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MODELLING THE TRANSITIONAL BOUNDARY LAYER

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ABSTRACT

Recent developments in the modelling of the transition zone in the boundary layer are reviewed (the zone being defined as extending from the station where intermittency begins to depart from zero to that where it is nearly unity). The value of using a new non-dimensional spot formation rate parameter, and the importance of allowing for so-called subtransitions within the transition zone, are both stressed. Models do reasonably well in constant pressure 2-dimensional flows, but in the presence of strong pressure gradients further improvements are needed. The linear combination approach works surprisingly well in most cases, but would not be so successful in situations where a purely laminar boundary layer would separate but a transitional one would not. Intermittency-weighted eddy viscosity methods do not predict peak surface parameters well without the introduction of an overshooting transition function whose connection with the spot theory of transition is obscure.

Suggestions are made for further work that now appears necessary for developing improved models of the transition zone.

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1. Introduction

There has been increasing interest in recent years in developing satisfactory models for the transition zone in a boundary layer, the zone being defined as one within which the flow changes from a purely laminar state at its upstream end to fully developed turbulence towards the downstream end. This interest is driven in part by technological applications where the design is governed by peak heat transfer rates (e.g., turbine blades, space shuttle) or seeks to utilize the benefits of extensive regions of laminar or transitional flow (e.g., laminar-flow aircraft); but there are in fact numerous other benefits of laminar flow technology, whose development demands better knowledge of transition phenomena in general and the transition zone in particular (Bushnell 1989). Furthermore, proper accounting of the transition zone could automatically provide the natural initial conditions for the computation of fully turbulent flow downstream, and should be a better alternative to the current practice of either arbitrary specification or appeal to experiment.

Early studies of the transition zone already indicated that while the appearance of short turbulent "bursts" was a fairly sudden phenomenon (Dryden 1939), the zone could be quite extensive in terms of the variation of mean flow properties (Prandtl 1935), and involved in some sense an alternation between laminar and turbulent flow (Liepmann 1943). The proposal by Emmons (1951) that transition occurred through the birth and growth of turbulent spots was confirmed by the experiments of Schubauer and Klebanoff (1955). It is the author's opinion that the most satisfactory models of the transition zone would have to be based on an understanding of the manner in which turbulent spots are created and grow, and the effect on these processes of the disturbance environment, pressure gradient, surface roughness, three-dimensionality, curvature, Mach number, and all the other factors that influence boundary layer development.

Figure 1 (Narasimha and Dey 1989) depicts in very broad terms the different stages in the transition from the laminar to a turbulent state in a two-dimensional boundary layer. When environmental disturbances are not high, two-dimensional Tollmien-Schlichting waves appear in the flow followed by the emergence of three-dimensionality and eventual breakdown to turbulent spots. The later stages of instability have been extensively discussed in a variety of reviews (Stuart 1986, Herbert 1988, Morkovin 1988). A quantitative measure of the progression of transition following breakdown is given by the intermittency γ, which is defined as the fraction of time that the flow is turbulent. The transition zone, in which γ varies from $0+$ to nearly unity $(1-)$, has been surveyed in some detail by Narasimha (1985). A remarkable feature of the current research scene is a sharp division of labor between workers studying the pre-onset ($\gamma = 0$) stage of the flow and those studying the post-onset stage ($\gamma > 0$). Few investigations *bridge* onset (the exception being Arnal et al. 1977). From the analysis in Narasimha (1985), it appears that the effective location of transition onset is very close to the well-known appearance of sharp spikes in the velocity signal in what is known as K-type breakdown. A general feature of boundary layer transition is that, if the environment is relatively quiet, the linearly unstable regime is the rate-controlling process, as the amplification factors of the most unstable waves are relatively low. We thus have a long period of weak instability, followed by rapid three-dimensional collapse due presumably to the creation of strongly unstable local flow features such as a high-shear layer. When the environment is noisy, the linear Tollmien-Schlichting stage may be bypassed (Morkovin

1977). In either case, however, the transition zone is intermittent, as the work of Suder et al. (1988) has shown, although the difficulty of distinguishing a spot from its disturbed surroundings has sometimes been commented upon.

In view of its direct relevance to understanding the transition zone, we begin with a brief review of studies relating to the behavior of turbulent spots.

2. Turbulent Spots

The classic experiments of Schubauer and Klebanoff (1955) established the basic facts of spot propagation. Since then the structure of a spot has been investigated by many workers (Cantwell, Coles, and Dimotakis 1978, Wygnanski et al. 1979, Gad-el-Hak et al. 1981; see in particular the review of Riley and Gad-el-Hak 1985).

