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# ON THE MANIPULABILITY OF DUAL COOPERATIVE ROBOTS

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**Abstract**

*In this paper the definition of manipulability ellipsoids for dual robot systems is given. A suitable kineto-static formulation for dual cooperative robots is adopted which allows for a global task space description of external and internal forces as well as absolute and relative velocities. The well-known concepts of force and velocity manipulability ellipsoids for a single robot are formally extended and the contributions of the two single robots to the cooperative system ellipsoids are evidenced. Duality properties are discussed. A practical case study is developed.*

## 1. Introduction

Cooperative robots have been recognized by the robotics research community as offering enhanced capabilities over current single robot structures. Dual robot cooperation allows for performing tasks such as handling large, heavy and non-rigid objects, assembly and mating mechanical parts, which could not be executed by a single robot. Another advantage is the enlargement of the reachable workspace. All the above features play a crucial role, for instance, in space robotics applications where cooperative manipulation is often considered as an essential requirement.

In spite of the potential benefits achievable with dual robots, the control problem becomes more complex due to the kinematic and dynamic interactions. A must for the solution of this kind of problem is constituted by an effective description of the kineto-static and dynamic relationship for a general dual robot system. To this purpose, the formulation proposed by Dauchez and Uchiyama [1] has been shown to be suitable to coordinated control schemes with equal importance attributed to the two robots performing a given task. Their approach is somewhat opposite to the master-slave strategy suggested by Luh and Zheng [2] which has been argued by Uchiyama et al. [3] to be unsuitable for practical position/force control of dual robots.

It is believed that an important issue is the definition of quantitative measures of the enhanced performance offered by dual robot cooperation. It is well-known that the manipulability ellipsoids introduced by Yoshikawa [4] represent one of such measures for a single robot. The contribution of this work is to provide a systematic way of extending the above concept [4] to the dual robot case. In order to accomplish this goal, the formulation dictated in [3] is followed here. The motivation behind this choice is that it leads to a natural, straightforward derivation of manipulability measures which be consistent with those proposed in [4] and susceptible of an immediate physical interpretation for the closed-chain system created by the dual robots tightly handling an object. A similar, parallel research effort has recently been produced by Lee and Bejczy [5], although the definition of manipulability measures for a dual robot system is obtained according to different criteria related to the effect of one robot on the other, instead of regarding the closed-chain as a whole.

A practical case study is worked out for two simple planar cooperative robots. Velocity and force static ellipsoids are obtained which show the correctness and functionality of the proposed approach.

## 2. Kineto-static formulation for two cooperative robots

In the following the formulation of task space coordinates required for describing cooperative tasks is briefly summarized from [2]. For the purpose of the present work, the case when the two robots are rigidly attached to the object is considered, i.e. a rigid grasp.

According to [2], the cooperative task is described in terms of a set of absolute coordinates and a set of relative coordinates. The static relationship between the generalized forces exerted by the two robots and the generalized forces acting on the object — external and internal — is presented first. The kinematic relationship will be derived by using the duality relation between forces and velocities. Fig. 1 illustrates two cooperative robots tightly grasping an object. Let $m$ be the common dimension of the task spaces of the two robots and

$$\mathbf{h}_1 = \begin{pmatrix} \mathbf{f}_1 \\ \mathbf{n}_1 \end{pmatrix} \qquad \mathbf{h}_2 = \begin{pmatrix} \mathbf{f}_2 \\ \mathbf{n}_2 \end{pmatrix} \tag{1}$$

denote the vectors of the generalized forces (forces $\mathbf{f}_1$, $\mathbf{f}_2$ and moments $\mathbf{n}_1$, $\mathbf{n}_2$) exerted by the two end-effectors, respectively. Let then

$$\mathbf{h}_a = \begin{pmatrix} \mathbf{f}_a \\ \mathbf{n}_a \end{pmatrix} \qquad \mathbf{h}_r = \begin{pmatrix} \mathbf{f}_r \\ \mathbf{n}_r \end{pmatrix} \tag{2}$$

denote the vectors of external and internal forces/moments acting on the object, respectively. Let be

$$\mathbf{f} = \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{pmatrix}. \tag{3}$$

