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# Direct Simulation of Compressible Turbulence

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The physics of turbulence remains one of the most challenging problems in fluid dynamics. Although more than a century of effort has been devoted to it, a lot of fundamental issues are still unresolved. This is particularly so in the case of turbulence in high speed flows because of the increased number of possible mode interactions due to compressibility effects. For example, the cubic non-linearities in the momentum equations allow the vorticity, acoustic and entropy modes to interact with each other. The dynamics are further complicated by the possible existence of non-stationary shocks and/or eddy shocklets.

In this paper, several direct simulations of 3-D homogeneous, compressible turbulence are presented with emphasis on the differences with incompressible turbulent simulations. A fully spectral collocation algorithm, periodic in all directions coupled with a 3rd order Runga-Kutta time discretization scheme is sufficient to produce well-resolved flows at Taylor Reynolds numbers below 40 on grids of 128x128x128. A Helmholtz decomposition of velocity is useful to differentiate between the purely compressible effects and those effects solely due to vorticity production. In the context of homogeneous flows, this decomposition is unique. Time-dependent energy and dissipation spectra of the compressible and solenoidal velocity components indicate the presence of localized small scale structures. These structures are strongly a function of the initial conditions. We concentrate on a regime characterized by very small fluctuating Mach numbers $Ma$ (on the order of 0.03) and density and temperature fluctuations much greater than $Ma^2$. This leads to a state in which more than 70% of the kinetic energy is contained in the so-called compressible component of the velocity. Furthermore, these conditions lead to the formation of curved weak shocks (or shocklets) which travel at approximately the sound speed across the physical domain. Various terms in the vorticity and divergence of velocity production equations are plotted versus time to gain some understanding of how small scales are actually formed. Possible links with Burger turbulence are examined.

To visualize better the dynamics of the flow, new graphic visualization techniques have been developed. The 3-D structure of the shocks are visualized with the help of volume rendering algorithms developed in house. A combination of stereographic projection and animation greatly increase the number of visual cues necessary to properly interpret the complex flow. The presence or absence of shocks is automatically detected by monitoring of the minimum and maximum divergence of the velocity field over the physical domain.

# NAVIER-STOKES EQUATIONS

Non-Dimensionalizations

Velocity, $U_0$, Temperature $T_0$, density $\rho_0$, pressure $\rho U_0^2$.

Viscosity and Prandl number are constant.

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

$$\rho\frac{D\vec{v}}{Dt} = -\frac{\epsilon_p}{\gamma M_\infty^2}\nabla P + \frac{1}{Re}\nabla\cdot\vec{\vec{\tau}}$$

$$\frac{DP}{Dt} = -\gamma P\nabla\cdot\vec{v} + \frac{\gamma}{\epsilon_p Pr Re}\nabla\cdot(\kappa\nabla T) + \frac{\gamma(\gamma-1)M_\infty^2}{\epsilon_p Re}\vec{\vec{\tau}} : \vec{\vec{e}}.$$

$$\epsilon_p P = \rho T$$

where the stress tensors and dissipation function are defined by

$$\vec{\vec{e}} = \frac{1}{2}(\nabla\vec{v} + \nabla\vec{v}^T),$$

$$\vec{\vec{\tau}} = 2\mu\vec{\vec{e}} - \frac{2}{3}\mu(\nabla\cdot\vec{v})\vec{\vec{I}},$$

and

$$\vec{\vec{\tau}} : \vec{\vec{e}} = \frac{\mu}{2}(\nabla\vec{v} + \nabla\vec{v}^T) : (\nabla\vec{v} + \nabla\vec{v}^T) - \frac{2}{3}\mu(\nabla\cdot\vec{v})^2.$$

# OBJECTIVE

- Understand if, when, and why shocks occur in homogeneous turbulence, shear layers and mixing layers in the supersonic and hypersonic regimes.
- Develop mixing enhancement strategies in supersonic shear layers (e.g. scramjets)
- The study of initial condition effects on the time-dependent characteristics of supersonic "isotropic" turbulence is the first step toward the aforementioned objectives.

