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Unstructured Mesh Solution of the Euler and Navier-Stokes Equations

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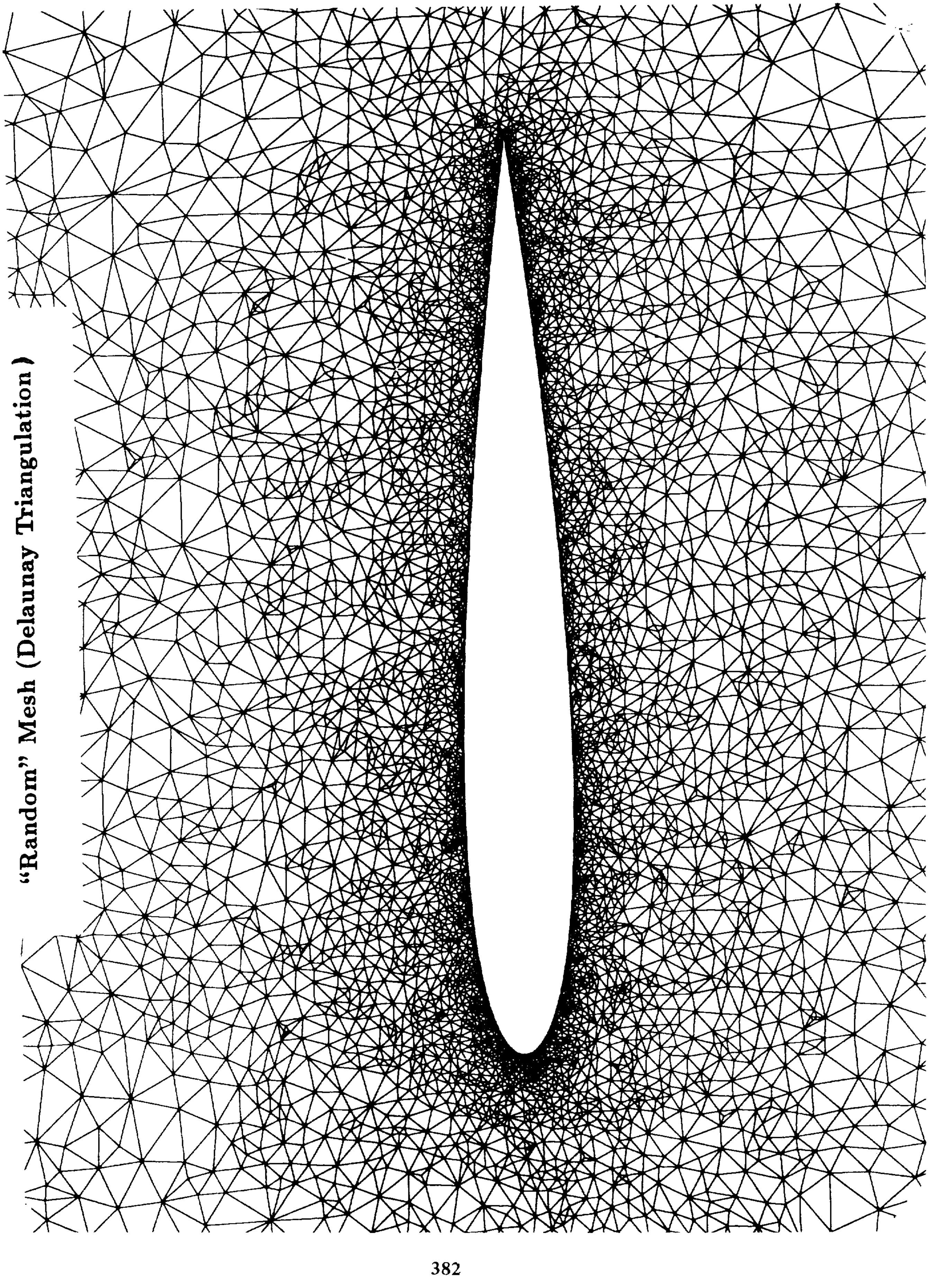
Mesh generation procedures as well as solution algorithms for solving the Euler and Navier-Stokes equations on unstructured meshes are presented. The solution algorithms discussed utilize approximate Riemann solver, upwind differencing to achieve high spatial accuracy. Numerical results for Euler flow over single and multi-element airfoils are presented.