It can be shown that

$$\mathbf{h}_a = W\mathbf{f} \tag{4}$$

where

$$W = \begin{pmatrix} I & 0 & I & 0 \\ -R_{1a} & I & -R_{2a} & I \end{pmatrix} \tag{5}$$

with $R_{1a}$, $R_{2a}$ defined by $\mathbf{f}_1 \times \mathbf{r}_{1a} = -R_{1a}\mathbf{f}_1$, $\mathbf{f}_2 \times \mathbf{r}_{2a} = -R_{2a}\mathbf{f}_2$, respectively. $I$ and 0 denote identity and null matrices of appropriate dimensions. Also it is

$$\mathbf{f} = V\mathbf{h}_r \tag{6}$$

where

$$V = \begin{pmatrix} I & 0 \\ R_{1a} & I \\ -I & 0 \\ -R_{2a} & -I \end{pmatrix}. \tag{7}$$

It can be recognized that the mapping $V$ in (7) spans the null space of the mapping $W$ in (5). This means that the external and internal force/moment vectors belong to orthogonal subspaces [6].

Once the static relationship has been established, the differential kinematic relationship is derived in a similar manner. Let

$$\dot{\mathbf{y}}_1 = \begin{pmatrix} \dot{\mathbf{x}}_1 \\ \omega_1 \end{pmatrix} \qquad \dot{\mathbf{y}}_2 = \begin{pmatrix} \dot{\mathbf{x}}_2 \\ \omega_2 \end{pmatrix} \tag{8}$$

denote the vectors of the velocities (translational $\dot{\mathbf{x}}_1$, $\dot{\mathbf{x}}_2$ and rotational $\omega_1$, $\omega_2$) at the two end-effectors, respectively. Let then

$$\dot{\mathbf{y}}_a = \begin{pmatrix} \dot{\mathbf{x}}_a \\ \omega_a \end{pmatrix} \qquad \dot{\mathbf{y}}_r = \begin{pmatrix} \dot{\mathbf{x}}_r \\ \omega_r \end{pmatrix} \tag{9}$$

denote the vectors of external (absolute) and internal (relative) velocities, respectively. Let be

$$\dot{\mathbf{z}} = \begin{pmatrix} \dot{\mathbf{y}}_1 \\ \dot{\mathbf{y}}_2 \end{pmatrix}. \tag{10}$$

In force of the duality relation between forces and velocities which is derived from the principle of virtual work in mechanics, it can be shown that

$$\dot{\mathbf{z}} = W^T \dot{\mathbf{y}}_a \tag{11}$$

and

$$\dot{\mathbf{y}}_r = V^T \dot{\mathbf{z}} \tag{12}$$

with $W$ and $V$ defined in (5) and (7), respectively.

## 3. Definition of manipulability ellipsoids

The idea of measuring the manipulating ability of robotic mechanisms was first introduced in [4]. According to that concept, a force manipulability ellipsoid and a velocity manipulability ellipsoid can be defined for a single robot. Assume that an $n$-DOF robot is given and an $m$-dimensional task space is of interest, usually with $m \leq n$. It is well-known that

$$\tau = J^T(\theta)\gamma \tag{13}$$

represents the static relationship between the task force vector $\gamma$ and the joint torque vector $\tau$ through the transpose of the Jacobian matrix $J(\theta)$, with $\theta$ denoting the joint displacement vector. Dually,

$$\mathbf{v} = J(\theta)\dot{\theta} \tag{14}$$

represents the kinematic relationship between the joint velocity vector $\dot{\theta}$ and the task velocity vector $\mathbf{v}$ through the Jacobian $J(\theta)$.

