NASA Technical Memorandum 103247

# Calculation of Weibull Strength Parameters, Batdorf Flaw Density Constants and Related Statistical Quantities Using PC-CARES

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October 1990



NASA

Finally, this manual is accompanied by a disk containing the following files:

```
INSTALL.BAT  - PC-CARES hard disk install batch file
PCCARES.INI  - PC-CARES initialization file
PCCARES.FOR  - PC-CARES FORTRAN source code
PCCARES.EXE  - PC-CARES DOS execution file
PCCARESB.EXE - PC-CARES DOS, OS/2 bound execution file
PCCARES.DAT  - PC-CARES sample problem input file
TEMPLET.INP  - PC-CARES input templet file
```

As with all new software one receives, the original disks should be backed up by making additional copies using the DOS or OS/2 COPY command. If this manual did not come with the above mentioned disk and/or you have a previous version of PC-CARES (called CARES.EXE compiled with Microsoft Quick BASIC or the Lahey FORTRAN compiler), it is recommended that you contact your source and get a copy of the newer version for which this manual was written. The older BASIC version does not calculate the Batdorf crack density coefficient and the Lahey FORTRAN version will only run on computers with math co-processors. Further, neither will execute under OS/2 protect mode, so it will be to your advantage to upgrade.

You may run PC-CARES off a floppy disk or install it on your hard disk. It is recommended that you put the PC-CARES code in its own separate directory when you install it. For your convenience a batch file, INSTALL.BAT, has been included on the distribution disk which is used as follows: At the DOS command prompt with the default drive set to the drive of the distribution disk simply type INSTALL C: where C: is the drive designator of your hard disk. The batch file will create the directory C:\PCCARES and place the contents of the distribution disk in this directory. The batch file assumes that there is no such file or directory called PCCARES in the root directory. For OS/2 users, INSTALL.BAT must be executed from the DOS compatability box or simply rename INSTALL.BAT to INSTALL.CMD and execute it from the OS/2 command prompt.

## 2.0 PROGRAM CAPABILITY AND DESCRIPTION

PC-CARES is a computer program written and compiled with Microsoft FORTRAN Version 5.0 compiler using the VAX FORTRAN extensions and dynamic array allocation supported by this compiler for the IBM/MS-DOS or OS/2 operating systems. The current version of this program will make use of a math co-processor if one is installed on your computer. It is not required, however.

Using the dynamic array allocation routines of the compiler allows the user to choose both the number of constant temperature fracture sets and the maximum number of test specimens per temperature set at runtime by setting these parameters in the keyword driven initialization file called PCCARES.INI. This initialization file also allows you to control the path and filenames of both the PC-CARES input and output files. Note that the number of constant temperature fracture sets (and the maximum number of fracture specimens per set) is limited only by the amount of memory available to the program.

Under DOS the maximum program size is 640 kB. This limit may be lower however, due to other memory

PRINT spooler, or other such utilities. Remember if you want to check out the amount of available memory under DOS, the CHKDSK command gives this information along with the report on the disk medium. For more information on CHKDSK consult your DOS reference manual. You may have to remove all the memory resident programs from memory. If you execute memory resident programs from your AUTOEXEC.BAT file, you may want to delete or comment out those commands before you reboot to achieve as much free memory as you can. If you are using a LIMS PC at NASA Lewis, it is recommended that you consult the LIMS manual or Computer Services before altering AUTOEXEC.BAT. If you should attempt to allocate more memory than is available, the program will simply halt and display an error message, so there is no harm in experimenting with the memory allocation.

Under OS/2 versions 1.1 and 1.2 you are limited only by the virtual memory capabilities of this operating system and the architecture of the 80286 chip which permits processes (programs) up to 16 mB of virtual address space. (Note that if you have a 80386 system with OS/2 versions 1.1 or 1.2, you are still limited to 16 mB as these versions operate the 386 chip in 286 mode.) In practice, you are limited only by the sum of the amount of physical RAM and the amount of hard disk space available on your system. If you are running other processes concurrently, this amount will be reduced accordingly.

The primary function of PC-CARES is statistical analysis of the data obtained from the fracture of simple, uniaxial tensile or flexural specimens and estimation of the Weibull and Batdorf material parameters from this data.

The weakest-link mechanism is expressed with the classical Weibull two-parameter formulation, which, for volume flaw reliability, is

$$P_{sV} = \exp\left[-\int_V \left(\frac{\sigma}{\sigma_{oV}}\right)^{m_V} dV\right] \tag{2.1}$$

and for surface flaw reliability is

$$P_{sS} = \exp\left[-\int_A \left(\frac{\sigma}{\sigma_{oS}}\right)^{m_S} dA\right] \tag{2.2}$$

where $P_s$ is the survival probability and $\sigma$ is the applied uniaxial tensile stress. Here $V$ is the volume of stressed material, and $A$ is the area. The subscripts $V$ and $S$ denote parameters that are a function of material volume and surface area, respectively. The scale parameter $\sigma_o$ has dimensions of stress x $(\text{volume})^{1/m_V}$ or stress x $(\text{area})^{1/m_S}$. The scale parameter corresponds to the stress level at which 63.2 percent of specimens with unit volume or area would fracture. The shape parameter (or Weibull modulus), denoted by $m$, is a dimensionless quantity and measures the degree of strength dispersion of the flaw distribution.

In general, the parameters are obtained from the fracture stresses of many specimens (30 or more are recommended) whose geometry and loading configurations are held constant. Solutions for the three-point modulus-of-rupture (MOR) bending bar, four-point MOR bending bar (ref. 4), and the pure tensile specimen (ref. 5) maintained at a constant specified temperature have been incorporated into the PC-CARES program. Since the material parameters are a function of temperature, different constant-temperature data sets can be input and the corresponding parameter estimates will be calculated. The amount of specimens per each constant temperature data set and the total number of these sets is only limited by the computer memory available to the program. In addition, each specimen can be identified by its mode of failure, either volume flaw, surface flaw, or some other mode so that parameter estimates for competing failure modes can be obtained. The statistical accuracy of the parameter estimates compared with the true material parameters depends on the number of specimens tested, assuming that the true distribution is a Weibull distribution.

Figure 2.1 shows the flowchart for the calculation of the statistical strength parameters of the two-parameter Weibull distribution for volume-flaw and surface-flaw-induced fracture, with complete (single mode) or censored (multiple mode) samples, and the calculation of other statistical quantities. Following the input of specimen geometry, fracture stresses, and respective flaw origins, PC-CARES will first identify any potential bad data (outliers). The outlier test developed by Stefansky (ref. 6) and subsequently used by Neal, Vangel, and Todt (ref. 7) is incorporated into the program. Although the technique is based on the normal distribution and, therefore, its application to the Weibull distribution is not rigorous, it serves as a guideline to the user. Data detected as outliers are flagged with a warning message, and any further action is left to the discretion of the user.

Weibull parameter estimates are obtained for the specimen surface and/or volume as requested by the user, taking into account the fracture origin data also supplied by the user. Biased estimates of the Weibull shape parameter and characteristic strength are obtained from either least-squares analysis or the maximum likelihood method for complete samples and/or censored samples. PC-CARES uses the Weibull log-likelihood equations given in Nelson (ref. 8) and the rank increment adjustment method described by Johnson (ref. 9), for complete and censored statistics.

Because the estimates of parameters are obtained from a finite amount of data, they contain an inherent uncertainty that can be characterized by bounds in which the true parameters are likely to lie. Methods have been developed to evaluate confidence limits that quantify this range with a level of probability as a function of sample size. For the maximum likelihood method with a complete sample, unbiasing factors for the shape parameter $m$, and 5 and 95 percentile confidence limits for $m$ and the characteristic strength $\sigma_\theta$, are provided (ref. 10). The characteristic strength, or characteristic modulus of rupture, is similar to the Weibull scale parameter except that it includes the effect of the total specimen volume or area. For a censored sample, an asymptotic approximation of the 90-percent confidence limits is calculated. No unbiasing of parameters or estimation of confidence limits is given when the least-squares option is requested.

The ability of the parameter estimates to reasonably fit the empirical data is measured with the Kolmogorov-Smirnov (K-S) and Anderson-Darling (A-D) goodness-of-fit tests. These tests are extensively discussed by D'Agostino and Stephens (ref. 11). The tests quantify discrepancies between the experimental data and the estimated Weibull distribution by a significance level associated with the hypothesis that the data were generated from the proposed distribution. The A-D test is more sensitive than the K-S test to discrepancies at low and high probabilities of failure. The Kanofsky-Srinivason 90-percent confidence band values (ref. 12) about the Weibull line are given as an additional test of the goodness-of-fit of the data to the Weibull distribution.

After the shape and characteristic strength parameters are estimated and analyzed, PC-CARES calculates the other material parameters. The biased estimate of the shape parameter $m$ and the estimated characteristic strength $\sigma_\theta$ are used along with the specimen geometry to calculate the Weibull scale parameter $\sigma_o$. The Batdorf normalized crack density coefficient $\bar{k}_B$, which is explained in the appendix section THEORY, is computed from the selected fracture criterion, crack geometry, and the biased estimate of the shape parameter. Figure 2.2 shows the fracture criteria and flaw geometries available to the user which must be specified in order to calculate the Batdorf crack density coefficient. If the user selects to calculate the Batdorf crack density coefficient by setting it to the value that is the solution for the normal stress fracture criterion, then the user need not specify the fracture criterion and the crack geometry.

The simple PIA fracture theory does not require a specific crack geometry, and for uniaxial stress states, it reduces to equation (2.1) or (2.2), whichever is appropriate. The Weibull normal stress averaging method is also independent of crack geometry, since it only considers impending mode I (opening mode) crack growth, and neglects mode II (sliding mode) and mode III (tearing mode) effects. Batdorf's fracture theory can be used with several different mixed-mode fracture criteria and crack geometries. The combination of a particular flaw shape and fracture criterion results in an effective stress equation involving far-field principal stresses in terms of normal and shear stresses acting on the crack plane. The coplanar crack extension criterion for shear-sensitive materials available in PC-CARES is the total strain energy release rate theory. Out-of-plane crack extension criteria are approximated by a simple semi-empirical equation (refs 13 and 14). This equation involves a parameter that can be varied to model the maximum tangential stress theory, the minimum strain energy density criterion, the maximum strain energy release rate theory, or experimental results . For comparison, Griffith's maximum tensile stress analysis for volume flaws is also included. The highlighted boxes in figure 2.2 show the recommended fracture criteria and flaw shapes.

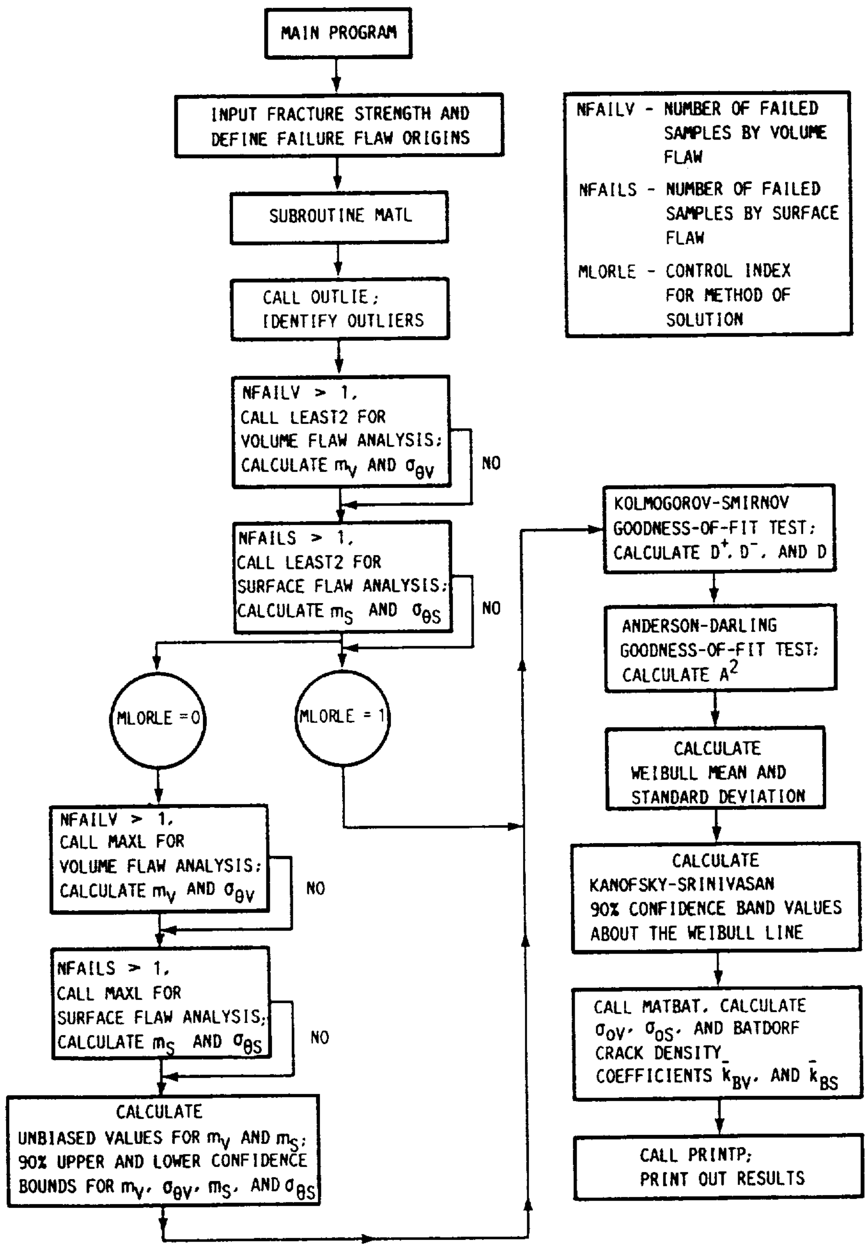


FIGURE 2.1. - FLOWCHART FOR CALCULATION OF MATERIAL STATISTICAL STRENGTH PARAMETERS.

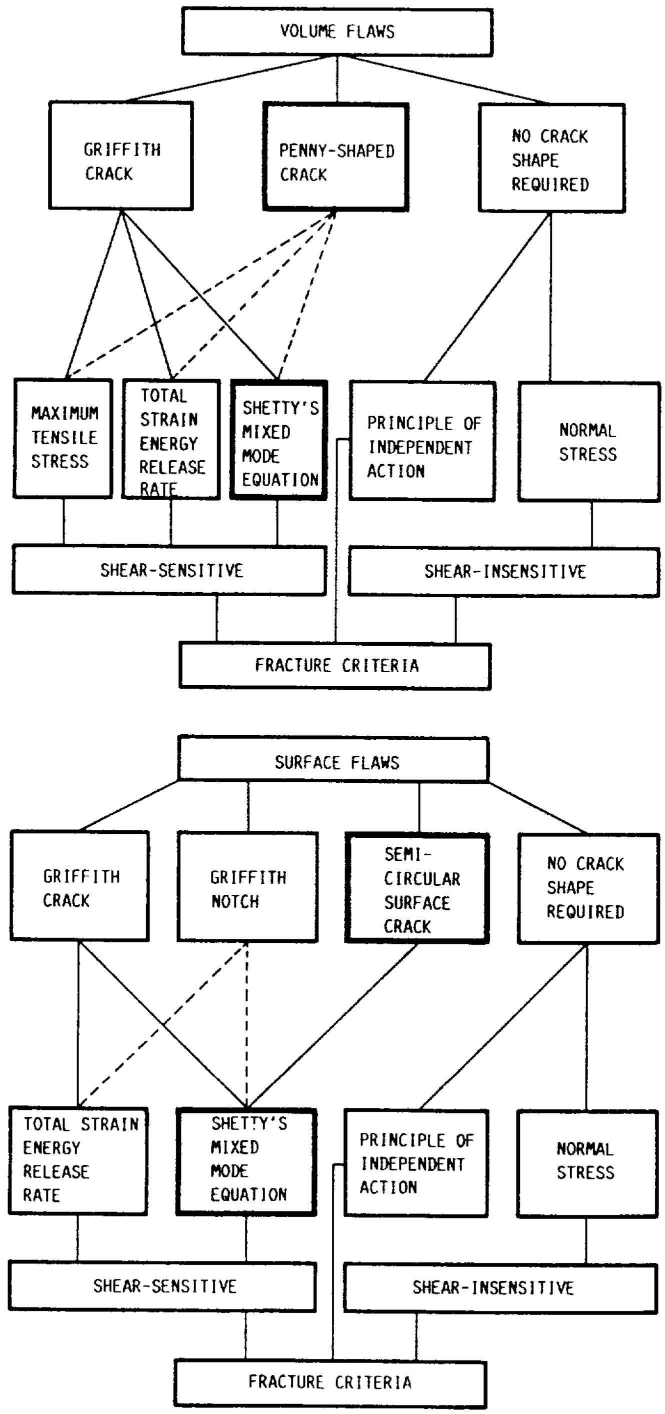


FIGURE 2.2. - AVAILABLE FAILURE CRITERIA AND CRACK SHAPES.

## 3.0 PC-CARES INPUT INFORMATION

PC-CARES uses two files during its execution, the initialization file (PCCARES.INI) and the input file (an example is included on the PC-CARES disk called PCCARES.DAT). Using the initialization file with PC-CARES is optional. For preparation of both the initialization file and the PC-CARES input file, it is assumed that the user has access to a DOS or OS/2 full screen text editor with block editing capabilities.

Input to PC-CARES for both the initialization file and the input file is keyword driven. The keywords can be present in any order within each input section, but they must start in the first column of the file. Examples of both the initialization file and the input file are given in figures 3.1 to 3.7. Note that underneath each keyword a location is given that specifies where the data value or values are input. An explanation of each keyword is provided to the right, and a list of available choices is given, if applicable. If integer input is required, then the input field is between two asterisks (*), and entries must be right justified. Real number input is read in an F10.4 format, and asterisks are not present to define the field width. A maximum of 20 lines between keywords is allowed before an error message is generated, and therefore, the user can insert short notes as desired.

### 3.1 PC-CARES Initialization File Description

Upon execution, PC-CARES first attempts to locate the file called PCCARES.INI in the directory from which PC-CARES was executed. If the file is not present a warning message is issued to the screen and the default parameters for filenames and array allocation sizes are used. Note that if you intend to make use of the initialization file, it must be located in the same directory as the PC-CARES execution file, otherwise the defaults are used.

The initialization file, PCCARES.INI, consists of two categories of input: (1) File Control Input and (2) Dynamic Memory Control Input. The File Control Input consists of two path/filenames (up to 20 characters) which PC-CARES uses as the input and output filenames, so that the user can specify different directory and/or filenames for the input and output files to/from the PC-CARES program. The Dynamic Memory Control Input can contain two indices: one to control the number of constant temperature data sets and the other to control the maximum number of fracture stress data per constant temperature data set. PC-CARES continues to read the initialization file until either it reads the $ENDI keyword or end of file is encountered. Note that keywords that are not found assume default values.

The user is advised to keep an unaltered copy of the PCCARES.INI file as a backup. Details on specific input preparation are described in 3.2.1 File Control Input and 3.2.2 Dynamic Memory Control Input sections of this manual. Each keyword is discussed briefly in these sections, and the format field for the input is denoted in parentheses next to the keyword as well as the default values.

## 3.2 PC-CARES Initialization File Preparation

### 3.2.1 File Control Input

The File Control Input optionally controls which files PC-CARES uses for input and output, overriding the default values. Each file/path name may be up to 20 characters in length. For example see figure 3.1. In this example, if PC-CARES was located in the C:\PCCARES directory the program accesses the input file PPCARES.DAT in the C:\PCCARES\DATA directory and produces the output file PCCARES.OUT in the C:\PCCARES\OUTPUT directory. Please note that the output directory must be created prior to program execution. The keywords pertaining to the File Control Input section of the PC-CARES Initialization file are defined as follows (the input format field is in parentheses):

INFILE (A20); Default : INFILE = 'CARES.DAT'

INFILE controls where PC-CARES reads its input.

OUTFILE (A20); Default : OUTFILE = 'CARES.OUT'

OUTFILE controls where PC-CARES sends its output.

### 3.2.2 Dynamic Memory Control Input

The Dynamic Memory Control Input optionally controls the maximum number of constant temperature test sets and the maximum number of fracture stresses per temperature set, and hence the amount of array space the program allocates. This dynamic array allocation makes the compiled program smaller and gives the user the flexibility to determine how much memory the program will use during execution. Setting IMAXT = 20 and IMAXF = 200 is about the maximum allocation for the PC-CARES running under DOS with its maximum 640K program size. Figure 3.1 shows an example of the Dynamic Memory Control Input section of the PC-CARES Initialization file. The keywords IMAXF and IMAXT are defined as follows (the input format field is in parentheses):

IMAXF (2X,I5); Default : IMAXF = 150

IMAXF controls the maximum number of fracture stress data per constant temperature set.

IMAXT (2X,I5); Default : IMAXT = 10

IMAXT controls the maximum number of constant temperature fracture stress sets.

## 3.3 PC-CARES Input File Description

For the PC-CARES input file, two categories of input are required for execution: (1) Master Control Input and (2) Material Control Input (which includes temperature-dependent material data). The Master Control Input is a set of control indices which directs the overall program execution. It specifies the number of brittle material statistical characterizations, the number of Guassian quadrature points to be used to perform numerical integration, and whether the fracture stresses should be reprinted in the output. The Material Control Input consists of control indices and either the data required to estimate the statistical material parameters or direct input of the Weibull statistical parameter values themselves, for various temperatures to calculate the Batdorf crack density coefficient only. This input category includes the choices of fracture criteria and flaw shapes shown in figure 2.2. The choice of fracture criteria and flaw shape is required to calculate the Batdorf crack density coefficient since it is a function of these conditions as well as the fracture stress data.

### 3.4 PC-CARES Input File Preparation

To control the execution of the PC-CARES program, the user must prepare an input file consisting of the Master Control Input and the Material Control Input. On the disk provided with the program is a file called TEMPLET.INP that can be used to construct an input file for a particular problem. The Master Control Input always comes at the beginning of a file.

After reading the initialization file (if one is present) PC-CARES continues execution by searching for the keywords associated with the Master Control Input in the PC-CARES input file. The end of the Master Control Input occurs when the $ENDX keyword is encountered. Following the Master Control Input, PC-CARES searches for keywords specific to the Material Control Input. The $ENDM and $ENDT keywords signal the end of two different sections of the Material Control Input. Keywords not found between $END intervals may assume default values. Because PC-CARES has a multiple material capability, each section of input for a particular material is separated by a $ENDT card. The TEMPLET.INP file has only two materials characterized. Modifying the file for more materials involves block copying sections of the original file, appending them to the end of the file, and modifying the copied input values accordingly. The user is advised to keep an unaltered copy of the TEMPLET.INP file as a backup. Details on specific input preparation are described in the Master Control Input and Material Control Input sections of this manual. Each keyword is discussed briefly in these sections, and the format field for the input is denoted in parentheses next to the keyword as well as the default value where applicable.

#### 3.4.1 Master Control Input

The Master Control Input section from the TEMPLET.INP file is reproduced in figure 3.2. If parameter keywords in the input file are omitted, they assume their default values.

NGP (4X,I2); Default : NGP = 15

NGP controls the number of Gaussian integration points that are used in the reliability calculations. An entry of 30 will give better accuracy but at the penalty of larger CPU requirements than an entry of 15. There are also other options for NGP in PC-CARES, but users should not specify less than 15 Gaussian points.

NMATS,NMATV (4X,I2); Defaults : NMATS = NMATV = 0

The keyword NMATS represents the number of materials for which surface flaw analysis is performed. NMATV represents the number of materials for which volume flaw analysis is performed. A component consisting of one material may have one set of statistical material parameters to characterize the surface and another set for the volume, for which NMATS = 1 and NMATV = 1. Statistical material parameters are a function of processing, microstructure, and environment. The PC-CARES program is capable of analyzing a single material with multiple statistical material characterizations or many materials with multiple statistical material characterizations. For example, if a single material has two different surface finishes, then NMATS = 2 is used because two different sets of statistical material parameters are required.

TITLE (72A1)

The input associated with the TITLE keyword is reproduced in the program output for problem identification.

$ENDX

$ENDX signifies the end of the MASTER CONTROL INPUT.

### 3.4.2 Material Control Input

A sample of the Material Control Input section from the TEMPLET.INP file is reproduced in figures 3.3 and 3.4. The figures are an example of the input required for PC-CARES to estimate the volume flaw statistical material parameters from experimental fracture data. In figures 3.5 and 3.6 an example of the input needed to estimate the surface flaw statistical material parameters from experimental fracture data is shown. Note that the Material Control Input actually consists of two different data partitions. Figures 3.3 and 3.4, for example, make up a single section of the Material Control Input. In figure 3.3, the control indices, material constants, and geometric variables necessary to calculate volume flaw statistical parameters are shown. In figure 3.4, the temperature-dependent fracture data are given. The temperature-dependent fracture data (MOR), or temperature-dependent values of the Weibull shape and scale parameters (PARAM), are always placed immediately following the control indices for that material.

The total number of Material Control Input sections is equal to the sum of NMATS + NMATV from the Master Control Input. Note that keywords that are not found assume default values.

#### 3.4.2.1 Material and Specimen Dependent Data

The following keywords are the control indices, material indices, and geometric variables necessary for calculation of volume and surface flaw statistical parameters as shown in figures 3.3 and 3.5.

C (F10.4); Default : NONE

If ID2S = 5 or ID2V = 5 (i.e., if Shetty's mixed-mode fracture criterion is selected), then the value of the empirical constant $\bar{C}$, denoted by the keyword C, must be specified. If this criterion is not selected, this input is ignored and can be deleted.

DL1, DL2, DH, DW (F10.4); Default : NONE

If ID1 = 2 or 5 (i.e., if statistical material parameters are to be determined from four-point MOR fracture specimens), then the specimen dimensions must be input. DL1 represents the length between the two outer symmetrical loads. DL2 is the length between the two inner central loads. DH is the total height of the test specimen cross section. DW is the total width of the test specimen cross section. By setting DL2 equal to zero, data obtained from three-point bend tests can also be used to obtain appropriate Weibull parameters.

ID1 (4X,I1); Default : NONE

ID1 is a control index for specifying the form of the data to be input for obtaining the statistical material parameters. Either the Weibull shape and scale parameters are directly specified or experimental fracture data are input. The fracture data can be either from four-point modulus-of-rupture bend bars or from tensile test specimens. If the fracture data are assumed to be all from one failure mode (all volume flaws or all surface flaws), then ID1 = 1 or 2 can be chosen. If ID1 = 1 or 2, then fracture origins are not to be input with the specimen fracture stresses, and PC-CARES assumes that the fracture origins are consistent with the ID4 input index. If ID1 = 4 or 5, then fracture origins must be supplied with the fracture data.

ID2S,ID2V (4X,I1); Default : NONE

These control indices are for selection of a fracture criterion. ID2S is for a surface flaw fracture criterion (see fig. 3.5). ID2V is for a volume flaw fracture criterion (see fig. 3.3). If ID4 = 1, then ID2V should be specified. If ID4 = 2, then ID2S should be specified. If both ID2S and ID2V are specified in the same input section, the entry not consistent with the ID4 index is ignored. Shetty's mixed-mode fracture criterion is recommended for both surface and volume flaw analysis.

ID3S,ID3V (4X,I1); Default : NONE

The ID3S and ID3V control indices are for selection of a crack geometry. ID3S is for surface flaw geometry (see fig. 3.5). ID3V is for volume flaw geometry (see fig. 3.3). If ID4 = 1, then ID3V should be specified. If ID4 = 2, then ID3S should be specified. If both ID3S and ID3V are specified in the same input section, the entry not consistent with the ID4 index is ignored. The penny-shaped crack is recommended for volume flaw analysis and the semicircular crack is recommended for surface flaw analysis.

ID4 (4X,I1); Default : NONE

ID4 controls the calculation of volume- or surface-based statistical material parameters. From the fracture data supplied, the Weibull shape and scale parameters along with the normalized Batdorf crack density coefficient are estimated. If the Weibull shape and scale parameters are directly input, then the normalized Batdorf crack density coefficient is calculated.

IKBAT (4X,I1); Default : IKBAT = 0

IKBAT selects the method of calculating the normalized Batdorf crack density coefficient. If IKBAT = 0, then the crack density coefficient is set to the value that is the solution for the normal stress fracture criterion, regardless of the fracture criterion and crack geometry selected by the user, and therefore the ID2S and ID3S (or ID2V and ID3V) indices do not have to be specified for surface or volume flaw analysis, respectively. If IKBAT = 1, then the crack density coefficient is calculated based on the fracture criterion and crack geometry selected by ID2S and ID3S or by ID2V and ID3V. IKBAT = 0 gives more conservative reliability predictions and usually agrees more closely with test data than IKBAT = 1 does; it is therefore recommended as the best choice unless specific data exist that indicate otherwise.

MATID (1X,I7)

MATID is the material identification number that is associated with the statistical material parameter data.

MLORLE (4X,I1); Default : MLORLE = 0

MLORLE is the control index for the method of estimation of the Weibull shape parameter $m$ and characteristic strength $\sigma_\theta$ from experimental fracture data. MLORLE is ignored if the Weibull shape and scale parameters are directly input.

PR (F10.4); Default : PR = 0.25

PR is Poisson's ratio. It is assumed to be temperature independent.

TITLE (72A1)

The input associated with the TITLE keyword is reproduced in the program output for material identification.

VAGAGE (F10.4); Default : NONE

VAGAGE is the gage volume or area of a tensile test specimen. If ID4 = 2 and ID1 = 1 or 4 (i.e., if surface flaw analysis is specified and the statistical material parameters are to be determined from simple tension tests), then the gage surface area of the specimen must be specified. If ID4 = 1 and ID1 = 1 or 4 (i.e., if volume flaw analysis is specified and the statistical material parameters are to be determined from simple tension tests), then the gage volume of the specimen must be specified.

$ENDM

$ENDM signifies the end of a section of the Material Control Input. The temperature-dependent specimen fracture data or the Weibull shape and scale parameters are assumed to immediately follow.

3.4.2.2 Temperature-Dependent Fracture or Statistical Material Parameters Data

Immediately following the $ENDM keyword, which signals the end of the material and specimen dependent data, the temperature-dependent experimental fracture data or Weibull shape and scale parameters are input. Data for up to IMAXT different temperatures can be specified, where IMAXT is the maximum number of constant temperature data sets as described in section 3.2.2 Dynamic Memory Control Input, of this manual. Figures 3.4 and 3.6 show examples of the input for experimental fracture stress data. Figure 3.7 shows an example of the input for the Weibull shape and scale parameters.

