. They are associated with the linear and angular velocities including the influence of the base motion.

Both  $\mathbf{V}_E$  and  $\boldsymbol{\Omega}_E$  what we use here are relative to *the inertial coordinate frame*,  $\Sigma_I$ , and they are expressed in terms of *the base coordinate frame*,  $\Sigma_B$ . This representation of velocities is unnatural, because the value of them cannot be directly derived from the data of the endtip measured with the sensor on the base. Using them, however, the generalized Jacobians,  $\hat{J}_L$  and  $\hat{J}_A$ , can be expressed as functions only of joint variable  $\mathbf{q}$ , and this clarifies the differences from the conventional Jacobian. Equations(26)and(27)show that the generalized Jacobian is summation of the conventional one and additional terms, and the additional term depend on the mass and inertia tensor of the each body as well as their dimensions. If the inertia of the base is very large, namely  $m_B \rightarrow \infty, I_B \rightarrow \infty$ , the additional term becomes zero and the generalized Jacobian is equal to the conventional one, because  $J_C \rightarrow O, \mathbf{r}_C \rightarrow O$ , and  $H_B \rightarrow \infty$ .

### 3 Dynamics

In this section, we derive the equation of motion that is required to discuss the stability of the control schemes proposed later by using the Lagrangian formulation.

#### 3.1 Kinetic Energy

The total kinetic energy of the whole system is defined by summing up the translational energy and rotational energy of each body.

$$T \stackrel{\text{def}}{=} \frac{1}{2} \sum_{i=0}^n \{ \boldsymbol{\Omega}_i^T {}^B I_i \boldsymbol{\Omega}_i + m_i \mathbf{V}_i^T \mathbf{V}_i \}. \quad (28)$$

Substituting equation(28) into equations(2)and(3) yields

$$T = \frac{1}{2} [ \mathbf{V}_B^T \quad \boldsymbol{\Omega}_B^T \quad \dot{\mathbf{q}}^T ] \begin{pmatrix} H_V & H_{V\Omega} & H_{Vq} \\ H_{V\Omega}^T & H_\Omega & H_{\Omega q} \\ H_{Vq}^T & H_{\Omega q}^T & H_q \end{pmatrix} \begin{bmatrix} \mathbf{V}_B \\ \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix}, \quad (29)$$

where matrix  $H_q$  is given by

$$H_q \stackrel{\text{def}}{=} \sum_{i=0, i \neq B}^n \{ J_{A_i}^T {}^B I_i J_{A_i} + m_i J_{L_i}^T J_{L_i} \} \in R^{n \times n} \quad (30)$$

that is the inertia matrix of the manipulator whose base is fixed [6].

In case that the linear and angular momenta are conserved at zero, the relevant equation(19) holds, and substituting it into equation(29), we obtain the reduced form

$$T = \frac{1}{2} \dot{\mathbf{q}}^T \hat{H}(\mathbf{q}) \dot{\mathbf{q}}, \quad (31)$$

where

$$\hat{H} \stackrel{\text{def}}{=} H_q - m_C J_C^T J_C - H_M^T H_B^{-1} H_M \in R^{n \times n}. \quad (32)$$

Using the fact that the  $(n+6) \times (n+6)$  original inertia matrix in equation (29) is positive definite, it is easy to prove the reduced inertia matrix,  $\hat{H}$ , to be symmetric and positive definite. Moreover each entry of  $\hat{H}$  is a function only of the joint variable,  $\mathbf{q}$ . Therefore the equation of motion for the reduced form can be derived in the same way used in conventional robotics.

### 3.2 Equation of Motion

Since there is no potential energy in non-gravitational environments, the Lagrangian is equal to the kinetic energy. Using the Lagrangian formulation, we can derive the equation of motion of the system

$$\hat{H}\ddot{\mathbf{q}} + \hat{H}\dot{\mathbf{q}} - \frac{\partial}{\partial \mathbf{q}} \left\{ \frac{1}{2} \dot{\mathbf{q}}^T \hat{H} \dot{\mathbf{q}} \right\} = \boldsymbol{\tau}, \quad (33)$$

where vector  $\boldsymbol{\tau} \stackrel{\text{def}}{=} [\tau_1, \tau_2, \dots, \tau_n]^T$  is composed of the control inputs into joint actuators.

## 4 Sensory Feedback Control

In this section, a new positioning control scheme for the endtip of manipulator whose base is uncontrolled during manipulation.