# PHYSICS

- Solve time-dependent, 3-D full Navier-Stokes equations
- Code set up for variable viscosity, but set to a constant
- $\mathrm{Pr} = 0.7$ (air)
- ideal gas
- no chemistry
- initial energy and thermodynamics autocorrelation spectrums identical

# INITIAL CONDITIONS (1)

- Generate random $\vec{v}, \delta\rho, \delta T$ subject to

$$\begin{aligned} \nabla\cdot\vec{v}_s &= 0 \\ \nabla\times\vec{v}_c &= 0 \end{aligned}$$

- Impose autocorrelation spectrum on $\vec{v}, \delta\rho, \delta T$

$$E(k) = k^4 e^{-k^2/2k_0^2}$$

- Specify rms levels for $\delta\rho, \delta T$
- Calculate

$$\begin{aligned} \rho &= 1 + \delta\rho \\ T &= 1 + \delta T \end{aligned}$$

# INITIAL CONDITIONS (2)

- Specify fluctuating Mach number $M_a$, and compute

$$M_0^2 = \frac{u_0^2}{\gamma R T_0}$$

according to the approximate formula

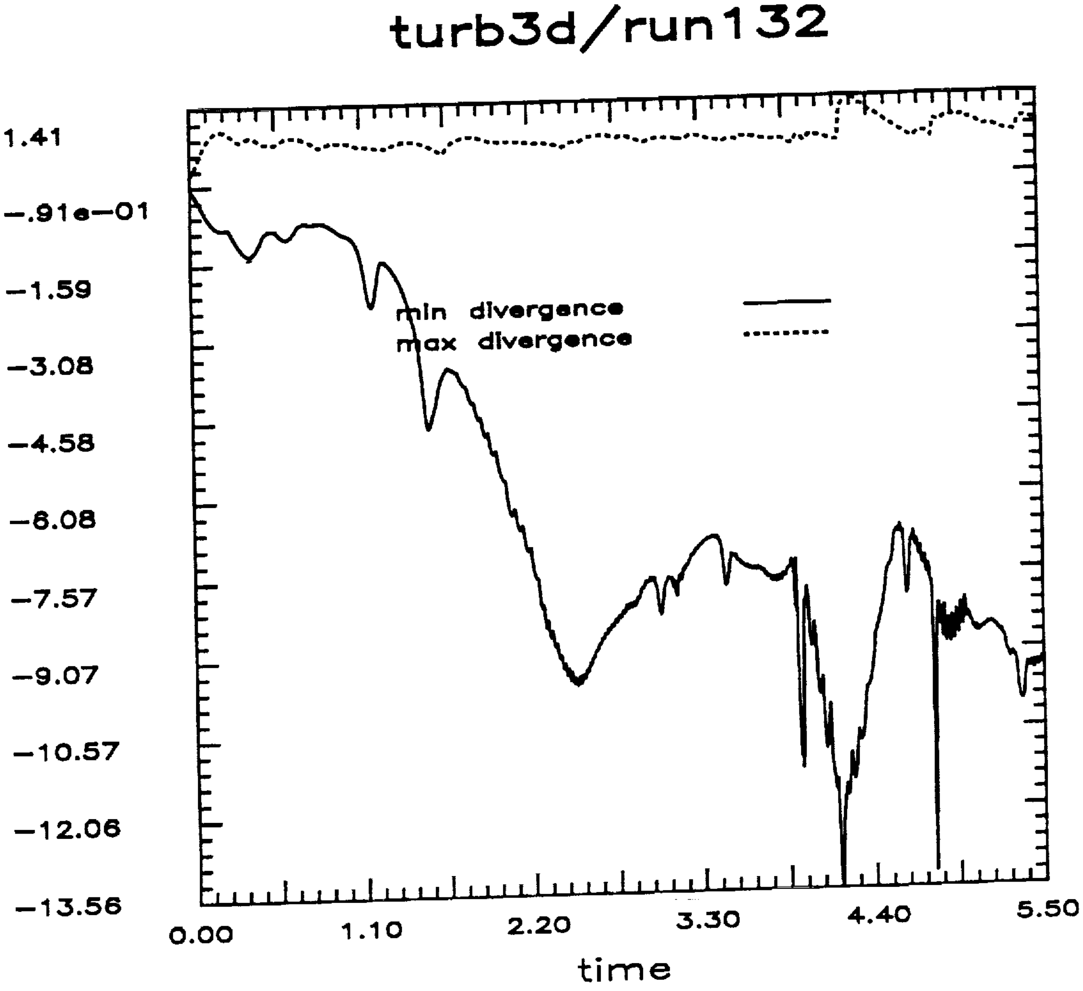
$$M_a = M_0 \, u_{rms}$$

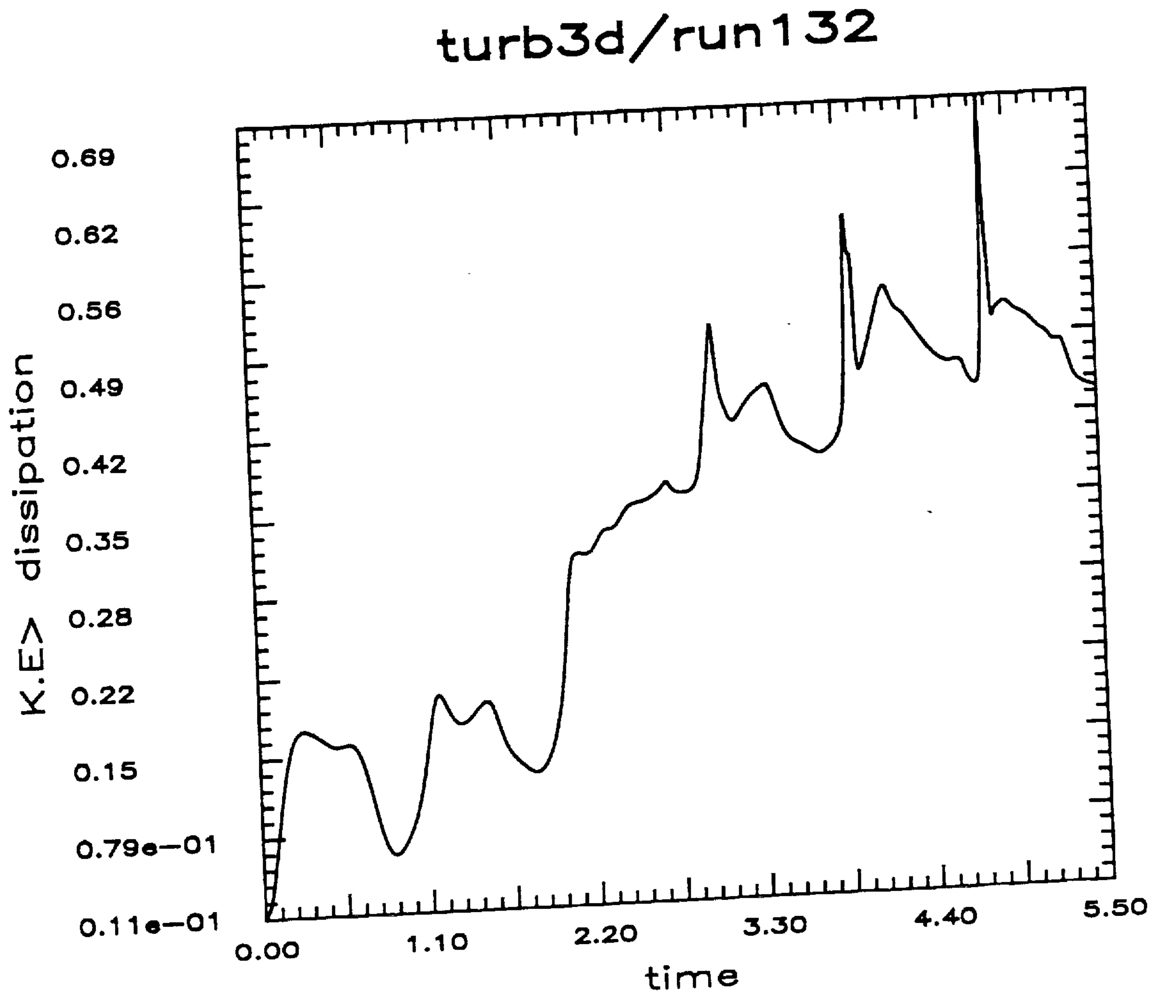
- Specify initial level of compressibility

