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Symbols

A_P	mass number of projectile nucleus
A_T	mass number of target nucleus
B	slope parameter
$\mathbf{b}$	impact parameter vector
$C(\mathbf{q} + \mathbf{q}')$	correlation function
C_T	target form-factor parameter
$F^{(1)}$	projectile i-particle form factor
F_i	projectile double-scattering structure function
$\hat{f}$	nucleus-nucleus scattering operator
f_{NN}	nucleon-nucleon scattering amplitude
$G^{(1)}$	target i-particle form factor
G_i	target double-scattering structure function
$K(\mathbf{b}, \mathbf{b}')$	correlation term for single scattering
$\mathbf{k}$	initial wave vector of projectile
$\mathbf{k}_F$	final wave vector of projectile
k_{NN}	two-body relative wave number
ℓ_C	correlation length
$M_{\alpha j}$	Glauber first-order matrix element
$|n_P>$	projectile state vector
$|O_P>$	projectile initial state vector
$|O_T>$	target initial state vector
$\mathbf{q}$	momentum transfer vector
R_P	projectile matter radius
R_T	target matter radius
$\mathbf{r}$	internal nuclear coordinate vector
$\mathbf{s}$	projection of internal coordinate onto impact parameter plane
T_D	double inelastic scattering term
T_S	single inelastic scattering term
t	four-momentum transfer to projectile
α	projectile constituent index
Γ	nucleus-nucleus profile function
γ_T	target form-factor parameter
μ_T, ν_T	target states
ρ_P	projectile ground-state single-particle density
ρ_T	target ground-state single-particle density
$\rho^{(1)}$	target one-body density

$\rho^{(2)}$	target two-body density
σ	cross section
$\frac{d\sigma}{d\Omega}$	differential cross section in angle
Υ	complete inelastic eikonal phase
$\overline{\chi}$	eikonal phase matrix
χ	first-order elastic eikonal phase
$\widehat{\chi}$	first-order eikonal phase operator
Ω	inelastic eikonal phase
Ω_C	inelastic correlation eikonal phase

Subscripts and superscripts:

C	correlation
P	projectile
T	target

Barred quantities represent matrices.

Summary

Calculations of inclusive inelastic scattering distributions for heavy-ion collisions are considered within the high-energy optical model. Using ground-state sum rules, the inclusive projectile and complete projectile-target inelastic angular distributions are treated in both independent particle and correlated nuclear models. Comparisons between the models introduced are made for ^{4}He particles colliding with ^{4}He, ^{12}C, and ^{16}O targets and for protons colliding with ^{16}O targets. Results indicate that correlations contribute significantly, at small momentum transfers, to the inelastic sum. Correlation effects are hidden, however, when total scattering distributions are considered because of the dominance of elastic scattering at small momentum transfers.

Introduction

Research efforts that consider the effects of high-energy nuclei, such as cosmic rays, on physical systems require a large nuclear cross-section data base as input. The importance of the accuracy of this data base for space-radiation transport calculations has recently been discussed by Townsend and Wilson (refs. 1 and 2). The energy dependence and normalization of nuclear fragmentation parameters were shown to have a large effect on the prediction of particle differential flux and absorbed dose in tissue and aluminum shielding. An accurate fragmentation data base is therefore essential in order to assess the effects on astronauts from these radiations.

For high-energy, charged-particle transport, the straight ahead approximation is seen to be accurate (refs. 3 and 4), and the fragmentation inputs reduce to energy distributions for the scattered primaries and projectile/target fragments. The large number of nuclei and the extended energy range of importance in these transport studies require theoretical predictions of these distributions that are accurate and comprehensive. The high-energy optical model, as derived from Watson's multiple-scattering series (refs. 5 and 6) or the Glauber approximation (refs. 7 and 8), gives reliable predictions for both total and absorption cross sections (ref. 9) and includes the energy dependence explicitly in the optical potential. In this report, we extend that work by evaluating angular distributions for inelastically scattered primaries using the high-energy optical model. We also consider the effects of nuclear correlations on these distributions.

The Glauber model has been used to study inclusive scattering for "elementary" projectiles (refs. 8 and 10) and for heavy-ion scattering in the rigid-projectile approximation (refs. 11 and 12). Semiclassical cascade equations have also been derived with the Glauber formalism (refs. 13–15). We generalize the Glauber result for inclusive inelastic scattering in the independent particle model to the heavy-ion case. The eikonal-coupled channel (ECC) amplitude (refs. 5 and 6) can be considered a matrix representation of the Glauber amplitude (refs. 16 and 17) when correct or equivalent kinematics are assumed. Using the ECC, the inclusion of correlated nuclear basis functions for evaluation of inelastic sums is handled in a straightforward manner. Correlation effects will contribute to leading order in the inelastic sum rule and therefore may become important for small momentum transfers. We consider a nonperturbative approach for including the effects of two-particle correlations in the inclusive sum. Comparisons between the models introduced will be made for protons (p) and alphas (α) scattering on ^{4}He, ^{12}C, and ^{16}O targets at several energies.

Inclusive Inelastic Scattering

We consider nucleus-nucleus scattering at high energies for the case where an inclusive measurement of the projectile final state is made,

$$P + T \rightarrow P + X \tag{1}$$

with P and T denoting the projectile and target, respectively, and X denoting some final state of the target that is not measured. In equation (1) the projectile scatters elastically, and meson production is not considered. In the overall center-of-mass (CM) frame, with the projectile and target states denoted by $|n_P>$ and $|\nu_T>$, respectively, the angular distribution for equation (1) is found by summing (TOT) the nuclear-scattering operator over all final states of the target,

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{TOT} = \sum_{\nu_T} | < \nu_T O_P | \hat{f}(\mathbf{q}) | O_T O_P > |^2 \tag{2}$$

where $\hat{f}$ is the scattering operator and $\mathbf{q}$ is the momentum transfer to the projectile defined by

$$\mathbf{q} \equiv \mathbf{k} - \mathbf{k}_F \tag{3}$$

In equation (3) $\mathbf{k}$ and $\mathbf{k}_F$ are the initial and final projectile wave vectors, respectively. In equation (2) the phase space is approximated by a two-body phase space that is expected to be accurate at high energies. Equation (2) can be separated into elastic (EL) and inelastic (IN) contributions given by

$$\left. \frac{d\sigma}{d\Omega} \right)_{EL} = | < O_P O_T | \hat{f}(\mathbf{q}) | O_P O_T > |^2 \tag{4}$$

and

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{IN} = \sum_{\nu_T \neq 0} | < O_P \nu_T | \hat{f}(\mathbf{q}) | O_P O_T > |^2 \tag{5}$$

respectively. The summation in equation (5) includes all excited states, bound and continuum, of the target. This infinite summation can be reduced to a single matrix element through the use of closure on the target states:

$$\sum_{\nu_T \neq 0} |\nu_T > < \nu_T| = 1 - |O_T > < O_T| \tag{6}$$

Inserting equation (6) into equation (5), gives

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{IN} = \left. \frac{d\sigma^P}{d\Omega} \right)_{TOT} - \left. \frac{d\sigma}{d\Omega} \right)_{EL} \tag{7}$$

where

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{TOT} = < O_T | < O_P | \hat{f}(\mathbf{q}) | O_P > < O_P | \hat{f}^+(\mathbf{q}) | O_P > | O_T > \tag{8}$$

The great advantage of equation (7) over equation (5) is that only the ground-state wave function of the target is needed.

A second reaction that we consider is complete inelastic scattering

$$P + T \rightarrow X + Y \tag{9}$$

where the projectile and target are both left in excited states (denoted by PT). The angular distribution for equation (9) is given by

$$\left. \frac{d\sigma^{PT}}{d\Omega} \right)_{IN} = \sum_{\nu_T \neq 0} \sum_{n_P \neq 0} | < \nu_T n_P | \hat{f}(\mathbf{q}) | O_P O_T > |^2 \tag{10}$$

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which is written, using closure on both the target and projectile states, as

$$\frac{d\sigma^{PT}}{d\Omega}\Bigg)_{\text{IN}} = \frac{d\sigma}{d\Omega}\Bigg)_{\text{TOT}} + \frac{d\sigma}{d\Omega}\Bigg)_{\text{EL}} - \frac{d\sigma^{P}}{d\Omega}\Bigg)_{\text{TOT}} - \frac{d\sigma^{T}}{d\Omega}\Bigg)_{\text{TOT}} \tag{11}$$

where

$$\frac{d\sigma}{d\Omega}\Bigg)_{\text{TOT}} = <O_P O_T| \left(|\hat{f}(\mathbf{q})|^2\right)|O_P O_T> \tag{12}$$

Equation (11) may be written as

$$\frac{d\sigma^{PT}}{d\Omega}\Bigg)_{\text{IN}} = \frac{d\sigma}{d\Omega}\Bigg)_{\text{TOT}} - \frac{d\sigma}{d\Omega}\Bigg)_{\text{EL}} - \frac{d\sigma^{P}}{d\Omega}\Bigg)_{\text{IN}} - \frac{d\sigma^{T}}{d\Omega}\Bigg)_{\text{IN}} \tag{13}$$

The distributions considered above are evaluated when models for the nuclear-scattering operator and ground-state wave functions are introduced.