The problem here is to position the endtip of manipulator at a target on an object floating still in the inertial frame. It is assumed that the position and attitude of the base is initially arranged so that the endtip of the manipulator can reach the target.

### 4.1 Deviation of the Endtip

While the base is moved by the reaction force and moment from the manipulator, the position and orientation of the target seen from the view point on the base change. In order to represent the orientation of a certain coordinate frame  $\Sigma_*$ , we introduce a matrix  $A_*$  that plays the role of rotational coordinate transformation from frame  $\Sigma_*$  to  $\Sigma_B$ . This matrix consists of the three unit vectors along the coordinate axes of  $\Sigma_*$ , namely  $\mathbf{n}_*$ ,  $\mathbf{s}_*$ , and  $\mathbf{a}_*$  orthogonal to each other;

$$A_* \stackrel{\text{def}}{=} [\mathbf{n}_* \ \mathbf{s}_* \ \mathbf{a}_*] \in R^{3 \times 3}. \quad (34)$$

Let  $\mathbf{r}_D$  and  $A_D$  be respectively the position and orientation of the target that the endtip is desired to approach,  $\mathbf{r}_E$  and  $A_E$  be respectively the position and orientation of the endtip. One can easily derive them from the data of an external sensor such as a vision device mounted on the base.

Now let us define position and orientation deviations,  $\mathbf{e}_p$  and  $\mathbf{e}_o$ , which represent the difference in position and orientation between the target and the endtip of the manipulator;

$$\mathbf{e}_p \stackrel{\text{def}}{=} \mathbf{r}_D - \mathbf{r}_E \in R^3 \quad (35)$$

$$\mathbf{e}_o \stackrel{\text{def}}{=} \frac{1}{2} (\mathbf{n}_E \times \mathbf{n}_D + \mathbf{s}_E \times \mathbf{s}_D + \mathbf{a}_E \times \mathbf{a}_D) \in R^3. \quad (36)$$

They are employed in the control scheme of the next sub-section. The orientational deviation,  $\mathbf{e}_o$ , is the same as Luh's introduced in resolved-acceleration control [7].

### 4.2 Control Scheme

We represent a control scheme that feeds back the above-mentioned deviations to the input torque of each joint  $\boldsymbol{\tau}$ , which is given by

$$\boldsymbol{\tau} = k_p \hat{J}_L^T(\mathbf{q}) \mathbf{e}_p + k_o \hat{J}_A^T(\mathbf{q}) \mathbf{e}_o - K_V \dot{\mathbf{q}}, \quad (37)$$

where  $k_p$  and  $k_o$  are the positive scalar gains for the position and orientation respectively, and symmetric positive definite  $K_V \in R^{n \times n}$  is the gain matrix for joint velocities. The block diagram of this scheme is shown in **Fig.3**. In this scheme, the controller can determine easily the inputs torque of each joint from the deviations of the endtip,  $\mathbf{e}_p$  and  $\mathbf{e}_o$ , multiplied by gains  $k_p$  and  $k_o$  and the transpose of the generalized Jacobian  $\hat{J}_L$  and  $\hat{J}_A$ . Moreover the position and attitude of the base is not required to measure, because they are never used in the whole process of deriving the control input.

Since the external sensor is mounted on the free-motion base, the manipulator must track the moving target that is actually floating stationary in inertial space. Generally it is difficult to discuss the global stability of the tracking system of a moving target. In our case, however, we can prove the stability because the seeming

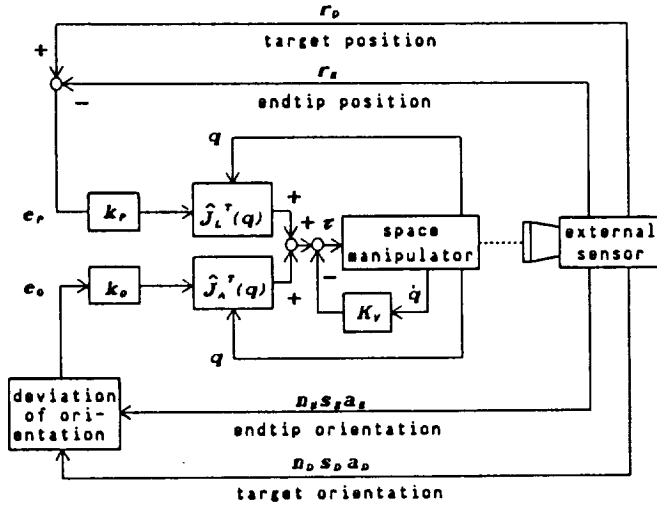


Fig.3: Block diagram of the control scheme

link	base	1	2	3	4	5	6
$m_i$ (kg)	2000	20.00	50.00	50.00	10.000	5.000	5.000
$I_i$ ( $\text{kgm}^2$ )							
$x$	1400	0.15	0.25	0.25	0.050	0.025	0.025
$y$	1400	0.10	26.00	26.00	0.075	0.025	0.025
$z$	2040	0.15	26.00	26.00	0.075	0.019	0.025

