N91-10305

AN ANALYTICAL SOLUTION FOR THE ELASTOPLASTIC RESPONSE OF A CONTINUOUS FIBER COMPOSITE UNDER UNIAXIAL LOADING

Jong-Won Lee, Research Assistant
David H. Allen, Professor

Aerospace Engineering Department
Texas A&M University
College Station, Texas 77843

ABSTRACT

A continuous fiber composite is modelled by a two-element composite cylinder in order to predict the elastoplastic response of the composite under a monotonically increasing tensile loading parallel to fibers. The fibers and matrix are assumed to be elastic-perfectly plastic materials obeying Hill's and Tresca's yield criteria, respectively. The present paper investigates the composite behavior when the fibers yield prior to the matrix.

INTRODUCTION

The elastoplastic response of fibrous composites has been the subject of a number of theoretical studies[1-4]. When a composite is subjected to uniaxial tension loading parallel to the fibers, a two-element composite cylinder(Figure 1-a) has been frequently utilized to model the composite response. The loading direction together with the axisymmetric geometry of the representative volume element simplify the mathematical difficulties associated with the equilibrium equations. By implementing a traction-free boundary condition to the outermost surface of the representative volume element, it becomes possible to construct a well-posed boundary value problem when the fibers and the matrix are assumed to obey Hill's and Tresca's yield criteria, respectively. A closed form analytical solution requires further simplifications such as elastic-perfectly plastic constituents, perfect interfacial bonding, etc. When the composite constituents are assumed to obey the modified yield criterion proposed by Hill[1], the hardening effect of the matrix can be taken into account without mathematical difficulties. However, the present study focuses on a composite with non-hardening constituents.

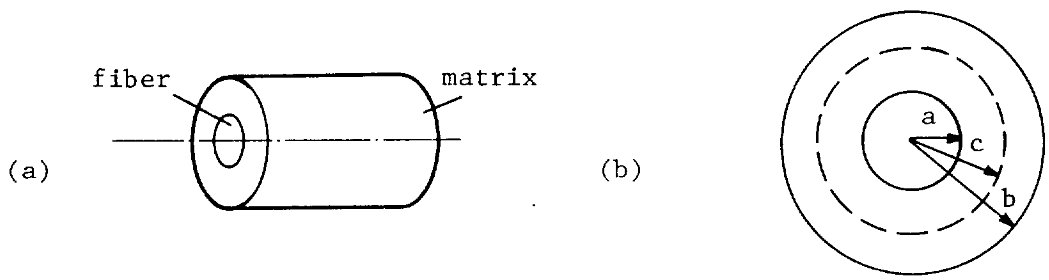

Figure 1. Continuous Fiber Composite

(a) Representative Volume Element
(b) Cross section

Hill proposed a relatively simple yield criterion which assumes the difference between the axial stress and the arithmetic mean value of the radial and circumferential stresses to be equal to the yield stress[1]. When this yield condition is implemented to a composite with elastic fibers surrounded by an elastic-perfectly plastic matrix, the entire matrix yields simultaneously, resulting in a discontinuity in the slope of the effective stress-strain curve.

Mulhern, Roger, and Spencer[2] proposed a rigorous analytical solution for a two-element composite cylinder under cyclic loading. Their study models an elastic core fiber surrounded by an elastic-perfectly plastic matrix tube obeying Tresca's yield criterion. The resulting composite behavior is almost as bilinear as Hill's solution. However, the slope of the effective stress-strain curve is continuous because the plastically deformed matrix zone propagates from the fiber -matrix interface to the outer surface of the matrix.

Ebert and Gadd[3] studied a similar problem for an elastic-perfectly plastic core fiber surrounded by elastic matrix. Ebert, et al.[4] extended this to an elastic-perfectly plastic matrix. However, the application of their numerical solution is restricted to a composite in which the Poisson's ratios of the fiber and the matrix are equivalent.

The present paper extends the study of Ebert and other authors[3,4] to a two-element composite cylinder representing a transversely isotropic fiber surrounded by an isotropic matrix in which the Poisson's ratios of the core fiber and the matrix need not be identical.