MOR (3A1,3E18.10) or (3E18.10)

MOR indicates that experimental fracture stresses will be input. Fracture stresses can be input in any order, with a maximum of IMAXF specimen failure stresses input for each temperature. There are two styles of input. If ID1 = 1 or 2 (i.e., if the fracture data are assumed to be a complete sample), then fracture stresses only are input and the input format is 3E18.10 as shown in figure 3.6. Referring to figure 3.4, if ID1 = 4 or 5 (i.e., if the fracture origins and the fracture stresses are to be input), then the input format is 3A1, 3E18.10. The 3A1 represents three fields of single alphanumeric characters. This field is for fracture origin input. An "S" indicates a surface flaw origin. A "V" represents a volume flaw origin. A "U" indicates an unknown flaw origin. Each fracture stress has a corresponding fracture origin. In figure 3.4, each line of fracture data consists of three fracture origins followed by their respective failure stresses. Fracture data values should be unique, and multiple identical values should not be input (change the values slightly).

NUT (3X,I3)

NUT is the sample size of the experimental fracture data for the temperature indicated by TDEG. NUT is specified if ID1 does not equal 3 (statistical material parameters are not being directly input). Different numbers of specimens are permitted at different temperatures.

PARAM (2E18.10)

PARAM signals that the Weibull shape and scale parameter will be input for the temperature indicated by TDEG. Referring to figure 3.7, the Weibull shape and then the Weibull scale parameters are entered at the indicated space with a format of 2E18.10. The Weibull shape parameter is dimensionless. The Weibull scale parameter has units of stress x $(\text{volume})^{1/m_V}$ for volume flaw analysis and units of stress x $(\text{area})^{1/m_S}$ for surface flaw analysis. Note that ID1 = 3 must be specified.

TDEG (F10.4)

TDEG is the input keyword for the temperature of the fracture data or of the statistical material parameters that immediately follow. Temperature can be specified in any units desired and is used for identification purposes only in the PC-CARES code.

$ENDT

$ENDT signals the end of the temperature-dependent data.

Another section of the MATERIAL CONTROL INPUT follows, if required.

```
                    --- PC-CARES INITIALIZATION FILE ---

************************************************************************
************************************************************************
                         FILE CONTROL INPUT

------------------------------------------------------------------------

INFILE     : PATH AND FILE NAME FOR CARES MASTER CONTROL INPUT FILE (LUA)
---------      (LENGTH OF PATH/FILE NAME MUST BE < 20 CHARACTERS)
DATA\PCCARES.DAT
---------      (DEFAULT: 'PCCARES.DAT')

OUTFILE    : PATH AND FILE NAME FOR CARES GENERAL OUTPUT FILE (LUB)
---------      (LENGTH OF PATH/FILE NAME MUST BE < 20 CHARACTERS)
DATA\PCCARES.OUT
---------      (DEFAULT: 'PCCARES.OUT')

------------------------------------------------------------------------

                     DYNAMIC MEMORY CONTROL INPUT

------------------------------------------------------------------------

IMAXF      : MAXIMUM NUMBER OF FRACTURE STRESSES THAT CAN BE INPUT PER RUN
---------      (IMAXF > 0. NOTE - THIS PARAMETER CONTROLS THE AMOUNT OF
 *00150*         ARRAY SPACE DYNAMICALLY ALLOCATED AT RUN-TIME.)
---------      (DEFAULT: 150)

IMAXT      : MAXIMUM NUMBER OF TEMPERATURE SETS THAT CAN BE INPUT PER RUN
---------      (IMAXT > 0. NOTE - THIS PARAMETER CONTROLS THE AMOUNT OF
 *00010*         ARRAY SPACE DYNAMICALLY ALLOCATED AT RUN-TIME.)
---------      (DEFAULT:  10)

------------------------------------------------------------------------
************************************************************************
$ENDI      : END OF INITIALIZATION FILE
************************************************************************
```

FIGURE 3.1. - PC-CARES INITIALIZATION FILE.

```
                --- PC-CARES TEMPLET INPUT FILE ---

**************************************************************************
**************************************************************************

               MASTER CONTROL INPUT

TITLE        : PROBLEM TITLE (ECHOED IN PC-CARES OUTPUT)
--------------------------------------------------------------------------
SAMPLE INPUT FOR PC-CARES USERS MANUAL
--------------------------------------------------------------------------

NMATS        : NO. OF MATERIALS FOR SURFACE FLAW ANALYSIS
----------     (NMATS+NMATV < 101)
   *01*        (DEFAULT: NMATS = 0)
----------

NMATV        : NO. OF MATERIALS FOR VOLUME FLAW ANALYSIS
----------     (NMATS+NMATV < 101)
   *01*        (DEFAULT: NMATV = 0)
----------

IPRINT       : CONTROL INDEX FOR STRESS OUTPUT
---------      (DEFAULT: IPRINT = 0)
   *1*         0 : DO NOT PRINT FRACTURE DATA
---------      1 : PRINT FRACTURE DATA

NGP          : NO. OF GAUSSIAN QUADRATURE POINTS (15 OR 30)
----------     (DEFAULT: NGP = 15)
   *15*
----------

**************************************************************************
$ENDX        : END OF MASTER CONTROL INPUT
**************************************************************************
```

FIGURE 3.2. - PC-CARES MASTER CONTROL INPUT.

MATERIAL CONTROL INPUT

```
TITLE        : MATERIAL TITLE (ECHOED IN PC-CARES OUTPUT)
------------------------------------------------------------------------
SAMPLE INPUT FOR PC-CARES USERS MANUAL
------------------------------------------------------------------------

MATID        : MATERIAL IDENTIFICATION NUMBER
---------      (NO DEFAULT)
*0000300*
---------

ID1          : CONTROL INDEX FOR EXPERIMENTAL DATA
---------      (NO DEFAULT)
   *5*         1 : UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
---------      2 : FOUR-POINT BEND TEST DATA
               3 : DIRECT INPUT OF THE WEIBULL PARAMETERS, M AND SP
                   (SHAPE PARAMETER AND SCALE PARAMETER)
               4 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
               5 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   FOUR-POINT BEND TEST DATA

ID4          : CONTROL INDEX FOR VOLUME OR SURFACE FLAW ANALYSIS
---------      (NO DEFAULT)
   *1*         1 : VOLUME
---------      2 : SURFACE

ID2V         : CONTROL INDEX FOR VOLUME FRACTURE CRITERION
---------      (NO DEFAULT)
   *5*         1 : NORMAL STRESS FRACTURE CRITERION
---------          (SHEAR-INSENSITIVE CRACK)
               2 : MAXIMUM TENSILE STRESS CRITERION
               3 : COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
                   (G SUB T)
               4 : WEIBULL PIA MODEL
               5 : SHETTY'S SEMI-EMPIRICAL CRITERION

ID3V         : CONTROL INDEX FOR SHAPE OF VOLUME CRACKS
---------      (NO DEFAULT)
   *2*         1 : GRIFFITH CRACK
---------      2 : PENNY-SHAPED CRACK

IKBAT        : CONTROL INDEX FOR METHOD OF CALCULATING BATDORF CRACK
---------      DENSITY COEFFICIENT (K SUB B) FROM TEST DATA
   *0*         (DEFAULT: IKBAT = 0)
---------      0 : SHEAR-INSENSITIVE METHOD (MODE I FRACTURE ASSUMED)
               1 : SHEAR-SENSITIVE METHOD (FRACTURE ASSUMED TO OCCUR
                   ACCORDING TO THE FRACTURE CRITERION AND CRACK SHAPE
                   SELECTED BY THE ID2 AND ID3 INDICES)

PR           : POISSON'S RATIO
----------     (DEFAULT: PR = 0.25)
00000.2500
----------

C            : CONSTANT FOR SHETTY'S SEMI-EMPIRICAL MIXED-MODE FRACTURE
----------     CRITERION (KI/KIC)+(KII/(C*KIC))**2 = 1
00000.8000     OBSERVED VALUES RANGE FROM 0.8 TO 2. (REF. D.K. SHETTY)
----------     NOTE: AS C APPROACHES INFINITY, PREDICTED FAILURE
```

FIGURE 3.3. - PC-CARES MATERIAL CONTROL INPUT FOR VOLUME FLAW ANALYSIS.

```
             PROBABILITIES APPROACH NORMAL STRESS CRITERION VALUES
             (DEFAULT C = 1.0)

MLORLE     : CONTROL INDEX FOR METHOD OF CALCULATING WEIBULL
---------    PARAMETERS FROM THE EXPERIMENTAL FRACTURE DATA
   *1*       (DEFAULT: MLORLE = 0)
---------    0 : MAXIMUM LIKELIHOOD
             1 : LEAST-SQUARES LINEAR REGRESSION

DH         : HEIGHT OF THE FOUR-POINT BEND BAR
----------   (NO DEFAULT)
00000.0710
----------

DL1        : OUTER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------   (NO DEFAULT)
00001.0000
----------

DL2        : INNER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------   (NO DEFAULT)
00000.5000
----------

DW         : WIDTH OF THE FOUR-POINT BEND BAR
----------   (NO DEFAULT)
00000.1477
----------

************************************************************************
$ENDM      : END OF TEMPERATURE INDEPENDENT MATERIAL CONTROL INPUT
************************************************************************
```

FIGURE 3.3. - CONCLUDED.

```
                    TEMPERATURE DEPENDENT MATERIAL CONTROL INPUT DATA
                    FOR THE ABOVE MATERIAL

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
 PLEASE NOTE THE FOLLOWING:
  1. FRACTURE STRESSES FOR A GIVEN TEMPERATURE CAN BE INPUT IN
     ARBITRARY ORDER.
  2. THE DEFAULT MAXIMUM NUMBER OF TEMPERATURE SETS IS 10.
  3. THE DEFAULT MAXIMUM NUMBER OF FRACTURE SPECIMENS PER TEMPERATURE IS
     150.
  4. REGARDLESS OF THE FRACTURE ORIGIN LOCATION, THE FRACTURE STRESS
     INPUT VALUE IS THE EXTREME FIBER STRESS WITHIN THE INNER LOAD SPAN
     OF THE MOR BAR.
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

TDEG        : TEMPERATURE OF THIS SET
----------
00070.0000
----------

NUT         : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *015*
----------

MOR         : S-URFACE, V-OLUME, OR U-NKNOWN FLAW AND RESPECTIVE STRESS
---*-----------------*-----------------*-----------------*
VVV     0.457500E+05      0.461000E+05      0.481000E+05
VVV     0.481250E+05      0.491250E+05      0.491880E+05
VVV     0.495000E+05      0.496250E+05      0.496500E+05
VSV     0.497500E+05      0.498500E+05      0.498900E+05
SSU     0.506250E+05      0.516250E+05      0.522500E+05
---*-----------------*-----------------*-----------------*
              END OF DATA FOR THE ABOVE TEMPERATURE

TDEG        : TEMPERATURE OF THIS SET
----------
00500.0000
----------

NUT         : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *015*
----------

MOR         : S-URFACE, V-OLUME, OR U-NKNOWN FLAW AND RESPECTIVE STRESS
---*-----------------*-----------------*-----------------*
VVV     0.407500E+05      0.411000E+05      0.431000E+05
VVV     0.431250E+05      0.441250E+05      0.441880E+05
VVV     0.445000E+05      0.446250E+05      0.446500E+05
VVV     0.447500E+05      0.448500E+05      0.448900E+05
VVV     0.456250E+05      0.466250E+05      0.472500E+05
---*-----------------*-----------------*-----------------*
              END OF DATA FOR THE ABOVE TEMPERATURE

TDEG        : TEMPERATURE OF THIS SET
----------
01000.0000
----------
```

FIGURE 3.4. - TEMPERATURE-DEPENDENT FRACTURE DATA FOR VOLUME FLAW ANALYSIS (CENSORED DATA OPTION).

```
NUT          : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *015*
----------

MOR          : S-URFACE, V-OLUME, OR U-NKNOWN FLAW AND RESPECTIVE STRESS
---*-----------------*-----------------*-----------------*
UVV     0.357500E+05      0.361000E+05      0.381000E+05
VVV     0.381250E+05      0.391250E+05      0.391880E+05
VVV     0.395000E+05      0.396250E+05      0.396500E+05
VVS     0.397500E+05      0.398500E+05      0.398900E+05
VSS     0.406250E+05      0.416250E+05      0.422500E+05
---*-----------------*-----------------*-----------------*
             END OF DATA FOR THE ABOVE TEMPERATURE

**************************************************************************
$ENDT        : END OF DATA FOR THE ABOVE MATERIAL
**************************************************************************
```

FIGURE 3.4. - CONCLUDED.

```
                    MATERIAL CONTROL INPUT

TITLE        : MATERIAL TITLE (ECHOED IN PC-CARES OUTPUT)
------------------------------------------------------------------------
SAMPLE INPUT FOR PC-CARES USERS MANUAL
------------------------------------------------------------------------

MATID        : MATERIAL IDENTIFICATION NUMBER
---------      (NO DEFAULT)
*0000300*
---------

ID1          : CONTROL INDEX FOR EXPERIMENTAL DATA
---------      (NO DEFAULT)
   *2*         1 : UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
---------      2 : FOUR-POINT BEND TEST DATA
               3 : DIRECT INPUT OF THE WEIBULL PARAMETERS, M AND SP
                   (SHAPE PARAMETER AND SCALE PARAMETER)
               4 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
               5 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   FOUR-POINT BEND TEST DATA

ID4          : CONTROL INDEX FOR VOLUME OR SURFACE FLAW ANALYSIS
---------      (NO DEFAULT)
   *2*         1 : VOLUME
---------      2 : SURFACE

ID2S         : CONTROL INDEX FOR SURFACE FRACTURE CRITERION
---------      (NO DEFAULT)
   *5*         1 : NORMAL STRESS FRACTURE CRITERION
---------          (SHEAR-INSENSITIVE CRACK)
               3 : COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
                   (G SUB T)
               4 : WEIBULL PIA MODEL
               5 : SHETTY'S SEMI-EMPIRICAL CRITERION

ID3S         : CONTROL INDEX FOR SHAPE OF SURFACE CRACKS
---------      (NO DEFAULT)
   *4*         1 : GRIFFITH CRACK
---------          (ASSOCIATED WITH STRAIN ENERGY RELEASE RATE CRIT.)
                   (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)
               3 : GRIFFITH NOTCH
                   (ASSOCIATED WITH STRAIN ENERGY RELEASE RATE CRIT.)
                   (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)
               4 : SEMICIRCULAR CRACK
                   (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)

IKBAT        : CONTROL INDEX FOR METHOD OF CALCULATING BATDORF CRACK
---------      DENSITY COEFFICIENT (K SUB B) FROM TEST DATA
   *0*         (DEFAULT: IKBAT = 0)
---------      0 : SHEAR-INSENSITIVE METHOD (MODE I FRACTURE ASSUMED)
               1 : SHEAR-SENSITIVE METHOD (FRACTURE ASSUMED TO OCCUR
                   ACCORDING TO THE FRACTURE CRITERION AND CRACK SHAPE
                   SELECTED BY THE ID2 AND ID3 INDICES)

PR           : POISSON'S RATIO
----------     (DEFAULT: PR = 0.25)
00000.2500
----------
```

FIGURE 3.5. - PC-CARES MATERIAL CONTROL INPUT FOR SURFACE FLAW ANALYSIS.

```
C           : CONSTANT FOR SHETTY'S SEMI-EMPIRICAL MIXED-MODE FRACTURE
----------    CRITERION (KI/KIC)+(KII/(C*KIC))**2 = 1
00000.8000    OBSERVED VALUES RANGE FROM 0.8 TO 2. (REF. D.K. SHETTY)
----------    NOTE: AS C APPROACHES INFINITY, PREDICTED FAILURE
              PROBABILITIES APPROACH NORMAL STRESS CRITERION VALUES
              (DEFAULT: C = 1.0)

MLORLE      : CONTROL INDEX FOR METHOD OF CALCULATING WEIBULL
---------     PARAMETERS FROM THE EXPERIMENTAL FRACTURE DATA
   *1*        (DEFAULT: MLORLE = 0)
---------     0 : MAXIMUM LIKELIHOOD
              1 : LEAST-SQUARES LINEAR REGRESSION

DH          : HEIGHT OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
00000.0710
----------

DL1         : OUTER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
00001.0000
----------

DL2         : INNER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
00000.5000
----------

DW          : WIDTH OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
00000.1477
----------

************************************************************************
$ENDM       : END OF TEMPERATURE INDEPENDENT MATERIAL CONTROL INPUT
************************************************************************
```

FIGURE 3.5. - CONCLUDED.

```
                 TEMPERATURE DEPENDENT MATERIAL CONTROL INPUT DATA
                 FOR THE ABOVE MATERIAL

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
 PLEASE NOTE THE FOLLOWING:
  1. FRACTURE STRESSES FOR A GIVEN TEMPERATURE CAN BE INPUT IN
     ARBITRARY ORDER.
  2. THE DEFAULT MAXIMUM NUMBER OF TEMPERATURE SETS IS 10.
  3. THE DEFAULT MAXIMUM NUMBER OF FRACTURE SPECIMENS PER TEMPERATURE IS
     150.
  4. REGARDLESS OF THE FRACTURE ORIGIN LOCATION, THE FRACTURE STRESS
     INPUT VALUE IS THE EXTREME FIBER STRESS WITHIN THE INNER LOAD SPAN
     OF THE MOR BAR.
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

TDEG         : TEMPERATURE OF THIS SET
----------
00070.0000
----------

NUT          : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *015*
----------

MOR          : FRACTURE STRESSES
*-----------------*-----------------*-----------------*
      0.457500E+05      0.461000E+05      0.481000E+05
      0.481250E+05      0.491250E+05      0.491880E+05
      0.495000E+05      0.496250E+05      0.496500E+05
      0.497500E+05      0.498500E+05      0.498900E+05
      0.506250E+05      0.516250E+05      0.522500E+05
*-----------------*-----------------*-----------------*
              END OF DATA FOR THE ABOVE TEMPERATURE

TDEG         : TEMPERATURE OF THIS SET
----------
00500.0000
----------

NUT          : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *015*
----------

MOR          : FRACTURE STRESSES
*-----------------*-----------------*-----------------*
      0.407500E+05      0.411000E+05      0.431000E+05
      0.431250E+05      0.441250E+05      0.441880E+05
      0.445000E+05      0.446250E+05      0.446500E+05
      0.447500E+05      0.448500E+05      0.448900E+05
      0.456250E+05      0.466250E+05      0.472500E+05
*-----------------*-----------------*-----------------*
              END OF DATA FOR THE ABOVE TEMPERATURE

TDEG         : TEMPERATURE OF THIS SET
----------
01000.0000
----------
```

FIGURE 3.6. - TEMPERATURE-DEPENDENT FRACTURE DATA FOR SURFACE FLAW ANALYSIS (COMPLETE SAMPLE OPTION).

```
NUT          : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *015*
----------

MOR          : FRACTURE STRESSES
*-----------------*-----------------*-----------------*
      0.357500E+05      0.361000E+05      0.381000E+05
      0.381250E+05      0.391250E+05      0.391880E+05
      0.395000E+05      0.396250E+05      0.396500E+05
      0.397500E+05      0.398500E+05      0.398900E+05
      0.406250E+05      0.416250E+05      0.422500E+05
*-----------------*-----------------*-----------------*
               END OF DATA FOR THE ABOVE TEMPERATURE

***********************************************************************
$ENDT        : END OF DATA FOR THE ABOVE MATERIAL
***********************************************************************
```

FIGURE 3.6. - CONCLUDED.

```
                TEMPERATURE DEPENDENT MATERIAL CONTROL INPUT DATA
                FOR THE ABOVE MATERIAL

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
 PLEASE NOTE THE FOLLOWING:
  1. THE DEFAULT MAXIMUM NUMBER OF TEMPERATURE SETS IS 10.
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

TDEG          : TEMPERATURE OF THIS SET
----------
00070.0000
----------

PARAM         : WEIBULL MODULUS (SHAPE PARAMETER) AND SCALE PARAMETER
*-WEIBULL MODULUS-*-SCALE PARAMETER-*
      0.765000E+01        0.878910E+05
*-----------------*-----------------*
                END OF DATA FOR THE ABOVE TEMPERATURE

***************************************************************************
$ENDT         : END OF DATA FOR THE ABOVE MATERIAL
***************************************************************************
```

FIGURE 3.7. - DIRECT INPUT OF TEMPERATURE-DEPENDENT STATISTICAL MATERIAL PARAMETERS.

## 4.0 EXECUTION OF THE PC-CARES PROGRAM

Prior to the execution of the PC-CARES program the user must prepare the initialization file (optional) and the PC-CARES input file as per the instructions of section 3.0 PC-CARES INPUT INFORMATION. Further, if one intends to use a special directory other than the default directory for the PC-CARES output file, then that directory must be created prior to execution of the program.

As can be noted from the distribution disk list in section 1.0 INTRODUCTION, there are two executable files included, PCCARES.EXE and PCCARESB.EXE. The former execution file is the PC-CARES code linked for DOS mode (or the OS/2 DOS compatability box) execution only. The latter code is the PC-CARES bound execution file, which means that this file will run in either DOS (real) mode or OS/2 (protected) mode. The reason for including the real mode execution file when the bound execution file will run in either mode is that the real mode execution file is smaller and will allow the user to have more array space for fracture test data which may make a significant difference given the DOS 640K memory limitation. For users running DOS only, it is recommended that you use the PCCARES.EXE execution file.

If you do have OS/2 version 1.1 or higher you can take advantage of the larger address space of protect mode and multiasking by executing PCCARESB.EXE from the OS/2 command prompt.

Execution of PC-CARES is straight forward. After setting up the necessary input files and setting the default directory to the location of the execution files, simply type PCCARES (or PCCARESB) at the DOS (or OS/2) command prompt. Immediately the message 'EXECUTING PC-CARES...' should appear on the screen. Except for any warning or error messages the program will run to completion without any user interaction. When the program finishes the message

PC-CARES EXECUTION COMPLETE

Stop - Program Terminated

will appear on the screen followed by the command prompt. You then may view the output of the program by printing the file, using the TYPE command, or simply using your screen editor to view it.

The program will generate two types of error messages: (1) Program Control Errors and (2) Data Control Errors. The former will always appear on the screen and immediately halt the program. These errors are generated by the inability of the program to either open the necessary files or allocate the array space dynamically; these are essentially errors associated with the initialization file. In the latter case the program will appear to terminate normally but upon viewing the output file an error message will be found. The Data Control Errors result from missing or inconsistent input from either the Master Control Input or the Material Control Input of the PC-CARES input file.

## 5.0 PC-CARES OUTPUT INFORMATION

The first part of the PC-CARES output is an echo of the choices selected (or default values) from the Master Control Input. The PRINTA subroutine echoes these data.

Then PC-CARES proceeds to the PRINTB subroutine to echo the user inputs for each section of the Material Control Input. The results of the analysis of the data from the Material Control Input are output in the PRINTP subroutine. If statistical material parameters are directly input, then output pertaining to calculated values of the normalized Batdorf crack density coefficient will follow. If statistical material parameters are determined from experimental fracture data, then the output will identify the method of solution, the control index used for experimental data, the number of specimens in each batch, and the temperature of each test. In addition, the output echoes the input values of all specimen fracture stresses with proper failure mode identification if the user has set IPRINT = 1 in the Master Control Input. Any data value that deviates greatly from the rest of the sample is detected as an outlier, and its corresponding significance level is printed. Three levels of significance are available for outliers: 1, 5, or 10 percent. The lower the significance level, the more extreme is the deviation of the data point from the rest of the distribution. A 1-percent significance level indicates that there is a 1-in-100 chance that the data point is actually a member of the same population as the other data, assuming a normal distribution.

Next, the biased and the unbiased value of the shape parameter, the specimen characteristic strength, the upper and lower bound values at 90-percent confidence level for both the shape parameter and the specimen characteristic strength, the specimen Weibull mean value, and the corresponding standard deviation are printed for each specified temperature. For censored statistics, these values are generated first for the volume flaw analysis and subsequently for the surface flaw analysis. Not all of this information is available for all methods of material parameter estimation, and section 2.0 PROGRAM CAPABILITY AND DESCRIPTION of this manual should be consulted for further information.

The Kolmogorov-Smirnov (K-S) goodness-of-fit test is done for each data point, and the corresponding K-S statistics ($D+$ and $D-$) and significance level are listed. Similarly, the K-S statistic $D$ for the overall population is printed along with the significance level. This overall statistic is the absolute maximum of individual specimen data $D+$ and $D-$ factors. For the Anderson-Darling (A-D) goodness-of-fit test, the A-D statistic $A^2$ is determined for the overall population and its associated significance level is printed. The lower the significance level, the worse is the fit of the experimental data to be proposed distribution. For these tests, a 1-percent level of significance indicates that there is a 1-in-100 chance that the specimen fracture data is from the estimated distribution.

The next table generated by PC-CARES from the PRINTP subroutine contains data to construct Kanofsky-Srinivasan 90-percent confidence bands about the Weibull distribution. The table includes fracture stress data, the corresponding Weibull probability of failure values, the 90-percent upper and lower confidence band values about the Weibull line, and the median rank value for each data point. These statistical quantities are calculated with either tabular values or approximating polynomial functions. Experimental fracture data lying outside these bands are an indication of poor fit to the Weibull distribution.

The last table from the PRINTP subroutine summarizes the material parameters listed as a function of temperature. These include the biased Weibull modulus, the Batdorf crack density coefficient, and the material Weibull scale parameter (unit volume or unit area characteristic strength). The values given correspond to the experimental temperatures input. Information on the selected fracture criterion and crack shape is printed for shear-sensitive fracture models. Crack shape is not required for the shear-insensitive fracture criterion or for the PIA model, and it need not be identified for those cases.

## 6.0 EXAMPLE PROBLEM

The following example and discussion of estimating the statistical material parameters was, like the THEORY section in the appendix, obtained from the original CARES manual. The CARES input file for this problem is included on the distribution disk in the file called PCCARES.DAT to allow the user to run the analysis in order to familiarize himself (herself) with PC-CARES operation. Following the text below is a copy of both the input file, PCCARES.DAT and the PC-CARES generated output.

### Example - Statistical Material Parameter Estimation

To validate the methods used to estimate statistical material parameters, we compared results from the fracture of four-point bend bars broken at NASA Lewis and analyzed by CARES with results independently obtained by Bruckner-Foit and Munz (ref. 15) for the International Energy Agency (IEA) Annex II, Subtask 4 (ref. 16). The IEA Annex II agreement is focused on cooperative research and development among the United States, West Germany, and Sweden in the areas of structural ceramics. Subtask 4 of the agreement addresses mechanical property measurement methods with initial research concentrating solely on four-point flexure testing. Three different materials were analyzed, namely a hot isostatic pressed (HIPed) silicon carbide (SiC) from Elektroschmelzwerke Kempten (ESK), West Germany, a HIPed silicon nitride ($Si_3N_4$) from ASEA CERAMA, Sweden; and a sintered silicon nitride from GTE WESGO, USA, although only results from the ESK and ASEA materials are discussed herein.

In November 1986, 400 HIPed SiC flexure bars from West Germany were distributed by Oak Ridge National Laboratory (ORNL) (Oakridge, Tennessee) to the five participating U S. laboratories, including NASA Lewis. The bars were fractured at these laboratories and the fracture stress data sets were returned to ORNL as complete data without censoring four different failure modes. Shortly thereafter, 400 $Si_3N_4$ bars from Sweden were also received by ORNL and subsequently distributed to the same U.S. laboratories for fracture testing. Again, the fracture stress data sets were returned to ORNL as complete samples. The number of specimens of a particular material given to each U.S. laboratory was 80. The specimens had cross-sectional dimensions of 3.5 mm (0.138 in.) in width and 4.5 mm (0.177 in.) in height. The specimens were tested in four-point bending with an outer span of 40 mm (1.57 in.) and an inner span of 20 mm (0.787 in.). The nominal loading rate was 0.5 mm/min (0.020 in./min), and the testing temperature was approximately 20 °C (68 °F).

Details of the statistical analyses of these data sets are given in references 15 and 16. The results of the 80 silicon carbide flexure bars tested at NASA Lewis, which are shown in table I, were analyzed with the CARES code to calculate the maximum likelihood estimates (MLE's) of the Weibull parameters. The Weibull parameter values from CARES, summarized in table II, match the predictions from reference 15 reasonably well. The SiC fracture data are plotted in figure 6.1 along with the proposed Weibull line and the Kanofsky-Srinivasan 90-percent confidence bands. Since all of the data are within the 90-percent bands and the goodness-of-fit significance levels are high, it is concluded that the fracture data show good Weibull behavior.

ASEA CERAMA HIPed $Si_3N_4$ bars (ref. 16) from Sweden were also fractured at NASA Lewis, and subsequently, the statistical material parameters were estimated with CARES by using the maximum likelihood method. A comparison of the $Si_3N_4$ results with those in reference 15 is also shown in table II. Agreement between estimates from the two sources is excellent. When the 80 ASEA silicon nitride bars were analyzed by the CARES code as a complete sample, the significance levels of 54 and 35 percent from the Kolmogorov-Smirnov and Anderson-Darling goodness-of-fit tests, respectively, were relatively low, indicating a questionable fit to the proposed Weibull distribution. The lower significance level for the Anderson-Darling test indicated greater deviation occurring in the low strength region of the distribution. From the outlier test included in the CARES code analysis package, the highest strength fracture stress was detected to be an outlier at the 1-percent significance level. Several of the lower strengths were flagged as outliers at various significance levels (1, 5, or 10 percent). Figure 6.2 shows a Weibull plot of the data. From the figure it appears that the data are bimodal with an outlier point at the highest strength.

Because of the observed trends, the data were re-analyzed assuming a censored distribution and removing the highest strength outlier point ($\sigma_f$ = 817.2 MPa (1.185×$10^5$ psi)) as bad data. Although it is possible that both failure modes were surface induced, for the sake of this example it is assumed that the low-strength failures were predominantly due to volume flaws and that the high-strength specimens fractured predominantly because of surface flaws. Since results from fractography of the individual specimens to identify the various failure modes were not available, the fracture origins had to be arbitrarily assigned prior to parameter estimation. Note that identifying individual specimen flaw origins is especially important for small sample sizes where a plot of the data does not yield clear trends. However, for the NASA Lewis $Si_3N_4$ data, the sample size was large, and clear trends could be observed, although extra care would be required to determine if the trends were surface flaw or volume flaw based. From inspection of figure 6.2, we decided to assign the lowest nine strengths as due to volume flaws and the remainder as due to surface flaws. The cracks were arbitrarily assumed to be Griffith cracks, and the total strain energy release rate fracture criterion was used. This assumption was used only in the calculation of $k_{BS}$ and $k_{BV}$. The K-S significance level increased from 0.54 to 0.68, and the A-D significance level increased from 0.35 to 0.58. This improvement supports the initial assumption of bimodal behavior. The value of $\hat{m}$ (the superscript $\hat{}$ indicates an estimated parameter) changed from 13.4 for the complete sample to $\hat{m}_S$ = 22.8 and $\hat{m}_V$ = 4.13. The value of $\hat{\sigma}_\theta$ changed from 686 MPa (9.950×$10^4$ psi) for the complete sample to $\hat{\sigma}_{\theta S}$ = 692 MPa ($1.004\times10^5$ psi) and $\hat{\sigma}_{\theta V}$ = 1128 MPa ($1.636\times10^5$ psi) for the surface and volume flaw distributions, respectively. Further improvements in the goodness-of-fit scores may be gained by correctly identifying the location of fracture origins.