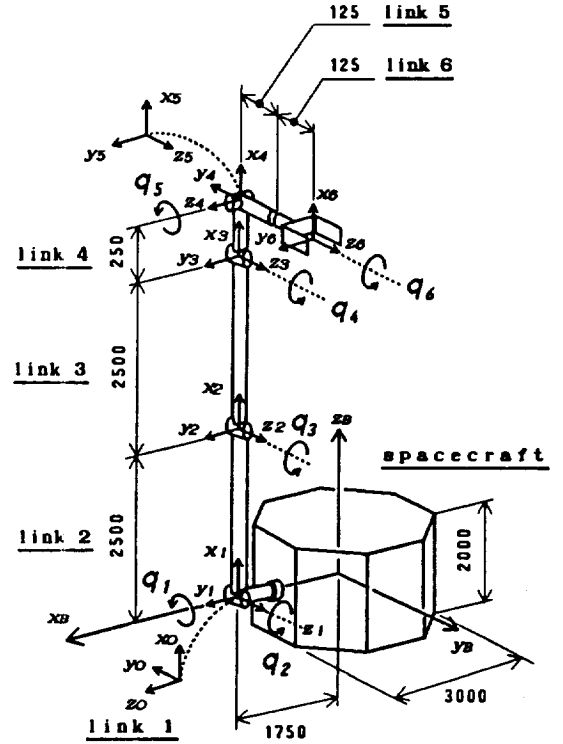


Fig.4: 6-link manipulator mounted on spacecraft

motion of the target object depends on only the motion of joints. If the following conditions hold during the manipulation,

$$\text{rank} \left( \hat{J}_L^T \hat{J}_A^T \right) \geq 6, \quad (38)$$

$$n_E^T n_D + s_E^T s_D + a_E^T a_D > -1, \quad (39)$$

the closed loop system applied this scheme is asymptotically stable. In other words, the position and orientation of the endtip converge to the target as  $t \rightarrow \infty$ . It should be noted that the condition(39) means that the orientation of  $\Sigma_D$  is not diametrically directed to that of  $\Sigma_E$ .

**Proof :** To represent the whole system in our case, the state variable must include of the orientation the base  $\phi$  as well as  $q$  and  $\dot{q}$  of the joint. Then the state variable vector  $z$  is defined as;

$$z \stackrel{\text{def}}{=} [\phi^T \ q^T \ \dot{q}^T]^T \in R^{2n+3}. \quad (40)$$

The closed loop system including the control scheme is written by the differential equation

$$\dot{z} = F(z), \quad (41)$$

where function  $F(z)$  is defined by

$$F(z) \stackrel{\text{def}}{=} \begin{bmatrix} G(\phi, q) \dot{q} \\ \dot{q} \\ \hat{H}^{-1}(q) b(\phi, q, \dot{q}) \end{bmatrix} \in R^{2n+3}, \quad (42)$$

$$G \stackrel{\text{def}}{=} -N^{-1} H_B^{-1} H_M \in R^{3 \times 3}, \quad (43)$$

$$\mathbf{b} \stackrel{\text{def}}{=} -\hat{H} \dot{\mathbf{q}} + \frac{\partial}{\partial \mathbf{q}} \left\{ \frac{1}{2} \dot{\mathbf{q}}^T \hat{H} \dot{\mathbf{q}} \right\} + \boldsymbol{\tau} \in R^n. \quad (44)$$

For this proof, let us introduce another orientational deviation vector;

$$\mathbf{E}_o \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{n}_D - \mathbf{n}_E \\ \mathbf{s}_D - \mathbf{s}_E \\ \mathbf{a}_D - \mathbf{a}_E \end{bmatrix} \in R^9. \quad (45)$$