MODEL FORMULATION

Consider a metal matrix composite reinforced by continuous fibers under uniaxial tension loading parallel to the fibers. The globally averaged stress state of the representative volume element is assumed to be one dimensional. The elastic-plastic response of the bar can be analytically predicted by solving an equivalent boundary value problem of a single core fiber which is perfectly bonded to the surrounding matrix tube. The volume element representing the equivalent boundary value problem is illustrated in Figure 1. The uniaxial tension loading in the fiber direction produces a three dimensional stress state in both the core fiber and the surrounding matrix. When the tension loading increases monotonically, either the fiber or matrix yields at a certain magnitude of the applied tension loading. Further increment of the tension loading induces the yielding everywhere in the composite. The possible yield sequences for the composite constituents of the representative volume element can be categorized into three cases as shown in Figure 2. The present study provides analytical solutions to the first case under monotonically increasing tension loading.

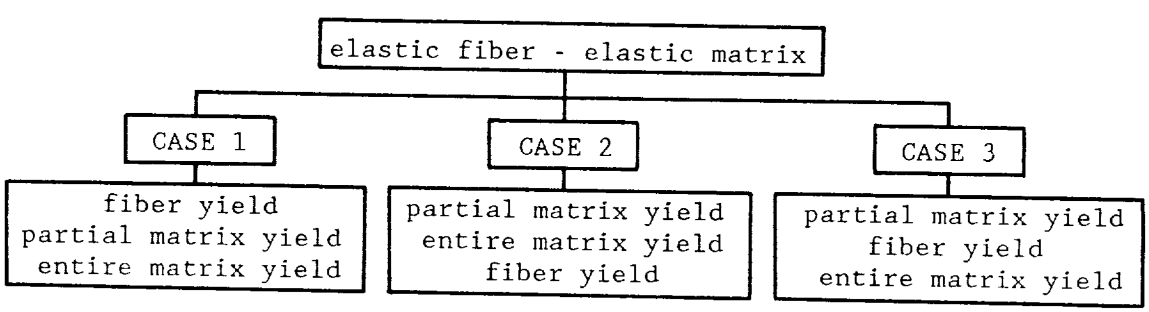

Figure 2. Possible Composite Yield Sequences

The entire mathematical formulation of the present study is based on the following key assumptions:

1. The core is assumed to be a transversely isotropic fiber surrounded by an isotropic matrix.
2. Both constituents are assumed to be elastic-perfectly plastic.
3. The interfacial bonding between the core fiber and the matrix is assumed to be perfect throughout deformation.
4. The core fiber is assumed to obey Hill's yield criterion.
5. The matrix tube is assumed to obey Tresca's yield criterion.
6. The axial strain is assumed to be spatially homogeneous.

Since the geometry of the representative volume element is axisymmetric and the loading direction is parallel to the core fiber, the only non-trivial equilibrium equation is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (1)$$

The constitutive equations for the transversely isotropic core fiber and the isotropic matrix are

$$\begin{Bmatrix} \epsilon_r^f - \epsilon_r^p \\ \epsilon_\theta^f - \epsilon_\theta^p \\ \epsilon_z^f - \epsilon_z^p \end{Bmatrix} = \begin{bmatrix} 1/E_T & -\nu_{TT}/E_T & -\nu_{LT}/E_L \\ -\nu_{TT}/E_T & 1/E_T & -\nu_{LT}/E_L \\ -\nu_{LT}/E_L & -\nu_{LT}/E_L & 1/E_L \end{bmatrix} \begin{Bmatrix} \sigma_r^f \\ \sigma_\theta^f \\ \sigma_z^f \end{Bmatrix} \qquad (2\text{-a})$$

and

$$\begin{Bmatrix} \sigma_r^m \\ \sigma_\theta^m \\ \sigma_z^m \end{Bmatrix} = \frac{E_m}{(1+\nu_m)(1-2\nu_m)} \begin{bmatrix} 1-\nu_m & \nu_m & \nu_m \\ \nu_m & 1-\nu_m & \nu_m \\ \nu_m & \nu_m & 1-\nu_m \end{bmatrix} \begin{Bmatrix} \epsilon_r^m - \epsilon_r^p \\ \epsilon_\theta^m - \epsilon_\theta^p \\ \epsilon_z^m - \epsilon_z^p \end{Bmatrix} \qquad (2\text{-b})$$

respectively.
Since the plastic strain is incompressible,

$$\epsilon_r^p + \epsilon_\theta^p + \epsilon_z^p = 0 \qquad (3)$$

It can be shown that Hill's yield criterion becomes the following yield condition for the transversely isotropic core fiber under a transversely isotropic loading:

$$| \sigma_z^f - \sigma_r^f | = | \sigma_z^f - \sigma_\theta^f | = Y_L \qquad (4\text{-a})$$