From equation (A.82) the normalized Batdorf crack density coefficient for volume flaws is $(m_V + 1) = 4.13 + 1 = 5.13$, and from equation (A.72) the scale parameter $\sigma_{oV}$ is 17.9 MPa $(m)^{3/4.13}$ ($3.742\times10^4$ psi $(in.)^{3/4.13}$). For surface flaws the normalized Batdorf crack density coefficient is 6.05, whereas $\sigma_{oS}$ calculated by using equation (A.87a) is 461.3 MPa $(m)^{2/22.8}$ ($9.234\times10^4$ psi $(in.)^{2/22.8}$).

For the $Si_3N_4$ fracture data from NASA Lewis, we have obtained goodness-of-fit significance levels as high as 0.78 and 0.88 for the K-S and A-D tests, respectively, by assuming a particular bimodal flaw distribution. For this case, 13 volume flaws were assumed, and the MLE's were $\hat{m}_S = 21.00$, $\hat{m}_V = 6.79$, $\hat{\sigma}_{\theta S}$ = 693 MPa ($1.005\times10^5$ psi), and $\hat{\sigma}_{\theta V}$ = 876 MPa ($1.271\times10^5$ psi). The 13 volume flaws did not correspond to the 13 lowest fracture strengths. On the basis of these goodness-of-fit scores, it is concluded that the data show good bimodal Weibull behavior.

It should be noted from figure 6.2 that the assumed volume flaw distribution dominates the failure response at low probabilities of failure. Therefore, in component design, it is essential to properly account for competing failure modes; otherwise nonconservative design predictions may result.

TABLE I. - EXTREME FIBER FRACTURE STRESSES

OF ESK HIPed SILICON CARBIDE (SiC) BARS

| Flexure bar | Strength, MPa | Flexure bar | Strength, MPa |
|---|---|---|---|
| 1 | 281.2 | 41 | 516.2 |
| 2 | 291.0 | 42 | 519.8 |
| 3 | 358.2 | 43 | 527.6 |
| 4 | 385.4 | 44 | 530.7 |
| 5 | 389.0 | 45 | 530.7 |
| 6 | 390.8 | 46 | 545.7 |
| 7 | 391.8 | 47 | 548.8 |
| 8 | 402.8 | 48 | 552.7 |
| 9 | 412.5 | 49 | 559.6 |
| 10 | 413.3 | 50 | 562.4 |
| 11 | 413.9 | 51 | 563.3 |
| 12 | 417.8 | 52 | 566.1 |
| 13 | 418.2 | 53 | 566.5 |
| 14 | 426.9 | 54 | 570.1 |
| 15 | 437.6 | 55 | 572.8 |
| 16 | 440.0 | 56 | 575.0 |
| 17 | 441.0 | 57 | 576.1 |
| 18 | 442.5 | 58 | 580.0 |
| 19 | 443.8 | 59 | 582.6 |
| 20 | 444.9 | 60 | 588.0 |
| 21 | 446.2 | 61 | 588.6 |
| 22 | 451.5 | 62 | 591.0 |
| 23 | 452.1 | 63 | 591.0 |
| 24 | 452.7 | 64 | 593.3 |
| 25 | 470.4 | 65 | 598.7 |
| 26 | 474.1 | 66 | 599.6 |
| 27 | 475.5 | 67 | 610.0 |
| 28 | 475.5 | 68 | 612.7 |
| 29 | 479.2 | 69 | 619.9 |
| 30 | 483.5 | 70 | 619.9 |
| 31 | 484.8 | 71 | 622.2 |
| 32 | 486.2 | 72 | 622.3 |
| 33 | 488.6 | 73 | 640.5 |
| 34 | 492.5 | 74 | 649.0 |
| 35 | 493.2 | 75 | 657.2 |
| 36 | 496.0 | 76 | 660.0 |
| 37 | 505.7 | 77 | 664.3 |
| 38 | 511.9 | 78 | 673.5 |
| 39 | 512.5 | 79 | 673.9 |
| 40 | 513.8 | 80 | 725.3 |

TABLE II. - WEIBULL PARAMETERS, KOLMOGOROV-SMIRNOV, AND ANDERSON-DARLING TEST RESULTS FOR FOUR-POINT BEND BAR FRACTURE DATA DETERMINED BY MAXIMUM LIKELIHOOD METHOD

[All estimates are biased estimates; 80 complete samples per material.]

| Material type | Source of data | Shape parameter, $\hat{m}$ | 90-Percent confidence limits, $\hat{m}$ | | Characteristic strength, $\hat{\sigma}_\theta$, MPa | 90-Percent confidence limits, $\hat{\sigma}_\theta$, MPa | | K-S test statistic, D | K-S test significance level, α, percent | A-D test significance level, α, percent |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Upper | Lower | | Upper | Lower | | | |
| SiC (ESK) | CARES | 6.48 | 7.38 | 5.52 | 556 | 573 | 539 | 0.070 | 83 | 86 |
| SiC (ESK) | Ref. 15 | 6.59 | 7.65 | 5.61 | 556 | 574 | 539 | .063 | (a) | (a) |
| $Si_3N_4$ (ASEA) | CARES | 13.39 | 15.25 | 11.42 | 686 | 696 | 676 | .0901 | 54 | 35 |
| $Si_3N_4$ (ASEA) | Ref. 15 | 13.40 | 15.3 | 11.4 | 686 | 696 | 676 | .088 | (a) | (a) |

[a]Not available.

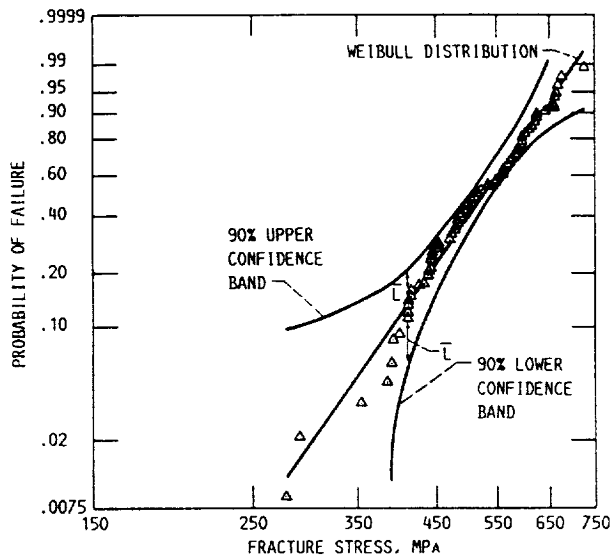


FIGURE 6.1. - 90-PERCENT CONFIDENCE BANDS ABOUT THE WEIBULL LINE FOR ESK HIPPED SILICON CARBIDE (SiC). (FRACTURE STRESS DATA GENERATED AT NASA-LEWIS; NOT ALL DATA POINTS SHOWN; $\bar{L}$ = 0.0829.)

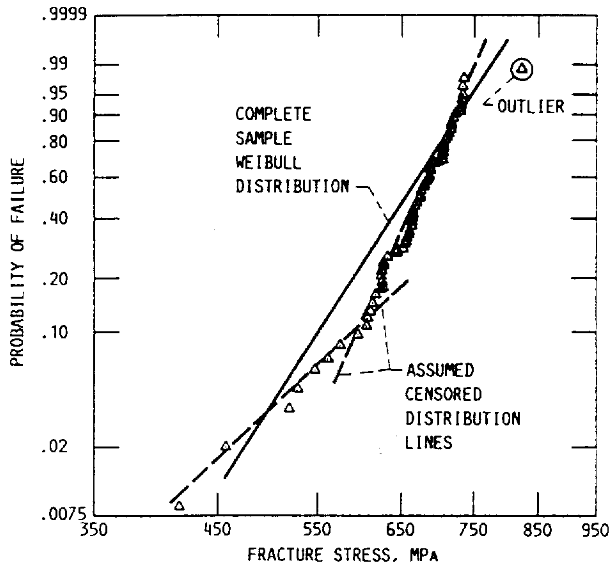


FIGURE 6.2. - COMPLETE SAMPLE, ASSUMED CENSORED SAMPLE WEIBULL DISTRIBUTIONS, AND OUTLIER IN ASEA CERAMA HIPPED SILICON NITRIDE ($Si_3N_4$). (FRACTURE STRESS DATA GENERATED AT NASA-LEWIS; NOT ALL DATA POINTS SHOWN.)

PC-CARES Templet Input File

```
********************************************************************
********************************************************************

            MASTER CONTROL INPUT

TITLE        : PROBLEM TITLE (ECHOED IN PC-CARES OUTPUT)
-----------------------------------------------------------------------
EXAMPLE PROBLEM : STATISTICAL MATERIAL PARAMETER ESTIMATION
-----------------------------------------------------------------------

NMATS        : NO. OF MATERIALS FOR SURFACE FLAW ANALYSIS
----------     (NMATS+NMATV < 101)
  *01*         (DEFAULT: NMATS = 0)
----------

NMATV        : NO. OF MATERIALS FOR VOLUME FLAW ANALYSIS
----------     (NMATS+NMATV < 101)
  *01*         (DEFAULT: NMATV = 0)
----------

IPRINT       : CONTROL INDEX FOR STRESS OUTPUT
---------      (DEFAULT: IPRINT = 0)
  *1*          0 : DO NOT PRINT FRACTURE DATA
---------      1 : PRINT FRACTURE DATA

NGP          : NO. OF GAUSSIAN QUADRATURE POINTS (15 OR 30)
----------     (DEFAULT: NGP = 15)
  *30*
----------

**********************************************************************
$ENDX        : END OF MASTER CONTROL INPUT
**********************************************************************

            MATERIAL CONTROL INPUT

TITLE        : MATERIAL TITLE (ECHOED IN PC-CARES OUTPUT)
-----------------------------------------------------------------------
SI3N4 SPECIMEN DATA FROM ASEA CERAMA FOR VOLUME FLAW ANALYSIS
-----------------------------------------------------------------------

MATID        : MATERIAL IDENTIFICATION NO. FROM THE FINITE ELEMENT
---------      MATERIAL PROPERTY CARD (IF POSTPROCESSING IS NOT
*0000001*      BEING PERFORMED THIS ENTRY SHOULD BE SOME UNIQUE NO.)
---------      (NO DEFAULT)

ID1          : CONTROL INDEX FOR EXPERIMENTAL DATA
---------      (NO DEFAULT)
  *5*          1 : UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
---------      2 : FOUR-POINT BEND TEST DATA
               3 : DIRECT INPUT OF THE WEIBULL PARAMETERS, M AND SP
                   (SHAPE PARAMETER AND SCALE PARAMETER)
               4 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
               5 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   FOUR-POINT BEND TEST DATA
```

```
ID4          : CONTROL INDEX FOR VOLUME OR SURFACE FLAW ANALYSIS
---------      (NO DEFAULT)
   *1*         1 : VOLUME
---------      2 : SURFACE

ID2V         : CONTROL INDEX FOR VOLUME FRACTURE CRITERION
---------      (NO DEFAULT)
   *3*         1 : NORMAL STRESS FRACTURE CRITERION
---------          (SHEAR-INSENSITIVE CRACK)
               2 : MAXIMUM TENSILE STRESS CRITERION
               3 : COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
                   (G SUB T)
               4 : WEIBULL PIA MODEL
               5 : SHETTY'S SEMI-EMPIRICAL CRITERION

ID3V         : CONTROL INDEX FOR SHAPE OF VOLUME CRACKS
---------      (NO DEFAULT)
   *1*         1 : GRIFFITH CRACK
---------      2 : PENNY-SHAPED CRACK

IKBAT        : CONTROL INDEX FOR METHOD OF CALCULATING BATDORF CRACK
---------      DENSITY COEFFICIENT (K SUB B) FROM TEST DATA
   *1*         (DEFAULT: IKBAT = 0)
---------      0 : SHEAR-INSENSITIVE METHOD (MODE I FRACTURE ASSUMED)
               1 : SHEAR-SENSITIVE METHOD (FRACTURE ASSUMED TO OCCUR
                   ACCORDING TO THE FRACTURE CRITERION AND CRACK SHAPE
                   SELECTED BY THE ID2 AND ID3 INDICES)

PR           : POISSON'S RATIO
----------     (DEFAULT: PR = 0.25)
00000.2500
----------

MLORLE       : CONTROL INDEX FOR METHOD OF CALCULATING WEIBULL
---------      PARAMETERS FROM THE EXPERIMENTAL FRACTURE DATA
   *0*         (DEFAULT: MLORLE = 0)
---------      0 : MAXIMUM LIKELIHOOD
               1 : LEAST-SQUARES LINEAR REGRESSION

DH           : HEIGHT OF THE FOUR-POINT BEND BAR
----------     (NO DEFAULT)
0000.00350
----------

DL1          : OUTER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------     (NO DEFAULT)
0000.04000
----------

DL2          : INNER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------     (NO DEFAULT)
0000.02000
----------

DW           : WIDTH OF THE FOUR-POINT BEND BAR
----------     (NO DEFAULT)
0000.00450
----------
************************************************************************
$ENDM        : END OF TEMPERATURE INDEPENDENT MATERIAL CONTROL INPUT
************************************************************************
```

```
              TEMPERATURE DEPENDENT MATERIAL CONTROL INPUT DATA
              FOR THE ABOVE MATERIAL

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
 PLEASE NOTE THE FOLLOWING:
  1. FRACTURE STRESSES FOR A GIVEN TEMPERATURE CAN BE INPUT IN
     ARBITRARY ORDER.
  2. THE DEFAULT MAXIMUM NUMBER OF TEMPERATURE SETS IS 10.
  3. THE DEFAULT MAXIMUM NUMBER OF FRACTURE SPECIMENS PER TEMPERATURE IS
     150.
  4. REGARDLESS OF THE FRACTURE ORIGIN LOCATION, THE FRACTURE STRESS
     INPUT VALUE IS THE EXTREME FIBER STRESS WITHIN THE INNER LOAD SPAN
     OF THE MOR BAR.
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

TDEG          : TEMPERATURE OF THIS SET
----------
00070.0000
----------

NUT           : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *079*
----------

MOR           : S-URFACE, V-OLUME, OR U-NKNOWN FLAW AND RESPECTIVE STRESS
---*-----------------*-----------------*-----------------*
SVV  0.6257900000E+03  0.6034700000E+03  0.5272300000E+03
SSS  0.6911000000E+03  0.6831100000E+03  0.6854900000E+03
SSS  0.6950100000E+03  0.6721000000E+03  0.6569200000E+03
SSS  0.6616800000E+03  0.7270500000E+03  0.7259500000E+03
SSS  0.6707500000E+03  0.7029700000E+03  0.6114900000E+03
SSS  0.6401800000E+03  0.7045300000E+03  0.7254400000E+03
VSS  0.5725000000E+03  0.6251400000E+03  0.6741300000E+03
SSS  0.6597400000E+03  0.6501400000E+03  0.7164300000E+03
SSS  0.6621400000E+03  0.6997700000E+03  0.6645400000E+03
SSS  0.7156300000E+03  0.6045300000E+03  0.6664400000E+03
VVS  0.5605800000E+03  0.4159200000E+03  0.6214600000E+03
SSS  0.6311300000E+03  0.7254400000E+03  0.6940100000E+03
SSS  0.6770800000E+03  0.7251700000E+03  0.7126000000E+03
SSS  0.7156300000E+03  0.7173200000E+03  0.7029200000E+03
VSS  0.5457700000E+03  0.7323300000E+03  0.6717300000E+03
SSS  0.6837300000E+03  0.6144200000E+03  0.6636100000E+03
SSS  0.6490400000E+03  0.6867500000E+03  0.7157700000E+03
SSS  0.7044700000E+03  0.6570300000E+03  0.6641400000E+03
SSS  0.6215900000E+03  0.7165100000E+03  0.7028800000E+03
SSS  0.6515900000E+03  0.7028810000E+03  0.7060500000E+03
SSV  0.7100500000E+03  0.6550600000E+03  0.4583600000E+03
SSS  0.6688200000E+03  0.6597400000E+03  0.6426400000E+03
SSS  0.6093200000E+03  0.6711600000E+03  0.6768300000E+03
SSS  0.6210000000E+03  0.6870000000E+03  0.6221500000E+03
SVS  0.6620500000E+03  0.5950400000E+03  0.6783500000E+03
SSS  0.7286600000E+03  0.6223700000E+03  0.6798500000E+03
V    0.5199000000E+03
---*-----------------*-----------------*-----------------*
              END OF DATA FOR THE ABOVE TEMPERATURE
*************************************************************************
$ENDT         : END OF DATA FOR THE ABOVE MATERIAL
*************************************************************************
```

MATERIAL CONTROL INPUT

```
TITLE        : MATERIAL TITLE (ECHOED IN PC-CARES OUTPUT)
-----------------------------------------------------------------------
SI3N4 SPECIMEN DATA FROM ASEA CERAMA FOR SURFACE FLAW ANALYSIS
-----------------------------------------------------------------------

MATID        : MATERIAL IDENTIFICATION NO. FROM THE FINITE ELEMENT
---------      MATERIAL PROPERTY CARD (IF POSTPROCESSING IS NOT
*0000002*      BEING PERFORMED THIS ENTRY SHOULD BE SOME UNIQUE NO.)
---------      (NO DEFAULT)

ID1          : CONTROL INDEX FOR EXPERIMENTAL DATA
---------      (NO DEFAULT)
  *5*          1 : UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
---------      2 : FOUR-POINT BEND TEST DATA
               3 : DIRECT INPUT OF THE WEIBULL PARAMETERS, M AND SP
                   (SHAPE PARAMETER AND SCALE PARAMETER)
               4 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
               5 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
                   FOUR-POINT BEND TEST DATA

ID4          : CONTROL INDEX FOR VOLUME OR SURFACE FLAW ANALYSIS
---------      (NO DEFAULT)
  *2*          1 : VOLUME
---------      2 : SURFACE

ID2S         : CONTROL INDEX FOR SURFACE FRACTURE CRITERION
---------      (NO DEFAULT)
  *3*          1 : NORMAL STRESS FRACTURE CRITERION
---------          (SHEAR-INSENSITIVE CRACK)
               3 : COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
                   (G SUB T)
               4 : WEIBULL PIA MODEL
               5 : SHETTY'S SEMI-EMPIRICAL CRITERION

ID3S         : CONTROL INDEX FOR SHAPE OF SURFACE CRACKS
---------      (NO DEFAULT)
  *1*          1 : GRIFFITH CRACK
---------          (ASSOCIATED WITH STRAIN ENERGY RELEASE RATE CRIT.)
                   (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)
               3 : GRIFFITH NOTCH
                   (ASSOCIATED WITH STRAIN ENERGY RELEASE RATE CRIT.)
                   (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)
               4 : SEMICIRCULAR CRACK
                   (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)

IKBAT        : CONTROL INDEX FOR METHOD OF CALCULATING BATDORF CRACK
---------      DENSITY COEFFICIENT (K SUB B) FROM TEST DATA
  *1*          (DEFAULT: IKBAT = 0)
---------      0 : SHEAR-INSENSITIVE METHOD (MODE I FRACTURE ASSUMED)
               1 : SHEAR-SENSITIVE METHOD (FRACTURE ASSUMED TO OCCUR
                   ACCORDING TO THE FRACTURE CRITERION AND CRACK SHAPE
                   SELECTED BY THE ID2 AND ID3 INDICES)
```

```
PR          : POISSON'S RATIO
----------    (DEFAULT: PR = 0.25)
00000.2500
----------

MLORLE      : CONTROL INDEX FOR METHOD OF CALCULATING WEIBULL
---------     PARAMETERS FROM THE EXPERIMENTAL FRACTURE DATA
   *0*        (DEFAULT: MLORLE = 0)
---------     0 : MAXIMUM LIKELIHOOD
              1 : LEAST-SQUARES LINEAR REGRESSION

DH          : HEIGHT OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
0000.00350
----------

DL1         : OUTER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
0000.04000
----------

DL2         : INNER LOAD SPAN OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
0000.02000
----------

DW          : WIDTH OF THE FOUR-POINT BEND BAR
----------    (NO DEFAULT)
0000.00450
----------

************************************************************************
$ENDM       : END OF TEMPERATURE INDEPENDENT MATERIAL CONTROL INPUT
************************************************************************

              TEMPERATURE DEPENDENT MATERIAL CONTROL INPUT DATA
              FOR THE ABOVE MATERIAL

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
 PLEASE NOTE THE FOLLOWING:
  1. FRACTURE STRESSES FOR A GIVEN TEMPERATURE CAN BE INPUT IN
     ARBITRARY ORDER.
  2. THE DEFAULT MAXIMUM NUMBER OF TEMPERATURE SETS IS 10.
  3. THE DEFAULT MAXIMUM NUMBER OF FRACTURE SPECIMENS PER TEMPERATURE IS
     150.
  4. REGARDLESS OF THE FRACTURE ORIGIN LOCATION, THE FRACTURE STRESS
     INPUT VALUE IS THE EXTREME FIBER STRESS WITHIN THE INNER LOAD SPAN
     OF THE MOR BAR.
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

TDEG        : TEMPERATURE OF THIS SET
----------
00070.0000
----------

NUT         : NUMBER OF FRACTURE SPECIMENS AT THIS TEMPERATURE
----------
  *079*
----------
```

```
MOR         : S-URFACE, V-OLUME, OR U-NKNOWN FLAW AND RESPECTIVE STRESS
---*-----------------*-----------------*-----------------*
SVV  0.6257900000E+03  0.6034700000E+03  0.5272300000E+03
SSS  0.6911000000E+03  0.6831100000E+03  0.6854900000E+03
SSS  0.6950100000E+03  0.6721000000E+03  0.6569200000E+03
SSS  0.6616800000E+03  0.7270500000E+03  0.7259500000E+03
SSS  0.6707500000E+03  0.7029700000E+03  0.6114900000E+03
SSS  0.6401800000E+03  0.7045300000E+03  0.7254400000E+03
VSS  0.5725000000E+03  0.6251400000E+03  0.6741300000E+03
SSS  0.6597400000E+03  0.6501400000E+03  0.7164300000E+03
SSS  0.6621400000E+03  0.6997700000E+03  0.6645400000E+03
SSS  0.7156300000E+03  0.6045300000E+03  0.6664400000E+03
VVS  0.5605800000E+03  0.4159200000E+03  0.6214600000E+03
SSS  0.6311300000E+03  0.7254400000E+03  0.6940100000E+03
SSS  0.6770800000E+03  0.7251700000E+03  0.7126000000E+03
SSS  0.7156300000E+03  0.7173200000E+03  0.7029200000E+03
VSS  0.5457700000E+03  0.7323300000E+03  0.6717300000E+03
SSS  0.6837300000E+03  0.6144200000E+03  0.6636100000E+03
SSS  0.6490400000E+03  0.6867500000E+03  0.7157700000E+03
SSS  0.7044700000E+03  0.6570300000E+03  0.6641400000E+03
SSS  0.6215900000E+03  0.7165100000E+03  0.7028800000E+03
SSS  0.6515900000E+03  0.7028810000E+03  0.7060500000E+03
SSV  0.7100500000E+03  0.6550600000E+03  0.4583600000E+03
SSS  0.6688200000E+03  0.6597400000E+03  0.6426400000E+03
SSS  0.6093200000E+03  0.6711600000E+03  0.6768300000E+03
SSS  0.6210000000E+03  0.6870000000E+03  0.6221500000E+03
SVS  0.6620500000E+03  0.5950400000E+03  0.6783500000E+03
SSS  0.7286600000E+03  0.6223700000E+03  0.6798500000E+03
V    0.5199000000E+03
---*-----------------*-----------------*-----------------*
             END OF DATA FOR THE ABOVE TEMPERATURE

************************************************************************
$ENDT       : END OF DATA FOR THE ABOVE MATERIAL
************************************************************************
```

PC-CARES Output File

```
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
00                                                                    00
00                                                                    00
00                                                                    00
00      CCCCCC         A        RRRRRRRR   EEEEEEEEE   SSSSSSS        00
00     C      C       A A       R       R  E          S       S       00
00    C              A   A      R       R  E          S               00
00    C             A     A     RRRRRRRR   EEEEEEE     SSSSSSS        00
00    C            AAAAAAAAA    R     R    E                  S       00
00     C      C   A         A   R      R   E          S       S       00
00      CCCCCC   A           A  R       R  EEEEEEEEE   SSSSSSS        00
00                                                                    00
00                                                                    00
00                                                                    00
00    CERAMICS ANALYSIS AND RELIABILITY EVALUATION OF STRUCTURES      00
00                                                                    00
00                                                                    00
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
```

```
**************************************************
*                                                *
*          ECHO OF MASTER CONTROL INPUT          *
*                                                *
**************************************************
```