Glauber Independent Particle Model

The Glauber scattering operator (ref. 8) is defined in terms of the nucleus-nucleus profile function as

$$\hat{f}(\mathbf{q}) = \frac{ik}{2\pi} \int \exp(i\mathbf{q} - \mathbf{b})\,\widehat{\Gamma}(\mathbf{b})\,d^2b \tag{14}$$

where $\mathbf{b}$ is the impact parameter, and the profile function is given by

$$\widehat{\Gamma}(\mathbf{b}) = 1 - \prod_{\alpha j}^{A_P, A_T} \left[1 - \Gamma_{\alpha j}(\mathbf{b} - \mathbf{s}_\alpha - \mathbf{s}_j)\right] \tag{15}$$

where $\Gamma_{\alpha j}$ is the two-body profile function, α and j label the projectile and target constituents, respectively, and $\mathbf{s}$ is the projection of the internal nuclear coordinate onto the impact parameter plane.

In the independent particle model (IPM), the nuclear transition density is approximated by the product of single-particle densities,

$$\rho_{\nu_T O_T}(\mathbf{r}_1 \cdots \mathbf{r}_{A_T}) = \prod_{j=1}^{A_T} \rho_{\nu_T O_T}(\mathbf{r}_j) \tag{16}$$

with

$$\int \rho_{\nu_T O_T}(\mathbf{r})d\mathbf{r} = \delta_{\nu_T O_T} \tag{17}$$

The ground-state single-particle densities ρ_{00} of the projectile and target are denoted by ρ_P and ρ_T, respectively.

We now consider the evaluation of the distributions introduced in the preceding section using the Glauber IPM. For elastic scattering, we have from equation (14),

$$<O_P O_T|\hat{f}(\mathbf{q})|O_P O_T> = \frac{ik}{2\pi} \int d^2b \, <O_P O_T|\widehat{\Gamma}(\mathbf{b})|O_P O_T> \exp(i\mathbf{q}\cdot\mathbf{b}) \tag{18}$$

Then, we find from equation (4),

$$\frac{d\sigma}{d\Omega}\Bigg)_{\text{EL}} = \left|\frac{ik}{2\pi}\right|^2 \int \exp\left[i\mathbf{q}\cdot(\mathbf{b} - \mathbf{b}')\right]d^2b\,d^2b'\left\{1 - \prod_{\alpha j}\left[1 - M_{\alpha j}(\mathbf{b})\right]\right\}\left\{1 - \prod_{\alpha j}\left[1 - M_{\alpha j}^+(\mathbf{b}')\right]\right\} \tag{19}$$

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where

$$M_{\alpha j}(\mathbf{b}) = \int d\mathbf{r}_\alpha \, d\mathbf{r}_j \, \rho_P(\mathbf{r}_\alpha) \, \rho_T(\mathbf{r}_j) \, \Gamma_{\alpha j}(\mathbf{b} - \mathbf{s}_\alpha - \mathbf{s}_j) \tag{20}$$

For inelastic scattering, we insert equations (14) and (15) into equation (8) to find

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{\text{TOT}} = |\frac{ik}{2\pi}|^2 \int d^2b \, d^2b' \exp\left[i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}')\right] < O_T| < O_P|\widehat{\Gamma}(\mathbf{b})|O_P >$$

$$\times < O_P|\widehat{\Gamma}^+(\mathbf{b}')|O_P > |O_T > \tag{21}$$

After completing some algebraic steps, we find

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{\text{TOT}} = |\frac{ik}{2\pi}|^2 \int d^2b \, d^2b' \exp\left[i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}')\right] \left\{ 1 - \prod_{\alpha j} \left[1 - M_{\alpha j}(\mathbf{b})\right] - \prod_{\alpha j} \left[1 - M_{\alpha j}^+(\mathbf{b}')\right] \right.$$

$$\left. + \prod_{\alpha j} \left[1 - M_{\alpha j}(\mathbf{b}) - M_{\alpha j}^+(\mathbf{b}') + \Omega_{\alpha j}(\mathbf{b}, \mathbf{b}')\right] \right\} \tag{22}$$

where

$$\Omega_{\alpha j}(\mathbf{b}, \mathbf{b}') = \int d\mathbf{r}_j \, \rho_T(\mathbf{r}_j) \int d\mathbf{r}_\alpha \rho_P(\mathbf{r}_\alpha) \, \Gamma_{\alpha j}(\mathbf{b} - \mathbf{s}_\alpha - \mathbf{s}_j) \times \prod_{\alpha'} \int d\mathbf{r}'_{\alpha'} \rho_P^+(\mathbf{r}'_{\alpha'}) \, \Gamma_{\alpha' j}^+(\mathbf{b}' - \mathbf{s}_{\alpha'} - \mathbf{s}_j) \tag{23}$$

Using equations (19) and (22) in equation (7) gives the inclusive inelastic distribution as

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{\text{IN}} = \left|\frac{ik}{2\pi}\right|^2 \int d^2b \, d^2b' \exp\left[i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}')\right] \left\{ \prod_{\alpha j} \left[1 - M_{\alpha j}(\mathbf{b}) - M_{\alpha j}^+(\mathbf{b}') + \Omega_{\alpha j}(\mathbf{b}, \mathbf{b}')\right] \right.$$

$$\left. - \prod_{\alpha j} \left[1 - M_{\alpha j}(\mathbf{b})\right] \left[1 - M_{\alpha j}^+(\mathbf{b}')\right] \right\} \tag{24}$$

Thus, the form for the inclusive nucleus-nucleus distribution, as given by equation (24), is identical to the nucleon-nucleus case found in reference 8 with the exception of the form for $\Omega_{\alpha j}$ given by equation (23). Similar to the nucleon-nucleus case (ref. 8), equation (24) can be approximated in the large mass number and large momentum transfer limits by

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{\text{IN}} = \left|\frac{ik}{2\pi}\right|^2 \int d^2b \, d^2b' \exp\left[i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}')\right] \exp\left\{i\left[\chi(\mathbf{b}) - \chi^+(\mathbf{b}')\right]\right\} \left\{\exp\left[\Omega(\mathbf{b}, \mathbf{b}')\right] - 1\right\} \tag{25}$$

where

$$i\chi(\mathbf{b}) = -\sum_{\alpha j} M_{\alpha j}(\mathbf{b}) \tag{26}$$

and

$$\Omega(\mathbf{b}, \mathbf{b}') = \sum_{\alpha j} \Omega_{\alpha j}(\mathbf{b}, \mathbf{b}') \tag{27}$$

Next, we consider the evaluation of the total angular distribution (eq. (12)). Thus,

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$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{TOT}} = \left|\frac{ik}{2\pi}\right|^2 \int d^2b\, d^2b'\, \exp\left[i\mathbf{q}\cdot(\mathbf{b}-\mathbf{b}')\right] < O_P O_T\left|\left\{1 - \prod_{\alpha j}\left[1 - \Gamma_{\alpha j}\left(\mathbf{b}-\mathbf{s}_\alpha-\mathbf{s}_j\right)\right]\right\}\right.$$
$$\times \left\{1 - \prod_{\alpha j}\left[1 - \Gamma_{\alpha j}^+(\mathbf{b}'-\mathbf{s}_\alpha-\mathbf{s}_j)\right]\right\}|O_P O_T > \tag{28}$$

which reduces to

$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{TOT}} = \left|\frac{ik}{2\pi}\right|^2 \int d^2b\, d^2b'\, \exp\left[i\mathbf{q}\cdot(\mathbf{b}-\mathbf{b}')\right]\left\{1 - \prod_{\alpha j}\left[1 - M_{\alpha j}(\mathbf{b})\right] - \prod_{\alpha j}\left[1 - M_{\alpha j}^+(\mathbf{b}')\right]\right.$$
$$\left. + \prod_{\alpha j}\left[1 - M_{\alpha j}(\mathbf{b}) - M_{\alpha j}^+(\mathbf{b}') + \Upsilon_{\alpha j}(\mathbf{b},\mathbf{b}')\right]\right\} \tag{29}$$

where

$$\Upsilon_{\alpha j}(\mathbf{b},\mathbf{b}') \equiv \int d\mathbf{r}_\alpha\, d\mathbf{r}_j\, \rho_P(\mathbf{r}_\alpha)\rho_T\left(\mathbf{r}_j\right)\Gamma_{\alpha j}(\mathbf{b}-\mathbf{s}_\alpha-\mathbf{s}_j)\,\Gamma_{\alpha j}^+(\mathbf{b}'-\mathbf{s}_\alpha-\mathbf{s}_j) \tag{30}$$