This mathematical expression is identical to Tresca's yield criterion.

The surrounding matrix will yield according to one of the following conditions.

$$| \sigma_z^m - \sigma_r^m | = Y_m \tag{4-b}$$

$$| \sigma_z^m - \sigma_\theta^m | = Y_m \tag{4-c}$$

$$| \sigma_z^m - \sigma_r^m | = | \sigma_z^m - \sigma_\theta^m | = Y_m \tag{4-d}$$

The external boundary conditions and interior compatibility are

$$\sigma_r = 0 \text{ at } r = b \tag{5-a}$$

$$\sigma_r = \text{unique at } r = a \text{ or } r = a,\ c \tag{5-b}$$

$$u_r = 0 \text{ at } r = 0 \tag{5-c}$$

$$u_r = \text{unique at } r = a \text{ or } r = a,\ c \tag{5-d}$$

When both fiber and matrix responses are elastic, the stress state and the displacement field in the representative volume element are determined by matching the radial stress and displacement at the fiber-matrix interface. The problem solving procedure for this elastic deformation is simple and straightforward as discussed below.

Since the stress state in the fiber is always transversely isotropic,

$$\sigma_r^f = \sigma_\theta^f = -P \tag{6-a}$$

$$\sigma_z^f = -2\nu_{LT}P + E_L\epsilon_z \tag{6-b}$$

where P is an unknown constant to be determined. The radial displacement in the fiber is given by

$$u_r = C_1 r \tag{7-a}$$

The strain components become

$$\epsilon_r^f = \epsilon_\theta^f = C_1 \tag{7-b}$$

From (2-a),(6), and (7-b),

$$C_1 = -[(1-\nu_{TT}-2\nu_{LT}\nu_{TL})P/E_T + \nu_{LT}\epsilon_z]r \tag{8-c}$$

Within the matrix,

$$\sigma_r^m = \frac{Pa^2}{b^2-a^2}\left[1 - \frac{b^2}{r^2}\right] \tag{9-a}$$

$$\sigma_\theta^m = \frac{Pa^2}{b^2-a^2}\left[1 + \frac{b^2}{r^2}\right] \tag{9-b}$$

$$\sigma_z^m = \frac{2P\nu_m a^2}{b^2-a^2} + E_m\epsilon_z \tag{9-c}$$

$$u_r^m = C_2 r + C_3/r \tag{10-a}$$

The strain components become

$$\epsilon_r^m = u_{r,r} = C_2 - C_3/r^2 \tag{10-b}$$

$$\epsilon_\theta^m = u_r/r = C_2 + C_3/r^2 \tag{10-c}$$

From (5-a) and (5-b), C_2 and C_3 are determined as functions of P.

$$C_2 = \frac{Pa^2(1+\nu_m)(1-2\nu_m)}{E_m(b^2-a^2)} - \nu_m\epsilon_z \tag{11-a}$$

$$C_3 = \frac{Pa^2b^2(1+\nu_m)}{E_m(b^2-a^2)} \tag{11-b}$$

Then the radial displacement in the matrix becomes

$$u_r = \frac{Pa^2(1+\nu_m)}{E_m(b^2-a^2)}[(1-2\nu_m)r+b^2/r] - \nu_m\epsilon_z \tag{12}$$

The interfacial stress in the radial direction, -P, is determined from the displacement compatibility given by (5-d).

