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DISCRETE-TIME ADAPTIVE CONTROL OF ROBOT MANIPULATORS

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Abstract

A discrete-time model reference adaptive control scheme is developed for trajectory tracking of robot manipulators. Hyperstability theory is utilized to derive the adaptation laws for the controller gain matrices. It is shown that asymptotic trajectory tracking is achieved despite gross robot parameter variation and uncertainties. The method offers considerable design flexibility and enables the designer to improve the performance of the control system by adjusting free design parameters. The discrete-time adaptation algorithm is extremely simple and is therefore suitable for real-time implementation.

1. Introduction

It is recognized that adaptive schemes are effective means of robot control due to their ability to cope with the highly nonlinear, coupled and time-varying characteristics of robots. This is specially true in the case of direct drive robots and light weight manipulators where inertia changes and gravity effects are significant. Research efforts on adaptive control of manipulators have been concentrated on developing continuous-time control schemes [e.g. 1-9]. In practice however, robots are controlled by digital computers on discrete-time basis. Digital implementation of a solution based on continuous-time formulation can result in degradation of performance and the closed-loop system can even become unstable, especially when the sampling time is not small. Even if the sampling time could be made sufficiently small, digital implementation of a discrete-time adaptive scheme is more direct and straightforward.

In this paper, we develop a discrete-time model reference adaptive control scheme for trajectory tracking of robot manipulators. The present approach differs from the previously published results [e.g. 10-12] in that the discrete-time adaptive control is developed on the basis of a general coupled robot model, without linearizing the model or assuming negligible interactions among robot joints. Furthermore, instead of the conventional Lyapunov approach, hyperstability theory is utilized to obtain the adaptation laws. The use of hyperstability theory is more appealing than the Lyapunov approach since it is better suited to discrete-time systems and also offers more flexibility in design by providing additional free design parameters. These parameters can be adjusted by the designer to improve the response. Finally, the proposed discrete-time adaptive control algorithm is extremely simple and computationally fast, and is therefore suitable for real time digital control of robot manipulators.

2. Discrete-Time Robot Model

The equation of motion of an n-joint robot manipulator carrying a payload of mass m can be written as [4,9]

$$B_2(\theta,\dot{\theta},m)\,\ddot{\theta} + B_1(\theta,\dot{\theta},m)\,\dot{\theta} + B_0(\theta,\dot{\theta},m)\,\theta(t) = u(t) \tag{1}$$

where $\theta(t)$ and $u(t)$ are the $n\times 1$ joint angle and joint torque vectors respectively, and $B_2(.)$, $B_1(.)$ and $B_0(.)$ are $n\times n$ matrices whose elements are complex nonlinear functions of $\theta(t)$, $\dot{\theta}(t)$ and $m(t)$. Since $\theta(t)$, $\dot{\theta}(t)$ and $m(t)$ are functions of time, (1) can be expressed as

$$B_2(t)\,\ddot{\theta}(t) + B_1(t)\,\dot{\theta}(t) + B_0(t)\,\theta(t) = u(t) \tag{2}$$

where $B_2(t) \equiv B_2(\theta,\dot{\theta},m)$, $B_1(t) \equiv B_1(\theta,\dot{\theta},m)$ and $B_0(t) \equiv B_0(\theta,\dot{\theta},m)$ are $n\times n$ time- varying robot matrices.

Suppose that the robot is controlled by a digital controller. The inputs to the controller are the reference trajectory represented by the $n\times 1$ vector $\theta_r(k)$ and the actual joint angle vector $\theta(k)$, where $\theta_r(k)$ and $\theta(k)$ are obtained by sampling $\theta_r(t)$ and $\theta(t)$ at equally spaced time intervals T. The output of the digital controller is the vector $u(k)$, and is passed through a hold circuit to obtain the continuous-time signal $u(t)$ where $u(t)$ is constant over the time interval $(k-1)T \le t \le kT$. In order to obtain the equation relating $\theta(k)$ and $u(k)$, we must discretize the robot model (2). A simple method of discretization is by using the approximations

$$\dot{\theta}(t) \approx \frac{1}{T}\Big[\theta(k)-\theta(k-1)\Big] \;\; ; \;\; \ddot{\theta}(t) = \frac{d}{dt}\dot{\theta}(t) \approx \frac{1}{T^2}\Big[\theta(k)-2\theta(k-1)+\theta(k-2)\Big] \tag{3}$$