Motivation/Requirements

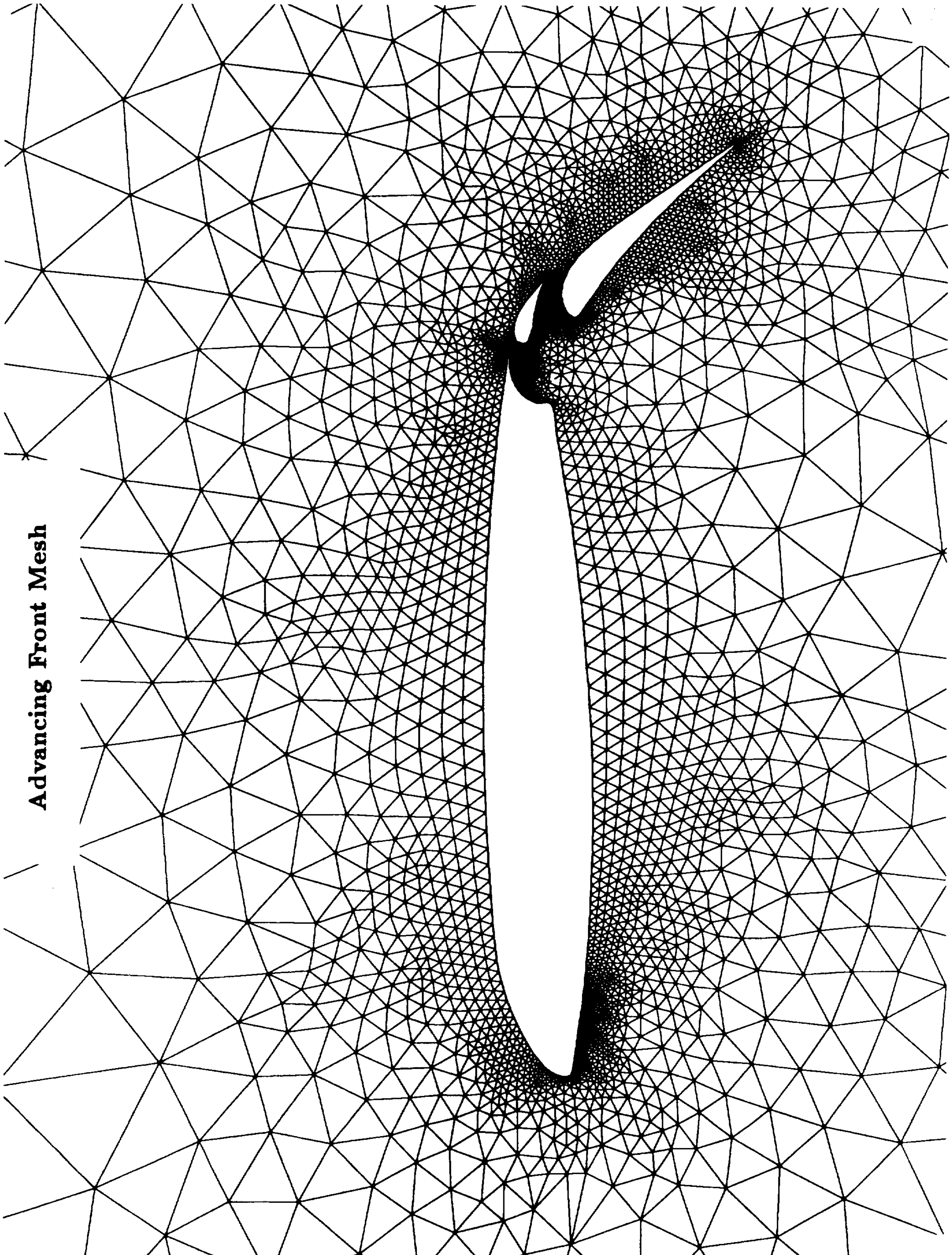
- Generality
  - Mesh Generation of Complex Geometries
  - Inviscid/Viscous, Subsonic/Transonic Flows
- Accuracy
  - High Spatial Accuracy for Smooth Flows
    - Mesh Adaptation
  - Shock Capturing for Discontinuous Flows
    - Generalized Upwind Scheme
- Efficiency
  - Edge Data Structure
    - Edge and Face "Coloring"

