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WAVE-INTERACTIONS IN SUPERSONIC AND HYPERSONIC FLOWS

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One of the most challenging problems in modern fluid mechanics is understanding the process of transition from laminar flow to turbulence. Not only is this a basic theoretical question, but a thorough understanding of the physical mechanisms by which instabilities of various types are initiated, interact, and grow is essential to the successful design of modern high–performance aerodynamic vehicles.

Transition is a highly complex process involving a number of stages and possible routes. In a typical scenario, the steady mean flow first becomes linearly unstable to small amplitude perturbations, e.g. two–dimensional Tollmien-Schlichting waves. As the amplitude of the T–S waves increase, the instability may itself begin to interact with other disturbances present in the flow field. Results of these wave–wave interactions are particularly dangerous in the instability context, and are the focus of the present investigation.

Work completed under the current grant comprises the start of a theoretical and computational attack on the subharmonic route to secondary instabilities in compressible flows. The total flow field in this problem is made up of the following components:

a) A steady streamwise mean boundary layer flow which depends only on the normal space component y,

b) A two–dimensional time dependent T–S wave which moves with wavespeed c and has no spanwise dependence, and

c) A fully three–dimensional, time dependent T–S wave whose streamwise wavenumber is half of the streamwise wavenumber associated with the two–dimensional T–S wave in b).

If a frame of reference is adopted which moves with the wavespeed c of the 2–D

T–S wave, the time dependence of this portion of the flow can be eliminated. The effective steady "mean" flow in this problem is now the sum of the original parallel steady mean flow and the initial 2–D T–S instability. Dependence on the streamwise coordinate x in this "mean" flow can be extracted by assuming normal mode expansions involving complex exponentials and the streamwise wavenumber α. However, it is important to note that, because this is a wave–wave interaction problem, unlike the usual linear instability case, both the complex exponential, and its complex conjugate, must be retained in descibing the 2–D T–S wave.

The role of the perturbation to the "steady mean flow" is now played by the 3–D time dependent T–S wave. In treating this wave, normal modes in the streamwise and spanwise directions and time may be used. Consistent with the subharmonic nature of this transition route, the streamwise wavenumber is $\alpha/2$, and complex conjugates of the complex exponential must be employed. This is not the case with the modes giving z and t dependence with wavespeed σ and spanwise wavenumber β as the effective "mean" flow quantities are independent of z and their time dependence is accounted for by the moving frame of reference. Consequently, the wave–wave interaction which will produce mean flow modification occurs through only through the streamwise exponentials.

Let u, v, and w denote velocity components in the x, y, and z directions, respectively, and let T denote temperature and ρ denote density. Then, if quantities associated with the original steady parallel mean boundary layer flow are unsubscripted while quantities associated with the two and three dimensional T–S waves of b) and c) above have the subscripts 2 and 3 respectively, the general form of the total flow fields is

$$\underline{u} = (u, v, w) = \overline{\underline{u}} + \underline{u}_2 + \underline{u}_3 \tag{1}$$

with

$$\begin{aligned} \overline{\underline{u}} &= (\overline{u}(y), 0, 0), \\ \underline{u}_2 &= (u_2(x, y, t),\ v_2(x, y, t),\ 0), \\ \underline{u}_3 &= (u_3(x, y, z, t),\ v_3(x, y, z, t),\ w_3(x, y, z, t)) \end{aligned} \tag{2}$$

and

$$\begin{aligned} \rho &= \overline{\rho}(y) + \rho_2(x, y, t) + \rho_3(x, y, z, t), \\ T &= \overline{T}(y) + T_2(x, y, t) + T_3(x, y, z, t). \end{aligned} \tag{3}$$

If q denotes either a velocity component, temperature, or density, then in the frame of reference moving with the wavespeed c, q_2 has the form

$$q_2 = q_2^+(y)\ E_{+1}(x)\ +\ q_2^-(y)\ E_{-1}(x) \tag{4}$$

so that $\frac{\partial q_2}{\partial t} = 0$. Also, q_3 is of the form

$$q_3 = \left[q_3^+(y)E_{+1/2}(x)\ +\ q_3^-(y)E_{-1/2}(x)\right] \exp\{i\beta z\ + \sigma t\}. \tag{5}$$

In both of the above expressions,

$$E_k(x) = \exp\{i\alpha k x\} \text{ and } E_{-k} = E_k^* \tag{6}$$

so thatE_{-k} is the complex conjugate of E_k.