TITLE = EXAMPLE PROBLEM : STATISTICAL MATERIAL PARAMETER ESTIMATION

```
  3 = CONTROL INDEX FOR OUTPUT OF ANALYSIS (ID4A)
       1 : VOLUME FLAW ANALYSIS OUTPUT ONLY
       2 : SURFACE FLAW ANALYSIS OUTPUT ONLY
       3 : VOLUME FLAW AND SURFACE FLAW ANALYSIS OUTPUT


  1 = NUMBER OF MATERIALS FOR SURFACE FLAW ANALYSIS (NMATS)


  1 = NUMBER OF MATERIALS FOR VOLUME FLAW ANALYSIS (NMATV)


 30 = NUMBER OF GAUSSIAN QUADRATURE POINTS, EITHER 15 OR 30 (NGP)


  1 = CONTROL INDEX FOR STRESS OUTPUT (IPRINT)
       0 : DO NOT PRINT ELEMENT STRESSES AND/OR FRACTURE DATA
       1 : PRINT ELEMENT STRESSES AND/OR FRACTURE DATA
```

```
**************************************************
*                                                *
*          ECHO OF MATERIAL CONTROL INPUT        *
*                                                *
**************************************************


TITLE = SI3N4 SPECIMEN DATA FROM ASEA CERAMA FOR VOLUME FLAW ANALYSIS

  1 = MATERIAL IDENTIFICATION NUMBER (MATID)

  1 = CONTROL INDEX FOR VOLUME OR SURFACE FLAW ANALYSIS (ID4)
       1 : VOLUME
       2 : SURFACE

  0 = CONTROL INDEX FOR METHOD OF CALCULATING WEIBULL PARAMETERS
      FROM THE EXPERIMENTAL FRACTURE DATA (MLORLE)
       0 : MAXIMUM LIKELIHOOD
       1 : LEAST-SQUARES LINEAR REGRESSION

  5 = CONTROL INDEX FOR EXPERIMENTAL DATA (ID1)
       1 : UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
       2 : FOUR-POINT BEND TEST DATA
       3 : DIRECT INPUT OF THE WEIBULL PARAMETERS, M AND SP
           (SHAPE PARAMETER AND SCALE PARAMETER)
       4 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
           UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
       5 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
           FOUR-POINT BEND TEST DATA

  3 = CONTROL INDEX FOR VOLUME FRACTURE CRITERION (ID2V)
       1 : NORMAL STRESS FRACTURE CRITERION
           (SHEAR-INSENSITIVE CRACK)
       2 : MAXIMUM TENSILE STRESS CRITERION
       3 : COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
       4 : WEIBULL PIA MODEL
       5 : SHETTY'S SEMI-EMPIRICAL CRITERION

  1 = CONTROL INDEX FOR SHAPE OF VOLUME CRACKS (ID3V)
       1 : GRIFFITH CRACK
       2 : PENNY-SHAPED CRACK

  1 = CONTROL INDEX FOR METHOD OF CALCULATING BATDORF CRACK DENSITY
   COEFFICIENT
      (K SUB B) FROM TEST DATA (IKBAT)
       0 : SHEAR-INSENSITIVE METHOD (MODE I FRACTURE ASSUMED)
       1 : SHEAR-SENSITIVE METHOD (FRACTURE ASSUMED TO OCCUR
           ACCORDING TO THE FRACTURE CRITERION AND CRACK
           SHAPE SELECTED BY THE ID2 AND ID3 INDICES)

   .2500 = POISSON'S RATIO (PR)
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*                                                 *
*  STATISTICAL ANALYSIS OF FRACTURE SPECIMEN DATA *
*                                                 *
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ECHO OF SPECIMEN INPUT DATA, IN ASCENDING ORDER OF FRACTURE STRESS

.4000E-01 = OUTER LOAD SPAN OF FOUR-POINT BEND BAR
.2000E-01 = INNER LOAD SPAN
.3500E-02 = DEPTH OF SPECIMEN
.4500E-02 = WIDTH OF SPECIMEN

79 = NUMBER OF SPECIMENS IN BATCH          70.000 = TEMPERATURE OF BATCH

"S"URFACE OR "V"OLUME OR "U"NKNOWN FLAW ORIGIN AND RESPECTIVE FAILURE STRESS

| | | | |
|---|---|---|---|
| VVV | .4159E+03 | .4584E+03 | .5199E+03 |
| VVV | .5272E+03 | .5458E+03 | .5606E+03 |
| VVV | .5725E+03 | .5950E+03 | .6035E+03 |
| SSS | .6045E+03 | .6093E+03 | .6115E+03 |
| SSS | .6144E+03 | .6210E+03 | .6215E+03 |
| SSS | .6216E+03 | .6222E+03 | .6224E+03 |
| SSS | .6251E+03 | .6258E+03 | .6311E+03 |
| SSS | .6402E+03 | .6426E+03 | .6490E+03 |
| SSS | .6501E+03 | .6516E+03 | .6551E+03 |
| SSS | .6569E+03 | .6570E+03 | .6597E+03 |
| SSS | .6597E+03 | .6617E+03 | .6621E+03 |
| SSS | .6621E+03 | .6636E+03 | .6641E+03 |
| SSS | .6645E+03 | .6664E+03 | .6688E+03 |
| SSS | .6708E+03 | .6712E+03 | .6717E+03 |
| SSS | .6721E+03 | .6741E+03 | .6768E+03 |
| SSS | .6771E+03 | .6784E+03 | .6799E+03 |
| SSS | .6831E+03 | .6837E+03 | .6855E+03 |
| SSS | .6867E+03 | .6870E+03 | .6911E+03 |
| SSS | .6940E+03 | .6950E+03 | .6998E+03 |
| SSS | .7029E+03 | .7029E+03 | .7029E+03 |
| SSS | .7030E+03 | .7045E+03 | .7045E+03 |
| SSS | .7061E+03 | .7101E+03 | .7126E+03 |
| SSS | .7156E+03 | .7156E+03 | .7158E+03 |
| SSS | .7164E+03 | .7165E+03 | .7173E+03 |
| SSS | .7252E+03 | .7254E+03 | .7254E+03 |
| SSS | .7260E+03 | .7271E+03 | .7287E+03 |
| S | .7323E+03 | | |

--- STEFANSKY OUTLIER TEST OF SPECIMEN FRACTURE STRESSES ---

RESULTS FROM THE STEFANSKY OUTLIER TEST FOR TEMP. = 70.0000

FAILURE STRESS

.4159E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 1% SIGNIFICANCE LEVEL
.4584E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 1% SIGNIFICANCE LEVEL
.5199E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 1% SIGNIFICANCE LEVEL
.5272E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 5% SIGNIFICANCE LEVEL
.5458E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 10% SIGNIFICANCE LEVEL

DEVIATION FROM THE MAIN TREND OF THE DATA MAY INDICATE BAD VALUES. MULTIPLE DEVIATIONS FROM THE SAME REGION OF THE DISTRIBUTION INDICATE THAT EITHER A CONCURRENT OR A PARTIALLY CONCURRENT FLAW POPULATION HAS BEEN DETECTED (NOTE THAT A CONCURRENT FLAW POPULATION MAY BE PRESENT BUT NOT BE DETECTED BY THE OUTLIER TEST). DEVIATIONS OCCURRING IN THE SAME REGION OF THE DISTRIBUTION WITH ALL THREE SIGNIFICANCE LEVELS (1%, 5% AND 10%) PRESENT INDICATE A CONCURRENT FLAW POPULATION. DEVIATIONS SHOULD BE EXAMINED AND TREATED ACCORDINGLY (I.E. IGNORE, CENSOR, ADJUST OR ELIMINATE STRESS). JUDGEMENT OF ACTION TAKEN CAN BE DETERMINED FROM THE GOODNESS-OF-FIT TESTS.
THE OUTLIER TEST IS NO SUBSTITUTE FOR GRAPHICAL EXAMINATION!!!

-- TEMP. DEP. WEIBULL MODULUS AND CHARACTERISTIC STRENGTH WITH 90% -- CONFIDENCE BOUNDS DETERMINED BY MAXIMUM LIKELIHOOD ANALYSIS

NOTE: 90% CONFIDENCE BOUNDS ON PARAMETERS DETERMINED FROM COMPETING FAILURE MODES (CENSORED DATA) ARE APPROXIMATE.
FOR CENSORED DATA THE UNBIASED VALUE OF THE PARAMETER "M" IS NOT GIVEN.
FOR SAMPLE SIZES LESS THAN 4, CONFIDENCE LIMITS ARE NOT GIVEN.

| | TEMP. | M BIASED | M UNBIASED | UP M | LOW M |
|---|---|---|---|---|---|
| | CHAR. STR. | UP C.S. | LOW C.S. | MEAN | STD. DEV. |
| VOLUME | 70.0000 | .4130E+01 | | .5666E+01 | .2179E+01 |
| | .1128E+04 | .1342E+04 | .9542E+03 | .1024E+04 | .2791E+03 |
| SURFACE | 70.0000 | .2281E+02 | | .2619E+02 | .1920E+02 |
| | .6917E+03 | .6981E+03 | .6853E+03 | .6755E+03 | .3684E+02 |

STATISTICS FROM THE GOODNESS-OF-FIT TESTS FOR TEMP. = 70.0000

KOLMOGOROV-SMIRNOV TEST

| ORDER | FRAC. STR. | WEIB. PROB. OF FAIL. | D+ FACTOR | D- FACTOR | SIGNIF. LEVEL |
|---|---|---|---|---|---|
| 1 | .4159E+03 | .0161 | -.0035 | .0161 | 99.0000 |
| 2 | .4584E+03 | .0240 | .0013 | .0114 | 99.0000 |
| 3 | .5199E+03 | .0414 | -.0034 | .0161 | 99.0000 |
| 4 | .5272E+03 | .0443 | .0064 | .0063 | 99.0000 |
| 5 | .5458E+03 | .0529 | .0104 | .0023 | 99.0000 |
| 6 | .5606E+03 | .0620 | .0140 | -.0013 | 99.0000 |
| 7 | .5725E+03 | .0714 | .0172 | -.0045 | 99.0000 |
| 8 | .5950E+03 | .0983 | .0029 | .0097 | 99.0000 |
| 9 | .6035E+03 | .1130 | .0009 | .0118 | 99.0000 |
| 10 | .6045E+03 | .1151 | .0114 | .0012 | 99.0000 |
| 11 | .6093E+03 | .1254 | .0139 | -.0012 | 99.0000 |
| 12 | .6115E+03 | .1305 | .0214 | -.0088 | 99.0000 |
| 13 | .6144E+03 | .1379 | .0267 | -.0140 | 99.0000 |
| 14 | .6210E+03 | .1567 | .0205 | -.0079 | 99.0000 |
| 15 | .6215E+03 | .1581 | .0317 | -.0191 | 99.0000 |
| 16 | .6216E+03 | .1585 | .0440 | -.0313 | 99.0000 |
| 17 | .6222E+03 | .1603 | .0549 | -.0422 | 97.1339 |
| 18 | .6224E+03 | .1610 | .0668 | -.0541 | 87.2422 |
| 19 | .6251E+03 | .1704 | .0701 | -.0575 | 83.1735 |
| 20 | .6258E+03 | .1726 | .0805 | -.0679 | 68.5012 |
| 21 | .6311E+03 | .1930 | .0728 | -.0602 | 79.6191 |
| 22 | .6402E+03 | .2346 | .0439 | -.0312 | 99.0000 |
| 23 | .6426E+03 | .2476 | .0435 | -.0309 | 99.0000 |
| 24 | .6490E+03 | .2853 | .0185 | -.0058 | 99.0000 |
| 25 | .6501E+03 | .2924 | .0241 | -.0114 | 99.0000 |
| 26 | .6516E+03 | .3020 | .0271 | -.0145 | 99.0000 |
| 27 | .6551E+03 | .3262 | .0156 | -.0030 | 99.0000 |
| 28 | .6569E+03 | .3399 | .0146 | -.0019 | 99.0000 |
| 29 | .6570E+03 | .3407 | .0264 | -.0137 | 99.0000 |
| 30 | .6597E+03 | .3616 | .0181 | -.0055 | 99.0000 |
| 31 | .6597E+03 | .3616 | .0308 | -.0181 | 99.0000 |
| 32 | .6617E+03 | .3773 | .0277 | -.0151 | 99.0000 |
| 33 | .6621E+03 | .3804 | .0374 | -.0247 | 99.0000 |
| 34 | .6621E+03 | .3811 | .0493 | -.0366 | 99.0000 |
| 35 | .6636E+03 | .3935 | .0496 | -.0369 | 99.0000 |
| 36 | .6641E+03 | .3980 | .0577 | -.0450 | 95.5201 |
| 37 | .6645E+03 | .4015 | .0669 | -.0542 | 87.1359 |
| 38 | .6664E+03 | .4182 | .0628 | -.0502 | 91.3909 |
| 39 | .6688E+03 | .4398 | .0538 | -.0412 | 97.5998 |
| 40 | .6708E+03 | .4580 | .0484 | -.0357 | 99.0000 |
| 41 | .6712E+03 | .4619 | .0571 | -.0445 | 95.8855 |
| 42 | .6717E+03 | .4674 | .0643 | -.0516 | 89.9719 |
| 43 | .6721E+03 | .4709 | .0734 | -.0607 | 78.8750 |
| 44 | .6741E+03 | .4909 | .0661 | -.0534 | 88.0606 |
| 45 | .6768E+03 | .5182 | .0515 | -.0388 | 98.4919 |
| 46 | .6771E+03 | .5207 | .0616 | -.0489 | 92.5627 |
| 47 | .6784E+03 | .5338 | .0611 | -.0484 | 92.9632 |
| 48 | .6799E+03 | .5495 | .0581 | -.0454 | 95.2717 |
| 49 | .6831E+03 | .5842 | .0360 | -.0234 | 99.0000 |
| 50 | .6837E+03 | .5909 | .0420 | -.0293 | 99.0000 |
| 51 | .6855E+03 | .6100 | .0356 | -.0229 | 99.0000 |
| 52 | .6867E+03 | .6237 | .0345 | -.0219 | 99.0000 |

| | | | | | |
|---|---|---|---|---|---|
| 53 | .6870E+03 | .6264 | .0445 | -.0318 | 99.0000 |
| 54 | .6911E+03 | .6712 | .0124 | .0003 | 99.0000 |
| 55 | .6940E+03 | .7027 | -.0065 | .0192 | 99.0000 |
| 56 | .6950E+03 | .7135 | -.0046 | .0173 | 99.0000 |
| 57 | .6998E+03 | .7635 | -.0419 | .0546 | 97.2613 |
| 58 | .7029E+03 | .7946 | -.0604 | .0731 | 79.2561 |
| 59 | .7029E+03 | .7946 | -.0478 | .0604 | 93.5064 |
| 60 | .7029E+03 | .7950 | -.0355 | .0482 | 99.0000 |
| 61 | .7030E+03 | .7955 | -.0233 | .0360 | 99.0000 |
| 62 | .7045E+03 | .8099 | -.0251 | .0378 | 99.0000 |
| 63 | .7045E+03 | .8105 | -.0130 | .0257 | 99.0000 |
| 64 | .7061E+03 | .8247 | -.0146 | .0272 | 99.0000 |
| 65 | .7101E+03 | .8598 | -.0370 | .0496 | 98.9973 |
| 66 | .7126E+03 | .8801 | -.0447 | .0574 | 95.7363 |
| 67 | .7156E+03 | .9022 | -.0541 | .0667 | 87.3420 |
| 68 | .7156E+03 | .9022 | -.0414 | .0541 | 97.5075 |
| 69 | .7158E+03 | .9031 | -.0297 | .0424 | 99.0000 |
| 70 | .7164E+03 | .9076 | -.0215 | .0341 | 99.0000 |
| 71 | .7165E+03 | .9081 | -.0094 | .0220 | 99.0000 |
| 72 | .7173E+03 | .9134 | -.0020 | .0146 | 99.0000 |
| 73 | .7252E+03 | .9549 | -.0309 | .0435 | 99.0000 |
| 74 | .7254E+03 | .9560 | -.0193 | .0320 | 99.0000 |
| 75 | .7254E+03 | .9560 | -.0067 | .0193 | 99.0000 |
| 76 | .7260E+03 | .9581 | .0039 | .0087 | 99.0000 |
| 77 | .7271E+03 | .9624 | .0123 | .0003 | 99.0000 |
| 78 | .7287E+03 | .9680 | .0193 | -.0067 | 99.0000 |
| 79 | .7323E+03 | .9786 | .0214 | -.0088 | 99.0000 |

KOLMOGOROV-SMIRNOV TEST YIELDS STATISTIC D = MAX (D+,D-) = .0805
WITH AN ASSOCIATED SIGNIFICANCE LEVEL OF 68.5%

ANDERSON-DARLING TEST YIELDS STATISTIC A**2 = .6725
WITH AN ASSOCIATED SIGNIFICANCE LEVEL OF 58.3%

KANOFSKY-SRINIVASAN 90% CONFIDENCE BANDS ABOUT THE WEIBULL DISTRIBUTION FOR
TEMP. = 70.0000

THE KANOFSKY-SRINIVASAN FACTOR FOR THIS DISTRIBUTION IS .0836 FOR A SAMPLE
SIZE OF 79

| ORD. | FRAC. STR. | WEIB. PROB. OF FAIL. | UPP.CONF.BAND | MED. RANK | LOW.CONF.BAND |
|---|---|---|---|---|---|
| 1 | .4159E+03 | .0161 | .0997 | .0088 | .0000 |
| 2 | .4584E+03 | .0240 | .1077 | .0214 | .0000 |
| 3 | .5199E+03 | .0414 | .1250 | .0340 | .0000 |
| 4 | .5272E+03 | .0443 | .1279 | .0466 | .0000 |
| 5 | .5458E+03 | .0529 | .1365 | .0592 | .0000 |
| 6 | .5606E+03 | .0620 | .1456 | .0718 | .0000 |
| 7 | .5725E+03 | .0714 | .1551 | .0844 | .0000 |
| 8 | .5950E+03 | .0983 | .1820 | .0970 | .0147 |
| 9 | .6035E+03 | .1130 | .1967 | .1096 | .0294 |
| 10 | .6045E+03 | .1151 | .1988 | .1222 | .0315 |
| 11 | .6093E+03 | .1254 | .2090 | .1348 | .0418 |
| 12 | .6115E+03 | .1305 | .2141 | .1474 | .0469 |
| 13 | .6144E+03 | .1379 | .2215 | .1599 | .0542 |

| | | | | | |
|---|---|---|---|---|---|
| 14 | .6210E+03 | .1567 | .2403 | .1725 | .0731 |
| 15 | .6215E+03 | .1581 | .2418 | .1851 | .0745 |
| 16 | .6216E+03 | .1585 | .2422 | .1977 | .0749 |
| 17 | .6222E+03 | .1603 | .2440 | .2103 | .0767 |
| 18 | .6224E+03 | .1610 | .2447 | .2229 | .0774 |
| 19 | .6251E+03 | .1704 | .2540 | .2355 | .0867 |
| 20 | .6258E+03 | .1726 | .2563 | .2481 | .0890 |
| 21 | .6311E+03 | .1930 | .2766 | .2607 | .1094 |
| 22 | .6402E+03 | .2346 | .3182 | .2733 | .1510 |
| 23 | .6426E+03 | .2476 | .3313 | .2859 | .1640 |
| 24 | .6490E+03 | .2853 | .3690 | .2985 | .2017 |
| 25 | .6501E+03 | .2924 | .3760 | .3111 | .2088 |
| 26 | .6516E+03 | .3020 | .3856 | .3237 | .2183 |
| 27 | .6551E+03 | .3262 | .4098 | .3363 | .2425 |
| 28 | .6569E+03 | .3399 | .4235 | .3489 | .2562 |
| 29 | .6570E+03 | .3407 | .4243 | .3615 | .2571 |
| 30 | .6597E+03 | .3616 | .4453 | .3741 | .2780 |
| 31 | .6597E+03 | .3616 | .4453 | .3866 | .2780 |
| 32 | .6617E+03 | .3773 | .4609 | .3992 | .2937 |
| 33 | .6621E+03 | .3804 | .4640 | .4118 | .2967 |
| 34 | .6621E+03 | .3811 | .4647 | .4244 | .2975 |
| 35 | .6636E+03 | .3935 | .4771 | .4370 | .3098 |
| 36 | .6641E+03 | .3980 | .4816 | .4496 | .3144 |
| 37 | .6645E+03 | .4015 | .4851 | .4622 | .3178 |
| 38 | .6664E+03 | .4182 | .5018 | .4748 | .3345 |
| 39 | .6688E+03 | .4398 | .5234 | .4874 | .3562 |
| 40 | .6708E+03 | .4580 | .5416 | .5000 | .3743 |
| 41 | .6712E+03 | .4619 | .5455 | .5126 | .3782 |
| 42 | .6717E+03 | .4674 | .5510 | .5252 | .3837 |
| 43 | .6721E+03 | .4709 | .5546 | .5378 | .3873 |
| 44 | .6741E+03 | .4909 | .5745 | .5504 | .4073 |
| 45 | .6768E+03 | .5182 | .6018 | .5630 | .4345 |
| 46 | .6771E+03 | .5207 | .6043 | .5756 | .4371 |
| 47 | .6784E+03 | .5338 | .6175 | .5882 | .4502 |
| 48 | .6799E+03 | .5495 | .6332 | .6008 | .4659 |
| 49 | .6831E+03 | .5842 | .6679 | .6134 | .5006 |
| 50 | .6837E+03 | .5909 | .6745 | .6259 | .5073 |
| 51 | .6855E+03 | .6100 | .6936 | .6385 | .5263 |
| 52 | .6867E+03 | .6237 | .7073 | .6511 | .5401 |
| 53 | .6870E+03 | .6264 | .7100 | .6637 | .5428 |
| 54 | .6911E+03 | .6712 | .7548 | .6763 | .5875 |
| 55 | .6940E+03 | .7027 | .7864 | .6889 | .6191 |
| 56 | .6950E+03 | .7135 | .7971 | .7015 | .6298 |
| 57 | .6998E+03 | .7635 | .8471 | .7141 | .6798 |
| 58 | .7029E+03 | .7946 | .8782 | .7267 | .7110 |
| 59 | .7029E+03 | .7946 | .8782 | .7393 | .7110 |
| 60 | .7029E+03 | .7950 | .8786 | .7519 | .7114 |
| 61 | .7030E+03 | .7955 | .8791 | .7645 | .7119 |
| 62 | .7045E+03 | .8099 | .8936 | .7771 | .7263 |
| 63 | .7045E+03 | .8105 | .8941 | .7897 | .7269 |
| 64 | .7061E+03 | .8247 | .9083 | .8023 | .7411 |
| 65 | .7101E+03 | .8598 | .9434 | .8149 | .7761 |
| 66 | .7126E+03 | .8801 | .9638 | .8275 | .7965 |
| 67 | .7156E+03 | .9022 | .9858 | .8401 | .8185 |
| 68 | .7156E+03 | .9022 | .9858 | .8526 | .8185 |
| 69 | .7158E+03 | .9031 | .9867 | .8652 | .8195 |
| 70 | .7164E+03 | .9076 | .9912 | .8778 | .8239 |
| 71 | .7165E+03 | .9081 | .9917 | .8904 | .8245 |
| 72 | .7173E+03 | .9134 | .9970 | .9030 | .8297 |
| 73 | .7252E+03 | .9549 | 1.0000 | .9156 | .8713 |

| | | | | | |
|---|---|---|---|---|---|
| 74 | .7254E+03 | .9560 | 1.0000 | .9282 | .8724 |
| 75 | .7254E+03 | .9560 | 1.0000 | .9408 | .8724 |
| 76 | .7260E+03 | .9581 | 1.0000 | .9534 | .8745 |
| 77 | .7271E+03 | .9624 | 1.0000 | .9660 | .8787 |
| 78 | .7287E+03 | .9680 | 1.0000 | .9786 | .8844 |
| 79 | .7323E+03 | .9786 | 1.0000 | .9912 | .8950 |

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*                                                  *
*          VOLUME FLAW PARAMETER ANALYSIS          *
*                                                  *
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**** BATDORF MODEL --- CRACK ORIENTATION, CRACK SHAPE, ****
AND FRACTURE CRITERION ARE CONSIDERED

--- TEMPERATURE DEPENDENT MATERIAL PARAMETERS FOR MATERIAL NUMBER 1

WEIBULL MODULUS (SHAPE PARAMETER), M (DIMENSIONLESS)
NORMALIZED BATDORF CRACK DENSITY COEFFICIENT, K (DIMENSIONLESS)
SCALE PARAMETER, SP (UNITS OF STRESS*VOLUME**(1/M))

| TEMPERATURE | M BIASED | K | SP |
|---|---|---|---|
| 70.0000 | .4130E+01 | .5130E+01 | .1787E+02 |

FRACTURE CRITERION = COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
CRACK SHAPE = GRIFFITH CRACK

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****************************************************
*                                                  *
*          ECHO OF MATERIAL CONTROL INPUT          *
*                                                  *
****************************************************
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TITLE = SI3N4 SPECIMEN DATA FROM ASEA CERAMA FOR SURFACE FLAW ANALYSIS

2 = MATERIAL IDENTIFICATION NUMBER (MATID)

2 = CONTROL INDEX FOR VOLUME OR SURFACE FLAW ANALYSIS (ID4)
 1 : VOLUME
 2 : SURFACE

```
0 = CONTROL INDEX FOR METHOD OF CALCULATING WEIBULL PARAMETERS
    FROM THE EXPERIMENTAL FRACTURE DATA (MLORLE)
     0 : MAXIMUM LIKELIHOOD
     1 : LEAST-SQUARES LINEAR REGRESSION


5 = CONTROL INDEX FOR EXPERIMENTAL DATA (ID1)
     1 : UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
     2 : FOUR-POINT BEND TEST DATA
     3 : DIRECT INPUT OF THE WEIBULL PARAMETERS, M AND SP
         (SHAPE PARAMETER AND SCALE PARAMETER)
     4 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
         UNIFORM UNIAXIAL TENSILE SPECIMEN TEST DATA
     5 : CENSORED DATA FOR SUSPENDED ITEM ANALYSIS OF
         FOUR-POINT BEND TEST DATA


3 = CONTROL INDEX FOR SURFACE FRACTURE CRITERION (ID2S)
     1 : NORMAL STRESS FRACTURE CRITERION
         (SHEAR-INSENSITIVE CRACK)
     3 : COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
     4 : WEIBULL PIA MODEL
     5 : SHETTY'S SEMI-EMPIRICAL CRITERION


1 = CONTROL INDEX FOR SHAPE OF SURFACE CRACKS (ID3S)
     1 : GRIFFITH CRACK
         (ASSOCIATED WITH STRAIN ENERGY RELEASE RATE CRITERION)
         (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)
     3 : GRIFFITH NOTCH
         (ASSOCIATED WITH STRAIN ENERGY RELEASE RATE CRITERION)
         (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)
     4 : SEMICIRCULAR CRACK
         (ASSOCIATED WITH SHETTY'S SEMI-EMPIRICAL CRITERION)


1 = CONTROL INDEX FOR METHOD OF CALCULATING BATDORF CRACK DENSITY
 COEFFICIENT
    (K SUB B) FROM TEST DATA (IKBAT)
     0 : SHEAR-INSENSITIVE METHOD (MODE I FRACTURE ASSUMED)
     1 : SHEAR-SENSITIVE METHOD (FRACTURE ASSUMED TO OCCUR
         ACCORDING TO THE FRACTURE CRITERION AND CRACK
         SHAPE SELECTED BY THE ID2 AND ID3 INDICES)

  .2500 = POISSON'S RATIO (PR)