Now we propose a function of  $\mathbf{z}$

$$W(\mathbf{z}) = \frac{1}{2} k_p \mathbf{e}_p^T \mathbf{e}_p + \frac{1}{4} k_o \mathbf{E}_o^T \mathbf{E}_o + \frac{1}{2} \dot{\mathbf{q}}^T \hat{H} \dot{\mathbf{q}} \quad (46)$$

as a candidate of Liapunov function. This function is zero in the set of desired state

$$\mathcal{E} \stackrel{\text{def}}{=} \{ \mathbf{z} \mid \mathbf{r}_E = \mathbf{r}_D, \mathbf{n}_E = \mathbf{n}_D, \mathbf{s}_E = \mathbf{s}_D, \mathbf{a}_E = \mathbf{a}_D, \dot{\mathbf{q}} = \mathbf{O}_n \}, \quad (47)$$

and is certainly positive except for the set  $\mathcal{E}$ . In space  $\mathbf{z}$ , set  $\mathcal{E}$  consists of not only an isolated point but also a set of connected points.

The time derivative of function  $W$  along the solution trajectory of equation(41) is

$$\dot{W} = -\dot{\mathbf{q}}^T K_V \dot{\mathbf{q}} \leq 0, \quad (48)$$

that is, negative semi-definite in term of state  $\mathbf{z}$ . To evaluate this equation, the following relations and the equation of the generalized Jacobian(24)and(25) are used;

$$\dot{\mathbf{q}}^T \frac{\partial}{\partial \mathbf{q}} \left\{ \frac{1}{2} \dot{\mathbf{q}}^T \hat{H} \dot{\mathbf{q}} \right\} = \frac{1}{2} \dot{\mathbf{q}}^T \dot{\hat{H}} \dot{\mathbf{q}}, \quad (49)$$

$$\dot{\mathbf{e}}_p = \mathbf{V}_E - [\boldsymbol{\Omega}_B \times] \mathbf{e}_p, \quad (50)$$

$$\dot{\mathbf{E}}_o = \begin{pmatrix} [\mathbf{n}_E \times] \\ [\mathbf{s}_E \times] \\ [\mathbf{a}_E \times] \end{pmatrix} \boldsymbol{\Omega}_E - \begin{pmatrix} [\boldsymbol{\Omega}_B \times] & O & O \\ O & [\boldsymbol{\Omega}_B \times] & O \\ O & O & [\boldsymbol{\Omega}_B \times] \end{pmatrix} \mathbf{E}_o, \quad (51)$$

If the time derivative of function  $\dot{W}$  is equal to zero,  $\dot{\mathbf{q}} = \mathbf{O}_n$ , all states in the invariant set satisfy the following equation;

$$\begin{pmatrix} \hat{J}_L^T & \hat{J}_A^T \end{pmatrix} \begin{bmatrix} k_p \mathbf{e}_p \\ k_o \mathbf{e}_o \end{bmatrix} = \mathbf{O}_n, \quad (52)$$

$$\dot{\boldsymbol{\phi}} = \mathbf{O}_3. \quad (53)$$

Condition(38) suggests that equation(52) is equivalent to  $\mathbf{e}_p = \mathbf{O}_3$  and  $\mathbf{e}_o = \mathbf{O}_3$ . Furthermore if condition(39) is satisfied, equation  $\mathbf{e}_o = \mathbf{O}_3$  is equivalent to  $\mathbf{E}_o = \mathbf{O}_9$ . Therefore the maximal invariant set whose entry satisfies  $\dot{W} = 0$  is equal to set  $\mathcal{E}$ . According to the theorem of LaSalle, set  $\mathcal{E}$  of system(41) is asymptotically stable.  $\square$

If conditions(38)and(39) are not satisfied, there are some equilibrium points except for the desired state in set  $\mathcal{E}$ , namely the singular points of Jacobian where the system will get stuck. However, this problem is not serious because the controller can make a temporary input to get the system out of the singular configuration.

Since this scheme is not a trajectory control, the endtip cannot always approach along a line to the target. However, making a virtual target on the line from the current endtip to the real target, the endtip moves nearly on the straight line. To realize this idea, one can use the following deviations  $\bar{\mathbf{e}}_p$  and  $\bar{\mathbf{e}}_o$  instead of  $\mathbf{e}_p$  and  $\mathbf{e}_o$ .

$$\bar{\mathbf{e}}_p \stackrel{\text{def}}{=} \begin{cases} \mathbf{e}_p & \|\mathbf{e}_p\| \leq e_{pmax} \\ \frac{\mathbf{e}_p}{\|\mathbf{e}_p\|} e_{pmax} & \|\mathbf{e}_p\| > e_{pmax} \end{cases} \quad (54)$$

$$\bar{\mathbf{e}}_o \stackrel{\text{def}}{=} \begin{cases} \mathbf{e}_o & \|\mathbf{e}_o\| \leq e_{omax} \\ \frac{\mathbf{e}_o}{\|\mathbf{e}_o\|} e_{omax} & \|\mathbf{e}_o\| > e_{omax} \end{cases} \quad (55)$$

Positive numbers  $e_{pmax}$  and  $e_{omax}$  are the maximal values of the norms of the deviations. This method does not influence the stability of the system.



