N91-14997

SIZE DISTRIBUTION OF DUST GRAINS - A PROBLEM OF SELF-SIMILARITY?

Th. Henning, J. Dorschner, and J. Gürtler
Universitäts-Sternwarte, Jena, DDR-6900, G.D.R.

Distribution functions describing the results of natural processes frequently show the shape of power laws, e.g. mass functions of stars and molecular clouds, velocity spectrum of turbulence, size distributions of asteroids, micrometeorites and also interstellar dust grains. It is an open question whether this behaviour is a result simply coming about by the chosen mathematical representation of the observational data or reflects a deep-seated principle of nature. We suppose the latter being the case.

Using a dust model consisting of silicate and graphite grains Mathis et al. (1977) showed that the interstellar extinction curve can be represented by taking a grain radii distribution of power law type

$$n(a) \propto a^{-p} \quad \text{with } 3.3 \leq p \leq 3.6 \qquad (1)$$

as a basis. Biermann and Harwit (1980) explained this finding by postulating grain-grain collisions in the extended atmospheres of those late-type stars where the mentioned grains originated. Size distribution functions of the above shape (1) with p within the mentioned range have been obtained as special solutions of a non-linear integro-differential equation describing fragmentation processes in a closed system of colliding particles (cf. Dorschner, 1981).

A totally different approach to understanding power laws like that in (1) becomes possible by the theory of self-similar processes (scale invariance). The β model of turbulence (Frisch et al., 1978) leads in an elementary way to the concept of the self-similarity dimension D, a special case of Mandelbrot's (1977) "fractal dimension". In the frame of this β model it is supposed that on each stage of a cascade the system decays to N clumps and that only the portion βN remains "active" further on. An important feature of this model is that the "active" eddies become less and less space-filling.

In the following we assume that grain-grain collisions are such a scale-invariant process and that the remaining grains are the inactive ("frozen") clumps of the cascade. In this way, a size distribution

$$n(a)\, da \propto a^{-(D+1)}\, da \qquad (2)$$

results. An analogous relation was obtained by Ferrini et al. (1982), who discussed the problem of molecular cloud fragmentation in this way. Mandelbrot (1977) gave several strong arguments in favour of a value of D ranging between 2 and 3 for such processes.

Assuming D=2.5 the power law with p=3.5 found by Mathis et al. (1977) is obtained. Although the exact value of D is a matter of further discussions it seems to be highly probable that the power law character of the size distribution of interstellar dust grains is the result of a self-similarity process. We can, however, not exclude that the process leading to the interstellar grain size distribution is not fragmentation at all. It could be, e.g., diffusion-limited growth discussed by Sander (1986), who applied the theory of fractal geometry to the classification of non-equilibrium growth processes. He received D=2.4 for diffusion-limited aggregation in 3d-space.

References

Dorschner, J.: 1982, Astrophys. Space Sci. 81, 323.

Biermann, P. and Harwit, M.: 1980 Astrophys. J. 241, L105.

Ferrini, F., Marchesoni, F., and Vulpiani, A.: 1982, Physics Lett. 92A, 47.

Frisch, U., Sulem, P.-L., and Nelkin, M.: 1978, J. Fluid Mech. 87, 719.

Mandelbrot, B.B.: 1977, Fractals: Form, Chance, and Dimension. W.H. Freeman and Co., San Francisco.

Mathis, J.S., Rumpl, W., and Nordsieck, K.H.: 1977, Astrophys. J. 217, 425.

Sander, L.M.: 1986, Nature 322, 789.