Substituting (3) into (2), we obtain

$$A_2(k,T)\,\theta(k-2) + A_1(k,T)\,\theta(k-1) + A_0(k,T)\,\theta(k) = u(k) \tag{4}$$

where $A_2(k,T) = \dfrac{B_2(k)}{T^2}$, $A_1(k,T) = \dfrac{B_1(k)}{T} - \dfrac{2B_2(k)}{T^2}$ and $A_0(k,T) = B_0(k) - \dfrac{B_1(k)}{T} + \dfrac{B_2(k)}{T^2}$ are $n\times n$ matrices, and $B_2(k)$, $B_1(k)$, $B_0(k)$ are the values of $B_2(t)$, $B_1(t)$, $B_0(t)$ respectively, evaluated at time $t=kT$. Note that $A_2(k,T)$ is a symmetric positive definite (SPD) matrix since $B_2(t)$ is always SPD [13]. Equation (4) is an accurate discrete-time representation of (1) provided that T is sufficiently small so that (3) can be used.

For exact discretization, we must find the response $\theta(t)$ of the continuous model (2) at time $t=kT$ and equate it with the response $\theta(k)$ of the discrete model (4), [14]. This will ensure that the two models describe the same robot motion at the sampling times $t=kT$, $k=0,1,2,\ldots$. Although this procedure provides structural information about the robot discrete-time model, it is extremely complex and will not be pursued here.

In the analysis to follow, we assume that the equation of the robot with a sampler in its output and a hold circuit in its input can be described by the discrete-time model (4), where $A_0(k,T)$ is invertible and the robot matrices are unknown. Since the sampling period is constant , we drop it for convenience and write (4) as

$$A_2(k)\,\theta(k-2) + A_1(k)\,\theta(k-1) + A_0(k)\,\theta(k) = u(k) \tag{5}$$

3. Adaptive Control Scheme

In this section, we describe a method for the design of discrete-time adaptive controllers for the robot model (5) such that the robot joint angle vector $\theta(k)$ tracks the reference trajectory vector $\theta_r(k)$ despite variations in the payload and unknown robot model parameters.

Let the $n \times 1$ joint angle error vector be defined as

$$\theta_e(k) = \theta_r(k) - \theta(k) \tag{6}$$

Substituting (6) into (5), we obtain the equation of the joint angle error as

$$\theta_e(k) = A_0^{-1}\Big[-u(k) - A_1(k)\,\theta_e(k-1) - A_2(k)\,\theta_e(k-2) + A_0(k)\,\theta_r(k) + A_1(k)\,\theta_r(k-1) + A_2(k)\,\theta_r(k-2)\Big] \tag{7}$$

Equation (7) suggests that in order to completely influence the joint angle error, we require a control law of the general form

$$u(k) = P_1(k)\,\theta_e(k-1) + P_2(k)\,\theta_e(k-2) + Q_0(k)\,\theta_r(k) + Q_1(k)\,\theta_r(k-1) + Q_2(k)\,\theta_r(k-2) \tag{8}$$

where $P_1(k), P_2(k)$ are time-varying feedback matrices acting on the joint angle error, and $Q_0(k), Q_1(k), Q_2(k)$ are time-varying feedforward matrices acting on the reference trajectory, all to be determined. Note that the discrete-time control law (8) is analogous to the continuous-time control law using position-velocity feedback and position-velocity-acceleration feedforward [9].

Substituting (8) into (7), we obtain the joint angle error equation for the closed-loop system

$$\theta_e(k) + A_0^{-1}\Big[P_1(k) + A_1(k)\Big]\,\theta_e(k-1) + A_0^{-1}\Big[P_2(k) + A_2(k)\Big]\,\theta_e(k-2) \tag{9}$$

$$= A_0^{-1}\Big[A_0(k) - Q_0(k)\Big]\,\theta_r(k) + A_0^{-1}\Big[A_1(k) - Q_1(k)\Big]\,\theta_r(k-1) + A_0^{-1}\Big[A_2(k) - Q_2(k)\Big]\,\theta_r(k-2)$$

Suppose that the desired performance of the manipulator is represented by

$$\theta_{em}(k) + C_1\theta_{em}(k-1) + C_2\theta_{em}(k-2) = 0 \tag{10}$$

where $\theta_{em}(k)$ is the $n \times 1$ joint angle error vector of the reference model and C_1, C_2 are constant $n \times n$ matrices chosen such that joint angle errors are decoupled and decay with time. In the model reference adaptive control terminology [15], equations (9) and (10) describe the adjustable system and the reference model, respectively. For decoupling of the joint errors , we choose $C_1 = diag\{c_{1i}\}$ and $C_2 = diag\{c_{2i}\}$, $i=1,2,\ldots,n$. In order that the errors decay to zero, the roots λ_{1i} , λ_{2i} of the characteristic polynomial $\Delta(z)$ of the reference model (10) must lie inside the unit circle in the complex z-plane, where

$$\Delta(z) = \Big| I_n z^2 + C_1 z + C_2 \Big| = \prod_{i=1}^{n} \delta_i(z) \tag{11a}$$

and

$$\delta_i(z) = z^2 + c_{1i} z + c_{2i} = (z+\lambda_{1i})(z+\lambda_{2i}) \tag{11b}$$

Thus the diagonal elements of the matrices C_1 and C_2 are

$$c_{1i} = \lambda_{1i} + \lambda_{2i} \quad ; \quad c_{2i} = \lambda_{1i}\lambda_{2i} \quad , \; i=1,2,\ldots,n \tag{11c}$$

where $|\lambda_{1i}| < 1$, $|\lambda_{2i}| < 1$ for the stability of the reference model.