The unit sphere in the joint torque space

$$\tau^T \tau = 1 \tag{15}$$

maps into the task force space ellipsoid

$$\gamma^T (JJ^T)\gamma = 1 \tag{16}$$

which is called *force manipulability ellipsoid* [4]. Dually, the unit sphere in the joint velocity space

$$\dot{\theta}^T \dot{\theta} = 1 \tag{17}$$

maps into the task velocity space ellipsoid

$$\mathbf{v}^T (JJ^T)^{-1}\mathbf{v} = 1 \tag{18}$$

which is called *velocity manipulability ellipsoid* [4]. Note that the explicit dependence on $\theta$ has been dropped in $J$. A direct comparison of (16) with (18) indicates that the principal axes (related to the eigenvectors) of the two ellipsoids coincide, whilst the lengths of the axes (related to the eigenvalues) are in inverse proportion. This inverse velocity-force relation is consistent with regarding the manipulator as a mechanical transformer [7]. Conservation of energy dictates that amplification in velocity transmission must invariably be accompanied by reduction in force transmission, and vice-versa.

In the following the concepts of force and velocity ellipsoids defined in (16) and (18) are formally extended to a two robot system, based on the kineto-static formulation given in the previous section. Let $n_1$ and $n_2$ be the DOF's of the robots, respectively. The static relationship (13) can be written for a two robot system as

$$\mathbf{t} = J_{12}^T \mathbf{f} \tag{19}$$

with $\mathbf{f}$ defined in (3), where

$$\mathbf{t} = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} \tag{20}$$

denotes the extended joint torque vector in an $(n_1 + n_2)$-dimensional space, and

$$J_{12} = \begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix} \tag{21}$$

denotes the extended Jacobian matrix. Solving eq. (19) for $\mathbf{f}$ yields

$$\mathbf{f} = J_{12}^{T\dagger} \mathbf{t} \tag{22}$$

where the simbol "$\dagger$" denotes a pseudo-inverse of proper dimensions; in this case it is a left pseudo-inverse.

The external force manipulability ellipsoid and the absolute velocity manipulability ellipsoid are derived first. Plugging (22) into (4) gives

$$\mathbf{h}_a = J_a^{T\dagger} \mathbf{t} \tag{23}$$

which expresses the relationship between the extended joint torque vector and the external force vector, through the matrix

$$J_a^{T\dagger} \triangleq W J_{12}^{T\dagger} \tag{24}$$

which is analogous to the pseudo-inverse of the Jacobian $J^T$ in (13) for a single robot. At this point the formal definition of the external force manipulability ellipsoid can be given. The unit sphere in the extended joint torque space

$$\mathbf{t}^T \mathbf{t} = 1 \tag{25}$$

maps into

$$\mathbf{h}_a^T (J_a J_a^T) \mathbf{h}_a = 1 \tag{26}$$

which is defined here as ***external force manipulability ellipsoid***. Dually, the formal definition of the absolute velocity manipulability ellipsoid can be given. The unit sphere in the extended joint velocity space

$$\dot{\mathbf{q}}^T \dot{\mathbf{q}} = 1 \tag{27}$$

maps into

$$\dot{\mathbf{y}}_a^T (J_a J_a^T)^{-1} \dot{\mathbf{y}}_a = 1 \tag{28}$$

which is defined here as ***absolute velocity manipulability ellipsoid***. Notice that in (27)

$$\mathbf{q} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{29}$$

indicates the extended joint vector.