Combining the first two terms on the right-hand side of equation (13) gives

$$\left.\frac{d\sigma^{PT}}{d\Omega}\right)_{\text{IN}} = \left.\frac{d\sigma}{d\Omega}\right)_{\text{IN}} - \left.\frac{d\sigma^P}{d\Omega}\right)_{\text{IN}} - \left.\frac{d\sigma^T}{d\Omega}\right)_{\text{IN}} \tag{31}$$

which, using equations (19) and (20), yields

$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{IN}} = \left|\frac{ik}{2\pi}\right|^2 \int d^2b\, d^2b'\, \exp\left[i\mathbf{q}\cdot(\mathbf{b}-\mathbf{b}')\right]\left\{\prod_{\alpha j}\left[1 - M_{\alpha j}(\mathbf{b}) - M_{\alpha j}^+(\mathbf{b}') + \Upsilon_{\alpha j}(\mathbf{b},\mathbf{b}')\right]\right.$$
$$\left. - \prod_{\alpha j}\left[1 - M_{\alpha j}(\mathbf{b})\right]\left[1 - M_{\alpha j}^+(\mathbf{b}')\right]\right\} \tag{32}$$

Equation (32) may be approximated by

$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{IN}} = \left|\frac{ik}{2\pi}\right|^2 \int d^2b\, d^2b'\, \exp\left[i\mathbf{q}\cdot(\mathbf{b}-\mathbf{b}')\right]\exp\left\{i\left[\chi(\mathbf{b})-\chi^+(\mathbf{b}')\right]\right\}\left\{\exp\left[\Upsilon(\mathbf{b},\mathbf{b}')\right]-1\right\} \tag{33}$$

We can introduce momentum space representations into equations (20), (23), and (30) to find

$$\chi(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{\text{NN}}} \int d^2q\, \exp\left(i\mathbf{q}\cdot\mathbf{b}\right)F^{(1)}(-\mathbf{q})\, G^{(1)}(\mathbf{q})f_{\text{NN}}(\mathbf{q}) \tag{34}$$

$$\Omega(\mathbf{b},\mathbf{b}') = \frac{A_P^2 A_T}{(2\pi k_{\text{NN}})^2} \int d^2q\, d^2q'\, \exp(i\mathbf{q}\cdot\mathbf{b})\, \exp(i\mathbf{q}'\cdot\mathbf{b}')$$
$$\times f_{\text{NN}}(\mathbf{q})\, f_{\text{NN}}^+(-\mathbf{q}')\, F^{(1)}(-\mathbf{q})F^{(1)}(-\mathbf{q}')G^{(1)}(\mathbf{q}+\mathbf{q}') \tag{35}$$

and

$$\Upsilon(\mathbf{b},\mathbf{b}') = \frac{A_P A_T}{(2\pi k_{\text{NN}})^2} \int d^2q\, d^2q'\, \exp(i\mathbf{q}\cdot\mathbf{b})\, \exp(i\mathbf{q}'\cdot\mathbf{b}')$$
$$\times f_{\text{NN}}(\mathbf{q})\, f_{\text{NN}}^+(-\mathbf{q}')\, F^{(1)}(\mathbf{q}+\mathbf{q}')\, G^{(1)}(\mathbf{q}+\mathbf{q}') \tag{36}$$

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where $F^{(1)}$ and $G^{(1)}$ are the projectile and target one-body form factors, respectively. Equations (34)–(36) may be more convenient for the evaluation of these phases than previous expressions.

Correlations and Inclusive Scattering

The effects of short-range dynamical correlations and Pauli blocking in the nuclear wave function will be most pronounced in the inelastic distribution at small and medium momentum transfers. In order to include these effects in the inelastic scattering distributions, we consider the ECC model. Assuming correct or equivalent kinematics, the ECC can be considered the matrix representation of the Glauber amplitude. In the ECC the matrix of scattering amplitudes for all possible projectile-target transitions is given by (ref. 17)

$$\bar{f}(\mathbf{q}) = \frac{ik}{2\pi} \int d^2b \ \exp(i\mathbf{q} \cdot \mathbf{b}) \ \{\exp[i\overline{\chi}(\mathbf{b})] - \bar{1}\} \tag{37}$$

where barred quantities represent matrices and the elements of $\overline{\chi}$ are written as

$$< m_P \mu_T \,|\widehat{\chi}(\mathbf{b})|\, n_P \nu_T > = \frac{1}{2\pi k_{\mathrm{NN}}} \sum_{\alpha j} \int d^2q \ \exp(i\mathbf{q} \cdot \mathbf{b}) F^{(1)}_{m_P n_P}(-\mathbf{q}) \ G^{(1)}_{\mu_T \nu_T}(\mathbf{q}) \ f_{\mathrm{NN}}(\mathbf{q}) \tag{38}$$

Assuming that the off-diagonal terms in $\overline{\chi}$ are small compared with the diagonal terms, we separate $\overline{\chi}$ into diagonal ($\overline{\chi}_D$) and off-diagonal ($\overline{\chi}_O$) terms as

$$\overline{\chi}(\mathbf{b}) = \overline{\chi}_D(\mathbf{b}) + \overline{\chi}_O(\mathbf{b}) \tag{39}$$

We further assume that the nuclear density in the excited states is approximately the same as the ground state, such that the elements of the diagonal matrix $\overline{\chi}_D$ are all taken as the elastic element,

$$\chi(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{\mathrm{NN}}} \int d^2q \ F^{(1)}(-\mathbf{q}) \ G^{(1)}(\mathbf{q}) \ f_{\mathrm{NN}}(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{b}) \tag{40}$$

To treat off-diagonal scattering, we expand $\bar{f}$ in powers of $\overline{\chi}_O$,

$$\bar{f}(\mathbf{q}) = \frac{-ik}{2\pi} \int d^2b \ \exp(i\mathbf{q} \cdot \mathbf{b}) \ \exp[i\overline{\chi}_D(\mathbf{b})] \sum_{m=1} \frac{[i\overline{\chi}_O(\mathbf{b})]^m}{m!} \tag{41}$$

The inclusive distribution for the projectile then follows as

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{\mathrm{IN}} = \left| \frac{ik}{2\pi} \right|^2 \int d^2b \, d^2b' \exp\left[i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}')\right] \ \exp\left\{ i \left[\chi(\mathbf{b}) - \chi^+(\mathbf{b}') \right] \right\}$$
$$\times \sum_{\nu_T \neq 0} [T_S(\mathbf{b}, \mathbf{b}') + T_D(\mathbf{b}, \mathbf{b}') + \ldots] \tag{42}$$

where the single inelastic scattering terms are

$$T_S(\mathbf{b}, \mathbf{b}') = \ < O_P O_T \,|\widehat{\chi}(\mathbf{b})|\, O_P \nu_T > < \nu_T O_P \left| \widehat{\chi}^+(\mathbf{b}') \right| O_P O_T > \tag{43}$$

and the double inelastic scattering terms are

$$T_D(\mathbf{b}, \mathbf{b}') = \frac{1}{4} \sum_{\mu_T \neq 0} \sum_{n_P = 0} < O_P O_T \,|\widehat{\chi}(\mathbf{b})|\, \mu_T n_P > < \mu_T n_P \,|\widehat{\chi}(\mathbf{b})|\, O_P \nu_T >$$
$$\times \sum_{\mu_T' \neq 0} \sum_{n_P' = 0} < \nu_T O_P \left| \widehat{\chi}^+(\mathbf{b}') \right| \mu_T' n_P' > < \mu_T' n_P' \left| \widehat{\chi}^+(\mathbf{b}') \right| O_P O_T > \tag{44}$$