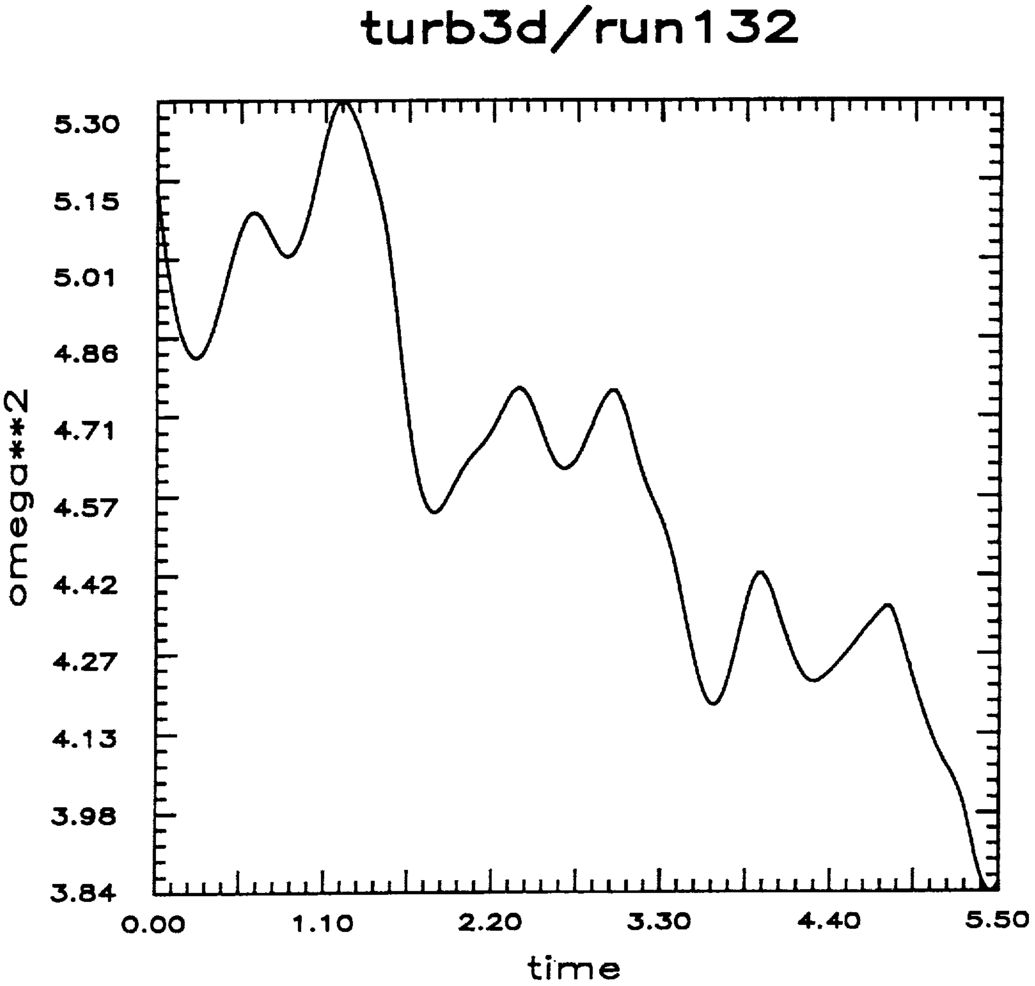
$$\chi = \frac{\int u_c^2 \, dV}{\int (u_s^2 + u_c^2) \, dV}$$