An attractive mathematical expression can be found for the matrix $J_a J_a^T$ constituting the core of the two manipulability ellipsoids just defined in (26) and (28), which is directly related to the Jacobians of the two robots defined in (21). As shown in Appendix A, one may obtain

$$J_a J_a^T = ((\tilde{J}_1 \tilde{J}_1^T)^{-1} + (\tilde{J}_2 \tilde{J}_2^T)^{-1})^{-1} \tag{30}$$

with

$$\tilde{J}_i^{T\dagger} = J_i^{T\dagger} + \begin{pmatrix} 0 \\ -R_{ia}(J_i^{T\dagger})_f \end{pmatrix} \qquad i = 1,2 \tag{31}$$

where the subscript $f$ refers to the upper block of the matrix $J_i^{T\dagger}$ which maps the joint torque vector $\tau_i$ into the sole force components of the task force vector $\mathbf{f}_i$ defined in (1) (i.e. excluding the moment components). It is worth noticing that in the particular case when $R_{ia} = 0$, the matrices $\tilde{J}_i^{T\dagger}$ simplify to $J_i^{T\dagger}$.

In the same formal manner as done above for the external force manipulability ellipsoid and the absolute velocity manipulability ellipsoid, the derivation of the internal force manipulability ellipsoid and the relative velocity manipulability ellipsoid is presented now. Plugging (22) into (6) gives

$$\mathbf{t} = J_r^T \mathbf{h}_r \tag{32}$$

with

$$J_r^T \triangleq J_{12}^T V. \tag{33}$$

It is to be remarked that, by virtue of the definitions (24) and (33) and of the structure of the matrices $W$ and $V$, it results

$$J_a^{T\dagger} J_r^T = 0. \tag{34}$$

The unit sphere (25) maps into

$$\mathbf{h}_r^T (J_r J_r^T) \mathbf{h}_r = 1 \tag{35}$$

which is defined as *internal force manipulability ellipsoid*. Dually, the unit sphere (27) maps into

$$\dot{\mathbf{y}}_r^T (J_r J_r^T)^{-1} \dot{\mathbf{y}}_r = 1 \tag{36}$$

which is defined as *relative velocity manipulability ellipsoid*. In this case too, an attractive mathematical expression can be found for the matrix $J_r J_r^T$ constituting the core of the two manipulability ellipsoids just defined in (35) and (36), which is directly related to the Jacobians of the two robots defined in (21). As shown in Appendix B, one may obtain

$$J_r J_r^T = \hat{J}_1 \hat{J}_1^T + \hat{J}_2 \hat{J}_2^T \tag{37}$$

with

$$\hat{J}_i^T = (-1)^{i-1} J_i^T + ((-1)^{i-1} (J_i^T)_n R_{1a} \quad 0) \qquad i = 1,2 \tag{38}$$

where the subscript $n$ refers to the lower block of the matrix $J_i$ which maps the sole moment components of the task force vector $\mathbf{f}_i$ defined in (1) into the joint torque vector $\tau_i$.

From eq. (34), it directly follows that

$$\tilde{J}_1^{T\dagger} \hat{J}_1^T - \tilde{J}_2^{T\dagger} \hat{J}_2^T = 0. \tag{39}$$

It is worth noticing that in the particular case when $R_{ia} = 0$, the matrices $\hat{J}_i^T$ simplify to $J_i^T$; in this case, thus, eq. (39) trivially holds.

Eqs. (30) and (37) suggest a nice interpretation of the way the Jacobians of the two robots combine to form the respective cores of the ellipsoids defined above. If each term of the type $JJ^T$ is regarded as a generalized impedance, eq. (30) resembles the mathematical expression of the parallel of two impedances, whilst eq. (37) resembles that of the series of two impedances. Therefore, one would naturally be driven to generalize these results to the multiple robot case; this topic is under investigation.

## 4. Case study

Two 3-DOF planar robots are considered for the purpose of illustrating the application of the concepts presented in this work to a practical two robot system. For the sake of simplicity, the end-effectors of the two robots are supposed to be located in the same point (i.e. the physical object is removed); this implies that $\tilde{J}_i = \bar{J}_i = J_i$. This assumption is not restrictive at all, as formally shown above. Moreover, a two-dimensional global task space is assumed, i.e. only forces and linear velocities are of interest; the system thus possesses two redundant DOF's.