$$-P = -\frac{(\nu_m-\nu_{LT})\epsilon_z}{\Lambda_1} \tag{13-a}$$

where

$$\Lambda_1 = \left[\frac{1-\nu_{TT}-\nu_{LT}\nu_{TL}}{E_T}\right] + \left[\frac{1+\nu_m}{E_m}\right]\left[\frac{a^2(1-2\nu_m+b^2/a^2)}{b^2-a^2}\right] \tag{13-b}$$

It can be shown that the effective axial Young's modulus of the volume element is given by

$$E_c = E_L(a/b)^2 + E_m(b^2-a^2)/b^2 + 2(a/b)^2(\nu_m-\nu_{LT})^2/\Lambda_1 \tag{14}$$

Under monotonically increasing axial strain, ϵ_z , either the core fiber or the surrounding matrix yields first. The yield strains, strengths, and the Poisson's ratios of the constituents govern the composite yield sequence. When the applied axial strain reaches a certain value, ϵ_{zY}, the core fiber yields first if the longitudinal yield strength of the fiber is much smaller than the yield strength of the matrix. The initial yield strain, ϵ_{zY}, at which the entire fiber yields is given by

$$\epsilon_{zY} = \frac{Y_L}{E_L}\left[1 - \frac{(\nu_m-\nu_{LT})(1-2\nu_{LT})}{E_L\Lambda_1 +(\nu_m-\nu_{LT})(1-2\nu_{LT})} \right] \tag{15}$$

After the core fiber yields, the surrounding matrix behaves elastically until matrix yield at the interface occurs. Since the plastic strain is incompressible and the stress state is transversely isotropic in the core fiber, stresses, total strains and plastic strains in the fiber become

$$\sigma_r^f = \sigma_\theta^f = -P, \quad \sigma_z^f = -P + Y_L \tag{16-a}$$

$$\epsilon_r^p = \epsilon_\theta^p = -\epsilon_z^p/2 \tag{16-b}$$

$$\epsilon_r^f = \epsilon_\theta^f = u_{r,r}^f = u_r^f/r \tag{16-c}$$

where -P is the unknown interfacial stress to be determined.

The first strain invariant, $\epsilon_r^f+\epsilon_\theta^f+\epsilon_z^f$, together with the uniqueness of the radial displacement at the material interface determine the magnitudes of the fiber stress components as functions of the applied axial strain. During this strain increment where the matrix still deforms elastically, the stress components and the radial displacement in the matrix given by equations (9-a), (9-b), and (9-c) are still valid if the interfacial stress in the radial direction is redefined as

$$P = \frac{-\eta_1\eta_2}{\eta_1+2\eta_2}\left[(1-2\nu_m)\epsilon_z-(1-2\nu_{LT})\frac{Y_L}{E_L}\right] \tag{17-a}$$

where

$$\eta_1 = \frac{E_m}{1+\nu_m}\left[\frac{(b^2-a^2)/a^2}{1-2\nu_m+(b/a)^2}\right] \tag{17-b}$$

$$\eta_2 = \frac{E_T}{2-2\nu_{TT}-4\nu_{TL}+\nu_{TL}/\nu_{LT}} \tag{17-c}$$

The radial displacement in the fiber becomes

$$u_r^f = -\left[\frac{\nu_m\eta_1+\eta_2}{\eta_1+2\eta_2}\right]\epsilon_z\, r + \left[\frac{(1-2\nu_{LT})\eta_2}{\eta_1+2\eta_2}\right]\frac{Y_L}{E_L}\, r \tag{18}$$

Further increase of the applied axial strain causes yielding of the matrix material. The plastically deformed region then propagates toward the outer surface of the matrix. During this strain increment, the elastically deformed matrix and the plastically deformed matrix exist together as shown in Figure 1-b. Outside the interface between the plastic matrix and elastic matrix, the stress state and deformation are given by the elasticity solution with an unknown internal pressure, P^*, acting on the interfacial surface between the plastic matrix and elastic matrix. Therefore, within the elastically deformed matrix,

$$\overset{m}{\sigma_r} = \frac{P^* c^2}{b^2-c^2}\left[1-\frac{b^2}{r^2}\right] \tag{19-a}$$

$$\overset{m}{\sigma_\theta} = \frac{P^* c^2}{b^2-c^2}\left[1+\frac{b^2}{r^2}\right] \tag{19-b}$$

$$\overset{m}{\sigma_z} = \frac{2P^*\nu_m c^2}{b^2-c^2} + E_m\epsilon_z \tag{19-c}$$

$$\overset{m}{u_r} = \frac{P^* c^2(1+\nu_m)}{E_m(b^2-c^2)}[(1-2\nu_m)r+b^2/r] - \nu_m\epsilon_z r \tag{20}$$

where P^* is determined from Tresca's yield criterion given below.