The solution to (10) is

$$\theta_{em}(k) = \Phi_m(k)\,\theta_{em}(0) \tag{12}$$

where $\Phi_m(k)$ is the transition matrix of the reference model (10) and $\theta_{em}(0)$ is the initial value of the reference model. If $\theta_{em}(0)$ is chosen to be zero, $\theta_{em}(k)$ becomes identically equal to zero, i.e. $\theta_{em}(k) \equiv 0$ for all $k \geq 0$, due to the stability of the reference model. The objective is now to devise an adaptation scheme such that the robot joint angle error dynamics $\theta_e(k)$ governed by (9) approaches that of the reference model dynamics (10) in which $\theta_{em}(k)\equiv 0$. In order to achieve this objective, we define the deviation between the ideal and the actual errors as

$$\varepsilon(k) = \theta_{em}(k) - \theta_e(k) \tag{13}$$

Combining (9), (10) and (13), we obtain

$$\varepsilon(k) + C_1\,\varepsilon(k-1) + C_2\,\varepsilon(k-2) + w(k) = 0 \tag{14a}$$

where

$$\begin{aligned} w(k) = &\left[C_1 - A_0^{-1}(A_1(k)+P_1(k))\right]\theta_e(k-1) + \left[C_2 - A_0^{-1}(A_2(k)+P_2(k))\right]\theta_e(k-2) \qquad (14b)\\ &+A_0^{-1}\left[A_0(k)-Q_0(k)\right]\theta_r(k) + A_0^{-1}\left[A_1(k)-Q_1(k)\right]\theta_r(k-1) + A_0^{-1}\left[A_2(k)-Q_2(k)\right]\theta_r(k-2)\\ &\equiv w_1(k) + w_2(k) + \cdots + w_5(k) \end{aligned}$$

The adaptation problem is to find the feedback gain matrices $P_1(k), P_2(k)$ and the feedforward gain matrices $Q_0(k), Q_1(k), Q_2(k)$ such that the adaptation error dynamic (14) is stable, i.e. $\varepsilon(k)$ approaches zero asymptotically. If this is achieved, the joint angle error vector $\theta_e(k)$ becomes equal to the reference model error vector $\theta_{em}(k)\equiv 0$, implying that $\theta_e(k)=0$, hence $\theta(k)=\theta_r(k)$ and trajectory tracking occurs.

The state-space representation of (14) is

$$\begin{bmatrix}\varepsilon(k-1)\\ \varepsilon(k)\end{bmatrix} = \begin{bmatrix}0 & I_n\\ -C_2 & -C_1\end{bmatrix}\begin{bmatrix}\varepsilon(k-2)\\ \varepsilon(k-1)\end{bmatrix} - \begin{bmatrix}0\\ I_n\end{bmatrix} w(k) \tag{15}$$

Now consider the adaptation algorithm

$$v(k) = D\begin{bmatrix}\varepsilon(k-1)\\ \varepsilon(k)\end{bmatrix} \tag{16}$$

$$w(k)=\Psi_1(v,\theta_e)\theta_e(k-1)+\Psi_2(v,\theta_e)\theta_e(k-2)+\Psi_3(v,\theta_r)\theta_r(k)+\Psi_4(v,\theta_r)\theta_r(k-1)+\Psi_5(v,\theta_r)\theta_r(k-2) \tag{17}$$

where $v(k)$ is an $n\times 1$ vector, D is a constant $n\times 2n$ matrix to be determined, and $\Psi_1(v,\theta_e), \ldots, \Psi_5(v,\theta_r)$ are $n\times n$ matrices, also to be determined. In order to ensure that the adaptation dynamics described by (15)-(17) is stable so that the adaptation error approaches zero asymptotically, we utilize the Popov hyperstability theory. This theory requires that the dynamic equations of the adaptation process be arranged in a feedback configuration. The forward block must contain only linear time-invariant dynamic equations while the feedback block can contain nonlinear time-varying dynamic equations. In the robot control problem under consideration, the forward block has the input $w(k)$, the output $v(k)$ and is described by (15)-(16). The nonlinear feedback block is described by (17).