         ****************************************************
         *                                                  *
         *  STATISTICAL ANALYSIS OF FRACTURE SPECIMEN DATA  *
         *                                                  *
         ****************************************************
```

ECHO OF SPECIMEN INPUT DATA, IN ASCENDING ORDER OF FRACTURE STRESS

.4000E-01 = OUTER LOAD SPAN OF FOUR-POINT BEND BAR
.2000E-01 = INNER LOAD SPAN
.3500E-02 = DEPTH OF SPECIMEN
.4500E-02 = WIDTH OF SPECIMEN

79 = NUMBER OF SPECIMENS IN BATCH 70.000 = TEMPERATURE OF BATCH

"S"URFACE OR "V"OLUME OR "U"NKNOWN FLAW ORIGIN AND RESPECTIVE FAILURE STRESS

| | | | |
|---|---|---|---|
| VVV | .4159E+03 | .4584E+03 | .5199E+03 |
| VVV | .5272E+03 | .5458E+03 | .5606E+03 |
| VVV | .5725E+03 | .5950E+03 | .6035E+03 |
| SSS | .6045E+03 | .6093E+03 | .6115E+03 |
| SSS | .6144E+03 | .6210E+03 | .6215E+03 |
| SSS | .6216E+03 | .6222E+03 | .6224E+03 |
| SSS | .6251E+03 | .6258E+03 | .6311E+03 |
| SSS | .6402E+03 | .6426E+03 | .6490E+03 |
| SSS | .6501E+03 | .6516E+03 | .6551E+03 |
| SSS | .6569E+03 | .6570E+03 | .6597E+03 |
| SSS | .6597E+03 | .6617E+03 | .6621E+03 |
| SSS | .6621E+03 | .6636E+03 | .6641E+03 |
| SSS | .6645E+03 | .6664E+03 | .6688E+03 |
| SSS | .6708E+03 | .6712E+03 | .6717E+03 |
| SSS | .6721E+03 | .6741E+03 | .6768E+03 |
| SSS | .6771E+03 | .6784E+03 | .6799E+03 |
| SSS | .6831E+03 | .6837E+03 | .6855E+03 |
| SSS | .6867E+03 | .6870E+03 | .6911E+03 |
| SSS | .6940E+03 | .6950E+03 | .6998E+03 |
| SSS | .7029E+03 | .7029E+03 | .7029E+03 |
| SSS | .7030E+03 | .7045E+03 | .7045E+03 |
| SSS | .7061E+03 | .7101E+03 | .7126E+03 |
| SSS | .7156E+03 | .7156E+03 | .7158E+03 |
| SSS | .7164E+03 | .7165E+03 | .7173E+03 |
| SSS | .7252E+03 | .7254E+03 | .7254E+03 |
| SSS | .7260E+03 | .7271E+03 | .7287E+03 |
| S | .7323E+03 | | |

--- STEFANSKY OUTLIER TEST OF SPECIMEN FRACTURE STRESSES ---

RESULTS FROM THE STEFANSKY OUTLIER TEST FOR TEMP. = 70.0000

FAILURE STRESS

.4159E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 1% SIGNIFICANCE LEVEL
.4584E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 1% SIGNIFICANCE LEVEL
.5199E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 1% SIGNIFICANCE LEVEL
.5272E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 5% SIGNIFICANCE LEVEL
.5458E+03 DEVIATES FROM THE MAIN TREND OF THE DATA AT THE 10% SIGNIFICANCE LEVEL

DEVIATION FROM THE MAIN TREND OF THE DATA MAY INDICATE BAD VALUES. MULTIPLE DEVIATIONS FROM THE SAME REGION OF THE DISTRIBUTION INDICATE THAT EITHER A CONCURRENT OR A PARTIALLY CONCURRENT FLAW POPULATION HAS BEEN DETECTED (NOTE THAT A CONCURRENT FLAW POPULATION MAY BE PRESENT BUT NOT BE DETECTED BY THE OUTLIER TEST). DEVIATIONS OCCURRING IN THE SAME REGION OF THE DISTRIBUTION WITH ALL THREE SIGNIFICANCE LEVELS (1%, 5% AND 10%) PRESENT INDICATE A CONCURRENT FLAW POPULATION. DEVIATIONS SHOULD BE EXAMINED AND TREATED ACCORDINGLY (I.E. IGNORE, CENSOR, ADJUST OR ELIMINATE STRESS). JUDGEMENT OF ACTION TAKEN CAN BE DETERMINED FROM THE GOODNESS-OF-FIT TESTS.
THE OUTLIER TEST IS NO SUBSTITUTE FOR GRAPHICAL EXAMINATION!!!

-- TEMP. DEP. WEIBULL MODULUS AND CHARACTERISTIC STRENGTH WITH 90% -- CONFIDENCE BOUNDS DETERMINED BY MAXIMUM LIKELIHOOD ANALYSIS

NOTE: 90% CONFIDENCE BOUNDS ON PARAMETERS DETERMINED FROM COMPETING FAILURE MODES (CENSORED DATA) ARE APPROXIMATE.
FOR CENSORED DATA THE UNBIASED VALUE OF THE PARAMETER "M" IS NOT GIVEN.
FOR SAMPLE SIZES LESS THAN 4, CONFIDENCE LIMITS ARE NOT GIVEN.

| | TEMP. | M BIASED | M UNBIASED | UP M | LOW M |
|---|---|---|---|---|---|
| | CHAR. STR. | UP C.S. | LOW C.S. | MEAN | STD. DEV. |
| VOLUME | 70.0000 | .4130E+01 | | .5666E+01 | .2179E+01 |
| | .1128E+04 | .1342E+04 | .9542E+03 | .1024E+04 | .2791E+03 |
| SURFACE | 70.0000 | .2281E+02 | | .2619E+02 | .1920E+02 |
| | .6917E+03 | .6981E+03 | .6853E+03 | .6755E+03 | .3684E+02 |

STATISTICS FROM THE GOODNESS-OF-FIT TESTS FOR TEMP. = 70.0000

KOLMOGOROV-SMIRNOV TEST

| ORDER | FRAC. STR. | WEIB. PROB. OF FAIL. | D+ FACTOR | D- FACTOR | SIGNIF. LEVEL |
|---|---|---|---|---|---|
| 1 | .4159E+03 | .0161 | -.0035 | .0161 | 99.0000 |
| 2 | .4584E+03 | .0240 | .0013 | .0114 | 99.0000 |
| 3 | .5199E+03 | .0414 | -.0034 | .0161 | 99.0000 |
| 4 | .5272E+03 | .0443 | .0064 | .0063 | 99.0000 |
| 5 | .5458E+03 | .0529 | .0104 | .0023 | 99.0000 |
| 6 | .5606E+03 | .0620 | .0140 | -.0013 | 99.0000 |
| 7 | .5725E+03 | .0714 | .0172 | -.0045 | 99.0000 |
| 8 | .5950E+03 | .0983 | .0029 | .0097 | 99.0000 |

| | | | | | |
|---|---|---|---|---|---|
| 9 | .6035E+03 | .1130 | .0009 | .0118 | 99.0000 |
| 10 | .6045E+03 | .1151 | .0114 | .0012 | 99.0000 |
| 11 | .6093E+03 | .1254 | .0139 | -.0012 | 99.0000 |
| 12 | .6115E+03 | .1305 | .0214 | -.0088 | 99.0000 |
| 13 | .6144E+03 | .1379 | .0267 | -.0140 | 99.0000 |
| 14 | .6210E+03 | .1567 | .0205 | -.0079 | 99.0000 |
| 15 | .6215E+03 | .1581 | .0317 | -.0191 | 99.0000 |
| 16 | .6216E+03 | .1585 | .0440 | -.0313 | 99.0000 |
| 17 | .6222E+03 | .1603 | .0549 | -.0422 | 97.1339 |
| 18 | .6224E+03 | .1610 | .0668 | -.0541 | 87.2422 |
| 19 | .6251E+03 | .1704 | .0701 | -.0575 | 83.1735 |
| 20 | .6258E+03 | .1726 | .0805 | -.0679 | 68.5012 |
| 21 | .6311E+03 | .1930 | .0728 | -.0602 | 79.6191 |
| 22 | .6402E+03 | .2346 | .0439 | -.0312 | 99.0000 |
| 23 | .6426E+03 | .2476 | .0435 | -.0309 | 99.0000 |
| 24 | .6490E+03 | .2853 | .0185 | -.0058 | 99.0000 |
| 25 | .6501E+03 | .2924 | .0241 | -.0114 | 99.0000 |
| 26 | .6516E+03 | .3020 | .0271 | -.0145 | 99.0000 |
| 27 | .6551E+03 | .3262 | .0156 | -.0030 | 99.0000 |
| 28 | .6569E+03 | .3399 | .0146 | -.0019 | 99.0000 |
| 29 | .6570E+03 | .3407 | .0264 | -.0137 | 99.0000 |
| 30 | .6597E+03 | .3616 | .0181 | -.0055 | 99.0000 |
| 31 | .6597E+03 | .3616 | .0308 | -.0181 | 99.0000 |
| 32 | .6617E+03 | .3773 | .0277 | -.0151 | 99.0000 |
| 33 | .6621E+03 | .3804 | .0374 | -.0247 | 99.0000 |
| 34 | .6621E+03 | .3811 | .0493 | -.0366 | 99.0000 |
| 35 | .6636E+03 | .3935 | .0496 | -.0369 | 99.0000 |
| 36 | .6641E+03 | .3980 | .0577 | -.0450 | 95.5201 |
| 37 | .6645E+03 | .4015 | .0669 | -.0542 | 87.1359 |
| 38 | .6664E+03 | .4182 | .0628 | -.0502 | 91.3909 |
| 39 | .6688E+03 | .4398 | .0538 | -.0412 | 97.5998 |
| 40 | .6708E+03 | .4580 | .0484 | -.0357 | 99.0000 |
| 41 | .6712E+03 | .4619 | .0571 | -.0445 | 95.8855 |
| 42 | .6717E+03 | .4674 | .0643 | -.0516 | 89.9719 |
| 43 | .6721E+03 | .4709 | .0734 | -.0607 | 78.8750 |
| 44 | .6741E+03 | .4909 | .0661 | -.0534 | 88.0606 |
| 45 | .6768E+03 | .5182 | .0515 | -.0388 | 98.4919 |
| 46 | .6771E+03 | .5207 | .0616 | -.0489 | 92.5627 |
| 47 | .6784E+03 | .5338 | .0611 | -.0484 | 92.9632 |
| 48 | .6799E+03 | .5495 | .0581 | -.0454 | 95.2717 |
| 49 | .6831E+03 | .5842 | .0360 | -.0234 | 99.0000 |
| 50 | .6837E+03 | .5909 | .0420 | -.0293 | 99.0000 |
| 51 | .6855E+03 | .6100 | .0356 | -.0229 | 99.0000 |
| 52 | .6867E+03 | .6237 | .0345 | -.0219 | 99.0000 |
| 53 | .6870E+03 | .6264 | .0445 | -.0318 | 99.0000 |
| 54 | .6911E+03 | .6712 | .0124 | .0003 | 99.0000 |
| 55 | .6940E+03 | .7027 | -.0065 | .0192 | 99.0000 |
| 56 | .6950E+03 | .7135 | -.0046 | .0173 | 99.0000 |
| 57 | .6998E+03 | .7635 | -.0419 | .0546 | 97.2613 |
| 58 | .7029E+03 | .7946 | -.0604 | .0731 | 79.2561 |
| 59 | .7029E+03 | .7946 | -.0478 | .0604 | 93.5064 |
| 60 | .7029E+03 | .7950 | -.0355 | .0482 | 99.0000 |
| 61 | .7030E+03 | .7955 | -.0233 | .0360 | 99.0000 |
| 62 | .7045E+03 | .8099 | -.0251 | .0378 | 99.0000 |
| 63 | .7045E+03 | .8105 | -.0130 | .0257 | 99.0000 |
| 64 | .7061E+03 | .8247 | -.0146 | .0272 | 99.0000 |
| 65 | .7101E+03 | .8598 | -.0370 | .0496 | 98.9973 |
| 66 | .7126E+03 | .8801 | -.0447 | .0574 | 95.7363 |
| 67 | .7156E+03 | .9022 | -.0541 | .0667 | 87.3420 |
| 68 | .7156E+03 | .9022 | -.0414 | .0541 | 97.5075 |

| | | | | | |
|---|---|---|---|---|---|
| 69 | .7158E+03 | .9031 | -.0297 | .0424 | 99.0000 |
| 70 | .7164E+03 | .9076 | -.0215 | .0341 | 99.0000 |
| 71 | .7165E+03 | .9081 | -.0094 | .0220 | 99.0000 |
| 72 | .7173E+03 | .9134 | -.0020 | .0146 | 99.0000 |
| 73 | .7252E+03 | .9549 | -.0309 | .0435 | 99.0000 |
| 74 | .7254E+03 | .9560 | -.0193 | .0320 | 99.0000 |
| 75 | .7254E+03 | .9560 | -.0067 | .0193 | 99.0000 |
| 76 | .7260E+03 | .9581 | .0039 | .0087 | 99.0000 |
| 77 | .7271E+03 | .9624 | .0123 | .0003 | 99.0000 |
| 78 | .7287E+03 | .9680 | .0193 | -.0067 | 99.0000 |
| 79 | .7323E+03 | .9786 | .0214 | -.0088 | 99.0000 |

KOLMOGOROV-SMIRNOV TEST YIELDS STATISTIC D = MAX (D+,D-) = .0805
WITH AN ASSOCIATED SIGNIFICANCE LEVEL OF 68.5%

ANDERSON-DARLING TEST YIELDS STATISTIC A**2 = .6725
WITH AN ASSOCIATED SIGNIFICANCE LEVEL OF 58.3%

KANOFSKY-SRINIVASAN 90% CONFIDENCE BANDS ABOUT THE WEIBULL DISTRIBUTION FOR
TEMP. = 70.0000

THE KANOFSKY-SRINIVASAN FACTOR FOR THIS DISTRIBUTION IS .0836 FOR A SAMPLE
SIZE OF 79

| ORD. | FRAC. STR. | WEIB. PROB. OF FAIL. | UPP.CONF.BAND | MED. RANK | LOW.CONF.BAND |
|---|---|---|---|---|---|
| 1 | .4159E+03 | .0161 | .0997 | .0088 | .0000 |
| 2 | .4584E+03 | .0240 | .1077 | .0214 | .0000 |
| 3 | .5199E+03 | .0414 | .1250 | .0340 | .0000 |
| 4 | .5272E+03 | .0443 | .1279 | .0466 | .0000 |
| 5 | .5458E+03 | .0529 | .1365 | .0592 | .0000 |
| 6 | .5606E+03 | .0620 | .1456 | .0718 | .0000 |
| 7 | .5725E+03 | .0714 | .1551 | .0844 | .0000 |
| 8 | .5950E+03 | .0983 | .1820 | .0970 | .0147 |
| 9 | .6035E+03 | .1130 | .1967 | .1096 | .0294 |
| 10 | .6045E+03 | .1151 | .1988 | .1222 | .0315 |
| 11 | .6093E+03 | .1254 | .2090 | .1348 | .0418 |
| 12 | .6115E+03 | .1305 | .2141 | .1474 | .0469 |
| 13 | .6144E+03 | .1379 | .2215 | .1599 | .0542 |
| 14 | .6210E+03 | .1567 | .2403 | .1725 | .0731 |
| 15 | .6215E+03 | .1581 | .2418 | .1851 | .0745 |
| 16 | .6216E+03 | .1585 | .2422 | .1977 | .0749 |
| 17 | .6222E+03 | .1603 | .2440 | .2103 | .0767 |
| 18 | .6224E+03 | .1610 | .2447 | .2229 | .0774 |
| 19 | .6251E+03 | .1704 | .2540 | .2355 | .0867 |
| 20 | .6258E+03 | .1726 | .2563 | .2481 | .0890 |
| 21 | .6311E+03 | .1930 | .2766 | .2607 | .1094 |
| 22 | .6402E+03 | .2346 | .3182 | .2733 | .1510 |
| 23 | .6426E+03 | .2476 | .3313 | .2859 | .1640 |
| 24 | .6490E+03 | .2853 | .3690 | .2985 | .2017 |
| 25 | .6501E+03 | .2924 | .3760 | .3111 | .2088 |
| 26 | .6516E+03 | .3020 | .3856 | .3237 | .2183 |
| 27 | .6551E+03 | .3262 | .4098 | .3363 | .2425 |
| 28 | .6569E+03 | .3399 | .4235 | .3489 | .2562 |
| 29 | .6570E+03 | .3407 | .4243 | .3615 | .2571 |

| | | | | | |
|---|---|---|---|---|---|
| 30 | .6597E+03 | .3616 | .4453 | .3741 | .2780 |
| 31 | .6597E+03 | .3616 | .4453 | .3866 | .2780 |
| 32 | .6617E+03 | .3773 | .4609 | .3992 | .2937 |
| 33 | .6621E+03 | .3804 | .4640 | .4118 | .2967 |
| 34 | .6621E+03 | .3811 | .4647 | .4244 | .2975 |
| 35 | .6636E+03 | .3935 | .4771 | .4370 | .3098 |
| 36 | .6641E+03 | .3980 | .4816 | .4496 | .3144 |
| 37 | .6645E+03 | .4015 | .4851 | .4622 | .3178 |
| 38 | .6664E+03 | .4182 | .5018 | .4748 | .3345 |
| 39 | .6688E+03 | .4398 | .5234 | .4874 | .3562 |
| 40 | .6708E+03 | .4580 | .5416 | .5000 | .3743 |
| 41 | .6712E+03 | .4619 | .5455 | .5126 | .3782 |
| 42 | .6717E+03 | .4674 | .5510 | .5252 | .3837 |
| 43 | .6721E+03 | .4709 | .5546 | .5378 | .3873 |
| 44 | .6741E+03 | .4909 | .5745 | .5504 | .4073 |
| 45 | .6768E+03 | .5182 | .6018 | .5630 | .4345 |
| 46 | .6771E+03 | .5207 | .6043 | .5756 | .4371 |
| 47 | .6784E+03 | .5338 | .6175 | .5882 | .4502 |
| 48 | .6799E+03 | .5495 | .6332 | .6008 | .4659 |
| 49 | .6831E+03 | .5842 | .6679 | .6134 | .5006 |
| 50 | .6837E+03 | .5909 | .6745 | .6259 | .5073 |
| 51 | .6855E+03 | .6100 | .6936 | .6385 | .5263 |
| 52 | .6867E+03 | .6237 | .7073 | .6511 | .5401 |
| 53 | .6870E+03 | .6264 | .7100 | .6637 | .5428 |
| 54 | .6911E+03 | .6712 | .7548 | .6763 | .5875 |
| 55 | .6940E+03 | .7027 | .7864 | .6889 | .6191 |
| 56 | .6950E+03 | .7135 | .7971 | .7015 | .6298 |
| 57 | .6998E+03 | .7635 | .8471 | .7141 | .6798 |
| 58 | .7029E+03 | .7946 | .8782 | .7267 | .7110 |
| 59 | .7029E+03 | .7946 | .8782 | .7393 | .7110 |
| 60 | .7029E+03 | .7950 | .8786 | .7519 | .7114 |
| 61 | .7030E+03 | .7955 | .8791 | .7645 | .7119 |
| 62 | .7045E+03 | .8099 | .8936 | .7771 | .7263 |
| 63 | .7045E+03 | .8105 | .8941 | .7897 | .7269 |
| 64 | .7061E+03 | .8247 | .9083 | .8023 | .7411 |
| 65 | .7101E+03 | .8598 | .9434 | .8149 | .7761 |
| 66 | .7126E+03 | .8801 | .9638 | .8275 | .7965 |
| 67 | .7156E+03 | .9022 | .9858 | .8401 | .8185 |
| 68 | .7156E+03 | .9022 | .9858 | .8526 | .8185 |
| 69 | .7158E+03 | .9031 | .9867 | .8652 | .8195 |
| 70 | .7164E+03 | .9076 | .9912 | .8778 | .8239 |
| 71 | .7165E+03 | .9081 | .9917 | .8904 | .8245 |
| 72 | .7173E+03 | .9134 | .9970 | .9030 | .8297 |
| 73 | .7252E+03 | .9549 | 1.0000 | .9156 | .8713 |
| 74 | .7254E+03 | .9560 | 1.0000 | .9282 | .8724 |
| 75 | .7254E+03 | .9560 | 1.0000 | .9408 | .8724 |
| 76 | .7260E+03 | .9581 | 1.0000 | .9534 | .8745 |
| 77 | .7271E+03 | .9624 | 1.0000 | .9660 | .8787 |
| 78 | .7287E+03 | .9680 | 1.0000 | .9786 | .8844 |
| 79 | .7323E+03 | .9786 | 1.0000 | .9912 | .8950 |

```
*****************************************************
*                                                   *
*          SURFACE FLAW PARAMETER ANALYSIS          *
*                                                   *
*****************************************************
```

```
****  BATDORF MODEL --- CRACK ORIENTATION, CRACK SHAPE,  ****
       AND FRACTURE CRITERION ARE CONSIDERED
```

--- TEMPERATURE DEPENDENT MATERIAL PARAMETERS FOR MATERIAL NUMBER 2

WEIBULL MODULUS (SHAPE PARAMETER), M (DIMENSIONLESS)
NORMALIZED BATDORF CRACK DENSITY COEFFICIENT, K (DIMENSIONLESS)
SCALE PARAMETER, SP (UNITS OF STRESS*AREA**(1/M))

| TEMPERATURE | M BIASED | K | SP |
|---|---|---|---|
| 70.0000 | .2281E+02 | .6050E+01 | .4613E+03 |

FRACTURE CRITERION = COPLANAR STRAIN ENERGY RELEASE RATE CRITERION
CRACK SHAPE = GRIFFITH CRACK

## 7.0 CODE DESCRIPTION

Included on the distribution disk is a file called PCCARES.FOR which contains the FORTRAN source code for the PC-CARES program. The code was written with VAX and Microsoft extensions and was compiled and linked with Microsoft FORTRAN 5.0. The specific extensions used include the DO...ENDDO, the DO WHILE...ENDDO, the ALLOCATE and DEALLOCATE routines and the EOF function.

Those wishing to modify the code and compile it with a compiler other than Microsoft FORTRAN 5.0 may have to alter the sections of the code which utilize these extensions. For the rest of this description, you may want to have a copy of the code in front of you.

The ALLOCATE and DEALLOCATE routines are called from the following PC-CARES routines: MAIN, MATL, CRACKV, CRACKS, and NORMAL. These calls should be removed. However note how they are dimensioned in the ALLOCATE call so that when you dimension them normally you will know to what size. You will have to remove the ALLOCATABLE attribute from the DIMENSION statements as well, when you insert the dimension numbers. For those arrays dimensioned using IMAXF and IMAXT parameters, it is suggested that you simply dimension the arrays using these labels and then set them to interger constants in the MAIN routine using the FORTRAN PARAMETER statement. The default PC-CARES values are IMAXF = 150 and IMAXT = 10. In addition the READINI subroutine should be altered to remove the assignment of the IMAXF and IMAXT parameters.

The EOF function is used in only two routines: the MAIN routine and the READINI subroutine. The code can be modified to support whatever end of file support your particular compiler supplies or the calls may simply be removed. Note that the $END keywords normally stop the FORTRAN file reading, however, if those keywords are missing then your code may attempt to read beyond the end of file and abort abnormally generating a run-time error.

The DO...ENDDO and DO WHILE...ENDDO extensions are used throughout the code, so if your compiler does not support these extensions then you will have to change these structures to standard FORTRAN 77 by using the DO label...label CONTINUE structure for the DO...END DO and by substituting an appropriately placed IF and GOTO statement combination for the DO WHILE...ENDDO. An example is given as follows:

| Microsoft/VAX Extension | Standard FORTRAN |
|---|---|
| DO I = 1, 1000 | DO 100 I = 1, 1000 |
| . | . |
| . | . |
| . | . |
| END DO | 100 CONTINUE |