$$\text{At} \quad r = c, \quad \begin{cases} |\overset{m}{\sigma_z}-\overset{m}{\sigma_r}| = Y_m, & \text{for } \nu_m>\nu_{LT} \quad (21\text{-a}) \\ |\overset{m}{\sigma_z}-\overset{m}{\sigma_\theta}| = Y_m, & \text{for } \nu_m<\nu_{LT} \quad (21\text{-b}) \end{cases}$$

Then,

$$P^* = \frac{(Y_m-E_m\epsilon_z)(b^2-c^2)/c^2}{2\nu_m - (b^2+c^2)/c^2} \tag{22}$$

where c is an unknown function of the applied axial strain, ϵ_z.

Within the plastically deformed matrix, Tresca's yield criterion given by eqs. (21-a) and (21-b) determines the stress state and displacement field. If the Poisson's ratio of the matrix is smaller than that of the fiber, the fiber-matrix interfacial stress in the radial direction is always tensile. On the other hand, when the Poisson's ratio of the matrix is greater than that of the fiber, the fiber-matrix interfacial stress may be either compressive or tensile. After the core fiber deforms plastically, the apparent Poisson's ratio of the fiber approaches 0.5 as the applied axial strain increases. The interfacial stress in the radial direction may be changed from compressive to tensile before the initiation of matrix yield. If the matrix yield strain is far greater than that of the core fiber, the interfacial stress in the radial direction at the onset of matrix yield becomes tensile. Then, from eq. (3) and (4-c),

$$\overset{p}{\epsilon_r} = 0, \quad \overset{p}{\epsilon_\theta} = -\overset{p}{\epsilon_z} \tag{23}$$

$$\overset{m}{\sigma_\theta} + \overset{m}{\sigma_z} = 2\overset{m}{\sigma_\theta} + Y_m = \frac{E_m}{(1+\nu_m)(1-2\nu_m)}(2\nu_m\overset{m}{\epsilon_r} + \overset{m}{\epsilon_\theta} + \overset{m}{\epsilon_z}) \tag{24}$$

From eq. (24) and the strain-displacement relationships, the radial and circumferential stresses are expressed in terms of the radial displacement and its gradient with respect to r.

$$\overset{m}{\sigma}_r = \frac{E_m}{(1+\nu_m)(1-2\nu_m)} [(1-\nu_m)\overset{m}{u}_{r,r} + \nu_m \overset{m}{u}_r/r + \nu_m \overset{m}{\epsilon}_z] \tag{25-a}$$

$$\overset{m}{\sigma}_\theta = \frac{E_m/2}{(1+\nu_m)(1-2\nu_m)} (2\nu_m \overset{m}{u}_{r,r} + \overset{m}{u}_r/r + \overset{m}{\epsilon}_z) - Y_m/2 \tag{25-b}$$

The equilibrium equation thus becomes

$$2(1-\nu_m)r^2\overset{m}{u}_{r,rr} + 2(1-\nu_m)r\overset{m}{u}_{r,r} - \overset{m}{u}_r = -r(1-2\nu_m)[(1+\nu_m)Y_m/E_m - \epsilon_z] \tag{26}$$

The solution to the above differential equation is given by

$$\overset{m}{u}_r = -[(1+\nu_m)Y_m/E_m - \epsilon_z]r + C_2 r^k + C_3 r^{-k} \tag{27}$$

where

$$k = [2(1-\nu_m)]^{-1/2}$$

Since the radial stress and displacement must be single-valued at r=c and r=a, the unknown constants are determined as

$$C_2 = \frac{-(1+\nu_m)(E_m\epsilon_z - Y_m)b^2 c^{-(k+1)}}{kE_m(1-2\nu_m+b^2/c^2)} \tag{28-a}$$

$$C_3 = \frac{(1+\nu_m)(E_m\epsilon_z - Y_m)b^2 c^{(k-1)}}{kE_m(1-2\nu_m+b^2/c^2)} \tag{28-b}$$