According to the hyperstability theory, the adaptation algorithm (15)-(17) is stable in the sense that $\lim\limits_{k\to\infty}\begin{bmatrix}\varepsilon(k-1)\\ \varepsilon(k)\end{bmatrix} = 0$ if the following two conditions are satisfied:

Condition 1 : The transfer function matrix of the forward block $H(z) = z\,D\,(zI_{2n} - C)^{-1}B$ is strictly positive real (SPR), where $C = \begin{bmatrix}0 & I_n\\ -C_2 & -C_1\end{bmatrix}$ and $B = \begin{bmatrix}0\\ I_n\end{bmatrix}$.

Condition 2 : The input-output of the feedback block satisfies the inequality $\sum_{k=0}^{k_1} v^T(k)\, w(k) \geq -\gamma^2$ for all k_1, where γ is an arbitrary finite constant and the superscript T denotes the transposition.

Using proportional plus integral type adaptation for the gain matrices, it is shown in the appendix that the following algorithm that satisfies conditions 1 and 2

$$P_1(k) = P_1(k-1) + \hat{\theta}_e(k)\, \theta_e^T(k-1) E_{1P} + \hat{\theta}_e(k-1)\, \theta_e^T(k-2)[E_{1I} - E_{1P}] \tag{18a}$$

$$P_2(k) = P_2(k-1) + \hat{\theta}_e(k)\, \theta_e^T(k-2) E_{2P} + \hat{\theta}_e(k-1)\, \theta_e^T(k-3)[E_{2I} - E_{2P}] \tag{18b}$$

$$Q_0(k) = Q_0(k-1) + \hat{\theta}_e(k)\, \theta_r^T(k) F_{0P} + \hat{\theta}_e(k-1)\, \theta_r^T(k-1)[F_{0I} - F_{0P}] \tag{18c}$$

$$Q_1(k) = Q_1(k-1) + \hat{\theta}_e(k)\, \theta_r^T(k-1) F_{1P} + \hat{\theta}_e(k-1)\, \theta_r^T(k-2)[F_{1I} - F_{1P}] \tag{18d}$$

$$Q_2(k) = Q_2(k-1) + \hat{\theta}_e(k)\, \theta_r^T(k-2) F_{2P} + \hat{\theta}_e(k-1)\, \theta_r^T(k-3)[F_{2I} - F_{2P}] \tag{18e}$$

and

$$\hat{\theta}_e(k) = R_2\, \theta_e(k-1) + R_3\, \theta_e(k) \tag{18f}$$

where $E_{0P}, E_{0I}, \ldots, F_{2P}$ and F_{2I} are SPD adaptation gain matrices and the subscripts P and I denote proportional and integral parts, respectively. R_2 and R_3 are $n \times n$ diagonal matrices whose diagonal elements r_{2i} and r_{3i} are obtained from

$$r_{2i} = \alpha_i\, \lambda_{1i}\, \lambda_{2i}\, (\lambda_{1i} + \lambda_{2i}) \tag{19a}$$

$$r_{3i} = \alpha_i\, (1 + \lambda_{1i}\, \lambda_{2i}) \qquad i = 1, 2, \ldots, n \tag{19b}$$

where α_i are positive constants and $\lambda_{1i}, \lambda_{2i}$ are the eigenvalues of the error reference model chosen such that $|\lambda_{1i}| < 1, |\lambda_{2i}| < 1$, as explained before. Note that the feedback gains depend only on the joint angle error vector, whereas the feedforward gains depend both on the joint angle vector and the reference trajectory vector. A block diagram of the adaptive control scheme is shown in Figure 1.

Equations (8), (18) and (19) constitute the adaptation control algorithm. The SPD matrices $E_{0P}, E_{0I}, \ldots, F_{2P}, F_{2I}$, the positive scalars α_i and the eigenvalues $\lambda_{1i}, \lambda_{2i}$ must be specified by the designer. A simple structure for the above matrices is the diagonal structure. Furthermore, a particularly simple expression for $\hat{\theta}_e(k)$ is obtained if the eigenvalues of the reference model are $\lambda_{1i} = \lambda_{2i} = 0$, $i = 1, 2, \ldots, n$. This corresponds to the so called "dead-beat control", and in this case (18f) simplies to

$$\hat{\theta}_e(k) = R_3 \theta_e(k) \quad ; \quad R_3 = diag\{\alpha_i\} \tag{20}$$

Larger values of the elements of the matrices $E_{0P}, \ldots, F_{2I}$ and the scalars α_i correspond to higher adaptation gains and make the errors decay faster. However, if unmodeled dynamics are present, high adaptation gains can excite unmodeled dynamics, resulting in instability. Thus the design parameters must be selected based on a compromise between speed of adaptation and stability considerations.