```
DO WHILE (CONDITION)              105  IF (.NOT.(CONDITION)) GOTO 110
     .                                   .
     .                                   .
     .                                   .
END DO                                 GOTO 105
                                  110  CONTINUE
```

Finally here is a list of descriptions of the routines used in PC-CARES.

The MAIN Routine

The PC-CARES main routine controls the logical flow of the PC-CARES program as diagrammed in figure 2.1. Specifically the MAIN routine initializes and dimensions the code's variables and arrays, after which it calls the READINI subroutine to read the initialization file. If the initialization file is not present then the initialization parameters are set to their default values. the routine then allocates the array space needed by the program by calling the Microsoft FORTRAN extension ALLOCATE, after which the PC-CARES input file is read to obtain the Master Control Input and the Material Control Input, echoing the input along the way by calling the subroutines PRINTA and PRINTB. The routine continues by reading the fracture stress data and their respective fracture origins if present. If the material parameters (specifically the shape and scale parameters) are supplied, they are read from the input file. Finally the subroutine MATL is called which performs all the statistical analysis of the material. Following the MATL subroutine call, the MAIN routine deallocates all of the allocated array space using the DEALLOCATE command from Microsoft FORTRAN, closes all the files and exits.

In addition to the main program, the following subroutines appear in the PC-CARES listings:

Subroutine ANGLE

This subroutine evaluates $\Omega(\Sigma,\sigma_{cr})$ or $\omega(\Sigma,\sigma_{cr})$ for volume and/or surface flaw analysis, respectively, when the Batdorf method is selected. ANGLE employs the quadratic solution procedure described in the appendix A.1 Volume Flaw Reliability and the appendix A.2 Surface Flaw Reliability sections of this manual. It determines the critical intervals where $\sigma_e \geq \sigma_{cr}$ for various angles of $\alpha$ and values of $\sigma_{cr}$ about the unit sphere for volume flaw analysis. For some specific stress states ($\sigma_2 = \sigma_3$), this evaluation is independent of $\beta$. For surface flaw analysis these intervals are determined about the unit circle. For volume flaw analysis the critical intervals correspond to the $\beta$ integral in equation (A.27). For surface flaw analysis the critical intervals correspond to the integral in equation (A.55). Figure 2.2 shows the fracture criteria and flaw geometries for which the coding has been developed. These correspond to the effective stress equations (A.18), (A.19), (A.22), (A.23), (A.25), (A.26), and (A.40) for volume flaw analysis and to equations (A.51) to (A.54) for surface flaw analysis. The equations listed in tables A.I to A.III are used to find when $\sigma_e = \sigma_{cr}$, and the procedure outlined in equations (A.36) and (A.37) is used to find the intervals where $\sigma_e \geq \sigma_{cr}$.

ANGLE is called from the MATBAT subroutine in order to aid in the calculation of the Batdorf crack density coefficient when IKBAT = 1 (in other words when the user specifies the fracture criterion and the crack geometry and wants shear sensitivity taken into account). Arguments R2 to R4 correspond to $\sigma_2$, $\sigma_3$, and $(\sigma_2 - \sigma_3)$, divided by $\sigma_1$ for the given stress state. Arguments P and Q represent squared trigonometric functions of the angles $\alpha$ or $\beta$, whereas argument H represents the values of $\sigma_{cr}$ at locations of the Gaussian quadrature points. These arguments are required within ANGLE to calculate intermediate variables $a_1$ to $a_3$ that are coefficients in the quadratic equation for $\cos^2\alpha$ or $\cos^2\beta$ listed in tables A.I to A.III.

Subroutine ANGLES

This subroutine is used with the Batdorf volume flaw model to integrate over the surface area of a quadrant of the unit sphere when the Shetty failure criterion is used and $\sigma_{e_{max}} > \sigma_1$. ANGLES determines the intervals where $\sigma_e \geq \sigma_{cr}$ for constant angles of $\alpha$ about the unit sphere and stores the limits of these intervals in the INTVAL array. The critical intervals correspond to the integral of $\beta$ described in equation (A.27). The limits of these intervals are determined for each transformed Gaussian value of $\sigma_{cr}$ and $\alpha$. Each consecutive pair of integers in the third index of array INTVAL represents an interval where $\sigma_e \geq \sigma_{cr}$ for an angle of $\alpha$ denoted by the second index. The first index corresponds to values of $\sigma_{cr}$ at locations of the Gaussian quadrature points. The limits of integration stored in INTVAL are integers representing 1° increments of angle $\beta$ counted from $-\pi/2$ to $\pi/2$.

Function CONLIC

This function performs a table lookup of the factors for obtaining 90 percent upper and lower confidence bounds of the MLE of $\sigma_\theta$. These factors have been taken from reference 10. They are obtained from a Monte-Carlo simulation by using maximum likelihood analysis and uncensored data. The confidence bound calculations are performed in subroutine MATL.

Function CONLIM

This function performs a table lookup of the factors for obtaining 90 percent upper and lower confidence bounds of the MLE of m. These factors have been taken from reference 10. They are obtained from a Monte-Carlo simulation by using maximum likelihood analysis and uncensored data. The confidence bound calculations are performed in subroutine MATL.

Subroutine CRACKS

Subroutine CRACKS is called from MATBAT and serves as an interface with the SORMAL, SNGLES, SVALP3, and FINDP subroutines for the calculation of the shear-sensitive (IKBAT = 1) normalized Batdorf surface crack density coefficient $\bar{k}_{BS}$ for the Shetty criterion when $\sigma_{e_{max}} > \sigma_1$.

Subroutine CRACKV

Subroutine CRACKV is called from MATBAT and serves as an interface with the NORMAL, ANGLES AND EVALP3 subroutines for the calculation of the shear-sensitive (IKBAT = 1) normalized Batdorf volume crack density coefficient $\bar{k}_{BV}$ for the Shetty criterion when $\sigma_{e_{max}} > \sigma_1$.

Subroutine EVALP3

Subroutine EVALP3 is used with the Batdorf model for volume flaw analysis with the Shetty failure criterion when $\sigma_{e_{max}} > \sigma_1$. It performs the integration

$$\int_0^{\sigma_{e_{max}}} \int_0^{\pi/2} \left(\int dB\right) \sin \alpha \sigma_{cr}^{m_V - 1} \, d\alpha \, d\sigma_{cr} \tag{7.1}$$

which is used in equations (A.27) and (A.38a). Legendre-Gauss quadrature is used for the numerical integrations of $d\alpha$ and $d\sigma_{cr}$. The stored values in the INTVAL array previously calculated in the ANGLES subroutine are used to perform the integration.

Function F

This function computes the polynomial approximation to the Gamma function as per the "Handbook of Mathematical Functions."

Subroutine FINDP

Subroutine FINDP is used with the Batdorf model for surface flaw analysis with the Shetty failure criterion when $\sigma_{e_{max}} > \sigma_1$. It calculates $P_{2S}$ as defined by equation (A.55). The interval is determined from transforming values stored in the INTVAL array into real numbers. FINDP is called from the SVALP3 subroutine.

Subroutine GAUSS

Subroutine GAUSS contains roots of the Legendre polynomials and the weight factors for the Gauss quadrature. It is employed in the calculation of the Batdorf crack density coefficient, when a closed-form solution is not available. The number of Gauss points (NGP) is specified by the user in the program input. Data are available in GAUSS for NGP = 2 to 10, 15, and 30 although only 15 and 30 are recommended. The weights and locations are contained in the W and H arrays, respectively.

Subroutine LEAST2

This subroutine calculates the Weibull strength parameters m and C (see eq. (A.59)) by using the least-squares analysis method for complete or censored samples. The slope m and the intercept (ln (C)) of the line of best fit are obtained by solving two simultaneous equations (ref. 17). For uncensored data, median rank regression analysis (eq. (A.61)) is used to calculate the failure probability, $P_f$. However, in case of censored data, the median rank regression analysis cannot be used directly because of the effect of competing failure modes. Instead, the rank increment technique (eq. (A.62)) is used to adjust rank values. These adjusted rank values are then used with median rank regression analysis to calculate the failure probability. The LEAST2 subroutine is called from the MATL subroutine.

Subroutine MATBAT

Subroutine MATBAT calculates the surface and/or volume scale parameters, $\sigma_{oS}$ and $\sigma_{oV}$, and the normalized Batdorf crack density coefficients, $\bar{k}_{BS}$ and $\bar{k}_{BV}$, respectively. For a given material, parameters are found for each temperature level that is input by the user. If $m_S$ and $\sigma_{oS}$ or $m_V$ and $\sigma_{oV}$ are directly input, then only $\bar{k}_{BS}$ or $\bar{k}_{BV}$ is calculated, respectively. If experimental fracture stresses are input for either four-point bend or uniaxial tensile specimens, then all required parameters are calculated. The scale parameter for volume flaws is calculated from equation (A.72), and for surface flaws, equation (A.87) is used. The scale parameter is determined from the specimen geometry and from the values of $m$ and $C$ estimated in the LEAST2 or MAXL subroutines. The coefficient $\bar{k}_{BS}$ is calculated from equation (A.91) or (A.94), and $\bar{k}_{BV}$ is calculated from equation (A.78) or (A.84). The ANGLE subroutine is called from MATBAT to evaluate $\omega(\Sigma,\sigma_{cr})/2\pi$ or $\Omega(\Sigma,\sigma_{cr})/4\pi$ for a uniaxial stress state to find $\bar{k}_{BS}$ or $\bar{k}_{BV}$. If the Shetty criterion is selected by the user with IKBAT = 1 option when $\sigma_{e_{max}} > \sigma_1$, then the CRACKS subroutine finds $\omega(\Sigma,\sigma_{cr})/2\pi$ and the CRACKV subroutine finds $\Omega(\Sigma,\sigma_{cr})/4\pi$ to calculate $\bar{k}_{BS}$ and $\bar{k}_{BV}$, respectively.

Subroutine MATL

Subroutine MATL controls the program logic flow for the determination of the statistical material parameters and other useful statistical quantities as shown in the flowchart of figure 2.1. Kolmogorov-Smirnov and Anderson-Darling goodness-of-fit tests, Kanofsky-Srinivasan 90-percent confidence bands, Weibull mean, Weibull variance, and 90-percent confidence bounds on the parameters are all calculated in this subroutine. Ancillary subroutines to detect outliers (OUTLIE), to perform least-squares (LEAST2) or maximum likelihood (MAXL) analysis, to calculate Weibull scale parameters and the Batdorf crack density coefficient (MATBAT), and to print out results of the analysis (PRINTP) are called from MATL.

Subroutine MAXL

This subroutine determines the MLE's of the Weibull strength Parameters $m$ and $\sigma_\theta$ by using the maximum likelihood method for both uncensored and censored data. The logarithm of the likelihood function is differentiated with respect to $m$ and $C$, and the resulting expressions are set equal to zero (eqs. (A.59), and (A.63) to (A.65)). The Newton-Raphson iterative technique is used to obtain the parameter MLE's by solving these nonlinear equations. The estimate for $m$ for the first iteration is obtained from least-squares analysis via the LEAST2 subroutine. If the convergence criterion it not met after 50 iterations, the maximum likelihood method is terminated, a warning message is printed, and the results from the least-squares analysis are subsequently used in the program. If the fracture data are a complete sample, then the unbiased estimate of the shape parameter is calculated with the factors stored in the UNBIAS subroutine. The unbiased estimate is passed to the MATL subroutine and is later printed out in the PRINTP subroutine. This parameter is not employed in any subsequent calculations. Reference 17 contains a detailed description of the method of calculation of these statistical quantities. The MAXL subroutine is called from the MATL subroutine.

Subroutine NORMAL

This subroutine is called when the Batdorf volume flaw model is used with the Shetty failure criterion and $\sigma_{e_{max}} > \sigma_1$. NORMAL calculates the normalized effective stress about the unit sphere as a function of the angles $\alpha$ and $\beta$, and stores these values in the SEANGL array. The effective stress is determined for 1° increments of $\beta$ and is stored in the second index of the array. The first index denotes angles of $\alpha$ with values corresponding to the transformed Gaussian points. The array values are normalized by the maximum effective stress $\sigma_{e_{max}}$ found for the stress state being evaluated. The effective stress is calculated from equations (A.25) and (A.26).

Subroutine OUTLIE

In this subroutine, the available specimen fracture stress data at each temperature level are examined for outliers or inconsistent data. At the start of the subroutine, the sample mean and sample standard deviation are calculated. From these values, the normed residual for each specimen is obtained (ref. 6). The normed residuals are normalized deviations of the data about the sample mean. The Weibull distribution is not symmetrical about its mean, and therefore this technique is only approximate. The absolute maximum of the normed residual (MNR) statistic is compared with the critical value (CV) at 1-, 5-, and 10-percent significance levels. If the MNR statistic is smaller than the three critical values, then no outliers are detected. However, if the MNR is larger than at least one of the three critical values, the corresponding data value with the MNR statistic is detected as an outlier with the appropriate significance level. The outlier test can only flag one point per trial as an outlier. If an outlier is detected at the 10-percent or less significance level, it is removed from the sample and the remaining points are then retested. This process is repeated until the sample is reduced such that no more outliers are detected. Once all such points are detected, each of these potential outliers is retested against the remaining "good" data, and those points that maintain significance levels at or below 10 percent are flagged with the appropriate significance level in the ISKIP array. The results of the outlier test are output in the PRINTP subroutine via the MATL subroutine. A discussion of the equations used with this method is given in reference 17.

Subroutine PRINTA

Subroutine PRINTA echoes the user input or default values from the Master Control Input.

Subroutine PRINTB

Subroutine PRINTB echoes the user input or default values from the Material Control Input.

Subroutine PRINTP

This subroutine prints the values of all of the statistical quantities calculated in MATL, OUTLIE, LEAST2, MAXL, and MATBAT subroutines at each discrete temperature. A detailed description of the information printed can be found in section 5.0 PC-CARES OUTPUT INFORMATION, of this manual.

Subroutine READINI

This routine reads the initialization file, PCCARES.INI, from the directory from which the PC-CARES program was executed and sets the INFILE, OUTFILE, IMAXF, and IMAXT parameters according to commands in the initialization file, if present. If the initialization file is not present then the default values of these parameters are used.

Subroutine SNGLES

This subroutine is used with the Batdorf surface flaw model to integrate over the contour of the unit circle when Shetty's failure criterion is used and $\sigma_{e_{max}} > \sigma_1$. SNGLES determines the intervals where $\sigma_e \geq \sigma_{cr}$ about the unit circle and stores the limits of these intervals as a function of $\sigma_{cr}$ in the INTVAL array. The critical intervals correspond to the integral described by equation (A.55) to determine $\omega(\Sigma,\sigma_{cr})$. The limits of these intervals are determined for each transformed Gaussian value of $\sigma_{cr}$. Each consecutive pair of integers in the second index of array INTVAL represents an interval where $\sigma_e \geq \sigma_{cr}$ for a value of $\sigma_{cr}$ denoted by the first index. The limits of integration stored in INTVAL are integers representing 1° increments of angle counted from $-\pi/2$ to $\pi/2$.

Subroutine SORMAL

This subroutine is called when the Batdorf surface flaw model is used with Shetty's failure criterion when $\sigma_{e_{max}} > \sigma_1$. SORMAL calculates the normalized effective stress about the unit circle as a function of the angle $\alpha$ and stores it in the SEANGL array. The effective stress is determined for 1° increments of $\alpha$. The array values are normalized by the maximum effective stress $\sigma_{e_{max}}$ found for the stress state being evaluated. The effective stress is calculated from equations (A.52) to (A.54).

Subroutine SORTRA

This subroutine sorts the experimental fracture stresses at a given temperature level into ascending order (IASEND = 1) along with the corresponding fracture origins. It is invoked at the beginning of subroutine MATL. Array D contains the fracture stresses to be sorted, and the parameter NSORT equals the number of fracture stresses. The alphanumeric AINDEX array contains fracture origins (S, V, or U) that correspond in position to the sorted stresses.

Subroutine SVALP3

Subroutine SVALP3 is used with the Batdorf model for surface flaw analysis with Shetty's criterion when $\sigma_{e_{max}} > \sigma_1$. It performs the integration

$$\int_0^{\sigma_{e_{max}}} \left(\int d\alpha\right) \sigma_{cr}^{m_S - 1} \, d\sigma_{cr} \tag{7.2}$$

which is used in equation (A.55). Gauss-Legendre quadrature is used for the numerical integration of $d\sigma_{cr}$. The stored values in the INTVAL array, which were previously calculated in the SNGLES subroutine, are used to perform the integration. The FINDP subroutine is called from SVALP3 to perform the evaluation of $P_{2S}$ for each transformed Gaussian value of $\sigma_{cr}$.

Function UNBFTR

This function performs a table lookup of the unbiasing factors for the estimated Weibull modulus $\hat{m}$. These factors are a function of sample size and are taken from reference 10. They are obtained from a Monte Carlo simulation of unimodal fracture data by using maximum likelihood analysis. The factors are based on the sample mean. The unbiased estimate of $m$ is obtained by multiplying the biased estimate of $m$ by the unbiasing factor.

# APPENDIX

## SYMBOL LIST

| | |
|---|---|
| A | surface area |
| A-D | Anderson-Darling |
| $A^2$ | Anderson-Darling goodness-of-fit test statistic |
| a | crack half length or penny-shaped crack radius or radius of semi-circular surface crack |
| $a_j$ | coefficients of quadratic equation used for calculating $\Phi_i$ where $j = 1,2,3$ |
| B | risk of rupture in Weibull's cumulative failure distribution |
| C | modified Weibull parameter $(C = 1/\sigma_\theta)^m$, or centigrade measure of temperature |
| c | contour of a unit radius circle in two-dimensional principal stress space |
| $\bar{C}$ | Shetty's constant in mixed-mode fracture criterion |
| cm | centimeter |
| CV | critical value |
| $D_j$ | constants used in calculating $P_2$ for $j = 1,2,3$ |
| $D^+,D^-$ | Kolmogorov-Smirnov goodness-of-fit test statistic |
| D | Kolmogorov-Smirnov goodness-of-fit test statistic defined as $D^+$ or $D^-$ whichever is the largest |
| E | Young's modulus of elasticity |
| EDF | empirical distribution function |
| F | Fahrenheit |
| F(x) | cumulative distribution function of a random variable |
| $F(\sigma)$ | Weibull cumulative distribution of material strength |
| $F_N(x)$ | empirical distribution function |
| G | strain energy release rate, or crack extension force |
| $G_C$ | critical value of strain energy release rate |

| | |
|---|---|
| $G_I$ | strain energy release rate for crack opening mode crack extension |
| $G_{II}$ | strain energy release rate for crack sliding mode crack extension |
| $G_{III}$ | strain energy release rate for crack tearing mode crack extension |
| $G_{max}$ | maximum strain energy release rate |
| $G_T$ | total strain energy release rate |
| GC | Griffith crack (flattened elliptical cylinder) |
| GN | Griffith notch |
| GPa | gigapascal |
| g | gram |
| h | total height of MOR bar with rectangular cross section |
| i | ranking of ordered fracture data in statistical analysis or any counter |
| in. | inch |
| $K_I$ | opening mode stress intensity factor |
| $K_{IC}$ | critical opening mode stress intensity factor |
| $K_{II}$ | sliding mode stress intensity factor |
| $K_{III}$ | tearing mode stress intensity factor |
| $K_\delta$ | $K_{II}$ or $K_{III}$ |
| K(N) | Kanofsky-Srinivasan confidence band factors |
| $k_B$ | Batdorf crack density coefficient or flaw distribution parameter |
| $\bar{k}_B$ | normalized Batdorf crack density coefficient |
| K-S | Kolmogorov-Smirnov |
| $k_w$ | Weibull crack density coefficient, $(1/\sigma_o)^m$ |
| $k_{wp}$ | polyaxial Weibull crack density coefficient |
| $L_1$ | length between outer loads in four-point bending |
| $L_2$ | length between symmetrically applied inner loads in four-point bending |
| lb | pound |

| | |
|---|---|
| l,m,n | direction cosines of oblique plane normal in principal stress space for the Cauchy infinitesimal tetrahedron |
| ln | natural logarithm |
| MLE | maximum likelihood estimate |
| MNR | maximum normed residual |
| MOR | modulus of rupture or extreme fiber fracture stress |
| $MOR_o$ | characteristic modulus of rupture or extreme fiber fracture stress at which 63.2 percent of MOR bars will fail |
| MPa | megapascal |
| m | Weibull modulus or shape parameter; also Batdorf crack density function exponent or flaw distribution parameter |
| mm | millimeter |
| N | number of MOR specimens at a given temperature |
| NGP | number of Gauss base points used in numerical integration |
| $N(\sigma)$ | Weibull crack density function or number of flaws per unit volume/area with strength $\leq\sigma$ in uniaxial stress state |
| $N(\sigma_{cr})$ | Batdorf crack density function which is a material property independent of stress state and is the number of cracks per unit volume/area with strength $\leq\sigma_{cr}$ |
| n | number of links in a structure |
| $\bar{n}$ | unit vector along oblique plane normal determined by angles $\alpha$ and $\beta$ in principal stress space |
| P | load applied to MOR bar specimen |
| $P_f$ | cumulative failure probability |
| $P_s$ | cumulative survival probability |
| PSC | penny-shaped crack (flattened oblate spheroid) |
| $P_1$ | probability of existence in incremental volume or area of a crack with strength between $\leq\sigma_{cr}$ |
| P2 | probability of crack with strength $\leq\sigma_{cr}$ being so oriented that $\sigma_e \geq \sigma_{cr}$ |
| PIA | principle of independent action |

| | |
|---|---|
| psi | pounds per square inch |
| $r$ | number of remaining specimens in censored data analysis |
| $r_i$ | inside radius |
| $r_o$ | outside radius |
| SC | semicircular crack |
| sec | second |
| $t$ | thickness |
| $V$ | volume |
| WLT | weakest link theory |
| $w$ | total width of MOR bar with rectangular cross section |
| $X$ | ordered statistics |
| $x$ | any variable |
| $x,y,z$ | Cartesian coordinate directions |
| $Z_i$ | predicted failure probability at the fracture strength of the $i^{th}$ specimen |
| $\alpha$ | angle between $\sigma_n$ and the maximum principal stress, $\sigma_1$ (figs. A.1 and A.2); also significance level |
| $\bar{\alpha}$ | defined as $\cos^{-1}\sqrt{\Phi}$, when $\sigma_1 > \sigma_2 = \sigma_3$ for volume flaws and also for surface flaws when $\sigma_1 > \sigma_2$ |
| $\beta$ | angle between $\sigma_n$ projection and the intermediate principal stress $\sigma_2$ in plane perpendicular to $\sigma_1$ (fig. A.1) |
| $\bar{\beta}$ | defined as root of $\cos^{-1}\sqrt{\Phi}$, when $\sigma_2 \neq \sigma_3$ |
| $\Delta$ | increment |
| $\Delta P_1$ | probability of existence in incremental volume or area of a crack with strength between $\sigma_{cr}$ and $\sigma_{cr} + \Delta\sigma_{cr}$ |
| $\Gamma$ | gamma function; tabulated in mathematical handbooks |
| $\nu$ | material Poisson's ratio |
| $\Pi$ | usual product notation |
| $\pi$ | 3.1416 |

| | |
|---|---|
| $\Sigma$ | applied multidimensional stress state or summation notation |
| $\sigma$ | applied stress distribution; also the traction or stress vector on oblique plane of Cauchy infinitesimal tetrahedron |
| $\sigma_{cr}$ | remote, macroscopic, uniaxial, normal fracture stress of a crack |
| $\sigma_e$ | effective stress acting on a crack plane, $\sigma_e = f(\sigma_n,\tau)$ |
| $\sigma_{e_{max}}$ | maximum effective stress for the particular stress state |
| $\sigma_f$ | extreme fiber fracture stress in MOR bar test |
| $\sigma_n$ | normal stress acting on oblique plane whose normal is determined by angles $\alpha$ and $\beta$ (figs. A.1 and A.2)) |
| $\bar{\sigma}_n$ | normal stress averaged about a unit radius sphere or unit radius circle |
| $\sigma_o$ | Weibull scale parameter or normalizing stress |
| $\sigma_u$ | Weibull location parameter or threshold stress |
| $\sigma_1,\sigma_2,\sigma_3$ | principal stresses ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) |
| $\sigma_\theta$ | volume or area characteristic strength or characteristic modulus of rupture, $MOR_o$. This is the stress or extreme fiber stress at which 63.21 percent of the specimens will fail |
| $\tau$ | shear stress acting on oblique plane whose normal is determined by angles $\alpha$ and $\beta$ (figs. A.1 and A.2)) |
| $\Phi$ | defined as $\cos^2\bar{\beta}$ or $\cos^2\bar{\alpha}$, depending on stress state, for which $\sigma_e - \sigma_{cr} = 0$ |
| $\Omega$ | solid angle in three-dimensional principal stress space for which $\sigma_e \geq \sigma_{cr}$ |
| $\omega$ | angle in two-dimensional principal stress space for which $\sigma_e \geq \sigma_{cr}$ |

Subscripts:

| | |
|---|---|
| B | Batdorf |
| cr | critical |
| e | effective |
| f | failure, fracture |
| g | gage |

| | |
|---|---|
| I | crack opening mode |
| II | crack sliding mode |
| III | crack tearing mode |
| n | normal |
| p | polyaxial |
| S | surface |
| s | survival |
| V | volume |
| w | Weibull |

Superscript:

| | |
|---|---|
| $\hat{}$ | estimated parameter |

## APPENDIX

## THEORY

The use of advanced ceramic materials in structural applications requiring high component integrity has led to the development of a probabilistic design methodology. This method combines three major elements: (1) linear elastic fracture mechanics theory which relates the strength of ceramics to the size, shape and orientation of critical flaws, (2) extreme value statistics to obtain the characteristic flaw size distribution function, which is a material property, and (3) material microstructure. Inherent to this design procedure is that the requirement of total safety must be relaxed and an acceptable failure probability must be specified.

The statistical nature of fracture in engineering materials can be viewed from two distinct models (ref. 18). The first was presented by Weibull and used the "weakest link" theory as originally proposed by Pierce (ref. 19). The second model was also analyzed by Pierce (ref. 19) and, in addition, by Daniels (ref. 20) and is referred to as the "bundle" or "parallel" model. In this model, a structure is viewed as a bundle of parallel fibers. Each fiber will support a load less than its breaking strength indefinitely but will break immediately under any load equal to or greater than its breaking strength. When a fiber fractures, a redistribution of load occurs and the structure may survive. Failure occurs when all of the fibers have fractured. The weakest link model assumes that the structure is analogous to a chain with "n" links. Each link may have a different limiting strength. When a load is applied to the structure such that the weakest link fails, then the structure fails. Observations show that advanced monolithic ceramics closely follow the weakest link hypothesis. A component fails when an equivalent stress at a flaw reaches a critical value which depends on a fracture mechanics criterion, crack configuration, crack orientation, and the crack density function of the material. In comparison with the bundle model, WLT is in most cases more conservative.

One of the important features of WLT is that it predicts a size effect. The number and severity of flaws present in a structure depends on the material volume and surface area. The largest flaw in a big specimen is expected to be more severe than the worst flaw in a smaller specimen. Another consequence of WLT is that component failure may not be initiated at the point of highest nominal stress (ref. 21), as would be true for ductile materials. A large flaw may be located in a region far removed from the most highly stressed zone. Therefore, the complete stress solution of the component must be obtained.

Classical WLT does not predict behavior in a multiaxial stress state. A number of concepts such as the PIA, Weibull's normal stress averaging method, and Batdorf's model have been applied to account for polyaxial stress state response. Batdorf's model (ref. 22) assumes the following: (1) microcracks in the material are the cause of fracture, (2) cracks do not interact, (3) each crack has a critical stress, $\sigma_{cr}$, which is defined as the stress normal to the crack plane which will cause fracture, and (4) fracture occurs under combined stresses when an effective stress, $\sigma_e$, acting on the crack is equal to $\sigma_{cr}$. For an assumed crack shape $\sigma_e$ may be obtained through the application of a fracture criterion. These concepts are applied in the PC-CARES code to obtain the normalized Batdorf crack density coefficient.

## A.1 Volume Flaw Reliability

Consider a stressed component containing many flaws and assume that failure is due to any number of independent and mutually exclusive mechanisms (links). Each link involves an infinitesimal probability of failure $\Delta P_{fV}$. Discretize the component into $n$ incremental links. The probability of survival, $P_{sV}$, of the $i^{th}$ link is

$$(P_{sV})_i = 1 - (\Delta P_{fV})_i \tag{A.1}$$

where the subscript $V$ denotes volume dependent terms. The resultant probability of survival of the whole structure is the product of the individual probabilities of survival

$$P_{sV} = \prod_{i=1}^{n} (P_{sV})_i = \prod_{i=1}^{n} \left[1 - (\Delta P_{fV})_i\right] \cong \prod_{i=1}^{n} \exp\left[-(\Delta P_{fV})_i\right] = \exp\left[-\sum_{i=1}^{n} (\Delta P_{fV})_i\right] \tag{A.2}$$

Assume the existence of a power function $N_V(\sigma)$, referred to as the crack density function, representing the number of flaws per unit volume having a strength equal to or less than $\sigma$. In uniform tension of magnitude $\sigma$, the probability of failure of the $i^{th}$ link, represented the incremental volume $\Delta V_i$, is

$$(\Delta P_{fV})_i = N_V(\sigma)\Delta V_i \tag{A.3}$$

and substituting into equation (A.2), the resultant probability of survival is

$$P_{sV} = \exp[-N_V(\sigma)V] \tag{A.4}$$

and the probability of failure is

$$P_{fV} = 1 - \exp[-N_V(\sigma)V] \tag{A.5}$$

where $V$ is the total volume. If the stress is a function of location then

$$P_{fV} = 1 - \exp\left[-\int_V N_V(\sigma)dV\right] = 1 - \exp[-B_V] \tag{A.6}$$

A term called the "risk of rupture" by Weibull and denoted by the symbol $B_V$ is commonly used in reliability analysis. Equations similar to (A.5) and (A.6) are applicable to surface distributed flaws where surface area replaces volume and the flaw density function is surface area dependent.

Weibull introduced a three-parameter power function for the crack density function $N_V(\sigma)$,

$$N_V(\sigma) = \left(\frac{\sigma - \sigma_{uV}}{\sigma_{oV}}\right)^{m_V} \tag{A.7}$$