The radius of the matrix yield front, c, can also be determined as a function of the applied axial strain, ϵ_z, and material properties. However, it is more convenient to express the applied axial strain as a function of the radius of the matrix yield front by satisfying the uniqueness of the radial displacement at the fiber-matrix interface. Until the yield front reaches the outermost surface of the matrix tube, the axial strain is given as

$$\epsilon_z = \frac{Y_m}{E_m} - \frac{(1-2\nu_m)\dfrac{Y_m}{E_m} - (1-2\nu_{LT})\dfrac{Y_L}{E_L}}{1-2\nu_m+2\phi_2(1+\nu_m) - \dfrac{E_m\phi_1}{\eta_2(1-2\nu_m)}} \tag{29-a}$$

where

$$\phi_1 = 1 - \frac{(b/c)^2[(1-\nu_m+\nu_m/k)(a/c)^{(k-1)}+(1-\nu_m-\nu_m/k)(a/c)^{-(k+1)}]}{1 - 2\nu_m + (b/c)^2} \tag{29-b}$$

$$\phi_2 = 1 - \frac{(b/c)^2[(a/c)^{(k-1)}-(a/c)^{-(k+1)}]/k}{1 - 2\nu_m + (b/c)^2} \tag{29-c}$$

After the entire matrix yields, it can be shown that eqns. (28-a) and (28-b) should be corrected for further axial strain increment as

$$C_2 = \frac{\left[\frac{1-(1-2\eta_2/\alpha_2)(a/b)^{-(k+1)}}{1-2\nu_m} - \frac{3\eta_2}{E_m}\right](E_m\epsilon_z-Y_m) - \alpha_3}{\alpha_1 a^{(k-1)}[(1-2\eta_2/\alpha_2)(a/b)^{-2k} - (1-2\eta_2/\alpha_1)]} \tag{30-a}$$

$$C_3 = \frac{\left[\frac{1-(1-2\eta_2/\alpha_1)(a/b)^{(k-1)}}{1-2\nu_m} - \frac{3\eta_2}{E_m}\right](E_m\epsilon_z-Y_m) - \alpha_3}{\alpha_2 a^{-(k+1)}[(1-2\eta_2/\alpha_1)(a/b)^{2k} - (1-2\eta_2/\alpha_2)]} \tag{30-b}$$

$$\alpha_1 = \frac{E_m[\nu_m+(1-\nu_m)k]}{(1+\nu_m)(1-2\nu_m)} \tag{30-c}$$

$$\alpha_2 = \frac{E_m[\nu_m-(1-\nu_m)k]}{(1+\nu_m)(1-2\nu_m)} \tag{30-d}$$

$$\alpha_3 = \frac{Y_m\eta_2}{E_m}\left[1-2\nu_m-(1-2\nu_{LT})\frac{Y_L E_m}{E_L Y_m}\right] \tag{30-e}$$

Then the stress components in the matrix become:

$$\sigma_r^m = \frac{E_m\epsilon_z-Y_m}{1-2\nu_m} + \alpha_1 C_2 r^{(k-1)} + \alpha_2 C_3 r^{-(k+1)} \tag{31-a}$$

$$\sigma_\theta^m = \frac{E_m\epsilon_z-Y_m}{1-2\nu_m} + \frac{E_m[(\nu_m k+1/2)C_2 r^{(k-1)}-(\nu_m k-1/2)C_3 r^{-(k+1)}]}{(1+\nu_m)(1-2\nu_m)} \tag{31-b}$$

$$\sigma_z^m = \sigma_\theta^m + Y_m \tag{31-c}$$

Further increase of the axial strain, as mentioned in ref. [2], may cause another type of plastically deformed matrix region in which the radial and circumferential stresses are identical. The present paper, however, does not consider this case because the infinitesimal strain assumption may not be valid for further increase of the applied axial strain.