Mesh Generation

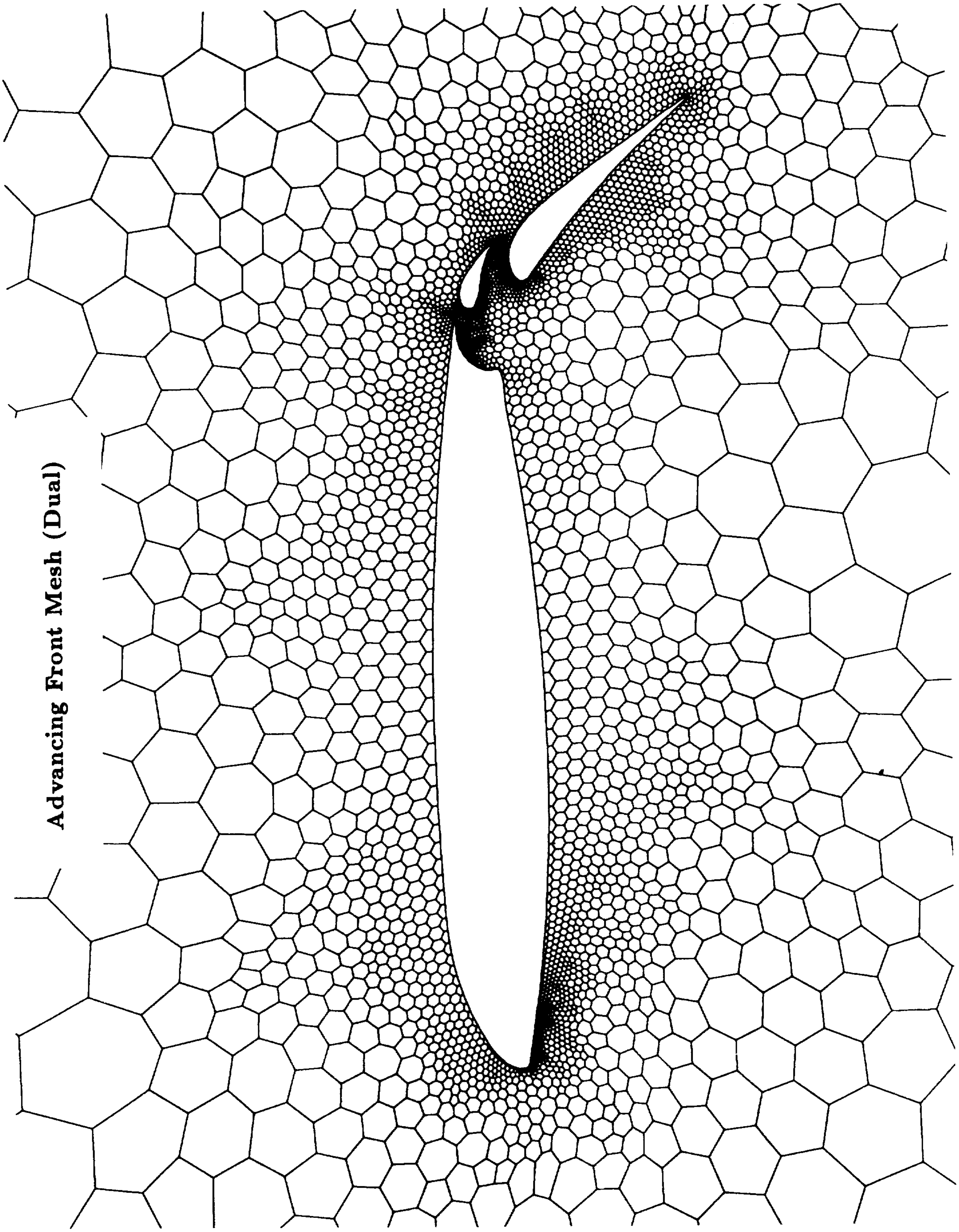
- Structured
    - Algebraic, Elliptic, Hyperbolic, Parabolic, ...
- Delaunay Triangulation
    - $O(n \log n)$
    - Maximize Minimum Angle
- Advancing Front
    - Probably $O(n \log n)$



"Random" Mesh (Delaunay Triangulation)



Advancing Front Mesh



Advancing Front Mesh (Dual)

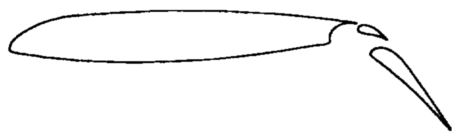


Fig. 2.1a) Three-component configuration.