A CAD tool has been developed which is articulated into the following steps. The contact point of the two end-effectors is input, then the two redundant DOF's are exploited to assign the orientation angles of the end-effectors. A software package for solving the inverse kinematics of general robot structures [8] is utilized to find the joint configurations and then the complete kineto-static characterization of the system. An option is provided to compute the ellipsoid of interest. The outputs are plotted by means of a graphic package. They illustrate the closed kinematic chain together with the principal axes of the ellipsoids of the two single robots and those of the dual robot system, in order that the manipulability of the cooperative system can be evaluated with respect to the single robot manipulabilities.

Two complete sets of results for two different configurations of the dual robot system are displayed in Figs. 2 and 3 respectively. The ellipsoids of each robot are included for a better comprehension of the effects of the cooperation. It is remarkable that, in the second configuration, the two robots are both proximal to singular configurations (see the shape of their ellipsoids). It can be recognized that the external force ellipsoids (Figs. 2a and 3a) are improved in that the ability of each robot to exert forces along a given direction is enhanced by the other, while the absolute velocity ellipsoids (Figs. 2b and 3b) show that the ability of each robot to perform motions along a given direction is penalized by the presence of the other. This result well agrees with practice, since it is intuitive that when two robots cooperate the static force is shared by them whereas the faster robot is slowed down by the other. Conversely, the ability of the system to absorbe forces along a direction is limited by the weaker robot of the chain (Figs. 2c and 3c), while the ability of the system to give rise to relative motions along a direction is supplied by both robots (Figs. 2d and 3d). All these conclusions reflect the concept of duality which is at the basis of the definition of the manipulability ellipsoids presented.

## 5. Conclusions

The concept of manipulability ellipsoids has formally been extended to the case of dual robot systems. A global kineto-static formulation of the closed chain created by two tightly cooperating robots has been exploited to define external and internal force manipulability ellipsoids. The corresponding absolute and relative velocity manipulability ellipsoids have been derived on the basis of the duality principle in mechanics. Functional expressions for these ellipsoids have been obtained through the Jacobians of the two robots and a practical rule of composition has been provided. The results achieved for a simple case study have validated the theoretical conclusions in view of the physical interpretation of the kineto-statics of a dual robot system. It has been conjectured that the proposed definitions can be extended to the multi-robot case, although the formulation of internal forces is not straightforward. This issue, along with the analysis of different types of cooperation (e.g. loose, soft) and the formulation of dynamic manipulability ellipsoids, will constitute the subject of further investigation.

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## Appendix A

The matrix ${J_a^T}^\dagger$ defined in (24), by substituting the expressions of $W$ in (5) and $J_{12}$ in (21), becomes

$$ {J_a^T}^\dagger = \begin{pmatrix} I & 0 & I & 0 \\ -R_{1a} & I & -R_{1a} & I \end{pmatrix} \begin{pmatrix} {J_1^T}^\dagger & 0 \\ 0 & {J_2^T}^\dagger \end{pmatrix}. \tag{A–1} $$

The matrices ${J_i^T}^\dagger$ can be partitioned by rows as

$$ {J_i^T}^\dagger = \begin{pmatrix} ({J_i^T}^\dagger)_f \\ ({J_i^T}^\dagger)_n \end{pmatrix} \qquad i = 1,2 \tag{A–2} $$

where the subscripts $f$ and $n$ refer to forces and moments respectively. Plugging (A–2) in (A–1) gives

$$ {J_a^T}^\dagger = \begin{pmatrix} {J_1^T}^\dagger & {J_2^T}^\dagger \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -R_{1a}({J_1^T}^\dagger)_f & -R_{2a}({J_2^T}^\dagger)_f \end{pmatrix} \tag{A–3} $$

which, by virtue of (31), can be compactly written as

$$ {J_a^T}^\dagger = \begin{pmatrix} {\tilde{J}_1^T}^\dagger & {\tilde{J}_2^T}^\dagger \end{pmatrix}. \tag{A–4} $$