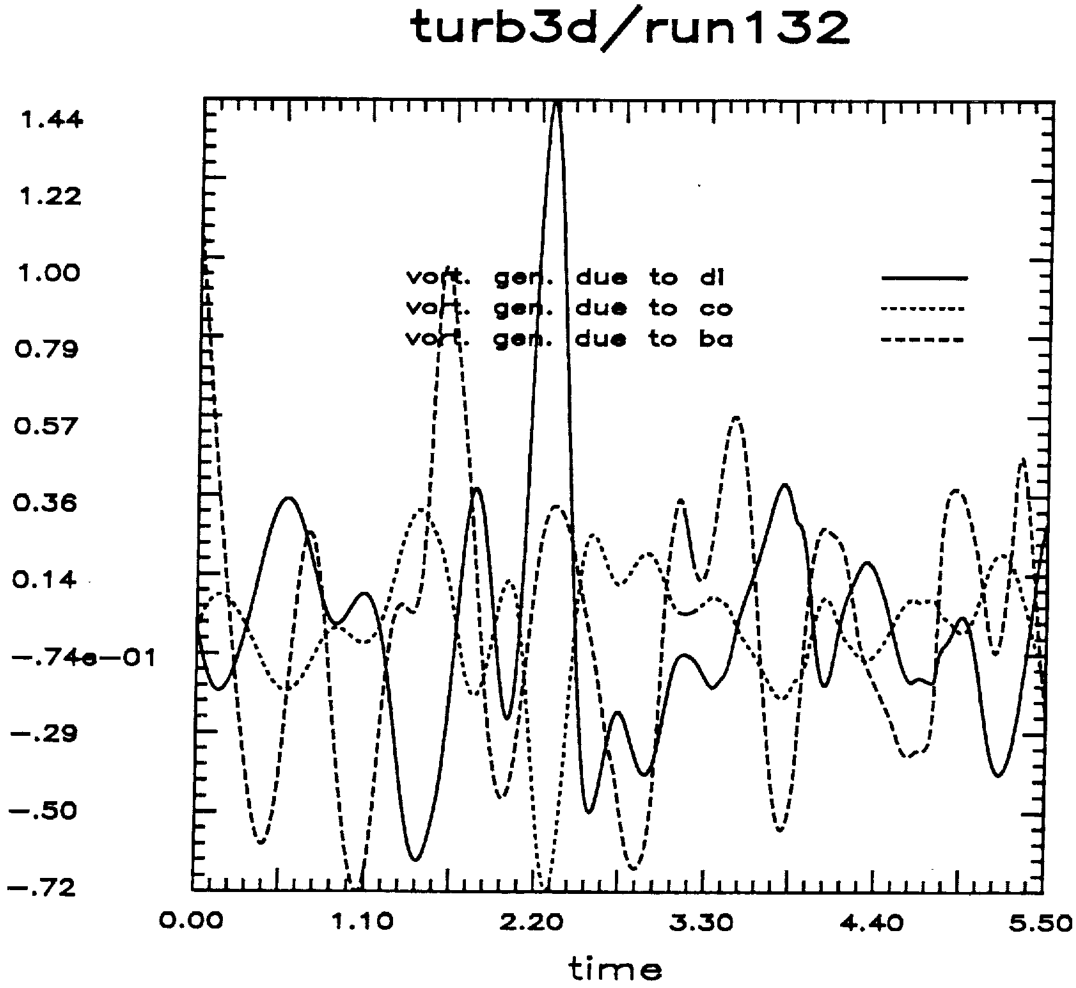
# NUMERICAL METHOD

- Fourier Collocation in both directions (periodic box)
- Conservative scheme in absence of time discretization errors
- Isotropic truncation in Fourier space at every iteration
- Splitting algorithm
  - 3rd order Runga-Kutta method in time on the explicit terms (1st step)
  - Acoustic terms treated implicitly (2nd step)