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Each term in the inelastic scattering expansion of equation (42) can be reduced through the use of closure to terms involving matrix elements of one-, two-, ..., etc., body operators over the ground state and thus includes the effects of two or more particle correlations. The single inelastic scattering terms may be reduced to

$$\sum_{\nu_T \neq 0} T_S(\mathbf{b}, \mathbf{b}') = \; <O_T\,|< O_P|\,\widehat{\chi}(\mathbf{b})|O_P><O_P\left|\widehat{\chi}^+(\mathbf{b}')\right|O_P> |O_T>$$

$$- <O_P O_T|\widehat{\chi}(\mathbf{b})|O_P O_T><O_P O_T|\widehat{\chi}^+(\mathbf{b}')|O_P O_T> \qquad (45)$$

which becomes (ref. 16)

$$\sum_{\nu_T \neq 0} T_S(\mathbf{b}, \mathbf{b}') = \frac{A_P^2 A_T}{(2\pi k_{NN})^2} \int d^2q \; d^2q' \; \exp(i\mathbf{q}\cdot\mathbf{b}) \; \exp(i\mathbf{q}'\cdot\mathbf{b}')$$

$$\times f_{NN}(\mathbf{q}) \; f_{NN}^+(-\mathbf{q}') \; F^{(1)}(\mathbf{q}) \; F^{(1)}(-\mathbf{q}')$$

$$\times \left[G^{(1)}(\mathbf{q}+\mathbf{q}') + (A_T - 1) \; G^{(2)}(\mathbf{q},\mathbf{q}') - A_T \; G^{(1)}(\mathbf{q}) \; G^{(1)}(\mathbf{q}') \right] \qquad (46)$$

where $G^{(2)}$ is the two-particle form factor of the target. A common approximation (ref. 18) is to neglect the renormalization of the one-body density to be consistent with the two-body density such that

$$G^{(2)}(\mathbf{q},\mathbf{q}') \cong G^{(1)}(\mathbf{q}) \; G^{(1)}(\mathbf{q}') \left[1 - C(\mathbf{q}+\mathbf{q}')\right] \qquad (47)$$

where $C(\mathbf{q}+\mathbf{q}')$ is a correlation factor. We then find

$$\sum_{\nu_T \neq 0} T_S(\mathbf{b}, \mathbf{b}') = \Omega(\mathbf{b}, \mathbf{b}') - \frac{1}{A_T} \chi(\mathbf{b}) \; \chi^+(\mathbf{b}') + K(\mathbf{b}, \mathbf{b}') \qquad (48)$$

where Ω is defined in equation (35) and K represents the correlation term. The second and third terms on the right-hand side of equation (48) represent corrections to the model given by equation (25). We note that the second term in equation (48) persists even if the IPM is assumed for the nuclear wave function.

The higher-order terms in equation (42) quickly become intractable as we go past the single-scattering term. Particle production multiplicities are generally small (< 3) for light-to-medium nuclei, suggesting that a perturbative approach would be useful, especially with the reasonableness of the distorted wave Born approximation (DWBA). A more fruitful approach is to look for a summation of the higher-order terms in a simplified, although approximate, manner. In order to make such a summation, we consider the double-scattering terms in equation (42). From equation (44) we have

$$\sum_{\nu_T \neq 0} T_D(\mathbf{b}, \mathbf{b}') = \frac{1}{4} \sum_{\nu_T \neq 0} \left\{ \left[<O_P O_T\,|\widehat{\chi}(\mathbf{b})\;\widehat{\chi}(\mathbf{b})\,|\,O_P \nu_T > \right. \right.$$

$$\left. - <O_P O_T\,|\widehat{\chi}(\mathbf{b})|\,O_T><O_T\,|\widehat{\chi}(\mathbf{b})|\,O_P \nu_T > \right]$$

$$\times \left[<\nu_T O_P\left|\widehat{\chi}^+(\mathbf{b}')\;\widehat{\chi}^+(\mathbf{b}')\right|O_P O_T > \right.$$

$$\left. \left. - <\nu_T O_P|\widehat{\chi}^+(\mathbf{b}')|O_T><O_T\left|\widehat{\chi}^+(\mathbf{b}')\right|O_P O_T > \right] \right\} \qquad (49)$$

which becomes

$$\sum_{\nu_T \neq 0} T_D(\mathbf{b}, \mathbf{b}') = \frac{1}{4} < O_P O_T | \Bigg\{ \widehat{\chi}(\mathbf{b}) \ \widehat{\chi}(\mathbf{b}) | O_P > < O_P | \widehat{\chi}^+(\mathbf{b}') \ \widehat{\chi}^+(\mathbf{b}')$$

$$- \widehat{\chi}(\mathbf{b}) \ \widehat{\chi}(\mathbf{b}) | O_P O_T > < O_P O_T | \widehat{\chi}^+(\mathbf{b}') \ \widehat{\chi}^+(\mathbf{b}')$$

$$- \widehat{\chi}(\mathbf{b}) \ \widehat{\chi}(\mathbf{b}) | O_P > < O_P | \widehat{\chi}^+(\mathbf{b}') | O_T > < O_T | \widehat{\chi}^+(\mathbf{b}')$$

$$+ \widehat{\chi}(\mathbf{b}) \ \widehat{\chi}(\mathbf{b}) | O_P O_T > < O_P O_T | \widehat{\chi}^+(\mathbf{b}') | O_T > < O_T | \widehat{\chi}^+(\mathbf{b}')$$

$$- \widehat{\chi}(\mathbf{b}) | O_T > < O_T | \widehat{\chi}(\mathbf{b}) | O_P > < O_P | \widehat{\chi}^+(\mathbf{b}') \ \widehat{\chi}^+(\mathbf{b}')$$

$$+ \widehat{\chi}(\mathbf{b}) | O_T > < O_T | \widehat{\chi}(\mathbf{b}) | O_P O_T > < O_P O_T | \widehat{\chi}^+(\mathbf{b}') \ \widehat{\chi}^+(\mathbf{b}')$$

$$+ \widehat{\chi}(\mathbf{b}) | O_T > < O_T | \widehat{\chi}(\mathbf{b}) | O_P > < O_P | \widehat{\chi}^+(\mathbf{b}') | O_T > < O_T | \widehat{\chi}^+(\mathbf{b}')$$

$$- \widehat{\chi}(\mathbf{b}) | O_T > < O_T | \widehat{\chi}(\mathbf{b}) | O_P O_T > < O_P O_T | \widehat{\chi}^+(\mathbf{b}') | O_T >$$

$$< O_T | \widehat{\chi}^+(\mathbf{b}') \Bigg\} | O_P O_T > \tag{50}$$

To further reduce these terms, we write

$$\sum_{\nu_T \neq 0} T_D(\mathbf{b}, \mathbf{b}') = \frac{1}{4} \sum_{i=1}^{8} T_{D_i}(\mathbf{b}, \mathbf{b}') \tag{51}$$

and write the general term as

$$T_{D_i}(\mathbf{b}, \mathbf{b}') = \left(\frac{1}{2\pi k_{\mathrm{NN}}} \right)^4 \int d^2 q_1 \ d^2 q_2 \ d^2 q_3 \ d^2 q_4$$
$$\times \exp\left[i(\mathbf{q}_1 + \mathbf{q}_2) \cdot \mathbf{b} \right] \ \exp\left[i(\mathbf{q}_3 + \mathbf{q}_4) \cdot \mathbf{b}' \right]$$
$$\times f_{\mathrm{NN}}(\mathbf{q}_1) \ f_{\mathrm{NN}}(\mathbf{q}_2) \ f_{\mathrm{NN}}^+(\mathbf{q}_3) \ f_{\mathrm{NN}}^+(\mathbf{q}_4)$$
$$\times F_i(\mathbf{q}_1, \ \mathbf{q}_2, \ \mathbf{q}_3, \ \mathbf{q}_4) \ G_i(\mathbf{q}_1, \ \mathbf{q}_2, \ \mathbf{q}_3, \ \mathbf{q}_4) \tag{52}$$

where F_i and G_i are functions of the projectile and target internal structure, respectively. The projectile function F_i is identical in each term of equation (50) and is given by

$$F_i(\mathbf{q}_1, \ \mathbf{q}_2, \ \mathbf{q}_3, \mathbf{q}_4) = \sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} < O_P | \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{\alpha_1}) \ \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{\alpha_2}) | O_P >$$
$$\times < O_P | \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{\alpha_3}) \ \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{\alpha_4}) | O_P > \tag{53}$$

which is written

$$F_i(\mathbf{q}_1 \ldots \mathbf{q}_4) = \left[A_P \ F^{(1)}(\mathbf{q}_1 + \mathbf{q}_2) + A_P(A_P - 1) \ F^{(2)}(\mathbf{q}_1, \mathbf{q}_2) \right]$$
$$\times \left[A_P \ F^{(1)}(\mathbf{q}_3 + \mathbf{q}_4) + A_P(A_P - 1) \ F^{(2)}(\mathbf{q}_3, \mathbf{q}_4) \right] \tag{54}$$