It is seen that the adaptive control laws given by (8), (18) and (19) are extremely simple and are suitable for real time control. Furthermore the complex robot dynamics or robot parameters are not required for the generation of control torques. The adaptation algorithm ensures that the closed-loop system remains stable and that trajectory tracking occurs provided the rate of adaptation of controller gains is higher than the rate of change of robot matrices. For example, the matrices of many industrial robots do not change appreciably over time intervals of about ten milliseconds, in which case the controller adaptation time can be a few milliseconds.

5. Conclusions

An adaptive control scheme is developed using a general discrete-time model of robot manipulators. The control scheme utilizes only joint position-velocity measurements and the reference position, and does not require knowledge of the payload or the robot characteristics. The adaptation laws are derived using hyperstability theory which guarantees asymptotic trajectory tracking despite gross robot parameter variations. The controller gains are independent of the robot parameters provided that the gain adaptation is sufficiently fast.

The method offers considerable flexibility in design by providing many free design parameters. These parameters can be adjusted by the designer to improve the response and to increase the speed of adaptation. The discrete-time adaptive control algorithm is extremely simple and computationally fast, and is therefore suitable for real time digital control of robot manipulators. Extensive computer simulation studies using a model of a direct drive manipulator have shown that the discrete-time adaptive scheme performs satisfactorily despite gross payload variations and unknown robot parameters.

References

1. Dubowsky, S. and DesForges, D.T., "The application of model reference adaptive control to robot manipulators", ASME Journal of Dynamics Systems, Measurement and Control, Vol. 101, 193-200, 1979.

2. Takegaki, M., and Arimoto, S., "An adaptive trajectory control of manipulators", Int. Journal of Control, Vol. 34, No. 2, 219-230, 1981.

3. Kim, B.K., and Shin, K.G., "An adaptive model following control of industrial manipulators", IEEE Trans. Aerospace and Electronic Systems, Vol. 19, No. 6, 805-813, 1983.

4. Balestrino, A., DeMaria, G., and Sciavicco, L., "An adaptive model following control for robot manipulators", ASME Journal of Dynamic Systems, Measurement and Control, Vol. 105, 143-151, 1983.

5. Nicosia, S., and Tomei, P., "Model reference adaptive control algorithms for industrial robots", Automatica, Vol. 20, No. 5, 635-644, 1984.

6. Singh, S.N., "Adaptive model following control of nonlinear robotic systems", IEEE Trans. Auto. Control, Vol. 30, No. 11, 1099-1100, 1985.

7. Lim, K.Y., and Eslami, M., "Adaptive controller design for robot manipulators systems using Lyapunov direct method", IEEE Trans. Auro. Control, Vol. 30, No. 12, 1229-1233, 1985.

8. Craig, J.J., Hsu, P. and Sastry, S.S., "Adaptive control of mechanical manipulators", Proc. IEEE Int. Conf. on Robotics and Automation, San Francisco, 190-195, 1986.

9. Seraji, H. "A new approach to adaptive control of robot manipulators", ASME Journal of Dynamic Systems, Measurement and Control, Vol. 109, 193-202, 1987.

10. Horowitz, R. and Tomizuka, M. "Discrete-time model reference adaptive control of mechanical manipulator", Computer in Engineering, Vol. 2, Robot and robotics, ASME, 107-112, 1982.

11. Koivo, A.J. and Guo, T.H., "Adaptive linear controller for robotic manipulators", IEEE Trans. Auto. Control, Vol. 28, No. 2, 162-170, 1983.

12. Lee, C.S.G., and Chung, M.J., "An adaptive control strategy for mechanical manipulators", IEEE Trans. Auto. Control, Vol. 29, No. 9, 837-840, 198.

13. Craig, J.J., Robotics- Mechanics and Control, Addison-Wesley, MA, 1986.

14. M. Tarokh, "The effect of discretization on the performance of robot control systems", Proc. Second Int. Symposium on Robotics and Manufacturing, Albuquerque, NM, ASME press, 719-926, 1988.