The matrix $J_a J_a^T$ in (26) can then be computed. First, one obtains

$$ J_a^T = \begin{pmatrix} J_1^\dagger({J_1^T}^\dagger J_1^\dagger + {J_2^T}^\dagger J_2^\dagger)^{-1} \\ J_2^\dagger({J_1^T}^\dagger J_1^\dagger + {J_2^T}^\dagger J_2^\dagger)^{-1} \end{pmatrix} \tag{A–5} $$

where the "$\tilde{}$"'s have been dropped without loss of generality. Eq. (A–5) leads to

$$J_a J_a^T = (J_1^{T\dagger} J_1^\dagger + J_2^{T\dagger} J_2^\dagger)^{-T} J_1^{T\dagger} J_1^\dagger (J_1^{T\dagger} J_1^\dagger + J_2^{T\dagger} J_2^\dagger)^{-1}$$
$$+ (J_1^{T\dagger} J_1^\dagger + J_2^{T\dagger} J_2^\dagger)^{-T} J_2^{T\dagger} J_2^\dagger (J_1^{T\dagger} J_1^\dagger + J_2^{T\dagger} J_2^\dagger)^{-1} \quad \text{(A–6)}$$

that can be compacted into

$$J_a J_a^T = (J_1^{T\dagger} J_1^\dagger + J_2^{T\dagger} J_2^\dagger)^{-T}. \quad \text{(A–7)}$$

By virtue of the property $J_i^{T\dagger} J_i^\dagger = (J_i J_i^T)^{-1}$, eq. (A–7) directly leads to (30).

## Appendix B

The matrix $J_r^T$ defined in (33), by substituting the expression of $J_{12}$ in (21) and $V$ in (7), becomes

$$J_r^T = \begin{pmatrix} J_1^T & 0 \\ 0 & J_2^T \end{pmatrix} \begin{pmatrix} I & 0 \\ R_{1a} & I \\ -I & 0 \\ -R_{2a} & -I \end{pmatrix}. \quad \text{(B–1)}$$

The matrices $J_i^T$ can be partitioned by columns as

$$J_i^T = ((J_i^T)_f \quad (J_i^T)_n) \qquad i = 1,2. \quad \text{(B–2)}$$

Plugging (B–2) in (B–1) gives

$$J_r^T = \begin{pmatrix} J_1^T \\ -J_2^T \end{pmatrix} + \begin{pmatrix} 0 & (J_1^T)_n R_{1a} \\ 0 & -(J_2^T)_n R_{2a} \end{pmatrix} \quad \text{(B–3)}$$

which, by virtue of (38), can be compactly written as

$$J_r^T = \begin{pmatrix} \hat{J}_1^T \\ -\hat{J}_2^T \end{pmatrix}. \quad \text{(B–4)}$$

Computing the matrix $J_r J_r^T$ in (35) directly leads to (37).