We note that in the IPM we have

$$F_i(\mathbf{q}_1 \ldots \mathbf{q}_4) \cong \left[A_P \, F^{(1)}(\mathbf{q}_1 + \mathbf{q}_2) + A_P(A_P - 1) \, F^{(1)}(\mathbf{q}_1) \, F^{(1)}(\mathbf{q}_2) \right]$$
$$\times \left[A_P \, F^{(1)}(\mathbf{q}_3 + \mathbf{q}_4) + A_P(A_P - 1) \, F^{(1)}(\mathbf{q}_3) \, F^{(1)}(\mathbf{q}_4) \right] \tag{55}$$

If the coherent approximation is made for the elastically scattered projectile, the summation over all projectile states in equation (44) is not considered, and then we have

$$F_i(\mathbf{q}_1 \ldots \mathbf{q}_4) \cong A_P^4 \, F^{(1)}(\mathbf{q}_1) \, F^{(1)}(\mathbf{q}_2) \, F^{(1)}(\mathbf{q}_3) \, F^{(1)}(\mathbf{q}_4) \tag{56}$$

Next, we list the target structure terms:

$$G_1(\mathbf{q}_1 \ldots \mathbf{q}_4) = - \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1}) \, \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2}) \, \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3}) \, \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T >$$
$$\tag{57a}$$

$$G_2(\mathbf{q}_1 \ldots \mathbf{q}_4) = \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1}) \, \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2})|O_T >$$
$$\times < O_T| \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3}) \, \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T > \tag{57b}$$

$$G_3(\mathbf{q}_1 \ldots \mathbf{q}_4) = \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1}) \, \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2}) \, \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3})|O_T >$$
$$\times < O_T| \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T > \tag{57c}$$

$$G_4(\mathbf{q}_1 \ldots \mathbf{q}_4) = \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1}) \, \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2})|O_T >$$
$$\times < O_T| \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3})|O_T >< O_T| \, \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T > \tag{57d}$$

$$G_5(\mathbf{q}_1 \ldots \mathbf{q}_4) = \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1})|O_T >$$
$$\times < O_T| \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2}) \, \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3}) \, \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T > \tag{57e}$$

$$G_6(\mathbf{q}_1 \ldots \mathbf{q}_4) = \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1})|O_T >< O_T| \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2}) \, |O_T >$$
$$\times < O_T| \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3}) \, \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T > \tag{57f}$$

$$G_7(\mathbf{q}_1 \ldots \mathbf{q}_4) = \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1})|O_T >< O_T| \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2}) \, \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3}) \, |O_T >$$
$$\times < O_T| \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T > \tag{57g}$$

$$G_8(\mathbf{q}_1 \ldots \mathbf{q}_4) = \sum_{j_1 \cdots j_4} < O_T| \exp(i\mathbf{q}_1 \cdot \mathbf{s}_{j_1})|O_T >< O_T| \exp(i\mathbf{q}_2 \cdot \mathbf{s}_{j_2}) \, |O_T >$$
$$\times < O_T| \exp(i\mathbf{q}_3 \cdot \mathbf{s}_{j_3})|O_T >< O_T| \exp(i\mathbf{q}_4 \cdot \mathbf{s}_{j_4})|O_T > \tag{57h}$$

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which become

$$G_1(\mathbf{q}_1 \ldots \mathbf{q}_4) = -A_T \Big\{ (A_T - 1)(A_T - 2)(A_T - 3) \, G^{(4)}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4)$$

$$+ (A_T - 1)(A_T - 2) \Big[G^{(3)}(\mathbf{q}_1 + \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4) + \text{Permutations} \Big]$$

$$+ (A_T - 1) \Big[G^{(2)}(\mathbf{q}_1 + \mathbf{q}_2, \mathbf{q}_3 + \mathbf{q}_4) + \text{Permutations} \Big]$$

$$+ (A_T - 1) \Big[G^{(2)}(\mathbf{q}_1, \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4) + \text{Permutations} \Big]$$

$$+ G^{(1)}(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4) \Big\} \tag{58a}$$

$$G_2(\mathbf{q}_1 \ldots \mathbf{q}_4) = A_T^2 \Big\{ \Big[G^{(1)}(\mathbf{q}_1 + \mathbf{q}_2) + (A_T - 1) \, G^{(2)}(\mathbf{q}_1, \mathbf{q}_2) \Big]$$

$$\times \Big[G^{(1)}(\mathbf{q}_3 + \mathbf{q}_4) + (A_T - 1) \, G^{(2)}(\mathbf{q}_3, \mathbf{q}_4) \Big] \Big\} \tag{58b}$$

$$G_3(\mathbf{q}_1 \ldots \mathbf{q}_4) = A_T^2 \, G^{(1)}(\mathbf{q}_4) \Big\{ (A_T - 1) \, (A_T - 2) \, G^{(3)}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$$

$$+ (A_T - 1) \Big[G^{(2)}(\mathbf{q}_1 + \mathbf{q}_2, \mathbf{q}_3) + \text{Permutations} \Big]$$

$$\times G^{(1)}(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3) \Big\} \tag{58c}$$

$$G_4(\mathbf{q}_1 \ldots \mathbf{q}_4) = -A_T^3 \, G^{(1)}(\mathbf{q}_3) \, G^{(1)}(\mathbf{q}_4) \Big[G^{(1)}(\mathbf{q}_1 + \mathbf{q}_2) + (A_T - 1) \, G^{(2)}(\mathbf{q}_1, \mathbf{q}_2) \Big] \tag{58d}$$

$$G_5(\mathbf{q}_1 \ldots \mathbf{q}_4) = G_3(\mathbf{q}_4, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_1) \tag{58e}$$

$$G_6(\mathbf{q}_1 \ldots \mathbf{q}_4) = G_4(\mathbf{q}_3, \mathbf{q}_4, \mathbf{q}_1, \mathbf{q}_2) \tag{58f}$$

$$G_7(\mathbf{q}_1 \ldots \mathbf{q}_4) = G_4(\mathbf{q}_3, \mathbf{q}_2, \mathbf{q}_1, \mathbf{q}_4) \tag{58g}$$

$$G_8(\mathbf{q}_1 \ldots \mathbf{q}_4) = A_T^4 \, G^{(1)}(\mathbf{q}_1) \, G^{(1)}(\mathbf{q}_2) \, G^{(1)}(\mathbf{q}_3) \, G^{(1)}(\mathbf{q}_4) \tag{58h}$$

where $G^{(3)}$ and $G^{(4)}$ are the target three- and four-particle form factors, respectively.

The reduction of the single inelastic scattering terms as given by equation (46) contains the two-particle form factor, whereas the double inelastic terms in equations (57) and (58) display the two-, three-, and four-particle form factors. Thus, correlation effects may lead to two-particle knockout for a single inelastic scattering on a target nucleon, and to three- or four-particle knockout for double inelastic scattering.

Upon identification of the double-scattering terms, and assuming the coherent approximation for the projectile, the approximation of equation (25) is seen to contain only a single term in equations (58). A model that retains the dominant contributions of two-particle correlations in the double and higher terms, while assuming the coherent approximation for the projectile, is to assume

$$\left. \frac{d\sigma^P}{d\Omega} \right)_{\text{IN}} = \left| \frac{ik}{2\pi} \right|^2 \int d\mathbf{b} \, d\mathbf{b}' \, \exp\left[i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}') \right] \, \exp\left\{ i \Big[\chi(\mathbf{b}) - \chi^+(\mathbf{b}') \Big] \right\}$$

$$\times \left(\exp\left[\Omega_C(\mathbf{b}, \mathbf{b}') \right] - 1 \right) \tag{59}$$

with

$$\Omega_C(\mathbf{b}, \mathbf{b}') = \sum_{\nu_T \neq 0} T_S(\mathbf{b}, \mathbf{b}') \tag{60}$$

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Upon comparison we find, within the coherent approximation,

$$\frac{1}{2}\Omega_C^2(\mathbf{b}, \mathbf{b}') = \frac{1}{2}\left(T_{D_2} + T_{D_4} + T_{D_6} + T_{D_8}\right) \tag{61}$$

The approximation of equation (59) will thus treat the double and higher inelastic scattering terms in an approximate manner but should be accurate if the inelastic scattering series converges quickly. A similar analysis for the complete inelastic distribution as defined in equation (13) could now be made using the coupled-channels approach, but it will be addressed in future work.