15. Landau, Y.D., Adaptive Control- Model Reference Approach, Marcel Dekker, N.Y., 1979.

Appendix

In this appendix, we derive the gain adaptation algorithm (18). Consider Condition 1 and write $H(z)$ as

$$H(z) = z\ D\ (zI_{2n} - C)^{-1}B = DB + DC\ (zI_{2n} - C)^{-1}B \tag{21}$$

Now, $H(z)$ is SPR if the exists $2n \times 2n$ symmetric positive definite (SPD) matrices R and M, $n \times n$ matrix K and $2n \times n$ matrix L such that [15, Lemma B.4-2]

$$C^T RC - R = -LL^T - M \tag{22}$$

$$B^T RC + K^T L^T = DC \tag{23}$$

$$K^T K = (DB) + (DB)^T - B^T RB \tag{24}$$

The problem is to choose the matrices R, M, K, L and D to satisfy (22)-(24). Let

$$R = \begin{bmatrix} R_1 & R_2 \\ R_2 & R_3 \end{bmatrix} \quad ; \quad R_1 = diag\{r_{1i}\}\ ,\ R_2 = diag\{r_{2i}\}\ ,\ R_3 = diag\{r_{3i}\}$$

where R_1, R_2 and R_3 are diagonal $n \times n$ matrices whose diagonal elements are $r_{1i} > 0, r_{2i} > 0$ and $r_{3i} > 0$; $i=1,2,\ldots,n$. This particular structure ensures that R is SPD and simplifies the derivations, as will be seen. Substituting R and B in (24), we have

$$K^T K = R_3^T = R_3$$

or $K = diag\ \{\sqrt{r_{3i}}\}$ and thus the matrix K is found to satisfy (24). Next we choose $D = (R_2 \quad R_3)$ and substitute for D, B, R, C and K in (23) to obtain $L = 0$. Thus L is also found and (23) is satisfied. Now we consider (22), and in order to obtain explicit relationship between the elements of R and the given matrices C and M, we select the following structures

$$M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \quad ; \quad M_1 = diag\{m_{1i}\}\ ,\ M_2 = diag\{m_{2i}\}\quad ; m_{1i}\ , m_{2i} > 0$$

$$C = \begin{bmatrix} 0 & I_n \\ -C_2 & -C_1 \end{bmatrix} \quad ; \ C_2 = diag\{c_{2i}\}\ \ ,\ \ C_1 = diag\{c_{1i}\}$$

Substituting for R ,C and M in (22) and solving, we obtain

$$r_{1i} = m_{1i} + \frac{m_i}{c_i} c_{2i}^2(1+c_{2i})\ \ ,\ \ r_{2i} = \frac{m_i}{c_i} c_{2i} c_{1i}\ \ ,\ \ r_{3i} = \frac{m_i}{c_i}(1+c_{2i}) \tag{25a}$$

where

$$m_i = (m_{1i} + m_{2i}) > 0 \ \ ; \ \ c_i = (1-c_{2i})\left[(1+c_{2i})^2 - c_{1i}^2 \right] \tag{25b}$$

The characteristic polynomial of the reference model (10) is

$$\Delta(z) = \left| z^2 I_n + C_1 z + C_2 \right| = \prod_{i=1}^{n} \delta_i(z) \tag{26a}$$

where

$$\delta_i(z) = z^2 + c_{1i} z + c_{2i} = (z+\lambda_{1i})(z+\lambda_{2i}) \tag{26b}$$

Since the reference model is stable, i.e. $|\lambda_{1i}| < 1$, $|\lambda_{2i}| < 1$, we must have

$$|\delta_i(0)|<1 \qquad \delta_i(1)>0 \qquad \delta_i(-1)>0$$

or

$$|c_{0i}|<1\,, \quad (1+c_{1i}+c_{2i})>0\,, \quad (1-c_{1i}-c_{2i})>0 \tag{27}$$

Inequalities (27) imply that c_i in (25b) is positive. Let $\alpha_i = \dfrac{m_{1i}+m_{2i}}{c_i}$ $,i=1,2,...,n$ where α_i are positive numbers, then using (25) and (26b) we have

$$r_{1i} = m_{1i}+\alpha_i(\lambda_{1i}\lambda_{2i})^2(1+\lambda_{1i}\lambda_{2i}) \tag{28a}$$

$$r_{2i} = \alpha_i\lambda_{1i}\lambda_{2i}(\lambda_{1i}+\lambda_{2i}) \tag{28b}$$

$$r_{3i} = \alpha_i(1+\lambda_{1i}\lambda_{2i}) \tag{28c}$$

Note that the acquired r_{1i} ,r_{2i} and r_{3i} are positive, and thus R_1, R_2, R_3 and consequently R are all SPD. We conclude that Condition 1 is satisfied by choosing D in the adaptation algorithm (16) as $D = (R_2 \quad R_3)$ where the elements of R_2 and R_3 are given by (28b) and (28c), respectively.