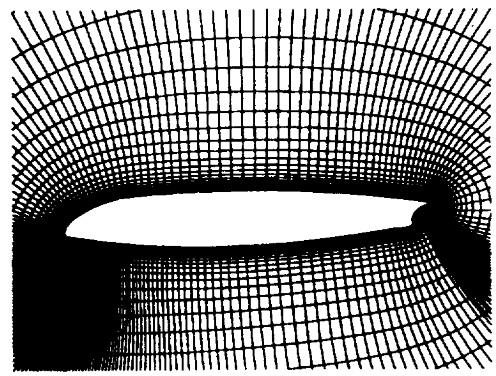


Fig. 2.1b) Grid about main element.

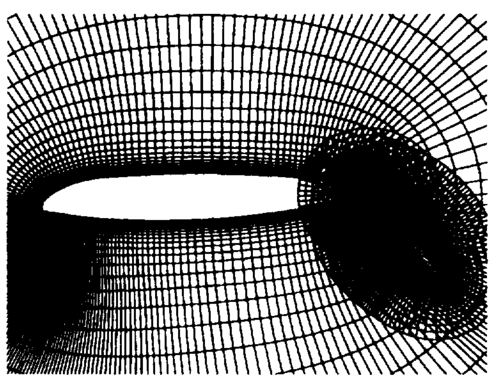


Fig. 2.1c) Grids plotted atop one another.

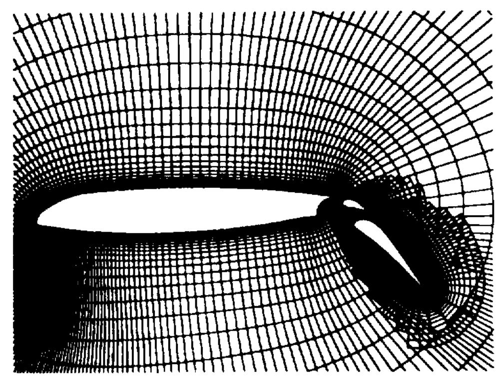


Fig. 2.1d) Grid after elimination of unwanted pts.

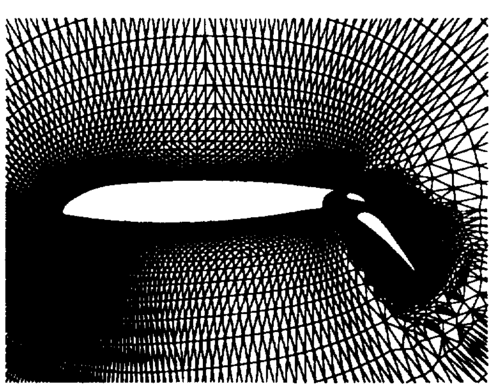


Fig. 2.1e) Grid after reconnection of points.

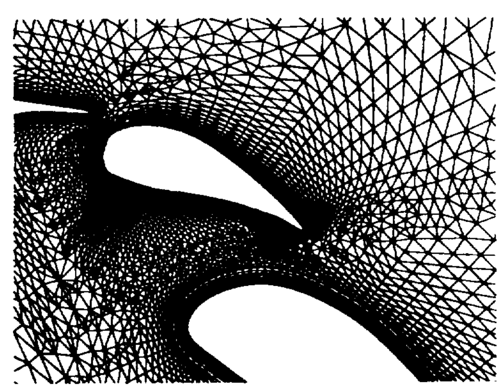


Fig. 2.1f) Grid after reconnection, detail.

Figure (2.1) Mesh generation synthesis of 3 element airfoil.

Finite-Volume Spatial Scheme

Euler equations in integral form:

$$\frac{\partial}{\partial t}\int_{\Omega} \mathbf{u}\, da + \oint_{\partial\Omega} \bar{\mathbf{f}}(\mathbf{n})\, dl = 0$$

where $\bar{\mathbf{f}}(\mathbf{n}) = n_x \mathbf{f} + n_y \mathbf{g}$

Numerical approximation about polygonal face:

$$\frac{\partial}{\partial t}\int_{\Omega_{f_j}} \mathbf{u}\, da = -\sum_{i=1}^{d(f_j)} \Delta l_i \, \mathbf{h}(\bar{\mathbf{u}}_i^+, \bar{\mathbf{u}}_i^-; \mathbf{n}_i), \quad j = 1, n(f)$$