Model Calculations

We now consider the evaluation of the inelastic distributions discussed above. Ignoring spin effects, we use an isospin-averaged, two-body amplitude given by

$$f_{\mathrm{NN}}(\mathbf{q}) = \frac{\sigma(\alpha + i)k_{\mathrm{NN}}}{4\pi} \exp(-Bq^2/2) \tag{62}$$

where the energy-dependent parameters σ, B, and α are listed in table I.

Table I. Parameters for Nucleon-Nucleon Scattering Amplitude

Reaction	σ, fm^2	B, fm^2	α
$\alpha - \alpha$ at 642A MeV	3.93	0.13	-0.39
$\alpha-{}^{12}\mathrm{C}$ at 3.64A GeV	4.2	.28	$-.43$
$p-{}^{16}\mathrm{O}$ at 1A GeV	4.3	.26	$-.22$
$\alpha - A_T$ at 1A GeV	4.3	.26	$-.23$

For the projectile, we use a one-body form factor

$$F^{(1)}(\mathbf{q}) = \exp(-R_P^2 q^2/4) \tag{63}$$

where R_P is the matter radius of the projectile. For the target one-body form factor, we use the harmonic-well form (ref. 19)

$$G^{(1)}(\mathbf{q}) = (1 - C_T q^2) \exp(-R_T^2 q^2/4) \tag{64}$$

where R_T is the matter radius of the target and

$$C_T = \frac{\gamma_T R_T^2}{4(1 + \frac{3}{2}\gamma_T)} \tag{65}$$

with values of γ_T from reference 19.

Correlation effects are included in the two-particle density through the approximation (ref. 18)

$$\rho^{(2)}(\mathbf{x}, \mathbf{y}) \approx \rho^{(1)}(\mathbf{x})\, \rho^{(1)}(\mathbf{y}) \left[1 - \exp\left[-(\mathbf{x} - \mathbf{y})^2/2\ell_C^2\right]\right] \tag{66}$$

where ℓ_C, an effective correlation length, is equal to 0.7 fm. With the inputs of equations (62)–(66), the functions $\chi(\mathbf{b})$, $\Omega_C(\mathbf{b}, \mathbf{b}')$, and $\Upsilon(\mathbf{b}, \mathbf{b}')$ are evaluated in analytic form. For comparison with experimental results, the inclusive invariant distribution is written as

$$\left.\frac{d\sigma^P}{dt}\right)_{\mathrm{IN}} \cong \frac{\pi}{k^2} \left.\frac{d\sigma^P}{d\Omega}\right)_{\mathrm{IN}} \tag{67}$$

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with

$$t \cong -q^2 \tag{68}$$

In figure 1 we show comparisons with experimental results from reference 11 for α–α scattering at 642A MeV. The solid line denotes the correlation model of equation (59), the dashed line denotes the IPM of equation (24), and the dotted line denotes the IPM of equation (25). The correlation model produces good agreement over the region of momentum transfers studied. The IPM results overestimate the data, thus indicating the importance of correlation effects in this reaction. Inclusion of more detailed two-body densities in the calculation should improve the predictions.

In figure 2 we show results for inclusive inelastic α–^{12}C scattering at 3.64A GeV. The correlation model shows a slight decrease in magnitude in comparison with the IPM results. Also apparent is a slight dip at small values of t. The increase in elastic coupling for increasing target or projectile mass number should mask correlation effects, a result that is seen to be true upon comparison with the data in figures 1 and 2. In figure 2 the approximation of equation (25) is seen to adequately represent the more exact IPM results of equation (24). Experimental results (ref. 20) for total inclusive α–^{12}C scattering at 3.64A GeV are shown (see data points) in figure 3. The dashed line denotes the inelastic results of equation (25); the dotted line, the elastic contribution calculated using the coherent approximation described in reference 17; and the solid line, the sum of inelastic and elastic contributions. Agreement with the data is fair, where calculations underestimate the data at larger values of t. Correlation effects in elastic scattering have been shown to increase the cross section in this region by a substantial amount (refs. 16 and 17) so that use of a second-order elastic scattering model should lead to improved agreement. The dominance of elastic scattering at small values of t, as seen in figure 3, indicates that the model of equation (29) is sufficient when total scattering distributions are considered.

In figures 4 and 5 we show results for inclusive inelastic p–^{16}O scattering at 1A GeV. In figure 4 the correlation model is seen to produce a substantial decrease in cross section in comparison with the IPM results. A much smaller decrease is seen for α–^{16}O results in figure 5.

In figures 6 and 7 we show results for complete inelastic scattering in α–α and α–^{12}C collisions at 1A GeV. The solid line denotes the exact IPM of equation (32); and the dashed line, the approximation of equation (33). This approximate form is noted to be inadequate at small momentum transfers. The complete inelastic distribution may provide a useful study of correlation effects at small values of q for identical projectile-target systems where the ground-state, two-particle form factor will play a dominant role in the leading-order terms.

Finally, we note that the results of this paper require large computational times (2–3 hours per figure on a MICRO-VAX TS 05) because of the four-dimensional integrations over the two required impact-parameter points. Computational times will be substantially reduced when equation (29) is accurate. Here, an approximate reduction of the two-point impact-parameter integration may be used.

Concluding Remarks

The inclusive inelastic scattering of heavy ions is discussed in terms of the high-energy optical model using ground-state sum rules. The multiple-scattering structure of these reactions is developed in terms of uncorrelated and correlated nuclear wave functions. Comparisons between the models are made and experimental data are introduced using simple models for ground-state, one-, and two-body densities. Results indicate that correlation effects may be important only for proton projectiles and light-ion scattering. Improved physical inputs and numerical evaluation techniques are necessary for further applications. Approximate laboratory energy spectra for inelastically scattered projectiles can be derived from the invariant momentum transfer distributions discussed.

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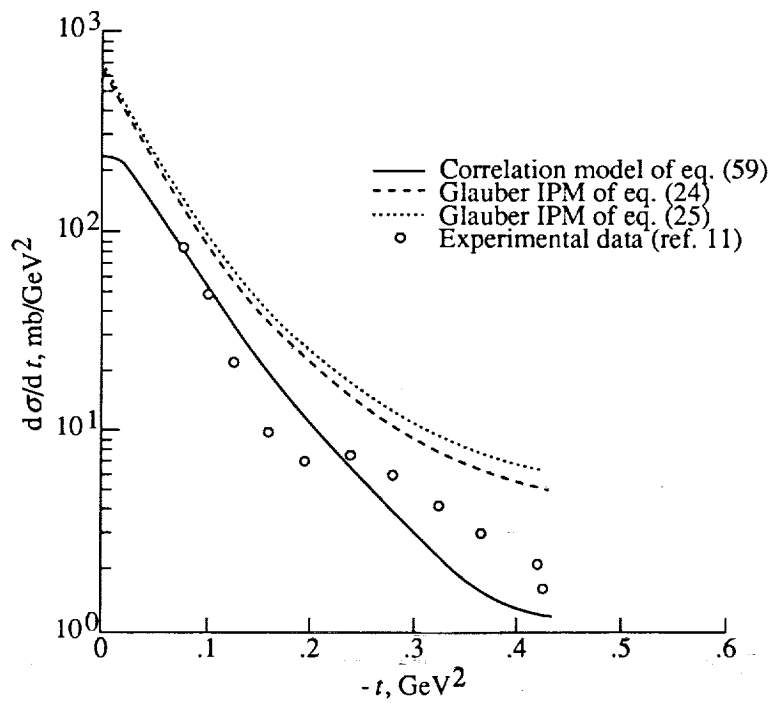
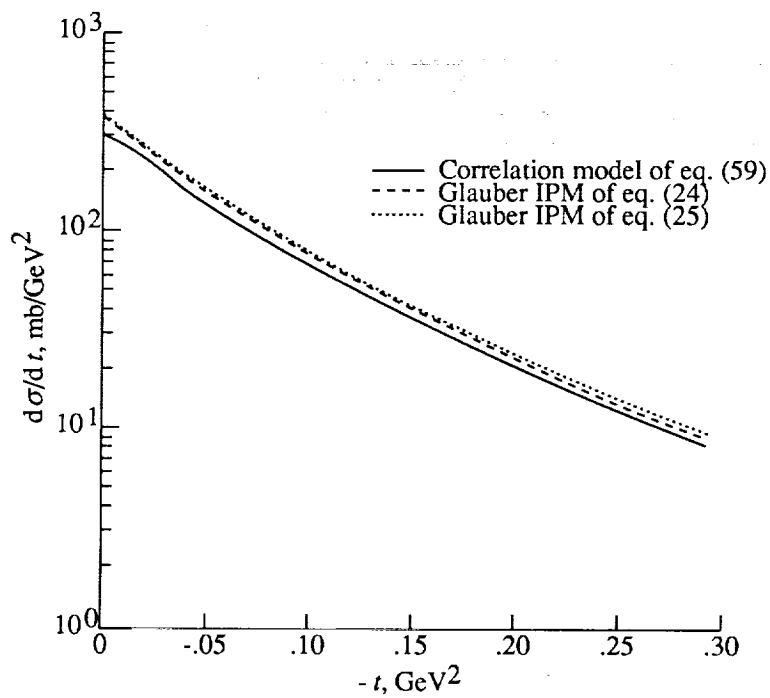Figure 1. Inclusive inelastic α–α scattering at 642A MeV.