where $\sigma_{uV}$ is the threshold stress (location parameter), usually taken as zero for ceramics. The location parameter is the value of applied stress below which the failure probability is zero. When the location parameter is zero, the two-parameter Weibull model is obtained. The scale parameter, $\sigma_{oV}$, then corresponds to the stress level where 63.2 percent of specimens with unit volumes would fracture. The scale parameter has dimensions of stress x(volume)$^{1/m_V}$. $m_V$ is the shape parameter (Weibull modulus) which measures the degree of strength variability. $m_V$ is a dimensionless quantity. As $m_V$ increases, the dispersion is reduced. For large values of $m_V$ ($m_V > 40$), such as those obtained for ductile metals, the magnitude of the scale parameter corresponds to the material ultimate strength. These three statistical parameters are material properties, which are temperature and processing dependent.

Three-parameter behavior is rarely observed in as-processed monolithic ceramics and statistical estimation of the three material parameters is very involved. Therefore, the PC-CARES program uses the two-parameter model. The subsequent reliability predictions are more conservative than for the three-parameter model since we have taken the minimum strength of the material as zero.

The two-parameter crack density function is expressed as

$$N_V(\sigma) = \left(\frac{\sigma}{\sigma_{oV}}\right)^{m_V} = k_{wV}\sigma^{m_V} \tag{A.8}$$

and substituting equation (A.8) into equation (A.6), the failure probability becomes

$$P_{fV} = 1 - \exp\left(-k_{wV}\int_V \sigma^{m_V}\, dV\right) \tag{A.9}$$

where $k_{wV} = \sigma_{oV}^{-m_V}$ is the uniaxial Weibull crack density coefficient. Various methods have been developed to calculate $\sigma_{oV}$ and $m_V$ for a given material using fracture strength data from simple uniaxial specimen tests (ref. 17).

The two most common techniques for using uniaxial data to calculate $P_{fV}$ in polyaxial stress states are the PIA (refs. 23 and 24) and the Weibull normal tensile stress averaging method (refs. 25 and 26). In the PIA model, the principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are assumed to act independently. If all principal stresses are tensile, the probability of failure according to this approach is

$$P_{fV} = 1 - \exp\left[-k_{wV}\int_V\left(\sigma_1^{m_V} + \sigma_2^{m_V} + \sigma_3^{m_V}\right)dV\right] \qquad (A.10)$$

Compressive principal stresses are assumed not to contribute to the failure probability. It has been shown that this equation yields nonconservative estimates of $P_{fV}$ in comparison with the Weibull normal stress method (ref. 27).

The failure probability using the Weibull normal tensile stress averaging method, which has been described through an integral formulation (ref. 28), can be calculated from

$$P_{fV} = 1 - \exp\left(-\int_V k_{wpV}\bar{\sigma}_n^{m_V}\,dV\right) \qquad (A.11)$$

where

$$\bar{\sigma}_n^{m_V} = \frac{\int_A \sigma_n^{m_V}\,dA}{\int_A dA}$$

The area integration is performed in principal stress space over the surface, A, of a sphere of unit radius for regions where $\sigma_n$, the projected normal stress on the surface, is tensile. The polyaxial Weibull crack density coefficient is $k_{wpV}$. The relationship between $k_{wpV}$ and $k_{wV}$ is found by equating the failure probability for uniaxial loading to that obtained for the polyaxial stress state when the latter is reduced to a uniaxial condition. The result is

$$k_{wpV} = (2m_V + 1)k_{wV} \qquad (A.12)$$

Batdorf and Crose (ref. 22) proposed a statistical theory in which attention is focused on cracks and their failure under stress. Flaws are taken to be uniformly distributed and randomly oriented in the material bulk. Fracture is assumed to depend only on the tensile stress acting normal to the crack plane, hence, shear-insensitivity is inherent to the model. Subsequently, Batdorf and Heinisch (ref. 29) included the detrimental effects of shear traction on a flaw plane. Their method applies fracture mechanics concepts by combining a crack geometry and a mixed-mode fracture criterion to describe the condition for crack growth. Adopting this approach, the PC-CARES program contains several fracture criteria and flaw shapes for volume and surface analyses (fig. 2.2).

Consider a small uniformly stressed material element of volume $\Delta V$. The incremental probability of failure under the applied state of stress, $\Sigma$, can be written as the product of two probabilities,

$$\Delta P_{fV}(\Sigma,\sigma_{cr},\Delta V) = \Delta P_{1V}P_{2V} \qquad (A.13)$$

where $\Delta P_{1V}$ is the probability of existence in $\Delta V$ of a crack having a critical stress between $\sigma_{cr}$ and $\sigma_{cr} + \Delta\sigma_{cr}$. As previously noted, critical stress is defined as the remote, uniaxial, fracture strength of a given crack in mode I loading. $P_{2V}$ denotes the probability that a crack of critical stress $\sigma_{cr}$ will be oriented in a direction such that an effective stress, $\sigma_e$ (function of fracture criterion, stress state, and crack configuration) satisfies the condition $\sigma_e \geq \sigma_{cr}$. The effective stress is defined as the equivalent mode I stress a flaw would experience when subjected to a multiaxial stress state which results in modes I, II, and III crack surface displacements.

Crack dimensions are related to crack strength, and crack size is never explicitly used in statistical fracture theories. Batdorf and Crose (ref. 22) describe $\Delta P_{1V}$ as

$$\Delta P_{1V} = \Delta V \frac{dN_V(\sigma_{cr})}{d\sigma_{cr}} d\sigma_{cr} \tag{A.14}$$

and $P_{2V}$ is expressed as

$$P_{2V} = \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} \tag{A.15}$$

Where $N_V(\sigma_{cr})$ is the Batdorf crack density function. $\Omega(\Sigma,\sigma_{cr})$ is the area of the solid angle projected on the unit radius sphere in principal stress space containing all the crack orientations for which $\sigma_e \geq \sigma_{cr}$. The constant $4\pi$ is the surface area of a unit radius sphere and corresponds to a solid angle containing all possible flaw orientations.

The probability of survival in a volume element $\Delta V_i$ is

$$(P_{sV})_i = \exp\left\{-\Delta V_i \left[\int_0^{\sigma_{e_{max}}} \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} \frac{dN_V(\sigma_{cr})}{d\sigma_{cr}} d\sigma_{cr}\right]\right\} \tag{A.16a}$$

where $\sigma_{e_{max}}$ is the maximum effective stress a randomly oriented flaw could experience from the given stress state. Hence, the component failure probability is

$$(P_{fV}) = 1 - \exp\left\{-\int_V \left[\int_0^{\sigma_{e_{max}}} \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} \frac{dN_V(\sigma_{cr})}{d\sigma_{cr}} d\sigma_{cr}\right] dV\right\} \tag{A.16b}$$

The Batdorf crack density function $N_V(\sigma_{cr})$ is a material property, independent of stress state, and is usually approximated by a power function (ref. 29). This leads to the Batdorf crack density function of the form

$$N_V(\sigma_{cr}) = k_{BV}\sigma_{cr}^{m_V} \tag{A.17}$$

where the material Batdorf crack density coefficient $k_{BV}$ and Weibull modulus $m_V$ can be evaluated from experimental fracture data. Batdorf and Crose (ref. 22) initially proposed a Taylor series expansion for $N_V(\sigma_{cr})$, but this method has computational difficulties. A more convenient integral equation approach was recently formulated and extended to the use of data from four-point MOR bar tests (ref. 30). Note that $N_V(\sigma_{cr})$ has units of inverse volume.

Although the Weibull (eq. (A.8)) and Batdorf (eq. (A.17)) crack density functions are similar in form, they are not the same. The Weibull function simply depends on the applied stress, $\sigma$, and is the only term other than the volume necessary to calculate $P_{fV}$. The Batdorf function depends on the mode I strength of the crack, $\sigma_{cr}$, which is probabilistic and must be integrated over a range of values for a given stress state. Furthermore, to obtain $P_{fV}$, a crack orientation function, $P_{2V}$, must be considered in addition to the density function and the volume. Finally, the Batdorf coefficient, $k_{BV}$, cannot be calculated from uniaxial data until a fracture criterion and crack shape are chosen, in contrast to the Weibull coefficient, $k_{wV}$, which depends only on the data itself.

Assuming a shear-insensitive condition, fracture occurs when $\sigma_n = \sigma_e \geq \sigma_{cr}$, where $\sigma_n$ is the normal tensile stress on the flaw plane. However, for a flat crack it is known from fracture mechanics analysis that a shear stress, $\tau$, applied parallel to the crack plane (mode II or III), also contributes to fracture. Therefore, for polyaxial stress states expressing the effective stress, $\sigma_e$, as a function of both $\sigma_n$ and $\tau$ is more accurate than assuming shear-insensitivity. Batdorf and Heinisch (ref. 29) give effective stress expressions for two flaw shapes using both Griffith's maximum tensile stress criterion and Griffith's critical coplanar strain energy release rate ($G_T$) criterion. Arranged in order of increasing shear-sensitivity, for the maximum tensile stress criterion the effective stress equations are

$$\sigma_e = \frac{1}{2}\left(\sigma_n + \sqrt{\sigma_n^2 + \tau^2}\right) \qquad \text{(Griffith flaw)} \qquad (A.18)$$

and

$$\sigma_e = \frac{1}{2}\left\{\sigma_n + \sqrt{\sigma_n^2 + \left[\frac{\tau}{(1 - 0.5\nu)}\right]^2}\right\} \qquad \text{(Penny-shaped flaw)} \qquad (A.19)$$

where $\nu$ is Poisson's ratio.

The total coplanar strain energy release rate criterion is calculated from

$$G_T = G_I + G_{II} + G_{III} \qquad (A.20)$$

where $G$ is the energy release rate for various crack extension modes. In terms of stress intensity factors, the effective stress equation can be derived from (plane strain condition assumed) enforcing the condition $G_T = G_C$, where $G_C$ is the critical strain energy release rate. Thus,

$$K_{IC}^2 = K_I^2 + K_{II}^2 + \frac{K_{III}^2}{1 - \nu} \qquad (A.21)$$

For a Griffith crack, assuming that modes I and II dominate the response with $K_I = \sigma_n \sqrt{\pi a}$ and $K_{II} = \tau \sqrt{\pi a}$, where $2a$ is the crack length, we have from equation (A.21)

$$\sigma_e = \sqrt{\sigma_n^2 + \tau^2} \tag{A.22}$$

For a penny-shaped crack at the critical point on the crack periphery, we have $K_I = 2\sigma_n \sqrt{a/\pi}$ and $K_{II} = [4\tau/(2 - \nu)]\sqrt{a/\pi}$ (ref. 31) where $a$ now is the crack radius. The resulting effective stress equation is

$$\sigma_e = \left\{\sigma_n^2 + \left[\frac{\tau}{(1 - 0.5\nu)}\right]^2\right\}^{1/2} \tag{A.23}$$

The equations given by Batdorf and Heinisch consider only self-similar (coplanar) crack extension. However, a flaw experiencing a multiaxial stress state usually undergoes crack propagation initiated at some angle to the flaw plane (noncoplanar crack growth). Shetty (ref. 14) performed experiments on polycrystalline ceramics and glass considering crack propagation as a function of an applied far field multiaxial stress state. He modified an equation proposed by Palaniswamy and Knauss (ref. 13) to empirically fit experimental data. This multimodal interaction equation takes the form

$$\frac{K_I}{K_{IC}} + \left(\frac{K_\delta}{\bar{C}K_{IC}}\right)^2 = 1 \tag{A.24}$$

where $K_\delta$ is either $K_{II}$ or $K_{III}$, whichever is dominant, and $\bar{C}$ is a constant adjusted to best fit the data. Shetty (ref. 14) found a range of values of $0.80 \leq \bar{C} \leq 2.0$ for the materials he tested which contained large induced flaws. As $\bar{C}$ increases, the response becomes progressively more shear-insensitive.

Using this relationship with assumed modes I and II dominance for the Griffith crack yields

$$\sigma_e = \frac{1}{2}\left[\sigma_n + \sqrt{\sigma_n^2 + \left(\frac{2\tau}{\bar{C}}\right)^2}\right] \tag{A.25}$$

and for a penny-shaped crack, we get

$$\sigma_e = \frac{1}{2}\left[\sigma_n + \sqrt{\sigma_n^2 + \left(\frac{4\tau}{\bar{C}(2 - \nu)}\right)^2}\right] \tag{A.26}$$

To determine a component probability of failure from equation (A.16), $P_{2V}$ has to be evaluated for each elemental volume $\Delta V_i$, within which a uniform stress state $\Sigma(\sigma_1,\sigma_2,\sigma_3)$ is assumed. The solid angle $\Omega(\Sigma,\sigma_{cr})$ depends on the selected fracture criterion, crack configuration and on the applied stress state. For multiaxial stress states, with few exceptions, $\Omega(\Sigma,\sigma_{cr})$ must be determined numerically. For a sphere of unit radius (fig. A.1), an elemental surface area of the sphere is $dA = \sin\alpha \, d\beta \, d\alpha$. Project onto the spherical surface the equivalent stress $\sigma_e(\Sigma,\alpha,\beta)$. The solid angle $\Omega(\Sigma,\sigma_{cr})$ is the area of the sphere containing all the projected equivalent stresses where $\sigma_e \geq \sigma_{cr}$. Noting the symmetry of $\sigma_e$, and addressing the first octant of the unit sphere, then

$$\Omega(\Sigma,\sigma_{cr}) = 4\pi P_{2V} = 8 \int_0^{\pi/2} \left(\int d\beta\right)\sin\alpha \, d\alpha \tag{A.27}$$

where $\beta$ is evaluated between 0 and $\pi/2$.

To obtain the limits of integration, $\bar{\beta}_1$ and $\bar{\beta}_2$, for the interval where $\sigma_e \geq \sigma_{cr}$, the principal stresses must first be transformed to normal and shear stresses. Selecting an arbitrary plane and imposing equilibrium of forces (fig. A.1), the following equations are obtained:

$$\sigma^2 = \sigma_1^2 \ell^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2 \tag{A.28}$$

$$\sigma_n = \sigma_1 \ell^2 + \sigma_2 m^2 + \sigma_3 n^2 \tag{A.29}$$

$$\tau^2 = \sigma^2 - \sigma_n^2 \tag{A.30}$$

where $\sigma$ is the total traction vector acting on the crack plane and the direction cosines $\ell$, $m$, and $n$ are given in figure A.1 in terms of trigonometric functions of $\alpha$ and $\beta$. From the selected fracture criterion and crack configuration $\sigma_e$ is obtained as a function of $\Sigma$, $\alpha$, and $\beta$.

By defining $\Phi = \cos^2\beta$ and enforcing the failure condition of $\sigma_e = \sigma_{cr}$, we obtain a quadratic equation in $\Phi$ satisfying either

$$\sigma_{cr}^2 - \left(\sigma_n^2 + D\tau^2\right) = 0 \tag{A.31}$$

or

$$\left(\sigma_{cr} - \frac{\sigma_n}{2}\right)^2 - \frac{1}{4}\left(\sigma_n^2 + D\tau^2\right) = 0 \tag{A.32}$$

where $D$ is some constant defined by the specific fracture criterion and crack geometry. Equation (A.31) is used with the effective stress equations (A.22) and (A.23). Equation (A.32) is used when the effective stress equations (A.18), (A.19), (A.25), and (A.26) are selected. The quadratic equation takes the form

$$a_1\Phi^2 + a_2\Phi + a_3 = 0 \tag{A.33}$$

and the roots $\Phi_1$ and $\Phi_2$ are

$$\Phi_{2,1} = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \tag{A.34}$$

where $\Phi_1 \leq \Phi_2$. The expressions for coefficients $a_1$ to $a_3$ are given in tables A.I and A.II. The values for $\bar{\beta}$ are then found as

$$\left.\begin{array}{ll}
\bar{\beta}_1 = \cos^{-1}\sqrt{\Phi_2} & 0 \leq \Phi_2 \leq 1 \\
\bar{\beta}_1 = 0 & \Phi_2 < 0 \text{ or } \Phi_2 > 1 \text{ or } \Phi_2 \text{ is a complex number} \\
\bar{\beta}_2 = \cos^{-1}\sqrt{\Phi_1} & 0 \leq \Phi_1 \leq 1 \\
\bar{\beta}_2 = \frac{\pi}{2} & \Phi_1 < 0 \text{ or } \Phi_1 > 1 \text{ or } \Phi_1 \text{ is a complex number}
\end{array}\right\} \tag{A.35}$$

After obtaining $\bar{\beta}_1$ and $\bar{\beta}_2$ for a given stress state $\Sigma$, $\alpha$, and $\sigma_{cr}$, care must be taken in evaluating the integral. The solution of the integral in equation (A.27) is either

$$\int_0^{\bar{\beta}_1} d\beta + \int_{\bar{\beta}_2}^{\pi/2} d\beta = \bar{\beta}_1 - \bar{\beta}_2 + \frac{\pi}{2} \tag{A.36}$$

or

$$\int_{\bar{\beta}_1}^{\bar{\beta}_2} d\beta = \bar{\beta}_2 - \bar{\beta}_1 \tag{A.37}$$

The correct solution is determined by checking if $\sigma_e - \sigma_{cr} \geq 0$ at some angle $\beta$ between 0 and $\pi/2$. In the PC-CARES program extensive logic has been devised to examine all possible permutations the roots may have, including imaginary roots.

An alternative approach to calculate $P_{2V}$ is to increment the angles $\alpha$ and $\beta$ over the surface of a unit radius sphere. By symmetry only one octant needs to be considered. At each discrete point on the surface, the effective stress is evaluated and the associated area element is summed depending on whether $\sigma_e \geq \sigma_{cr}$. This procedure is computationally intensive, and, whenever possible PC-CARES employs the more efficient approach described previously.

For a given stress state and value of $\sigma_{cr_i}$, $\alpha$ is varied from 0 to $\pi/2$ and $\Omega(\Sigma,\sigma_{cr_i})$ is evaluated. The values of $\sigma_{cr_i}$ vary from 0 to $\sigma_{e_{max}}$, for the Gauss-Legendre integration used by PC-CARES. The probability of survival in volume $\Delta V_i$ is obtained by substituting equation (A.17) into equation (A.16a) to get

$$(P_{sV})_i = \exp\left\{-\Delta V_i m_V k_{BV}\left[\int_0^{\sigma_{e_{max}}} \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi}\,\sigma_{cr}^{m_V-1}\,d\sigma_{cr}\right]\right\} \quad \text{(A.38a)}$$

and the component failure probability is

$$(P_{fV}) = 1 - \exp\left\{-m_V k_{BV}\int_V\left[\int_0^{\sigma_{e_{max}}} \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi}\,\sigma_{cr}^{m_V-1}\,d\sigma_{cr}\right]dV\right\} \quad \text{(A.38b)}$$

Consider the simple stress state $\sigma_1 > \sigma_2 = \sigma_3$. For this case $\sigma_e$ and $\Omega(\Sigma,\sigma_{cr})$ are independent of $\beta$ and equation (A.27) reduces to

$$\Omega(\Sigma,\sigma_{cr}) = 8\int\left(\int_0^{\pi/2} d\beta\right)\sin\alpha\, d\alpha = 4\pi\int \sin\alpha\, d\alpha \quad \text{(A.39)}$$

where $\alpha$ is integrated between 0 and $\pi/2$ in a manner similar to the integration of $\beta$ in equation (A.27). The quadratic equation (A.33) is reformulated as a function of $\alpha$, where now $\Phi = \cos^2\alpha$. Table A.II(a) contains the coefficients $a_1$ to $a_3$ for calculating $\Phi_i$ and thence $\Omega(\Sigma,\sigma_{cr})$ for various fracture criteria and crack shapes for this stress state. The logic for evaluating the $\alpha$ integral is the same as that for the $\beta$ integral, as described in equations (A.35) to (A.37). With the possible exception of the Shetty criterion or when $\sigma_3 < 0$, the quadratic equation can have only one root between 0 and 1 and $\Omega(\Sigma,\sigma_{cr})$ is simply $4\pi(1 - \cos\bar{\alpha})$ where $\bar{\alpha}$ corresponds to the single root. If both roots lie outside the range 0 to 1, then a sample point is required to determine whether $\Omega(\Sigma,\sigma_{cr}) = 0$ or $\Omega(\Sigma,\sigma_{cr}) = 4\pi$. Additional equations for calculating $P_{2V}$ are also listed in table A.II(b) for special stress states, such as the uniaxial, equibiaxial and equitriaxial loading conditions.

For certain stress states and crack plane orientations, the normal stress on the crack plane can be compressive. When this situation occurs in the PC-CARES program the normal stress is set to zero and only the shear stress is assumed to contribute to crack growth. This is generally a conservative assumption since friction between the crack faces is ignored. If friction were considered, the effective applied shear would be reduced.

For most fracture criteria, $\sigma_{e_{max}} = \sigma_1$, that is the maximum effective stress is equal to the maximum tensile principal stress. For noncoplanar crack extension using equations (A.25) and (A.26), if $1/\bar{C}$ and $2/\bar{C}(2 - \nu)$ are $\leq 1.0$, respectively, then $\sigma_{e_{max}} = \sigma_1$. If these terms are greater than 1, then $\sigma_{e_{max}} > \sigma_1$ is possible. Also $\sigma_{e_{max}} > \sigma_1$ is possible when $\sigma_3 < 0$. For these conditions, the values of $\bar{\beta}$ of equation (A.35) are found by a surface element sampling scheme.

For the special case of shear insensitivity, the projected equivalent stress on a unit radius sphere is equal to the normal stress, that is, $\sigma_e = \sigma_n$. Substituting for $\sigma_n$, we obtain

$$\sigma_e = \sigma_3 + (\sigma_1 - \sigma_3)\cos^2\alpha + (\sigma_2 - \sigma_3)\cos^2\beta \sin^2\alpha \qquad (A.40)$$

The value of $\bar{\beta}$ satisfying $\sigma_e - \sigma_{cr} = 0$ is obtained by defining $\Phi = \cos^2\beta$ and calculating the coefficients $a_i$ for equation (A.33). For this shear-insensitive case, we get

$$a_1 = 0$$

$$a_2 = (\sigma_2 - \sigma_3)\sin^2\alpha \qquad (A.41)$$

and

$$a_3 = (\sigma_1 - \sigma_3)\cos^2\alpha + \sigma_3 - \sigma_{cr} \qquad (A.42)$$

We can now solve for $\Phi$ to obtain

$$\Phi = \frac{-a_3}{a_2} = \frac{\sigma_{cr} - \sigma_3 - (\sigma_1 - \sigma_3)\cos^2\alpha}{(\sigma_2 - \sigma_3)\sin^2\alpha} \qquad (A.43)$$

It is obvious that only one value of $\Phi$ satisfies equation (A.43), from which the limits of integration become

$$\left.\begin{aligned}
\bar{\beta}_1 &= 0 \\
\bar{\beta}_2 &= \cos^{-1}\sqrt{\Phi} && \text{if} \quad 0 \le \Phi \le 1 \\
\bar{\beta}_2 &= 0 && \text{if} \quad \Phi > 1 \\
\bar{\beta}_2 &= \frac{\pi}{2} && \text{if} \quad \Phi \le 0
\end{aligned}\right\} \tag{A.44}$$

and equation (A.37) is used for all cases.

## A.2 Surface Flaw Reliability

For surface flaw analysis (ref. 2), many of the equations from section A.1 Volume Flaw Reliability remain the same, except that the statistical material parameters are a function of surface area instead of volume and the equivalent stress projections are onto the contour of a circle of unit radius rather than onto the surface of a unit radius sphere. The cracks are assumed to be randomly oriented in the plane of the external boundary with their planes normal to the surface.

For surface flaw induced failure in ceramic structures the probability of failure for the two-parameter Weibull distribution, which is analogous in form to equation (A.9) is

$$P_{fS} = 1 - \exp\left(-k_{wS}\int_A \sigma^{m_S}\, dA\right) \tag{A.45}$$

where $k_{wS} = (1/\sigma_{oS})^{m_S}$, is the Weibull surface crack density coefficient. The subscript S denotes the terms that are surface area dependent. Here $\sigma_{oS}$ is the surface scale parameter with units of stress $\times(\text{area})^{1/m_S}$ and A is the stressed surface area. For biaxial stress states, the Weibull distribution in combination with the PIA hypothesis yields

$$P_{fS} = 1 - \exp\left[-k_{wS}\int_A \left(\sigma_1^{m_S} + \sigma_2^{m_S}\, dA\right)\right] \tag{A.46}$$

where $\sigma_1$ and $\sigma_2$ are the principal tensile in-plane stresses acting on the surface of the structure. The failure probability using the Weibull normal stress averaging method can be calculated from

$$P_{fS} = 1 - \exp\left(-k_{wpS}\int_A \bar{\sigma}_n^{m_S}\, dA\right) \tag{A.47}$$

where

$$\bar{\sigma}_n^{m_S} = \frac{\int_c \sigma_n^{m_S} \, dc}{\int_c dc}$$

Here $k_{wpS}$ is the polyaxial Weibull crack density coefficient for surface flaws. The line integration is performed over the contour, c, of a unit radius circle where the projected normal stress, $\sigma_n$, is tensile. The relationship of $k_{wpS}$ to $k_{wS}$ is obtained by carrying out the integration in equation (A.47) for a uniaxial stress and equating the resultant failure probability to that of equation (A.45) (ref. 17). This results in

$$k_{wpS} = \frac{m_S \Gamma(m_S) \sqrt{\pi}}{\Gamma\left(m_S + \frac{1}{2}\right)} k_{wS} \qquad \text{(A.48)}$$

where $\Gamma$ is the gamma function. Equation (A.47) is the shear-insensitive case of the more general Batdorf polyaxial model.

For mixed-mode fracture due to surface flaws the Batdorf polyaxial failure probability equation (analogous to eq. (A.16b)) is

$$P_{fS} = 1 - \exp\left[-\int_A \int_0^{\sigma_{e_{max}}} \frac{\omega(\Sigma,\sigma_{cr})}{2\pi} \frac{dN_S(\sigma_{cr})}{d\sigma_{cr}} d\sigma_{cr} \, dA\right] \qquad \text{(A.49)}$$

where

$$\Delta P_{1S} = \Delta A \frac{dN_S(\sigma_{cr})}{d\sigma_{cr}} d\sigma_{cr}$$

and

$$P_{2S} = \frac{\omega(\Sigma,\sigma_{cr})}{2\pi}$$

For randomly oriented cracks $\omega(\Sigma,\sigma_{cr})$ is the total arc length on a unit radius circle in principal stress space on which the projection of the equivalent stress satisfies $\sigma_e \geq \sigma_{cr}$ and $2\pi$ is the total arc length of the circle. The same as for volume flaws, the Batdorf surface crack density function is approximated by the power function,

$$N_S(\sigma_{cr}) = k_{BS}\sigma_{cr}^{m_S} \tag{A.50}$$

where $k_{BS}$ is the Batdorf surface crack density coefficient.

Fracture occurs when the equivalent stress $\sigma_e \geq \sigma_{cr}$. For the shear-insensitive case fracture depends only on the value of the normal tensile stress such that $\sigma_e = \sigma_n$. For shear-sensitive cracks and colinear crack extension ($G_T$ criterion), assuming a Griffith crack with $K_I = \sigma_n\sqrt{\pi a}$ and $K_{II} = \tau\sqrt{\pi a}$ we obtain as before

$$\sigma_e = \sqrt{\sigma_n^2 + \tau^2} \tag{A.51a}$$

while for a Griffith notch subjected to plane strain conditions with $K_I = 1.1215\sigma_n\sqrt{\pi a}$ and $K_{III} = \tau\sqrt{\pi a}$ (ref. 31) we get

$$\sigma_e = \sqrt{\sigma_n^2 + \frac{0.7951}{(1 - \nu)}\tau^2} \tag{A.51b}$$

Note that the equivalent stress for the Griffith crack is dependent on modes I and II, while for the Griffith notch, the equivalent stress is dependent on modes I and III (ref. 2).

For noncoplanar crack growth, from equation (A.24) the effective stress equations for the Griffith crack and Griffith notch, respectively, are

$$\sigma_e = \frac{1}{2}\left[\sigma_n + \sqrt{\sigma_n^2 + 4\left(\frac{\tau}{\bar{C}}\right)^2}\right] \tag{A.52}$$

and

$$\sigma_e = \frac{1}{2}\left[\sigma_n + \sqrt{\sigma_n^2 + 3.1803\left(\frac{\tau}{\bar{C}}\right)^2}\right] \tag{A.53}$$

For a semicircular surface crack $K_I = 1.366\sigma_n\sqrt{a}$, $K_{II} = 1.241\tau\sqrt{a}$, and $K_{III} = 0.133\tau\sqrt{a}$ (refs. 32 and 33). Since the contribution of $K_{III}$ is small it is neglected, and thus, the effective stress for this case is

$$\sigma_e = \frac{1}{2}\left[\sigma_n + \sqrt{\sigma_n^2 + 3.301\left(\frac{\tau}{\bar{C}}\right)^2}\right] \tag{A.54}$$

For the same stress state and identical $\bar{C}$, the Griffith crack is the most shear-sensitive, while the Griffith notch and the semi-circular crack give almost identical predictions.

The solution procedure for $\omega(\Sigma,\sigma_{cr})$ is similar to the methods outlined in section A.1 Volume Flaw Reliability. The probability that the crack orientation is such that $\sigma_e \geq \sigma_{cr}$ can be calculated from

$$P_{2S} = \frac{\omega(\Sigma,\sigma_{cr})}{2\pi} = \frac{2}{\pi}\int d\alpha \tag{A.55}$$

where over the unit radius circle, $0 \leq \bar{\alpha}_i \leq \pi/2$. The limits of integration $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are obtained through the enforcement of the failure condition $\sigma_e = \sigma_{cr}$. The required normal and shear stresses are calculated from force equilibrium on a crack plane. As shown in figure A.2, the stress vector $\sigma$, the normal stress $\sigma_n$ and the shear stress $\tau$ can be expressed as

$$\sigma^2 = \left(\sigma_1^2 - \sigma_2^2\right)\cos^2\alpha + \sigma_2^2 \tag{A.56}$$

$$\sigma_n = \left(\sigma_1 - \sigma_2\right)\cos^2\alpha + \sigma_2 \tag{A.57}$$

$$\tau^2 = \sigma^2 - \sigma_n^2 = \left(\sigma_1 - \sigma_2\right)^2 \cos^2\alpha\left(1 - \cos^2\alpha\right) \tag{A.58}$$

Upon substitution of $\sigma_n$, $\tau$ and satisfaction of $\sigma_e = \sigma_{cr}$, equations (A.51) to (A.54) are reduced to a quadratic expression of the same form as equation (A.33) with $\Phi = \cos^2\alpha$. However, since $\beta = 0°$ (fig. A.1(a)) in the $\sigma_1 - \sigma_2$ plane, the constants $a_1$ to $a_3$ are dependent only on the two principal stresses, $\sigma_{cr}$, and in some cases on Poisson's ratio. Using the solution methods outlined in the previous section we obtain the roots of the quadratic equation. These values are in table A.III along with the coefficients $a_i$. For cases where the roots, $\Phi_i$, of the quadratic equation are not between 0 and 1, the calculation of $P_{2S}$ in equation (A.55) follows the same logic as has been given in equations (A.35) to (A.37), with $\bar{\alpha}$ replacing $\bar{\beta}$. Specific examples for this situation have been given in reference 2. For the equibiaxial surface stress state, we always have $\omega/2\pi = 1$ for $\sigma_{cr} \leq \sigma_{e_{max}}$, since the in-plane shear stress is zero and hence $\sigma_e = \sigma_1$ for all values of $\alpha$ and any effective stress equation.

When the normal stress is compressive ($\sigma_n < 0$) it is equated to zero and the shear stress alone contributes to crack growth. The maximum equivalent stress $\sigma_{e_{max}}$ for most cases is equal to $\sigma_1$, the maximum principal stress. However, for noncoplanar crack extension, using equation (A.24), $\sigma_{e_{max}}$ is dependent on the value of $\bar{C}$, and may exceed $\sigma_1$. Also when $\sigma_2 < 0$, then $\sigma_{e_{max}} > \sigma_1$ is possible. Again a sampling scheme is used to evaluate $\omega(\Sigma,\sigma_{cr})$ when this condition occurs.

## A.3 Estimation of Statistical Material Strength Parameters

Selected statistical theories and equations for parameter estimation are explained in detail in reference 17. The following is a brief description of these methods and how they are used in the PC-CARES code. Typically for brittle materials, the Weibull parameters are determined from simple specimen geometry and loading conditions, such as beams under flexure and either cylindrical or flat specimens under uniform uniaxial tension. The flexural test failure probability can be expressed in terms of the extreme fiber fracture stress, $\sigma_f$, or modulus of rupture, MOR, using the two-parameter Weibull form as

$$P_f = 1 - \exp\left(-C\sigma_f^m\right) = 1 - \exp\left[-C(MOR)^m\right]$$

$$= 1 - \exp\left[-\left(\frac{MOR}{\sigma_\theta}\right)^m\right] \qquad (A.59)$$

where $m$ is the volume or area Weibull modulus, $C$ is the modified Weibull parameter ($C = (1/\sigma_\theta)^m$) and $\sigma_\theta$ is the volume or area specimen characteristic strength or characteristic modulus of rupture, $MOR_0$. For uniform uniaxial tension tests $\sigma_f$ in equation (A.59) would just be replaced by $\sigma_1$. The Weibull scale parameter, $\sigma_0$, as defined in equations (A.7) and (A.45) for volume and surface cracks, respectively, is determined from $\sigma_\theta$, m, the specimen geometry, and the loading configuration. The scale parameter, $\sigma_0$, is based on a unit volume or area, whereas $\sigma_\theta$ includes the effects of the specimen dimensions. The characteristic strength $\sigma_\theta$ is defined as the uniform stress or extreme fiber stress at which the probability of failure is 0.632.

Before computing the estimates of the statistical material parameters, it is essential to carefully examine the available specimen data to screen them for outliers. Very often, a data set may contain one or more values which may not belong to the overall population. The statistical procedures to detect the outliers at different significance levels are explained in references 6 and 17. The outlier test assumes that the data is normally distributed and from a complete sample. Therefore, the application of this test to the Weibull distribution and censored statistics is only approximate.

Various methods are available to estimate the statistical material parameters from experimental data for the two-parameter Weibull distribution. The success of the statistical approach depends upon how well the probability density function fits the data. Two popular techniques used to evaluate the characteristic strength and shape parameter ($\sigma_\theta$ and $m$) are the least-squares analysis and the maximum likelihood method. Least-squares analysis is a special case of the maximum likelihood method where the error is normally distributed and has a zero mean and constant variance. The least-squares method is not suitable for calculating confidence intervals and unbiasing factors, which quantify the statistical uncertainties in the available data.

Equation (A.59) can be linearized by taking the natural logarithm twice yielding

$$\ln\left[\ln\left(\frac{1}{P_s}\right)\right] = \ln\left[\ln\left(\frac{1}{1 - P_f}\right)\right] = \ln C + m \ln \sigma_f \qquad (A.60)$$