In order to satisfy Condition 2, we select the matrices $\Psi_1(v,\theta_e), \ldots, \Psi_5(v,\theta_r)$ in (17 according to the following proportional plus integral (summation) adaptation law

$$\Psi_1(v,\theta_e) = C_1 - A_0^{-1}\left[A_1(k)+P_1(k)\right] = Gv(k)\theta_e^T(k-1)E_{1P} + G\sum_{l=0}^{k-1} v(l)\theta_e^T(l-1)E_{1I} \tag{29a}$$

$$\Psi_2(v,\theta_e) = C_2 - A_0^{-1}\left[A_2(k)+P_2(k)\right] = Gv(k)\theta_e^T(k-2)E_{2P} + G\sum_{l=0}^{k-1} v(l)\theta_e^T(l-2)E_{2I} \tag{29b}$$

$$\Psi_3(v,\theta_r) = A_0^{-1}\left[A_0(k)-Q_0(k)\right] = Gv(k)\theta_r^T(k)F_{0P} + G\sum_{l=0}^{k-1} v(l)\theta_r^T(l)F_{0I} \tag{29c}$$

$$\Psi_4(v,\theta_r) = A_0^{-1}\left[A_1(k)-Q_1(k)\right] = Gv(k)\theta_r^T(k-1)F_{1P} + G\sum_{l=0}^{k-1} v(l)\theta_r^T(l-1)F_{1I} \tag{29d}$$

$$\Psi_5(v,\theta_r) = A_0^{-1}\left[A_2(k)-Q_2(k)\right] = Gv(k)\theta_r^T(k-2)F_{2P} + G\sum_{l=0}^{k-1} v(l)\theta_r^T(l-2)F_{2I} \tag{29e}$$

where $G, E_{1P}, E_{1I}, \ldots, F_{2P}, F_{2I}$ are SPD matrices, and the subscripts P and I denote proportional and integral terms, respectively.

Consider the first term in the expression for $w(k)$, i.e. $w_1(k)$ given in (14b). Using (29a), we have

$$\sum_{k=0}^{k_1} v^T(k)w_1(k) = \sum_{k=0}^{k_1}\left[v^T(k)Gv(k)\theta_e^T(k-1)E_{1P}\theta_e(k-1) + v^T(k)G\sum_{l=0}^{k-1} v(l)\theta_e^T(l-1)E_{1I}\theta_e(k-1)\right] \tag{30}$$

It is seen that the proportional term produces two quadratic forms $v^T(k)Gv(k)$ and

$\theta_e(k-1)^T E_{1P} \theta_e(k-1)$ which are both positive for all $k \geq 0$. Similarly, it can be shown [15, Appendix D] that the integral term produces quadratic forms and thus $\sum_{k=0}^{k_1} v^T(k) w_1(k) > 0$. Since $w_2(k), \ldots, w_5(k)$ in (14b) and (17) have structures similar to $w_1(k)$, we have $\sum_{k=0}^{k_1} v^T(k) w_j(k) > 0$, $j=1,2,\ldots,5$. We conclude that Condition 2 is satisfied due to the particular choices in (29).

Let us chose $G = A_0^{-1}$, define the change in the gain matrices due to adaptation at time k as $\Delta P_1(k) = P_1(k) - P_1(k-1), \ldots, \Delta Q_2(k) = Q_2(k) - Q_2(k-1)$, and denote the corresponding changes in the robot matrices by $\Delta A_0(k)$, $\Delta A_1(k)$ and $\Delta A_2(k)$. Then after simplifications, we obtain from (29)

$$\Delta P_1(k) + \Delta A_1(k) - \Delta A_0(k) C_1 = \left[v(k-1)\, \theta_e^T(k-2) - v(k)\, \theta_e^T(k-1) \right] E_{1P} - v(k-1)\, \theta_e^T(k-2) E_{1I} \tag{31a}$$

$$\Delta P_2(k) + \Delta A_2(k) - \Delta A_0(k) C_2 = \left[v(k-1)\, \theta_e^T(k-3) - v(k)\, \theta_e^T(k-2) \right] E_{2P} - v(k-1)\, \theta_e^T(k-3) E_{1I} \tag{31b}$$

$$\Delta Q_0(k) - \Delta A_0(k) = \left[v(k-1)\, \theta_r^T(k-1) - v(k)\, \theta_r^T(k) \right] F_{0P} - v(k-1)\, \theta_r^T(k-1) F_{0I} \tag{31c}$$

$$\Delta Q_1(k) - \Delta A_1(k) = \left[v(k-1)\, \theta_r^T(k-2) - v(k)\, \theta_r^T(k-1) \right] F_{1P} - v(k-1)\, \theta_r^T(k-2) F_{1I} \tag{31d}$$

$$\Delta Q_2(k) - \Delta A_2(k) = \left[v(k-1)\, \theta_r^T(k-3) - v(k)\, \theta_r^T(k-2) \right] F_{2P} - v(k-1)\, \theta_r^T(k-3) F_{2I} \tag{31e}$$