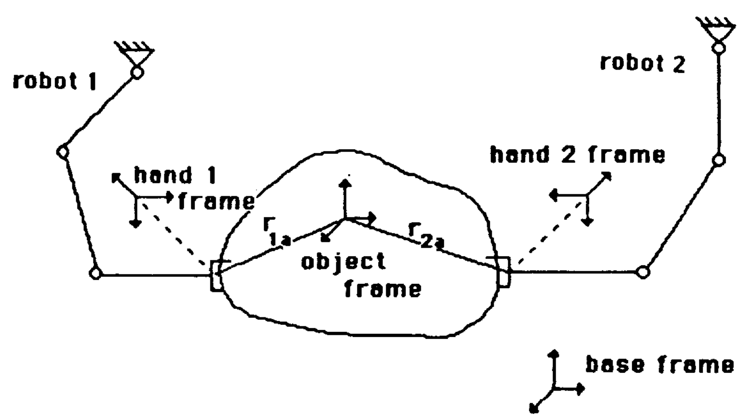


Fig. 1 – A dual cooperative robot system

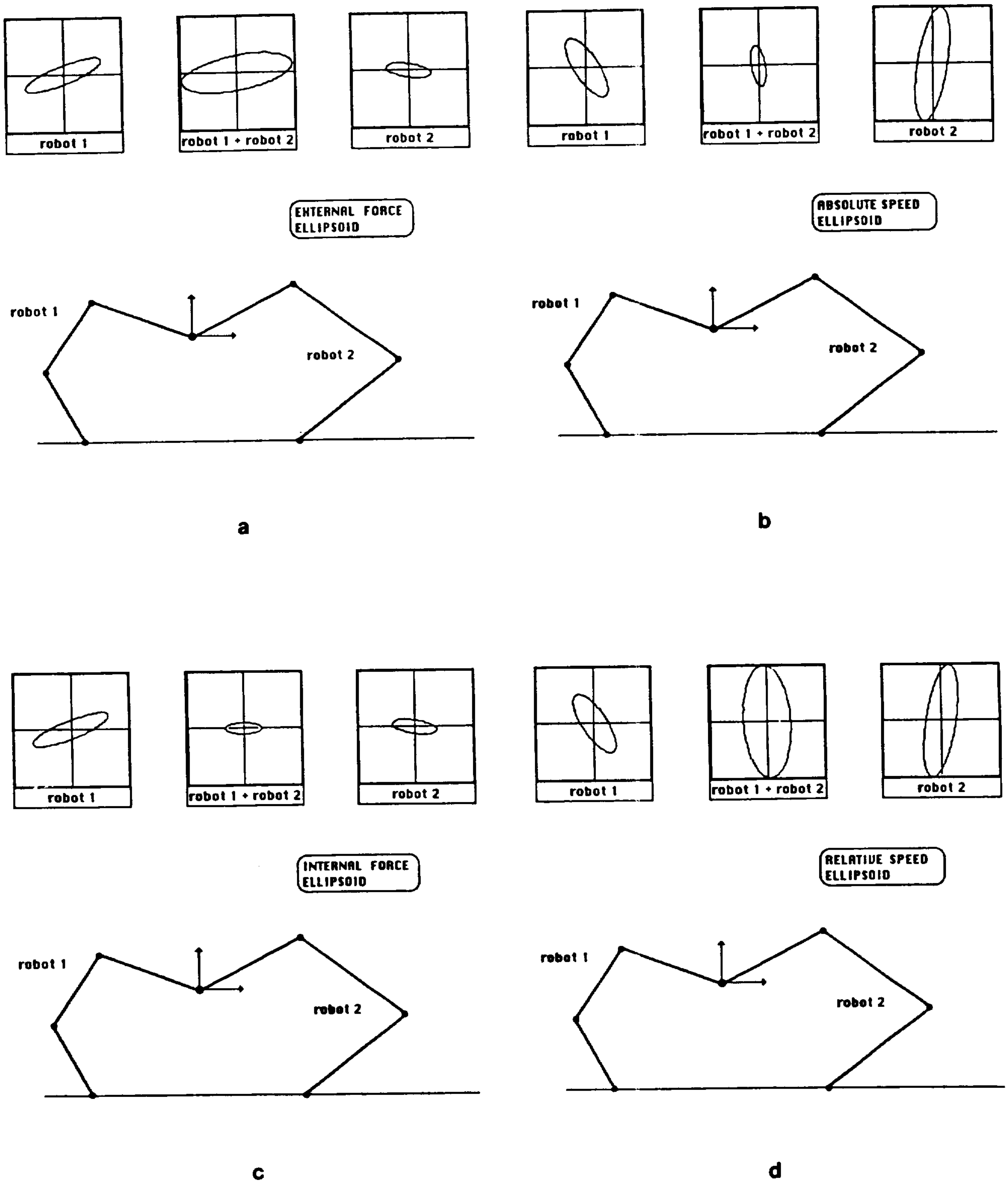