Roe Flux Function:

$$\begin{aligned}\mathbf{h}(\mathbf{u}^+, \mathbf{u}^-; \mathbf{n}) =& \frac{1}{2}(\bar{\mathbf{f}}(\mathbf{u}^+; \mathbf{n}) + \bar{\mathbf{f}}(\mathbf{u}^-; \mathbf{n})) \\ & - \frac{1}{2}|A(\mathbf{u}^+, \mathbf{u}^-; \mathbf{n})|(\mathbf{u}^+ - \mathbf{u}^-)\end{aligned}$$

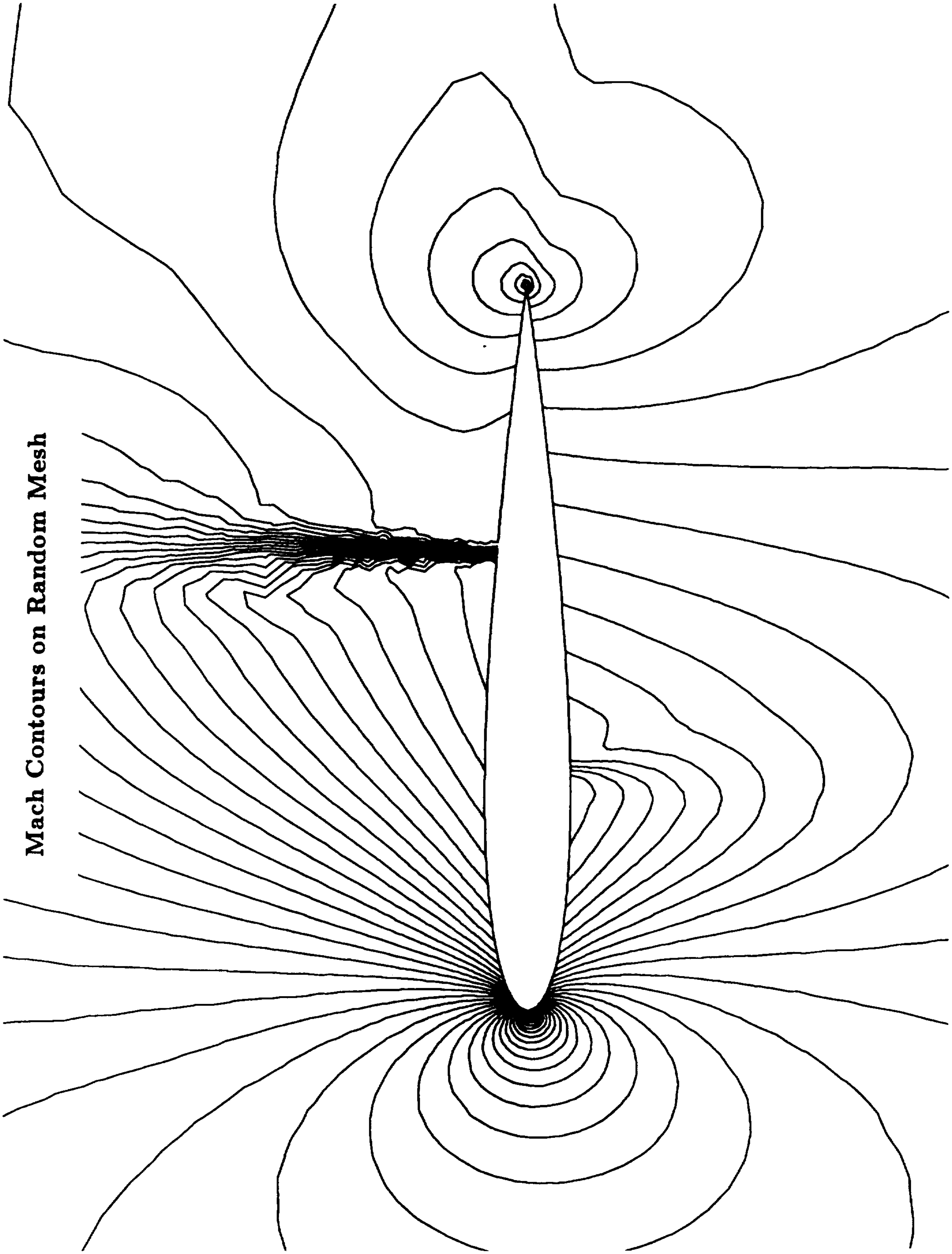
Reconstruction - Evolution

- Solution Reconstructions
  - Piecewise Constant
  - Piecewise Linear $\overline{u}(x,y) = u(x_0,y_0) + \Phi \nabla u \cdot \Delta \mathbf{r}$
    - Gradient Estimation in each face, $\nabla u = \frac{1}{a_\Omega} \int_{\partial\Omega} u\mathbf{n}\, d\,l$
    - Monotonicity Enforcement
      - Limiting (Multidimensional), $\Phi \in [0,1]$