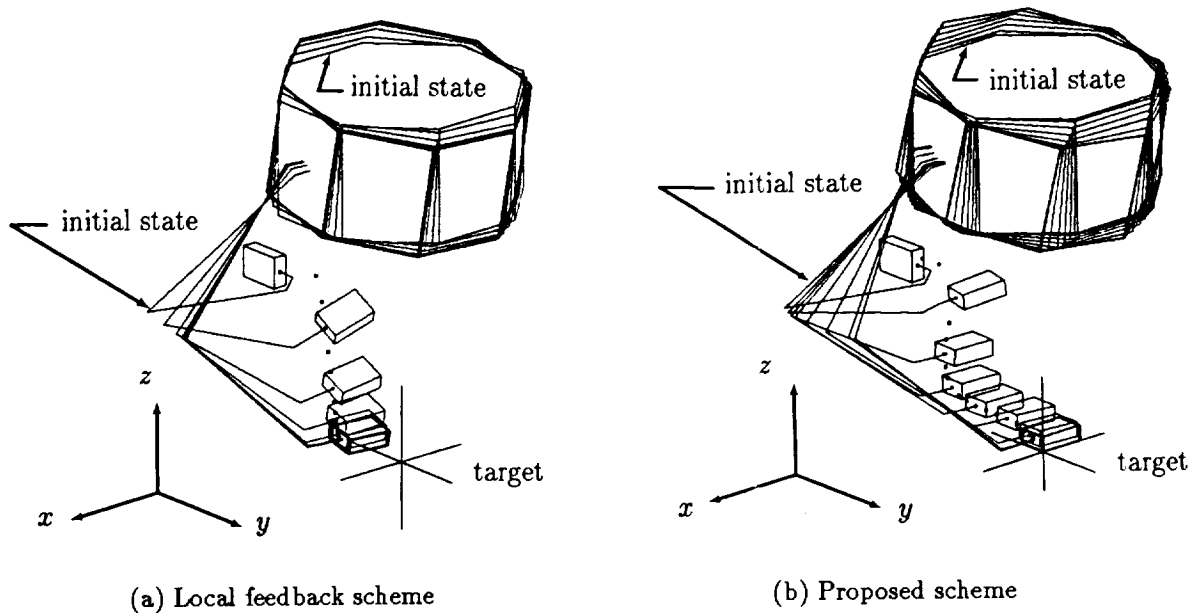


Fig.5: Results of the simulations (state every 3 seconds)

## 5 Computer Simulation

We verify the effectiveness of the control scheme proposed in the previous section through the computer simulation. We use a 140(kg) weight manipulator model mounted on a 2000(kg) weight spacecraft shown in Fig.4, that has six revolute joints.

First a result that suggest the local feedback schemes are not suitable is shown in Fig.5(a). In this figure, the endtip has a box to make its orientation clear. The endtip is positioned at a different point from the target, because the desired joint values are calculated on the assumption that the base is fixed. The inertia of the base in this model is quite large, however, the base's motion is not negligible for the endtip control.