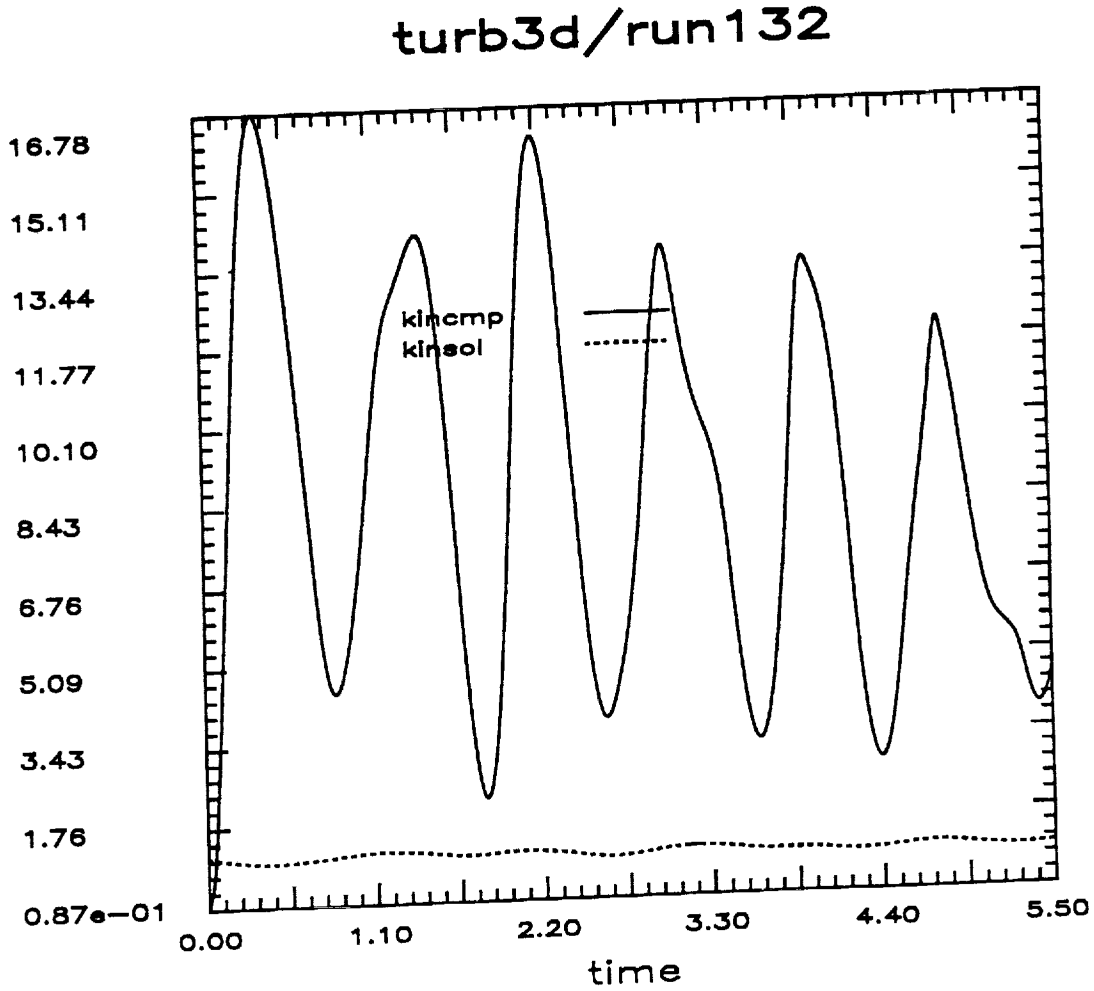

turb3d/run132
1.41
-.91e-01
-1.59
-3.08
-4.58
-6.08
-7.57
-9.07
-10.57
-12.06
-13.56
0.00
1.10
2.20
3.30
4.40
5.50
time
min divergence
max divergence


turb3d/run132
K.E> dissipation
0.69
0.62
0.56
0.49
0.42
0.35
0.28
0.22
0.15
0.79e-01
0.11e-01
0.00
1.10
2.20
3.30
4.40
5.50
time


turb3d/run132
omega**2
5.30
5.15
5.01
4.86
4.71
4.57
4.42
4.27
4.13
3.98
3.84
0.00
1.10
2.20
3.30
4.40
5.50
time