For the least-squares analysis, it is necessary to obtain the line of best fit with slope m and an intercept b which, as seen in equation (A.60), is equal to the natural log of C. The failure probability $P_f$ is determined by conducting fracture tests on N specimens. The fracture stresses are ranked such that $\sigma_{f1} < \sigma_{f2} < \ldots < \sigma_{fi} < \ldots < \sigma_{fN}$. For median rank regression analysis, the probability of failure of a specimen with rank i is

$$P_f(\sigma_{fi}) = \frac{i - 0.3}{N + 0.4} \tag{A.61}$$

By taking the partial derivative of the sum of the squared residuals with respect to m and C, and by equating the derivates to zero, values of m and C can be estimated.

With censored data, one cannot directly use the median rank regression analysis as given in equation (A.61) because of the competing failure modes. To take into account the influence of the suspended items, Johnson (ref. 9) developed the rank increment technique. For this technique, all observed fracture stresses are arranged in ascending order, and rank increment values are calculated for each failure stress from the following equation:

$$\text{Rank increment} = \frac{(N + 1) - (\text{previous adjusted rank})}{1 + (\text{number of items beyond present suspended item})} \tag{A.62}$$

In the PC-CARES program for volume flaw analysis, all fracture stresses designated as V's are considered as failure data; for surface flaw analysis, the S's are considered as failure data. The new adjusted rank values are obtained by adding the rank increment value to the previously adjusted rank. These adjusted rank values and the median rank regression analysis (i.e., eq. (A.61)) are then used to calculate the failure probability $P_f$. Finally, the Weibull parameters $\hat{m}$ and $\hat{C}$ are obtained.

Since the distribution of errors from the data is not normal, the maximum likelihood method is often preferred in Weibull analysis. This method has certain inherent properties. The likelihood equation from which the maximum likelihood estimates (MLE's) are obtained will have a unique solution. In addition, as the sample size increases the solution converges to the true values of the parameters. Another feature of the maximum likelihood method is that there are no ranking functions or linear regression analysis when complete or censored samples are analyzed. The likelihood equation for a complete sample is given by

$$L = \prod_{i=1}^{N} \left(\frac{m}{\sigma_\theta}\right)\left(\frac{\sigma_{fi}}{\sigma_\theta}\right)^{m-1} \exp\left[-\left(\frac{\sigma_{fi}}{\sigma_\theta}\right)^{m}\right] \tag{A.63}$$

The values of m and $\sigma_\theta$ which maximize the likelihood function L, are determined by taking the partial derivative of the logarithm of the likelihood function with respect to m and with respect to $\sigma_\theta$. The values of $\hat{m}$ and $\hat{\sigma}_\theta$ are obtained by equating the resulting expressions to zero and solving the simultaneous equations using the Newton-Raphson iterative technique. The MLE of m and $\sigma_\theta$ are designated by $\hat{m}_V$ and $\hat{\sigma}_{\theta V}$ and by $\hat{m}_S$ and $\hat{\sigma}_{\theta S}$ for volume flaw analysis and surface flaw analysis, respectively. For censored statistics we have

$$\frac{\sum_{i=1}^{N} \left(\sigma_{fi}\right)^{\hat{m}} \ln\left(\sigma_{fi}\right)}{\sum_{i=1}^{N} \left(\sigma_{fi}\right)^{\hat{m}}} - \frac{1}{r} \sum_{i=1}^{r} \ln\left(\sigma_{fi}\right) - \frac{1}{\hat{m}} = 0 \tag{A.64}$$

and

$$\hat{\sigma}_{\theta} = \left(\frac{\sum_{i=1}^{N} \sigma_{fi}^{\hat{m}}}{r}\right)^{1/\hat{m}} \tag{A.65}$$

where $r$ is the number of remaining specimens failed by the flaw mode for which parameters are being calculated. For a complete (uncensored) sample, $r$ is replaced by $N$ which is the total size of the sample.

The MLE of the shape parameter, m, is always a biased estimate that depends on the number of specimens in the sample. Unbiasing of the shape parameter estimate is desired to minimize the deviation between the sample and the true population. The unbiased estimate of $m$ is obtained by multiplying the biased estimate with an unbiasing factor (ref. 10). The confidence intervals for complete samples can also be obtained (ref. 10). For censored samples, a rigorous method for obtaining confidence intervals has not yet been developed due to the complexity of competing failure modes. Confidence bounds for censored statistics are instead estimated in the PC-CARES code from the factors obtained from complete samples (ref. 17). Confidence bounds enable the user to estimate the uncertainty in the parameters as a function of the number of specimens. Bounds at 90-percent confidence level and therefore, 5 and 95 percentage points of distribution of the MLE's of the parameters, have been incorporated into the PC-CARES program, with data taken from reference 10.

Subjective judgement is needed to test the goodness-of-fit of the data to the assumed distribution. When graphical techniques are used, it can be very difficult to decide if the hypothesized distribution is valid, especially for small sample sizes. Therefore, many statistical tests have been developed to quantify the degree of correlation of the experimental data to the proposed distribution.

In general, a statistic is a numerical value computed from a random sample of the total population. The difference between an empirical distribution function (EDF) and a hypothesized distribution function is called an EDF statistic. There are two major classes of EDF statistics and they differ in the manner in which the functional (vertical) difference between the EDF and the proposed distribution function $F(x)$ is considered. The Kolmogorov-Smirnov

(K-S) goodness-of-fit statistic D belongs to the supremum class and is very effective for small samples. It uses the largest vertical difference between the two distribution functions to determine the goodness-of-fit. For the K-S test, the sample is arranged in ascending order, and the empirical distribution function $F_N(x)$ is a step-function obtained from the following expressions:

$$\left.\begin{aligned} F_N(x) &= 0 && x < X_i \\ F_N(x) &= \frac{i}{N} && X_i \le x < X_{i+1} \qquad i = 1,2,\ldots,N-1 \\ F_N(x) &= 1 && X_N \le x \end{aligned}\right\} \tag{A.66}$$

where $X_1 < X_2 < \ldots X_i \ldots < X_N$ are the ordered fracture stresses from a sample of size N. The statistic D is obtained by initially evaluating two other statistics $D^+$ and $D^-$, the largest vertical differences when $F_N(x)$ is greater than F(x) and the largest vertical differences when $F_N(x)$ is smaller than F(x), respectively. All three statistics are calculated by using the following expressions:

$$\left.\begin{aligned} D^+ &= \left|\frac{i}{N} - F(x)_i\right| \\ D^- &= \left|F(x)_i - \frac{i-1}{N}\right| \qquad i = 1,2,\ldots,N \\ D &= \max(D^+,D^-) \end{aligned}\right\} \tag{A.67}$$

For ceramics design, the $F(x)_i$'s are equal to $P_f$'s and are calculated by using equation (A.59).

On the other hand, the Anderson-Darling statistic, $A^2$, belongs to the quadratic class and is a more powerful goodness-of-fit statistic. It evaluates the discrepancy between the two distributions through squared differences and the use of an appropriate weighting function. The statistic $A^2$ is given by

$$A^2 = -N - \left(\frac{1}{N}\right) \sum_{i+1}^{N} (2i - 1) \left\{\ell n\left[Z_{(i)}\right] + \ell n\left[1 - Z_{(N+1-i)}\right]\right\} \tag{A.68}$$

In this case, $Z_i$'s are the predicted failure probabilities obtained from equation (A.59). Corresponding significance levels $\alpha$ are calculated from the D and $A^2$ statistics. From previous surveys (ref. 17) there is no specific mention of an absolute accepted significance level. Therefore, the user has to be subjective, using his own judgement in either accepting or rejecting the hypothesis that the data fit a Weibull distribution. However, a higher value of $\alpha$ indicates that the data fit the proposed distribution to a greater extent.

For complete samples, the 90-percent Kanofsky-Srinivasan confidence band values about the proposed distribution are also calculated to ascertain the fit of the data. These values are similar to the K-S statistic $D$ centered around the EDF. The bands are generated by

$$\text{Confidence bands} = [F(x) - K(N),\ F(x) + K(N)] \tag{A.69}$$

where $F(x)$ is the failure probability obtained by substituting the Weibull parameters in equation (A.59). The Kanofsky functions, denoted by $K(N)$, are described in reference 34.

Some limitations are intrinsic to a purely statistical approach to design. One problem occurs when the design stress is well below the range of experimental data as shown in figure A.3. Extrapolation of the Weibull distribution into this regime may yield erroneous results if other phenomena are present. When two flaw populations exist concurrently, but only one (population A) is active in the strength regime tested, the predicted failure probability may be incorrect. Furthermore, if the threshold strength is not zero, the strength may be underestimated. Finally, an approach based only on statistics can allow for stress state effects only in an empirical fashion.

## A.4 Material Strength Characterization

Ceramic strength is an ambiguous entity since, for brittle materials, tensile strength, compressive strength, shear strength, flexural strength, and theoretical strength all have unique meanings and different values. The theoretical strength is defined as the tensile stress required to break atomic bonds, which typically ranges from one-tenth to one-fifth of the elastic modulus for ceramic materials. Because of processing flaws, this strength is never obtained. A much more meaningful strength measurement is the tensile strength in uniaxial tension or through flexural testing. In flexural strength testing the bend strength $\sigma_f$ of a ceramic is defined as the maximum tensile stress in the extreme fiber of a beam specimen (modulus of rupture, MOR). The main objective of the PC-CARES program is to characterize ceramic strength in terms of the MOR or pure uniaxial strength and to use this data with appropriate analysis to predict component response under complex multiaxial stress states. The PC-CARES program calculates required polyaxial statistical material strength parameters from uniaxial tensile specimen or four-point bend specimen fracture data. After evaluating the initial parameters as described in section A.3 Estimation of Statistical Material Strength Parameters, additional calculations are performed to determine the material scale parameter and Batdorf crack density coefficient for use in the reliability calculations.

For volume flaw analysis using four-point MOR bar data with known geometry (fig. A.4), the value of $C_V$ in equation (A.59) are obtained from the least-squares or maximum likelihood analysis. The tensile stress distribution in a four-point bend specimen is

$$\left.\begin{aligned}
\sigma_x &= \frac{4xy\sigma_f}{(L_1 - L_2)h} & 0 \le x \le \frac{L_1 - L_2}{2} \\
\sigma_x &= \frac{2y\sigma_f}{h} & \frac{L_1 - L_2}{2} \le x \le \frac{L_1 + L_2}{2} \\
\sigma_x &= \frac{4(L_1 - x)y\sigma_f}{(L_1 - L_2)h} & \frac{L_1 + L_2}{2} \le x \le L_1
\end{aligned}\right\} \qquad \text{(A.70)}$$

By equating the risk of ruptures of equations (A.9) and (A.59), we obtain

$$\int_V \left(\frac{\sigma_x}{\sigma_{oV}}\right)^{m_V} dV = C_V \sigma_f^{m_V} \qquad \text{(A.71)}$$

and, after integrating over the tensile portion of the bar, the scale parameter is

$$\sigma_{oV} = \left[\frac{wh}{2} \frac{(L_1 + m_V L_2)}{C_V (m_V + 1)^2}\right]^{1/m_V} = \left(\frac{V_e}{C_V}\right)^{1/m_V} \qquad \text{(A.72)}$$

where $V_e$ is the effective volume. For uniaxial tensile loading, the effective volume is equal to the gage volume $V_g$, which is the uniformly stressed region where fracture is expected to occur. Note when $L_2$ is zero the solution for the three-point MOR bar is obtained.

For the Batdorf model, using the shear-insensitive case from table A.II(b), we have

$$\frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} = 1 - \sqrt{\frac{\sigma_{cr}}{\sigma_x}} \qquad \text{(A.73)}$$

From equations (A.16b) and (A.17), after performing the $d\sigma_{cr}$ integration, the risk of rupture for the four-point-bend specimen is

$$\int_V \left[\int_0^{\sigma_{e_{max}}} \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} \left(m_V k_{BV} \sigma_{cr}^{m_V - 1}\right) d\sigma_{cr}\right] dV = \frac{k_{BV}}{2m_V + 1} \int_V \sigma_x^{m_V} \, dV \qquad \text{(A.74)}$$

Equating risk of ruptures from equations (A.74) and (A.59) gives

$$C_V \sigma_f^{m_V} = \frac{k_{BV}}{2m_V + 1} \int_V \sigma_x^{m_V} \, dV \tag{A.75}$$

from which, after integrating the stress over the tensile loaded volume with $\sigma_x$ defined in equation (A.70), we get

$$k_{BV} = (2m_V + 1)\left[\frac{2C_V(m_V + 1)^2}{wh(L_1 + m_V L_2)}\right] = (2m_V + 1)\left(\frac{C_V}{V_e}\right) \tag{A.76}$$

Using equation (A.72) and the previously defined Weibull crack density coefficient, we have

$$C_V = \frac{V_e}{\left(\sigma_{oV}\right)^{m_V}} = k_{wV} V_e \tag{A.77}$$

Substituting equation (A.77) into equation (A.76) and rearranging gives the normalized Batdorf crack density coefficient for the shear-insensitive case,

$$\bar{k}_{BV} = \frac{k_{BV}}{k_{wV}} = 2m_V + 1 \tag{A.78}$$

For the Batdorf shear-sensitive case, assuming a Griffith crack and coplanar strain energy release rate criterion, we obtain $\Omega(\Sigma,\sigma_{cr})/4\pi$ from table A.II(b). For uniaxial loading $\sigma$, after performing the indicated integration we get

$$\int_V \left[\int_0^{\sigma} \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} \frac{dN_V(\sigma_{cr})}{d\sigma_{cr}} \, d\sigma_{cr}\right] dV = \frac{k_{BV}}{m_V + 1} \int_V \sigma^{m_V} \, dV \tag{A.79}$$

Again equating the risk of ruptures from equations (A.59) and (A.79) in terms of the effective volume gives

$$C_V \sigma^{m_V} = \frac{k_{BV}}{m_V + 1} \sigma^{m_V} V_e \tag{A.80}$$

Using equation (A.77), we substitute for $C_V$ to get

$$k_{wV} = \frac{k_{BV}}{m_V + 1} \tag{A.81}$$

from which the normalized crack density coefficient for the selected shear-sensitive case is

$$\bar{k}_{BV} = \frac{k_{BV}}{k_{wV}} = m_V + 1 \qquad \text{(A.82)}$$

In the PC-CARES program, $k_{BV}$ is computed numerically for a shear-sensitive material for the general case where no closed-form solution exists. By using equations (A.16b) and (A.17) and equating the appropriate risk of ruptures we obtain

$$C_V \sigma_f^{m_V} = \frac{k_{BV} m_V}{4\pi} \int_V \left[ \int_0^{\sigma_{e_{max}}} \Omega(\Sigma, \sigma_{cr}) \sigma_{cr}^{m_V - 1} \, d\sigma_{cr} \right] dV \qquad \text{(A.83)}$$

or rearranging

$$k_{BV} = \frac{4\pi C_V \sigma_f^{m_V}}{m_V \int_V \left[ \int_0^{\sigma_{e_{max}}} \Omega(\Sigma, \sigma_{cr}) \sigma_{cr}^{m_V - 1} \, d\sigma_{cr} \right] dV} \qquad \text{(A.84)}$$

For surface flaw analysis, using data from four-point MOR bars with known geometry (fig. A.4), the value of $C_S$ and $m_S$ in equation (A.59) is obtained from the least-squares or maximum likelihood analysis. Equating the risk of ruptures of equations (A.45) and (A.59) gives

$$\int_A \left( \frac{\sigma_x}{\sigma_{oS}} \right)^{m_S} dA = C_S \sigma_f^{m_S} \qquad \text{(A.85)}$$

By using the tensile surface stress on the beam sides as given by equation (A.70), and in addition at $y = h/2$, where

$$\left.\begin{aligned}
\sigma_x &= \frac{2x\sigma_f}{(L_1 - L_2)} && 0 \le x \le \frac{L_1 - L_2}{2} \\
\sigma_x &= \sigma_f && \frac{L_1 - L_2}{2} \le x \le \frac{L_1 + L_2}{2} \\
\sigma_x &= \frac{2(L_1 - x)}{(L_1 - L_2)} \sigma_f && \frac{L_1 + L_2}{2} \le x \le L_1
\end{aligned}\right\} \qquad \text{(A.86)}$$

then substituting for $\sigma_x$ and performing the integration in equation (A.85), the scale parameter is obtained as

$$\sigma_{oS} = \left\{ \left[ \frac{\left(\frac{L_2}{L_1}\right) m_S + 1}{C_S (m_S + 1)^2} \right] \left( \frac{m_S w}{w + h} + 1 \right) (w + h) L_1 \right\}^{1/m_S} \tag{A.87a}$$

or

$$\sigma_{oS} = \left( \frac{A_e}{C_S} \right)^{1/m_S} \tag{A.87b}$$

where $A_e$ is the effective area. For uniaxial tensile loading, the effective area is equal to the specimen gage area $A_g$, which is the total specimen surface area of interest. Note when $L_2$ is zero the solution for the three-point MOR bar is obtained.

For surface flaw reliability analysis with the Weibull normal stress averaging method, we calculate the polyaxial crack density coefficient $k_{wpS}$ from the following equation (refs. 17 and 28):

$$k_{wpS} = \frac{m_S \sqrt{\pi}\, \Gamma(m_S) k_{wS}}{\Gamma\left(m_S + \frac{1}{2}\right)} \tag{A.88}$$

where $k_{wS}$ has been previously defined in equation (A.45).

By combining equations (A.49) and (A.50) for the Batdorf surface flaw model, we can express $P_{fS}$ as

$$P_{fS} = 1 - \exp\left\{ -m_S k_{BS} \int_A \left[ \int_0^{\sigma_{e_{max}}} \frac{\omega(\Sigma, \sigma_{cr})}{2\pi} \sigma_{cr}^{m_S - 1} \, d\sigma_{cr} \right] dA \right\} \tag{A.89}$$

For uniaxial tension with a shear-insensitive fracture criterion, substituting for $\omega(\Sigma,\sigma_{cr})/2\pi$ from table A.III ($\sigma_2 = 0$), we obtain

$$P_{fS} = 1 - \exp\left\{ - \frac{2 m_S k_{BS}}{\pi} \int_A \left[ \int_0^{\sigma_1} \cos^{-1}\sqrt{\frac{\sigma_{cr}}{\sigma_1}} \, \sigma_{cr}^{m_S - 1} \, d\sigma_{cr} \right] dA \right\} \tag{A.90}$$

Equating the risk of rupture in equation (A.90) with that of equation (A.45) results in

$$\bar{k}_{BS} = \frac{k_{BS}}{k_{wS}} = \frac{m_S \sqrt{\pi}\ \Gamma(m_S)}{\Gamma\left(m_S + \frac{1}{2}\right)} \tag{A.91}$$

Hence, for this special case the Batdorf crack density coefficient is identical to the Weibull polyaxial crack density coefficient: that is

$$k_{BS} = k_{wpS} \tag{A.92}$$

Similar results were obtained for volume flaw based analysis as well.

For the general shear-sensitive case, $k_{BS}$ is computed numerically since no closed-form solution exists. Thus, equating the risk of ruptures of equations (A.59) and (A.89) gives

$$C_S \sigma_f^{m_S} = m_S k_{BS} \int_A \left[ \int_0^{\sigma_{e_{max}}} \frac{\omega(\Sigma,\sigma_{cr})}{2\pi}\ \sigma_{cr}^{m_S - 1}\ d\sigma_{cr} \right] dA \tag{A.93}$$

from which we obtain

$$k_{BS} = \frac{2\pi C_S \sigma_f^{m_S}}{m_S \int_A \left[ \int_0^{\sigma_{e_{max}}} \omega(\Sigma,\sigma_{cr}) \sigma_{cr}^{m_S - 1}\ d\sigma_{cr} \right] dA} \tag{A.94}$$

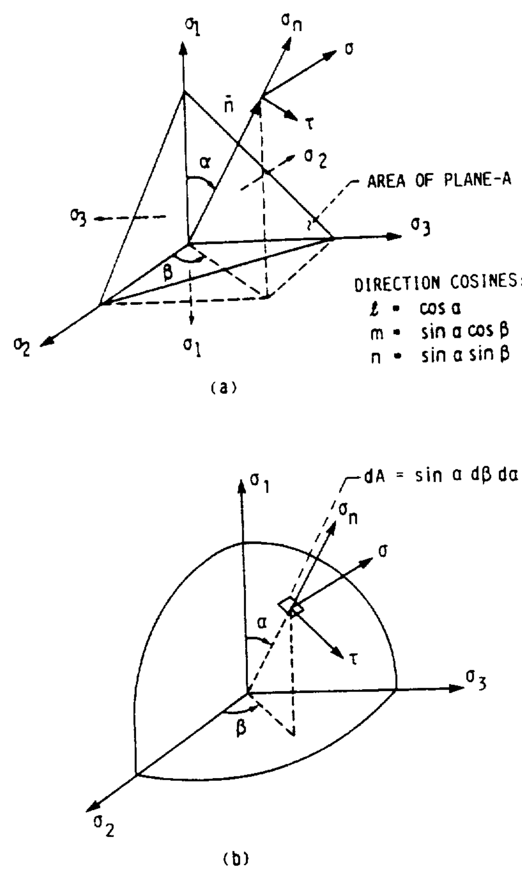


FIGURE A.1. - STRESSES ON CAUCHY INFINITESIMAL TETRAHEDRON IN PRINCIPAL STRESS SPACE (a), PROJECTED ONTO A TANGENT PLANE TO THE UNIT RADIUS SPHERE AS SHOWN IN (b).

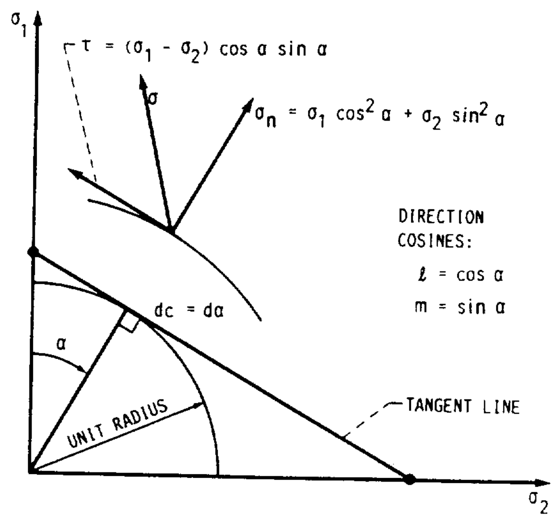


FIGURE A.2. - NORMAL AND SHEAR STRESS AS A FUNCTION OF $\alpha$ PROJECTED ONTO A TANGENT LINE TO THE UNIT RADIUS CIRCLE.

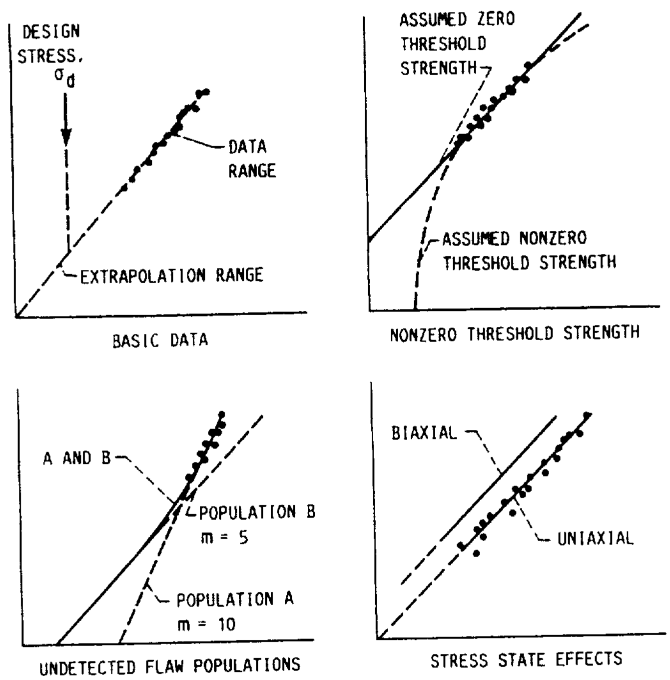


FIGURE A.3. - LIMITATIONS OF EXPERIMENTAL DATA EXTRAPOLATION AND STATISTICAL APPROACH TO DESIGN.

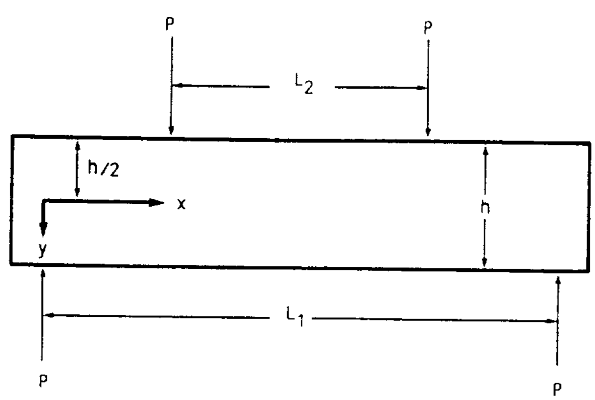


FIGURE A.4. - FOUR-POINT BEND SPECIMEN GEOMETRY (BEAM WIDTH IS w).

TABLE A.I. - FORMS OF $P_{2V}$ FOR VARIOUS FRACTURE CRITERIA AND SELECTED CRACK CONFIGURATIONS

[$\sigma_1 \geq \sigma_2 > \sigma_3$, $\sigma_3 \geq 0$ and $\sigma_e = \sigma_{cr}$.]

$$P_{2V} = \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} = \frac{2}{\pi}\int_0^{\pi/2}\int d\beta \sin\alpha \, d\alpha$$

$$\Phi_{2,1} = \cos^2\bar{\beta}_{1,2} = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \quad \text{or} \quad \Phi = -\frac{a_3}{a_2} \quad \text{when} \quad a_1 = 0$$

where $\Phi_1 \leq \Phi_2$

$$\bar{\beta}_1(\alpha,\sigma_{cr}) = \cos^{-1}\sqrt{\Phi_2}$$

$$\bar{\beta}_2(\alpha,\sigma_{cr}) = \cos^{-1}\sqrt{\Phi_1}$$

$$\bar{\beta}(\alpha,\sigma_{cr}) = \cos^{-1}\sqrt{\Phi}$$

After obtaining roots for a given stress state $\Sigma$, $\sigma_{cr}$ and varying $\alpha$, care must be taken in evaluating $\int d\beta$. The relation of $\sigma_e$ to $\sigma_{cr}$ in the neighborhood of $\bar{\beta}$ must be known to obtain the proper limits of the integral.

$$D_1 = \frac{1}{(1 - 0.5\nu)^2}, \qquad D_2 = \frac{\nu(1 - 0.25\nu)}{(1 - 0.5\nu)^2} = D_1 - 1$$

| Fracture criterion | Crack configuration | Quadratic equation coefficients for $\sigma_n(\Sigma,\alpha,\beta) \geq 0$ |
|---|---|---|
| Normal stress (shear-insensitive cracks) | Independent of crack shape | $a_1 = 0$<br>$a_2 = (\sigma_2 - \sigma_3)\sin^2\alpha$<br>$a_3 = (\sigma_1 - \sigma_3)\cos^2\alpha + \sigma_3 - \sigma_{cr}$ |
| Maximum tensile stress | Griffith crack (GC) | $a_1 = \left(\sigma_2 - \sigma_3\right)^2 \sin^4\alpha$<br>$a_2 = (\sigma_2 - \sigma_3)\sin^2\alpha\left[2\left(\sigma_1\cos^2\alpha + \sigma_3\sin^2\alpha\right) - 4\sigma_{cr} - \sigma_3 - \sigma_2\right]$<br>$a_3 = -\left(\sigma_1 - \sigma_3\right)^2 \sin^2\alpha \cos^2\alpha - 4\sigma_{cr}\left(\sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha\right) + 4\sigma_{cr}^2$ |
| | Penny-shaped crack (PSC) | $a_1 = D_1\left(\sigma_2 - \sigma_3\right)^2 \sin^4\alpha$<br>$\alpha_2 = D_1(\sigma_2 - \sigma_3)\sin^2\alpha\left[2\left(\sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha\right) - \frac{4}{D_1}\sigma_{cr} - \sigma_3 - \sigma_2\right]$<br>$\alpha_3 = -D_1\left(\sigma_1 - \sigma_3\right)^2 \sin^2\alpha \cos^2\alpha - 4\sigma_{cr}\left(\sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha\right) + 4\sigma_{cr}^2$ |

TABLE A.I. - Concluded.

| Fracture criterion | Crack configuration | Quadratic equation coefficients for $\sigma_n(\Sigma,\alpha,\beta) \geq 0$ |
|---|---|---|
| Strain energy release rate, $G_T$ | Griffith crack (GC) | $a_1 = 0$<br>$a_2 = -\left(\sigma_2^2 - \sigma_3^2\right)\sin^2\alpha$<br>$a_3 = -\sigma_1^2 \cos^2\alpha - \sigma_3^2 \sin^2\alpha + \sigma_{cr}^2$ |
| | Penny-shaped crack (PSC) | $a_1 = D_2\left(\sigma_2 - \sigma_3\right)^2 \sin^4\alpha$<br>$a_2 = -D_1\left(\sigma_2^2 - \sigma_3^2\right)\sin^2\alpha + 2D_2(\sigma_2 - \sigma_3)\sin^2\alpha\left(\sigma_3 \sin^2\alpha + \sigma_1 \cos^2\alpha\right)$<br>$a_3 = -D_1\left(\sigma_1^2 \cos^2\alpha + \sigma_3^2 \sin^2\alpha\right) + D_2\left(\sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha\right)^2 + \sigma_{cr}^2$ |
| Shetty | Griffith crack (GC) | $a_1 = \frac{1}{\bar{C}^2}\left(\sigma_2 - \sigma_3\right)^2 \sin^4\alpha$<br>$a_2 = (\sigma_2 - \sigma_3)\sin^2\alpha\left[-\sigma_{cr} + \frac{1}{\bar{C}^2}\left(2\sigma_1 \cos^2\alpha + 2\sigma_3 \sin^2\alpha - \sigma_2 - \sigma_3\right)\right]$<br>$a_3 = \sigma_{cr}^2 - \sigma_{cr}\left(\sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha\right) - \frac{1}{\bar{C}^2}\left[\left(\sigma_1 - \sigma_3\right)^2 \sin^2\alpha \cos^2\alpha\right]$ |
| | Penny-shaped crack (PSC) | $a_1 = \frac{D_1}{\bar{C}^2}\left(\sigma_2 - \sigma_3\right)^2 \sin^4\alpha$<br>$a_2 = (\sigma_2 - \sigma_3)\sin^2\alpha\left[-\sigma_{cr} + \frac{D_1}{\bar{C}^2}\left(2\sigma_1 \cos^2\alpha + 2\sigma_3 \sin^2\alpha - \sigma_2 - \sigma_3\right)\right]$<br>$a_3 = \sigma_{cr}^2 - \sigma_{cr}\left(\sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha\right) - \frac{D_1}{\bar{C}^2}\left[\left(\sigma_1 - \sigma_3\right)^2 \sin^2\alpha \cos^2\alpha\right]$ |

TABLE A.II(a). - FORMS OF $P_{2V}$ FOR VARIOUS FRACTURE CRITERIA, CRACK CONFIGURATIONS, AND STRESS STATES

[$\sigma_1 > \sigma_2 = \sigma_3$, $\sigma_3 \geq 0$ and $\sigma_e = \sigma_{cr}$.]

$$P_{2V} = \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi} = \int \sin\alpha \, d\alpha$$

$$\Phi_{2,1} = \cos^2\bar{\alpha}_{1,2} = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \quad \text{or} \quad \Phi = -\frac{a_3}{a_2} \quad \text{when} \quad a_1 = 0$$

where $\Phi_1 \leq \Phi_2$

$$\bar{\alpha}_1(\sigma_{cr}) = \cos^{-1}\sqrt{\Phi_2}$$

$$\bar{\alpha}_2(\sigma_{cr}) = \cos^{-1}\sqrt{\Phi_1}$$

$$\bar{\alpha}(\sigma_{cr}) = \cos^{-1}\sqrt{\Phi}$$

$$D_1 = \frac{1}{(1 - 0.5\nu)^2}, \quad D_2 = \frac{\nu(1 - 0.25\nu)}{(1 - 0.5\nu)^2} = D_1 - 1$$

| Fracture criterion | Crack configuration | Quadratic equation coefficients for $\sigma_n(\Sigma,\alpha) \geq 0$ |
|---|---|---|
| Maximum tensile stress | Griffith crack (GC) | $a_1 = \left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -\left(\sigma_1 - \sigma_2\right)^2 - 4(\sigma_1 - \sigma_2)\sigma_{cr}$<br>$a_3 = 4\sigma_{cr}(\sigma_{cr} - \sigma_2)$ |
| | Penny-shaped crack (PSC) | $a_1 = D_1\left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -D_1\left[\left(\sigma_1 - \sigma_2\right)^2 + \frac{4}{D_1}(\sigma_1 - \sigma_2)\sigma_{cr}\right]$<br>$a_3 = 4\sigma_{cr}(\sigma_{cr} - \sigma_2)$ |

TABLE A.II(a). - Concluded.

| Fracture criterion | Crack configuration | Quadratic equation coefficients for $\sigma_n(\Sigma,\alpha) \geq 0$ |
|---|---|---|
| Strain energy release rate, $G_T$ | Griffith crack (GC) | $a_1 = 0$<br>$a_2 = \sigma_2^2 - \sigma_1^2$<br>$a_3 = \sigma_{cr}^2 - \sigma_2^2$ |
| | Penny-shaped crack (PSC) | $a_1 = D_2\left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -D_1\left(\sigma_1^2 - \sigma_2^2\right) + 2D_2\sigma_2(\sigma_1 - \sigma_2)$<br>$a_3 = \sigma_{cr}^2 - \sigma_2^2$ |
| Shetty | Griffith crack (GC) | $a_1 = \frac{1}{\bar{C}^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -\sigma_{cr}(\sigma_1 - \sigma_2) - \frac{1}{\bar{C}^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_3 = \sigma_{cr}(\sigma_{cr} - \sigma_2)$ |
| | Penny-shaped crack (PSC) | $a_1 = \frac{D_1}{\bar{C}^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -\sigma_{cr}(\sigma_1 - \sigma_2) - \frac{D_1}{\bar{C}^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_3 = \sigma_{cr}(\sigma_{cr} - \sigma_2)$ |

TABLE A.II(b). - FORMS OF $P_{2V}$ FOR VARIOUS FRACTURE CRITERIA, CRACK CONFIGURATIONS, AND STRESS STATES

[$\sigma_3 \geq 0.$]

$$P_{2V} = \frac{\Omega(\Sigma,\sigma_{cr})}{4\pi}$$

| Fracture criterion | Crack configuration | Stress state | $P_{2V}$ |
|---|---|---|---|
| Normal stress (shear-insensitive cracks) | Independent of crack shape | $\sigma_2 = \sigma_3$<br>$\sigma_1 \neq \sigma_2$ | $\frac{\Omega}{4\pi} = 1 - \cos \alpha = 1 - \sqrt{\Phi}$<br>where<br>$\Phi = \cos^2\bar{\alpha} = \frac{\sigma_{cr} - \sigma_2}{\sigma_1 - \sigma_2}$ |
| | | Uniaxial<br>$\sigma_1 = \sigma$<br>$\sigma_2 = \sigma_3 = 0$ | $\frac{\Omega}{4\pi} = 1 - \cos \alpha = 1 - \sqrt{\Phi}$<br>where<br>$\Phi = \cos^2\bar{\alpha} = \frac{\sigma_{cr}}{\sigma}$<br>Note: CARES defaults to shear-insensitive crack for uniaxial loading when IKBAT = 0. |
| | | Equibiaxial<br>$\sigma_1 = \sigma_2 = \sigma$<br>$\sigma_3 = 0$ | $\frac{\Omega}{4\pi} = \sqrt{1 - \frac{\sigma_{cr}}{\sigma}}$ |
| Strain energy release rate, $G_T$ | Griffith crack (GC) | Uniaxial<br>$\sigma_1 = \sigma$<br>$\sigma_2 = \sigma_3 = 0$ | $\frac{\Omega}{4\pi} = 1 - \frac{\sigma_{cr}}{\sigma}$ |
| | | Equibiaxial<br>$\sigma_1 = \sigma_2 = \sigma$<br>$\sigma_3 = 0$ | $\frac{\Omega}{4\pi} = \sqrt{1 - \left(\frac{\sigma_{cr}}{\sigma}\right)^2}$ |
| Independent of fracture criterion | Independent of crack shape | Equitriaxial<br>$\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ | $\frac{\Omega}{4\pi} = 1.0$ |

TABLE A.III. - FORMS OF $P_{2S}$ FOR VARIOUS FRACTURE CRITERIA AND SELECTED CRACK CONFIGURATIONS

[$\sigma_1 > \sigma_2$, $\sigma_2 \geq 0$, $\sigma_3 = 0$ and $\sigma_e = \sigma_{cr}$.]

$$P_{2S} = \frac{\omega(\Sigma,\sigma_{cr})}{2\pi} = \frac{2}{\pi}\int d\alpha$$

$$\Phi_{2,1} = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \quad \text{or} \quad \Phi = -\frac{a_3}{a_2} \quad \text{when} \quad a_1 = 0$$

where $\Phi_1 \leq \Phi_2$

$$\bar{\alpha}_1(\sigma_{cr}) = \cos^{-1}\sqrt{\Phi_2}$$

$$\bar{\alpha}_2(\sigma_{cr}) = \cos^{-1}\sqrt{\Phi_1}$$

$$\bar{\alpha}(\sigma_{cr}) = \cos^{-1}\sqrt{\Phi}$$

After obtaining roots for a given stress state $\Sigma$ and $\sigma_{cr}$, care must be taken in evaluating $\int d\alpha$. The relation of $\sigma_e$ to $\sigma_{cr}$ in the neighborhood of $\bar{\alpha}$ must be known to obtain the proper limits of the integral.

$$D_3 = \frac{1}{1 - \nu}$$

| Fracture criterion | Crack configuration | Quadratic equation coefficients for $\sigma_n(\Sigma,\alpha) \geq 0$ |
|---|---|---|
| Normal stress (shear-insensitive cracks)[a] | Independent of crack shape | $a_1 = 0$<br>$a_2 = \sigma_1 - \sigma_2$<br>$a_3 = \sigma_2 - \sigma_{cr}$ |
| Strain energy release rate, $G_T$[a] | Griffith crack (GC) | $a_1 = 0$<br>$a_2 = \sigma_2^2 - \sigma_1^2$<br>$a_3 = \sigma_{cr}^2 - \sigma_2^2$ |
| | Griffith notch (GN) | $a_1 = (\sigma_1 - \sigma_2)^2\left[\frac{D_3}{(1.1215)^2} - 1\right]$<br>$a_2 = -2\sigma_2(\sigma_1 - \sigma_2) - \frac{D_3}{(1.1215)^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_3 = \sigma_{cr}^2 - \sigma_2^2$ |

[a]For cases where neither $\Phi_1$ nor $\Phi_2$ is between 0 and 1, see ref. 2.

TABLE A.III. - Concluded.

| Fracture criterion | Crack configuration | Quadratic equation coefficients for $\sigma_n(\Sigma,\alpha) \geq 0$ |
|---|---|---|
| Shetty | Griffith crack (GC) | $a_1 = \frac{1}{\bar{C}^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -\sigma_{cr}(\sigma_1 - \sigma_2) - \frac{1}{\bar{C}^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_3 = \sigma_{cr}(\sigma_{cr} - \sigma_2)$ |
| | Griffith notch (GN) | $a_1 = \frac{1}{(1.1215\bar{C})^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -\sigma_{cr}(\sigma_1 - \sigma_2) - \frac{1}{(1.1215\bar{C})^2}\left(\sigma_1 - \sigma_2\right)^2$<br>$a_3 = \sigma_{cr}(\sigma_{cr} - \sigma_2)$ |
| | Semicircular crack | $a_1 = \left(\frac{0.9085}{\bar{C}}\right)^2\left(\sigma_1 - \sigma_2\right)^2$<br>$a_2 = -\sigma_{cr}(\sigma_1 - \sigma_2) - \left(\frac{0.9085}{\bar{C}}\right)^2\left(\sigma_1 - \sigma_2\right)^2$<br>$a_3 = \sigma_{cr}(\sigma_{cr} - \sigma_2)$ |

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# Report Documentation Page

| 1. Report No. | 2. Government Accession No. | 3. Recipient's Catalog No. |
|---|---|---|
| NASA TM-103247 | | |

| 4. Title and Subtitle | 5. Report Date |
|---|---|
| Calculation of Weibull Strength Parameters, Batdorf Flaw Density Constants and Related Statistical Quantities Using PC-CARES | October 1990 |
| | 6. Performing Organization Code |
| 7. Author(s) | 8. Performing Organization Report No. |
| Steven A. Szatmary, John P. Gyekenyesi, and Noel N. Nemeth | E-5674 |
| | 10. Work Unit No. |
| | 505-63-1B |
| 9. Performing Organization Name and Address | 11. Contract or Grant No. |
| National Aeronautics and Space Administration<br>Lewis Research Center<br>Cleveland, Ohio 44135-3191 | |
| | 13. Type of Report and Period Covered |
| 12. Sponsoring Agency Name and Address | Technical Memorandum |
| National Aeronautics and Space Administration<br>Washington, D.C. 20546-0001 | 14. Sponsoring Agency Code |

15. Supplementary Notes

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16. Abstract

This manual describes the operation and theory of the PC-CARES (Personal Computer-Ceramic Analysis and Reliability Evaluation of Structures) computer program for the IBM PC and compatibles running PC-DOS/MS-DOS OR IBM/MS-OS/2 (version 1.1 or higher) operating systems. The primary purpose of this code is to estimate Weibull material strength parameters, the Batdorf crack density coefficient, and other related statistical quantities. Included in the manual is the description of the calculation of shape and scale parameters of the two-parameter Weibull distribution using the least-squares analysis and maximum likelihood methods for volume- and surface-flaw-induced fracture in ceramics with complete and censored samples. The methods for detecting outliers and for calculating the Kolmogorov-Smirnov and the Anderson-Darling goodness-of-fit statistics and 90-percent confidence bands about the Weibull line, as well as the techniques for calculating the Batdorf flaw-density constants are also described.

| 17. Key Words (Suggested by Author(s)) | 18. Distribution Statement |
|---|---|
| Ceramic strength; MOR bars; Weibull parameters; Least squares; Maximum likelihood; Confidence bands; Goodness-of-fit; Batdorf; CARES; Censored-statistics | Unclassified – Unlimited<br>Subject Category 39 |

| 19. Security Classif. (of this report) | 20. Security Classif. (of this page) | 21. No. of pages | 22. Price* |
|---|---|---|---|
| Unclassified | Unclassified | 110 | A06 |

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