In order to make the controller gain matrices independent of the robot matrices, we assume that the changes in the robot matrices is much smaller than the corresponding changes in the gain matrices due to adaptation, i.e.

$$\Delta P_1(k) \gg \Delta A_1(k) - \Delta A_0(k) C_1, \ldots, \Delta Q_2(k) \gg \Delta A_2(k) \tag{32}$$

This assumption is valid if the adaptation rate is sufficiently fast or equivalently, if the robot matrices are slowly time-varying. The vector $v(k)$ in (31) is obtained from (16) as

$$v(k) = R_2\, \varepsilon(k-1) + R_3\, \varepsilon(k) \tag{33}$$

which in view of (13) with $\theta_{em}(k) \equiv 0$, is

$$v(k) = -(R_2 \theta_e(k-1) + R_3 \theta_e(k)) \equiv -\hat{\theta}_e(k) \tag{34}$$

Finally, using (31), (32) and (34), we obtain the gain adaptation laws given by (18).

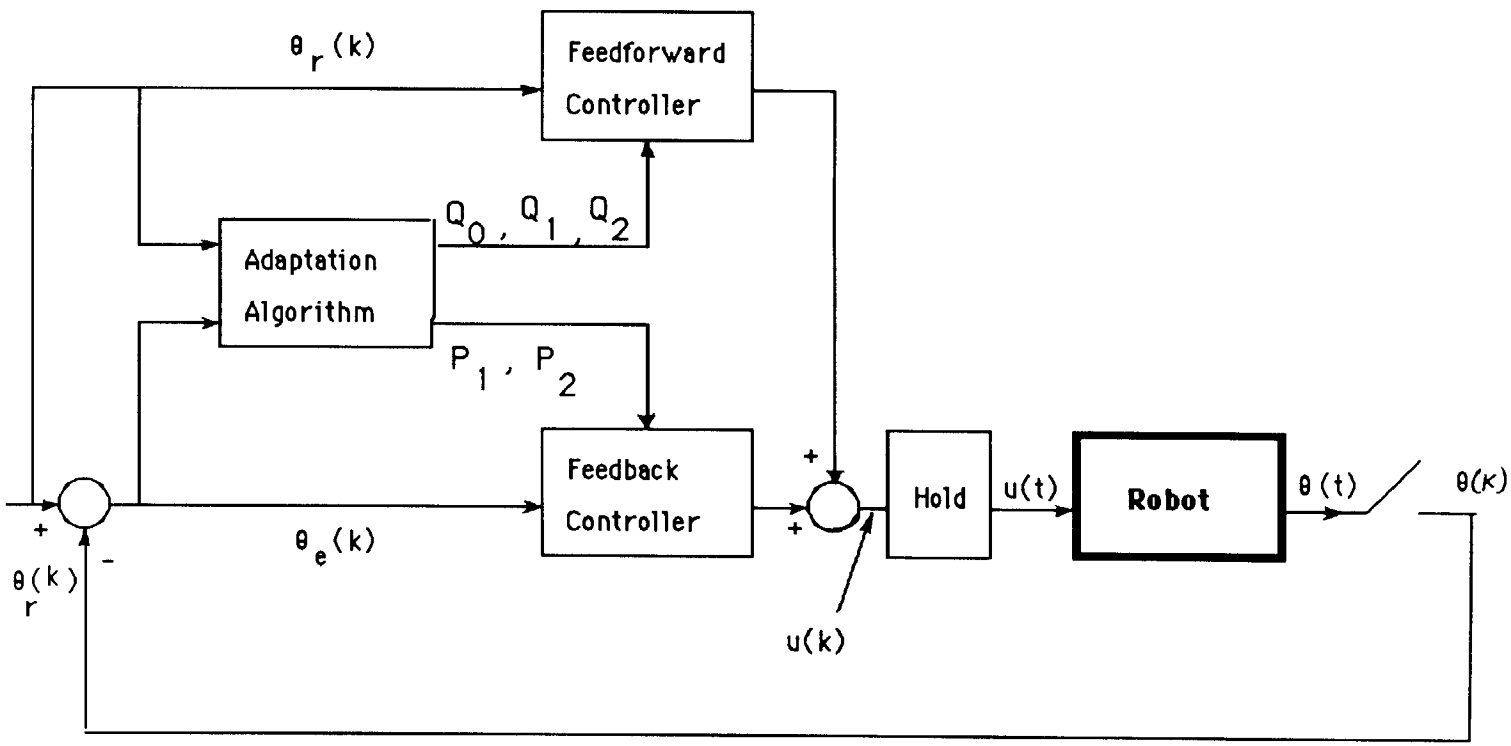

Figure 1 - Discrete-Time Adaptive Control Scheme