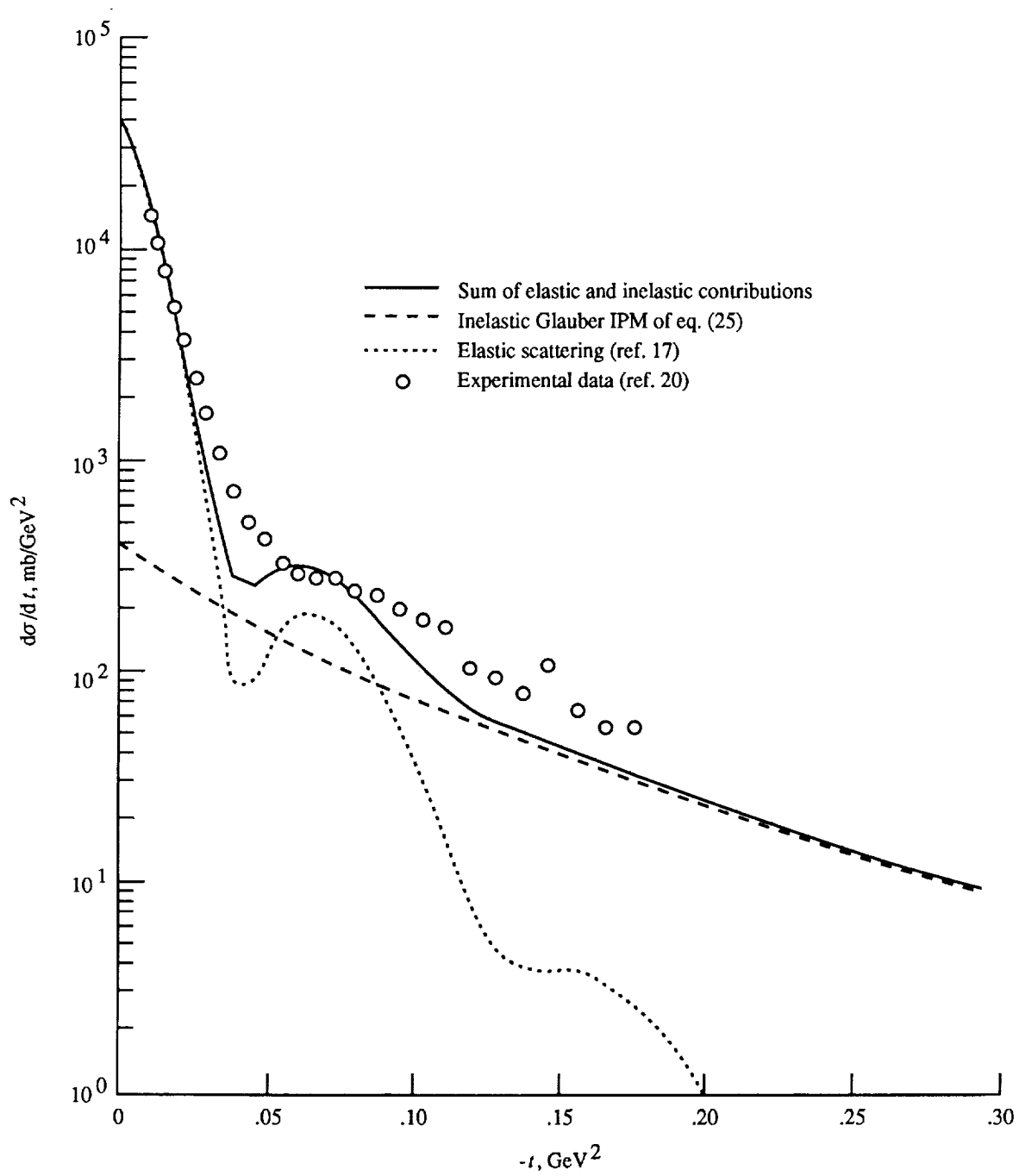Figure 2. Inclusive inelastic α–^{12}C scattering at 3.64A GeV.

Figure 3. Total inclusive α-^{12}C scattering at 3.64A GeV.

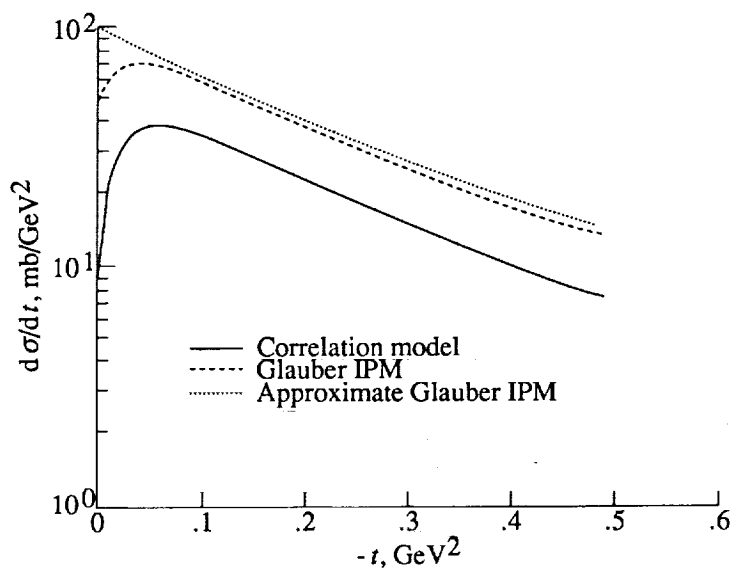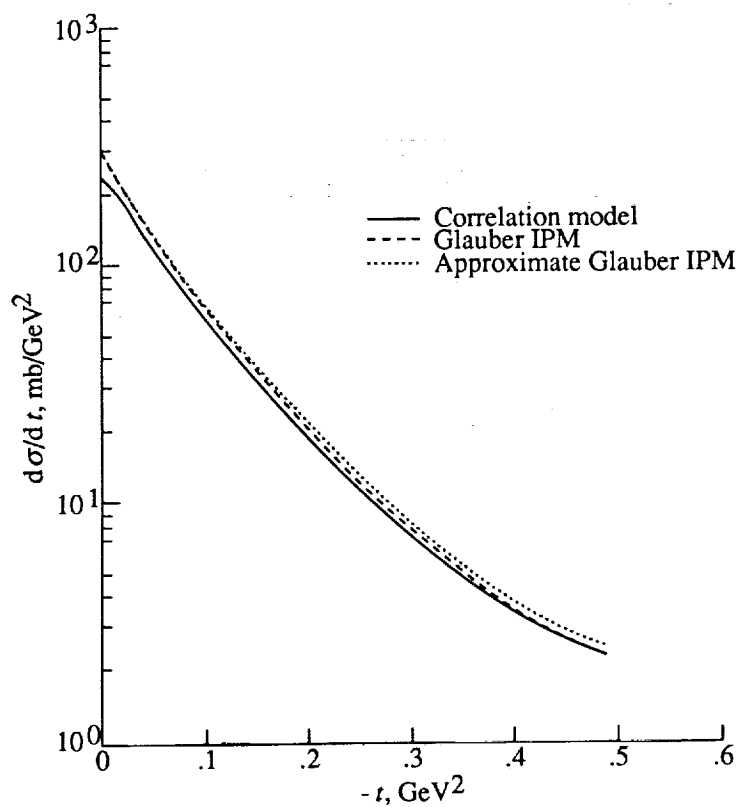Figure 4. Inclusive inelastic p–^{16}O scattering at 1 GeV.