RESULTS AND DISCUSSION

The effective stress-strain curve for a composite cylinder can be predicted by calculating the average value of the axial stress, σ_z, as a function of the applied axial strain, ϵ_z, and the mechanical properties of the composite constituents. As an example, the effective stress-strain curve of the composite studied by Ebert, et al.[4] is demonstrated in Figure 3. The mechanical properties of the composite constituents appear in Table 1. In Figure 3, the solid lines represent the present analysis. Within the straight line segment(OA), both the core fiber and the surrounding matrix tube are within their elastic limits. When the applied axial strain reaches ϵ_{zY}, the entire core fiber yields. The next line segment(AB) represents the hardening behavior of the composite with plastically deformed core surrounded by an elastic tube. When the applied axial strain reaches ϵ_{z1} in the same figure, the surrounding matrix starts yielding from the fiber-matrix interface. This strain can be calculated from eq. (29-a) by setting c=a. Then the plastically deformed matrix region propagates outward until the entire matrix tube yields. This smooth transient region is represented by the line segment BC. The applied axial strain, ϵ_{z2}, where this transient phenomenon terminates can be calculated from the same equation by setting c=b. The composite response to further axial strain increase then follows the remaining line segment. Within the transient region and for the higher value of the applied axial strain, the matrix tube material is assumed to be nonhardening even though the material hardens significantly(Figure 3 in ref. 4). The experimental results of Ebert, et al.[4] are also plotted in the same figure.

Table 1. Constituent Properties the Composite[4]

Material	Ultimate Strength (Ksi)	0.05% Offset Yield (Ksi)	Elastic Modulus (Msi)	Poisson's ratio
SAE 4140 (Core)	93	54.9	28.7	0.29
Maraging Steel (Matrix Tube)	318	288	25.5	0.29

In Figure 4, the radial variations of the radial, circumferential and axial stresses in the composite cylinder of which the core volume fraction is 0.5 are plotted for two distinctive axial strain values, ϵ_{z1} and ϵ_{z2}. The stresses in the core material decrease slightly as the axial strain increases from ϵ_{z1}. At the onset of the initiation of the matrix yield, the axial stress in the matrix tube is constant. As the applied axial strain increases beyond ϵ_{z1}, the axial stress in the matrix has its maximum value at the free surface.