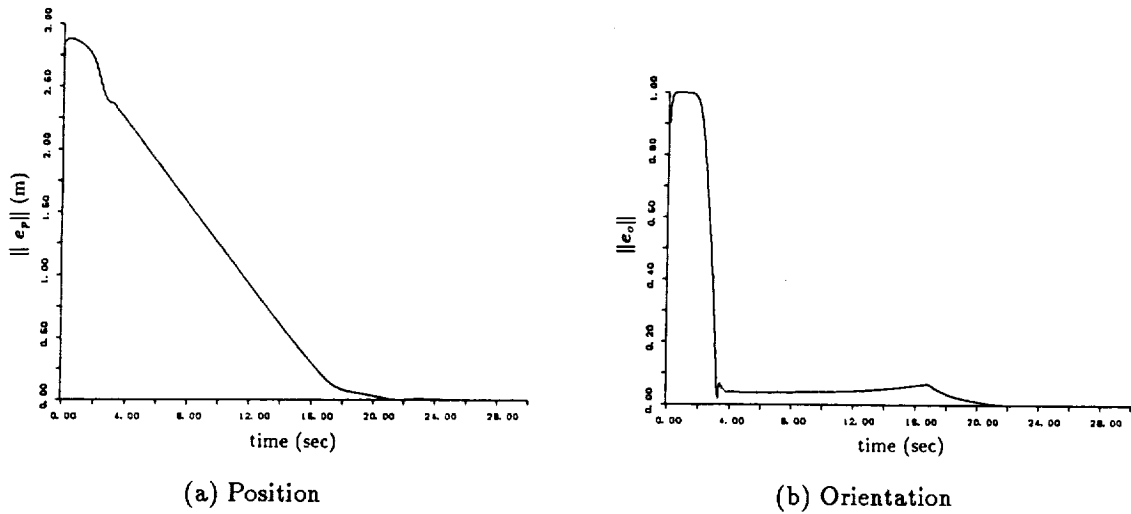
Next we show a result using the proposed scheme in case that the gains and the limits of deviations are given as;

$$\begin{aligned}
 k_p &= 75 \text{ (N)}, \\
 k_o &= 100 \text{ (Nm)}, \\
 K_V &= \text{diag} [400, 100, 200, 10, 2, 2] \text{ (Nmsec/rad)}, \\
 e_{pmax} &= 0.2 \text{ (m)}, \\
 e_{omax} &= 0.1.
 \end{aligned}$$

The state of the system is displayed every 3 seconds in Fig.5(b). Fig.6 represents the time responses of deviations  $\|e_p\|$  and  $\|e_o\|$ . When the endtip reach the target, the translational displacement of the base is  $x$ : 0.01 (m),  $y$ : 0.07 (m),  $z$ : 0.04 (m) and rotational displacement of the base is roll: 16 (deg), pitch: 10 (deg), yaw: 1 (deg). The translational displacement is small, on the other hand, rotational one is so large that have a quite influence on positioning of the endtip. Using the proposed scheme, the position and orientation of the endtip reach the target, nevertheless the base is moved as this results.

## 6 Conclusion

For space manipulators, global control scheme that feed directly information of external sensors back to joints actuators is indispensable. In this paper, we proposed one method to realize them.



**Fig.6:** Time responses of the norm of the endtip deviations

Computer technologies in space are developing slowly in comparison with those on the ground due to the influence of radiant rays and badness of heat radiation. Therefore the simple control scheme as equation(37) that does not burden computers is suited to use in space.

In this paper, we also show the method that makes up the generalized Jacobian from the conventional Jacobian. As shown in this process, obtaining the generalized Jacobian needs much more calculation than the conventional Jacobian. Moreover it needs identification of inertial parameters as well as dimensional parameters. To avoid these obstacles, we are examining the possibility to apply approximately the conventional Jacobians,  $J_L$  and  $J_A$ , in equation(37) instead of the generalized Jacobian,  $\hat{J}_L$  and  $\hat{J}_A$  [8].

## References

- [1] Z. Vafa and Z. Dubowsky. On the dynamics of manipulators in space using the virtual manipulator approach. In *Proceeding of the 1987 IEEE international Conference on Robotics and Automation*, pages 579–585, 1987.
- [2] H.L. Alexander and R.H. Cannon. Experimental on the control of a satellite manipulator. In *Proceeding of American Control Conference*, 1987.
- [3] Y. Umetani and K. Yoshida. Continuous path control of space manipulator mounted on OMV. *Acta Astronautica*, 15(12):981–986, 1987.
- [4] M. Takegaki and S. Arimoto. A new feedback method for dynamic control of manipulators. *ASME Journal of Dynamics, System, Measurement, and Control*, 103:119–125, 1981.
- [5] F. Miyazaki and S. Arimoto. Sensory feedback for robot manipulators. *Journal of Robotic Systems*, 2(1):53–71, 1985.
- [6] H. Asada and J.J.E. Slotine. *Robot Analysis and Control*. Wiley-Interscience, 1985.
- [7] J.Y.S. Luh, M.W. Walker, and P.R.C. Paul. Resolved-acceleration control of mechanical manipulators. *IEEE Transaction on Automatic Control*, 25(3):468–474, 1980.
- [8] F. Miyazaki, Y. Masutani, and S. Arimoto. Sensor feedback using approximate Jacobian. In *Proceedings of the USA-Japan Symposium on Flexible Automation*, pages 139–145, 1988.