- Solution Evolution
  - Three Stage Runge-Kutta Time-Stepping

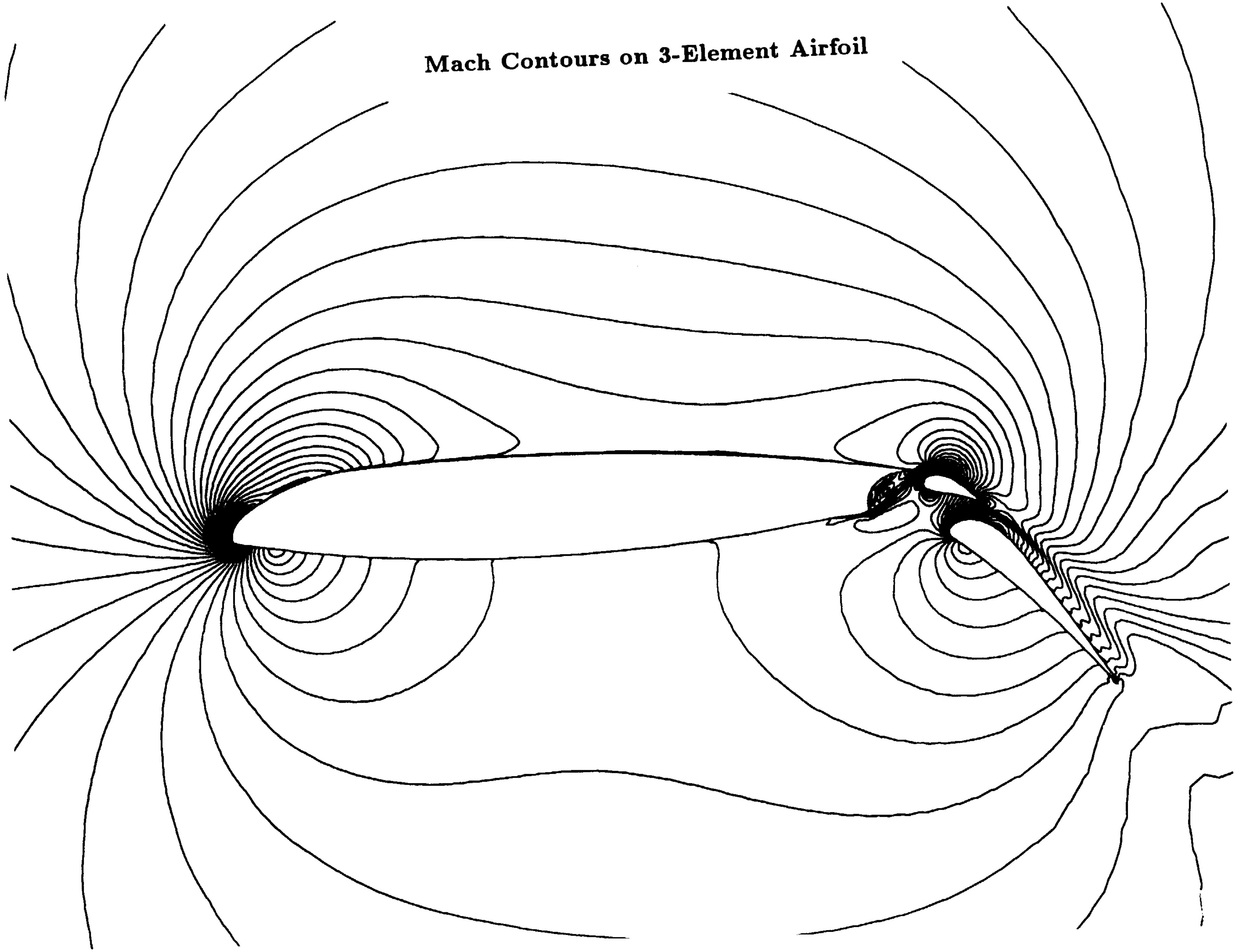
Numerical Examples

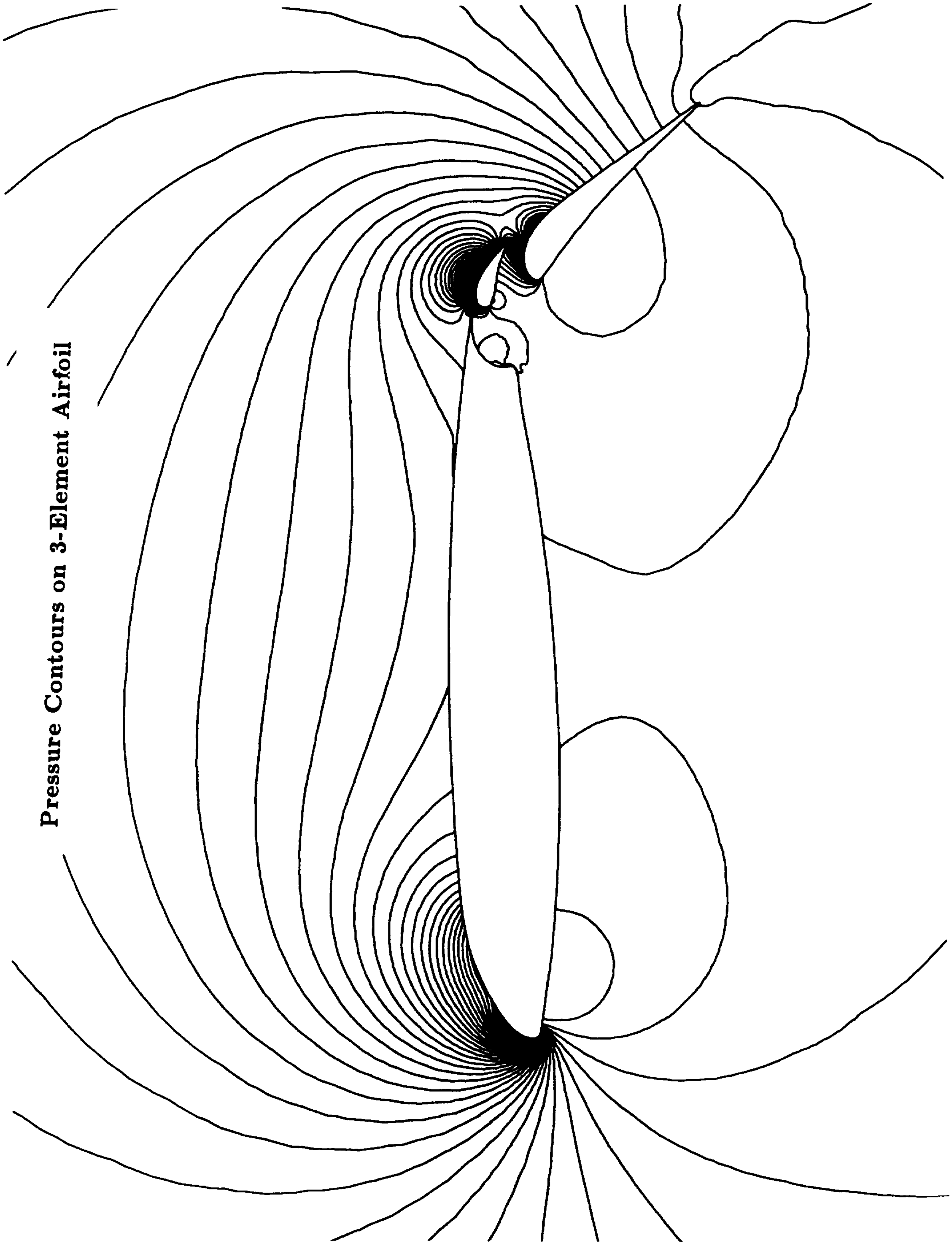
- NACA 0012 airfoil ($M_\infty = .8, \alpha = 1.25°$)
    - Subdivided Structured Mesh (193x33)
    - Irregular Mesh
- 3 Element Airfoil ($M_\infty = .2, \alpha = 0.0°$)