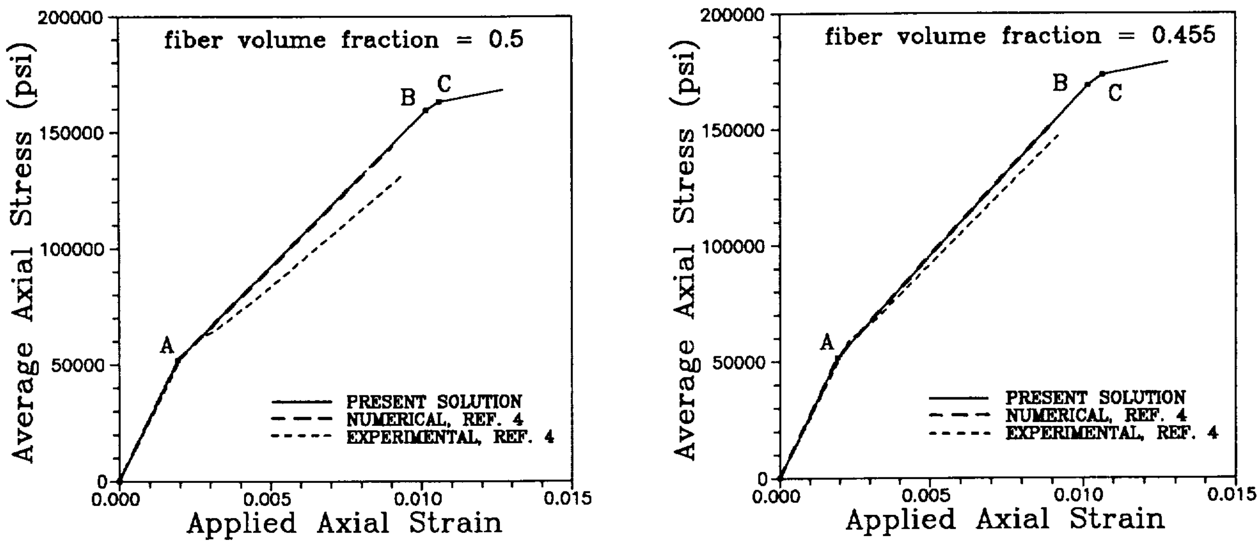

Figure 3. Composite stress-strain curve

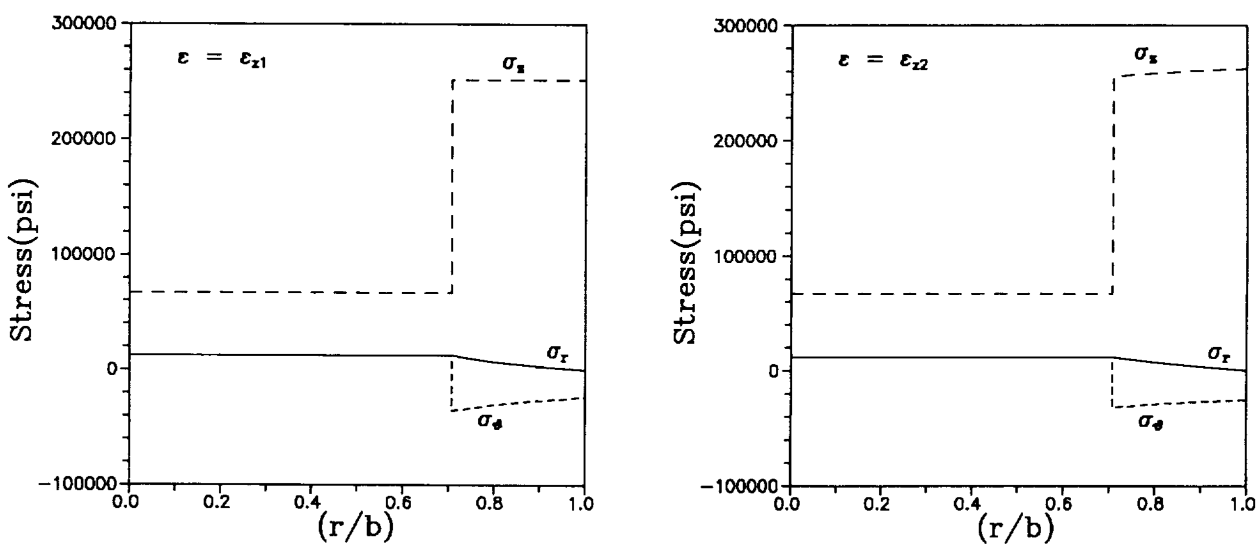

Figure 4. Stresses in the composite

CONCLUSIONS

The present study provides an analytical prediction of the elastoplastic response of continuous fiber composites with weaker fibers. The incremental form proposed by Ebert, et al.[4] must be replaced by the second order ordinary differential equation given by eq. (26). Furthermore, the present analysis can handle the mismatch of the Poisson's ratios as well as transversely isotropic fibers. The present analysis will be generalized for the same type of composites under cyclic loading for providing a comparison to the study of Mulhern, et al.[2]

REFERENCES

1. Hill, R., "Theory of Mechanical Properties of Fibre-Strengthened Materials: II. Inelastic Behavior," J. Mech. Phys. Solids, Pergamon Press Ltd., Vol. 12, pp. 213-218, 1964.
2. Mulhern, J. F., Rogers, T. G., and Spencer A. J. M., "Cyclic Extension of an Elastic Fiber with an Elastic-Plastic Coating," J. Inst. Maths Applics, Vol. 3, pp. 21-40, 1967.
3. Ebert, L. J. and Gadd, J. D., "A Mathematical Model for Mechanical Behavior of Interfaces in Composite Materials," <u>Fiber Composite Materials</u>, Papers Presented at a Seminar of the ASTM, October 17-18, 1964, pp. 89-113.
4. Ebert, L. J., Hecker, S. S., and Hamilton, C. H., "The Stress-Strain Behavior of Concentric Composite Cylinders," J. Composite Materials, Vol. 2, No. 4, pp. 458-476, Oct. 1968.