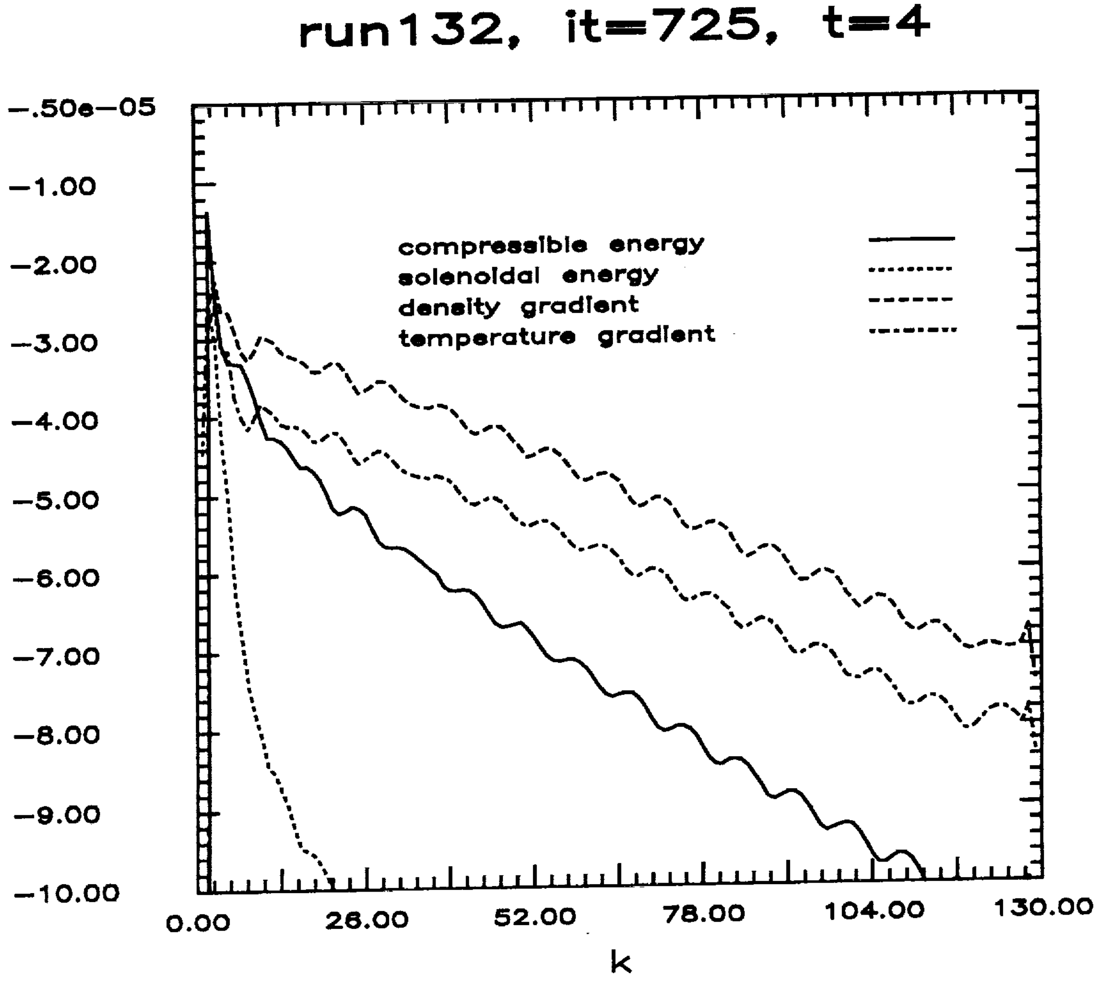
turb3d/run132


turb3d/run132
16.78
15.11
13.44
11.77
10.10
8.43
6.76
5.09
3.43
1.76
0.87e-01
kincmp
kinsol
0.00
1.10
2.20
3.30
4.40
5.50
time

DIVERGENCE OF VELOCITY

turb3d/run132, t=3.3, it=600

Re=150, Ma=.028, k0=1.85, urms=.1, chi=.07, rho,T=10%


run132, it=725, t=4
-.50e-05
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
0.00
26.00
52.00
78.00
104.00
130.00
k
compressible energy
solenoidal energy
density gradient
temperature gradient

# CONCLUSIONS

- Weak shocks can be generated by an initially random 2-D velocity and thermodynamic field.
- These shocks propagate at the speed of sound across the domain
- The presence of these shocks is reflected in the structure of the compressible energy spectrum
- Although the flow is isotropic at the onset, the compressible component of velocity quickly becomes anisotropic (as evidenced by the shock structure)
- Sophisticated visualization techniques are necessary to capture the essence of the dynamics in 3 dimensions

Future Work

- Mixing layers with shock interactions
- Three-dimensional isotropic turbulence (in progress)
- Turbulence modeling Testing