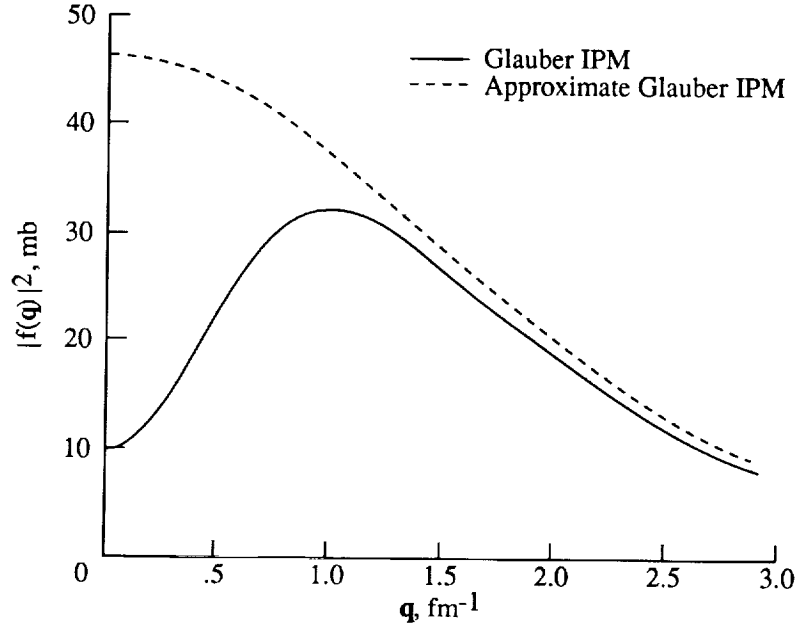Figure 5. Inclusive inelastic α–^{16}O scattering at 1A GeV.

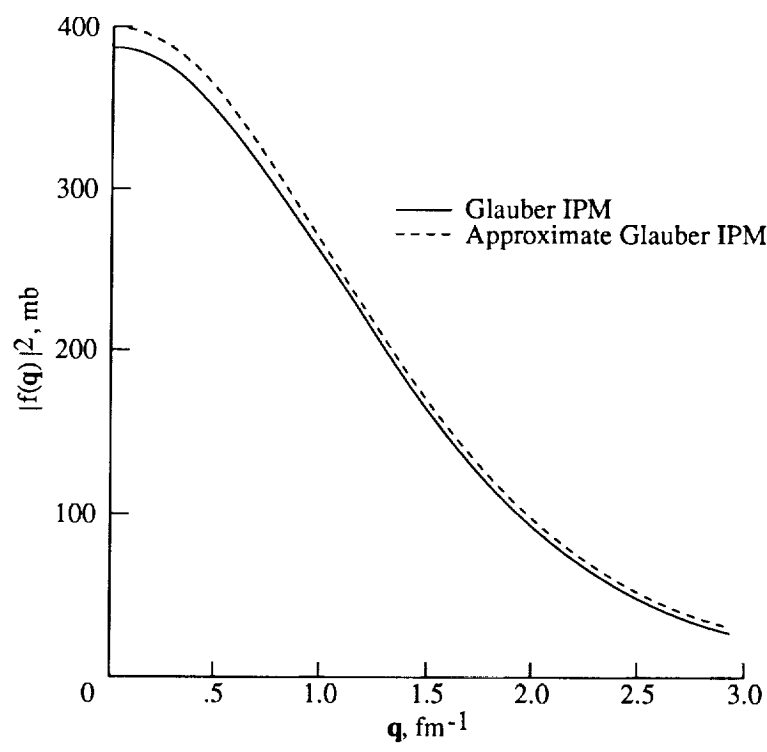Figure 6. Complete inelastic α-α scattering at 1A GeV.

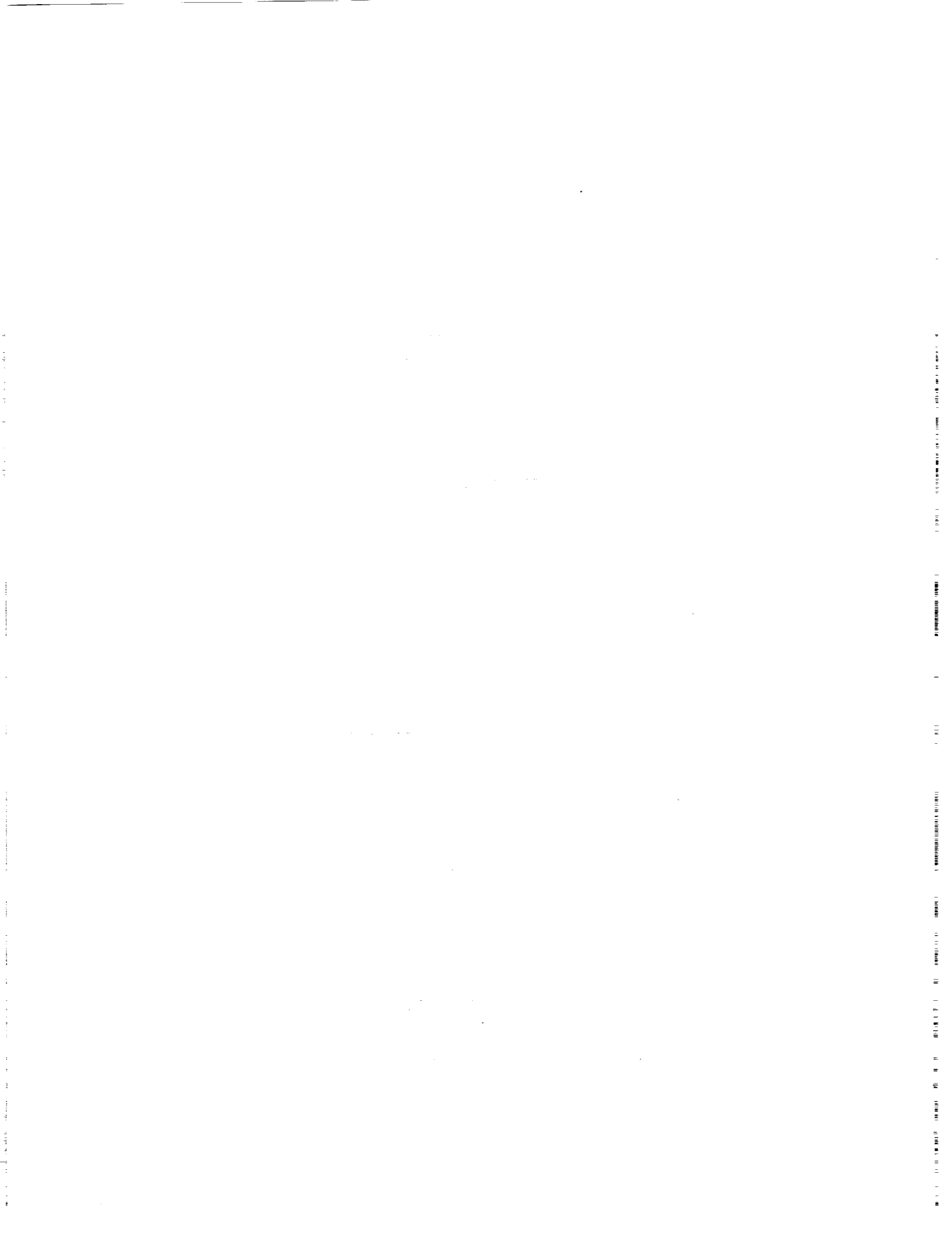Figure 7. Complete inelastic α ^{12}C scattering at 1A GeV.

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16. Abstract

Calculations of inclusive inelastic scattering distributions for heavy-ion collisions are considered within the high-energy optical model. Using ground-state sum rules, the inclusive projectile and complete projectile-target inelastic angular distributions are treated in both independent particle and correlated nuclear models. Comparisons between the models introduced are made for ^{4}He particles colliding with ^{4}He, ^{12}C, and ^{16}O targets and for protons colliding with ^{16}O targets. Results indicate that correlations contribute significantly, at small momentum transfers, to the inelastic sum. Correlation effects are hidden, however, when total scattering distributions are considered because of the dominance of elastic scattering at small momentum transfers.

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