Governing equations for the amplitude functions are obtained by substituting the total flow field into the compressible Navier–Stokes equations. Consistent with the concept of the effective steady mean flow, the equations are linearized so as to retain products of subscript 3 quantities with unsubscripted or subscript 2 quantities, while products of two or more subscript 3 quantities are neglected. Because of the normal mode expansions of the two and three dimensional T–S waves, the resulting equations for the 10 quantities q_3^+ and q_3^- are ordinary differential equations of total order 36 involving only derivatives with respect to y.

As might be expected, the governing equations in this secondary instability problem are long and highly complex. A set of notes containing both the full equations and details of their formal derivation has been communicated to the technical monitor for this project. Accordingly, for the sake of brevity, the equations themselves will not be duplicated here. However, it is useful to make some general comments about some aspects of their derivation.

The complexity in deriving the governing equations stems, in large part, from the need to include the complex conjugates of complex exponentials involving x so as to allow for wave–wave interactions. Formally, sets of equations associated with both E_+ and E_- must be considered separately. However, when considering products of flow quantities, both the portion of the product involving E_+ and the portion involving the complex conjugate E_- can be considered simultaneously through definition of a "pseudo–conjugation" operator which acts so as to change signs on both exponentials, amplitude functions, and constant multipliers which arise from the various partial derivatives. When combined with rules for commuting pseudo–conjugation and partial differentiation operators, a structure is created which simplifies derivation of the governing equations for the secondary instability and greatly reduces the possibility of undetected errors.

It would certainly be possible to use symbolic manipulation programs, such as MATHEMATICA, to derive complicated perturbation equations such as occur in the present work. However, it should also be noted that great potential for unsuspected errors exists if such programs are used as the sole basis of such derivations. "Hand" calculations of at least some equations are essential to validate automated derivations, and in this role the pseudo–conjugation operators and commuting rules should prove especially valuable in the future.

To obtain numerical solutions of the equations for the amplitude functions of the secondary instability, it was decided to make use of the nonlinear boundary value solver COLSYS. As written, COLSYS is a real–variable solver for finite intervals, and it does not determine eigenvalues. However, for application to earlier stability problems, the Principal Investigator has developed modifications to be placed in the driver for COLSYS which allow direct complex arithmetic. The use of asymptotic outer boundary condition allows generalization to boundary layer flows, and eigenvalues can be determined by appending a single scalar, but complex–valued, equation for the eigenvalue. The result of these modifications is a routine which has proven especially robust in the stability context. In the present context, the number of equations is such that it exceeds the maximum allowed by the standard version of COLSYS. This limitation was removed after discussions with COLSYS' author, Dr. Uri Ascher of the University of British Columbia, and driver routines appropriate to the present system of differential equations and boundary conditions were created and validated.

Initial tests of the developed code for this problem on the mainframe computers at NASA Langley Research Center indicated that the computational resources required by this code would require use of a supercomputer. Arrangements were then made for access to the Cray supercomputer at NASA Ames. Unfortunately, before the code could be ported to the Cray and made operational, the funding for this work expired.

The Principal Investigator for this project has now changed institutions and is currently a Professor of Applied Mathematics at the University of Vermont. Plans do exist to make the code for the subharmonic route to instability in compressible flows fully operational. When results of the calculations are reported in the professional literature, research funding received under the present NASA grant will be fully acknowledged.