Mach Contours on Random Mesh

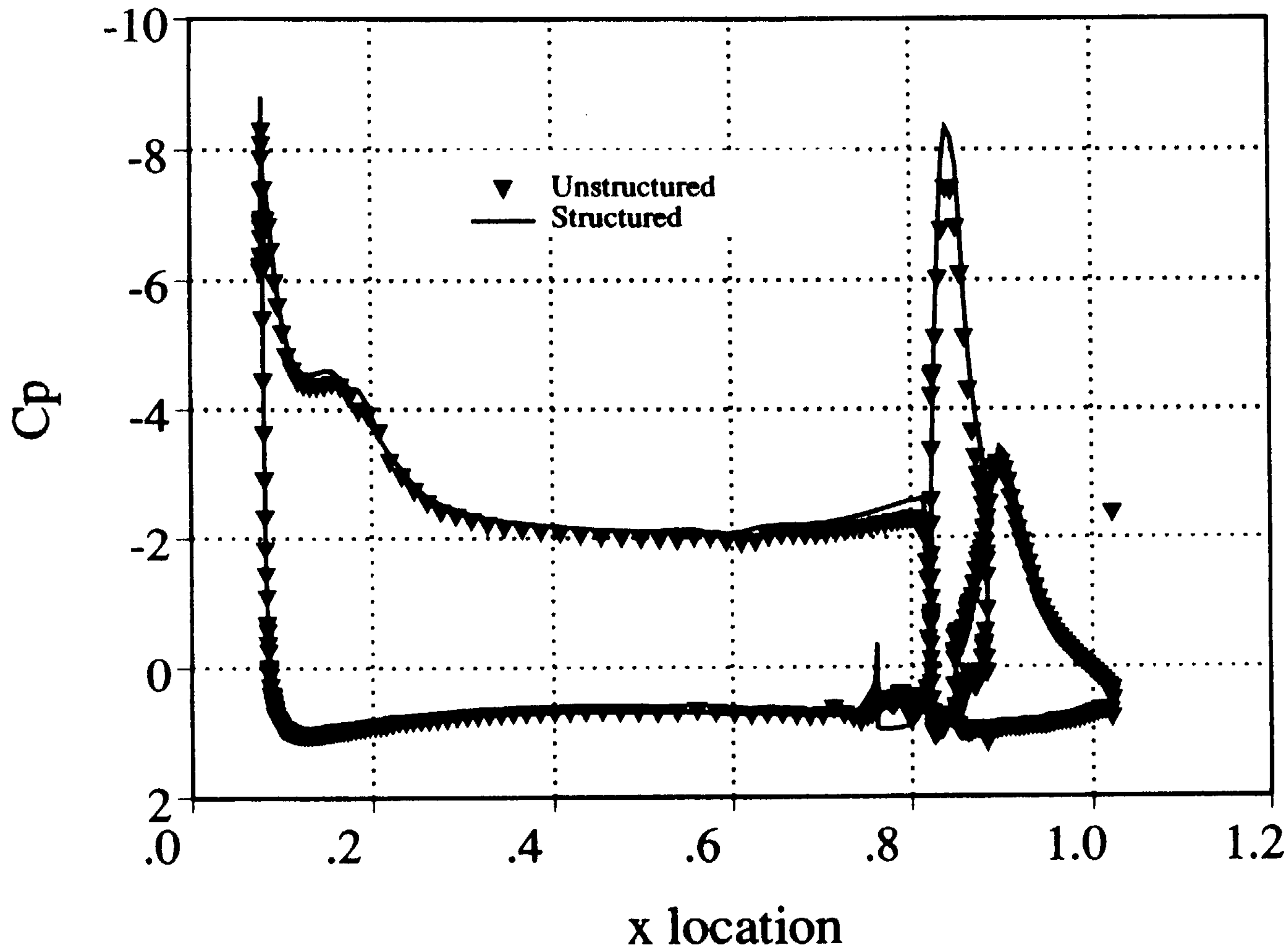
Mach Contours on 3-Element Airfoil





Pressure Contours on 3-Element Airfoil

3-Element Airfoil Cp Distribution



Current and Future Directions

- Quadratic Reconstructions
- Viscous Calculations with Turbulence Modelling
- Adaptive Mesh Refinement