Fig. 2 – Manipulability ellipsoids for a first configuration

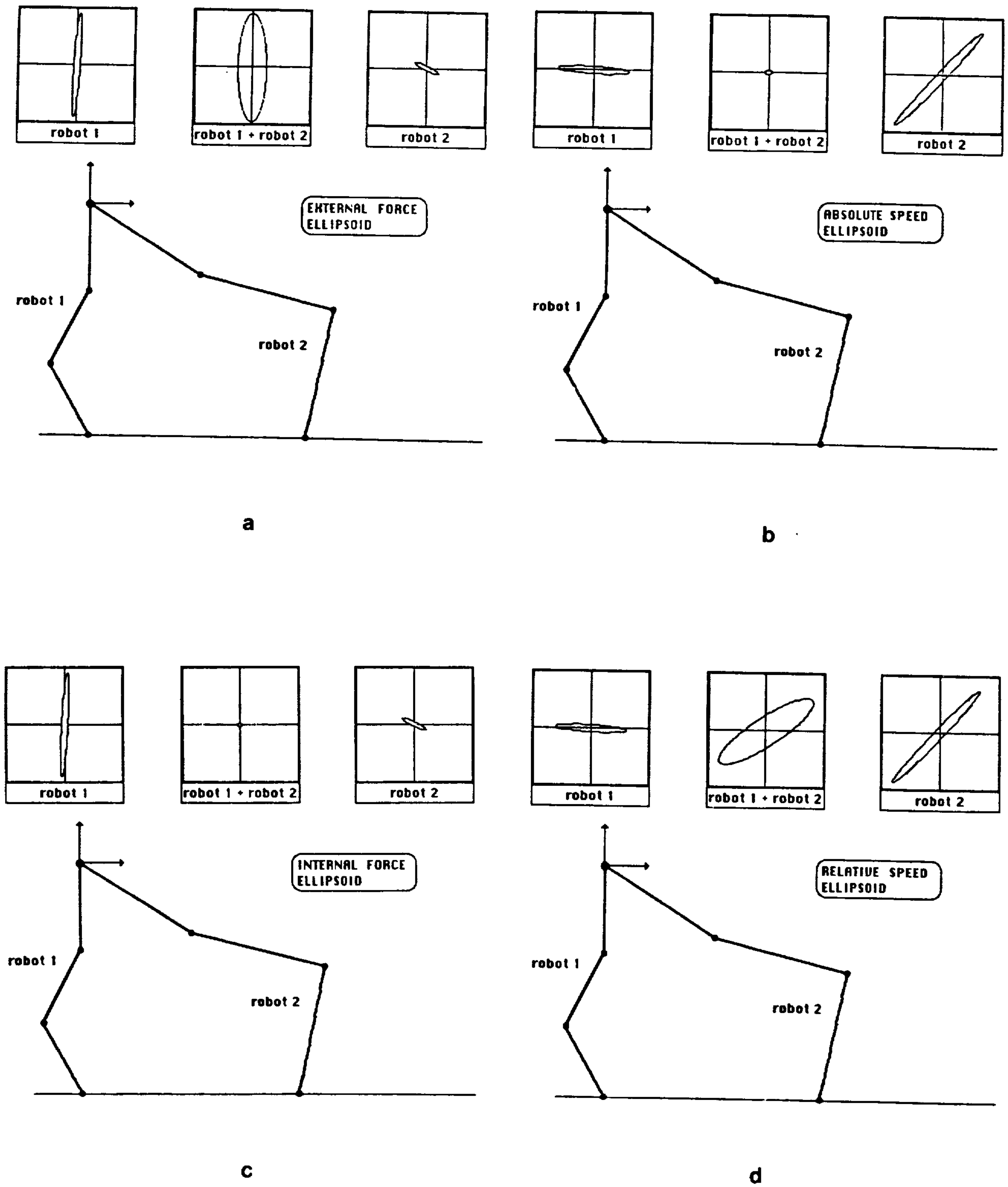


Fig. 3 – Manipulability ellipsoids for a second configuration