The present article is not the appropriate occasion to survey these interesting studies, but we need to note that the investigations necessary for improving models of the transition zone have not received enough attention. Thus, the interesting conclusion of Wygnanski (1981) in a favorable pressure gradient that spot propagation velocity is *not* proportional to local free stream velocity has not been followed up. Spot growth rates are affected by pressure gradient (Narasimha et al. 1984), low Reynolds numbers (Schubauer and Klebanoff 1955) and flow distortion (Dey et al. 1990); however, there is not enough data yet to make satisfactory quantitative estimates. Little is known about spot characteristics in 3-D flows. When pressure gradients vary downstream, there is a possibility of what has been called subtransitions (Narasimha 1984); but more systematic studies of this phenomenon need to be made yet.

3. Approaches to Modelling

A detailed survey of models proposed to date has been recently made by Narasimha and Dey (1989): Table 1 lists the models with some commentary.

As will be seen, the models fall into three broad classes: linear-combination, algebraic, and differential. The earliest models (e.g., Goldstein 1938) assumed transition to occur abruptly at some station $x = X$ (say), the fully turbulent flow at $x > X$ being so determined that the momentum thickness θ is continuous at X. Such abrupt-transition models are still used in some areas of engineering design. However, they yield unrealistically high values of peak wall stress and heat transfer. Transition zones do tend to be very short in adverse pressure gradients (see, e.g., Walker and Gostelow 1990), but it is now feasible to take this into account in a general scheme that we shall describe below.

Both linear-combination and algebraic models need the intermittency distribution as an input. In the former (e.g., Dey and Narasimha 1990), the laminar and turbulent boundary layers (the latter originating at a specified or predicted onset location x_t) are separately calculated, and then combined in the proportion $(1 - \gamma) : \gamma$ to obtain the transitional boundary layer (Figure 2). In the latter, the molecular viscosity ν_m is enhanced by an intermittency-weighted eddy viscosity, the effective total viscosity being taken as

$$\nu = \nu_m + \gamma \nu_T. \tag{1}$$

In either case, a suitable model for calculating the turbulent flow is required.

Differential models tackle directly the Reynolds-averaged equations of motion, usually with one- or two-equation turbulence closures. In these models, some initial disturbance has to be specified; McDonald and Fish (1973) use a source term in the kinetic energy equation, whereas Wilcox (1981) uses the linear stability solutions at the e^4 amplification point to provide initial profiles of turbulent energy and dissipation. Vancoillie (1984) formulates equations for conditional averages, using the $K-\varepsilon$ approach, but needs to introduce the intermittency explicitly.

It is thus seen that with the exception of some differential models, the intermittency appears as a key variable in many transition zone models. This is indeed logical, as γ provides a measure of progression towards full turbulence. We therefore review theories of intermittency briefly.

4. Intermittency Distributions

Emmons (1951) showed how the intermittency $\gamma(x)$ (defined as the fraction of time that the flow is turbulent at a given station x) could be related to a source function giving the rate at which turbulent spots are formed over the surface. It can be shown (Narasimha 1985) that this relation can be obtained by postulating "independence" and "orderlines" hypotheses that imply that spot formation is a Poisson process; in particular the propagation and growth of a spot is assumed not to depend on the presence of other spots in the neighborhood. These assumptions are sufficiently realistic that the resulting expression provides an effective means for analyzing transition-zone data.

Narasimha (1957) showed that measured intermittency distributions in 2D incompressible flow could be explained on the basis of the additional hypothesis of concentrated breakdown, which leads to the result

$$\begin{aligned} \gamma &= 0, x < x_t \\ &= 1 - \exp\left[-\frac{(x-x_t)^2 n\sigma}{U}\right] = 1 - \exp(-0.41\xi^2), x_t < x \end{aligned} \tag{2}$$

where x_t is the onset location, n is the spot formation rate (per unit time, per unit distance in the spanwise direction), σ is a spot propagation parameter ($\simeq 0.25$, see Narasimha 1978) and

$$\xi = (x - x_t)/\lambda \tag{3}$$

is a non-dimensional variable using the distance λ between the stations where $\gamma = 0.25$ and 0.75 to characterize the extent of the transition zone. Earlier assumptions on the probability of spot formation may be shown (Narasimha 1985) to be incompatible with the observed similarity in the measured γ.

The validity of (2) has been confirmed by a variety of measurements, including Dhawan and Narasimha (1958), Owen (1970), Fraser and Gardiner (1988), and Gostelow and Walker (1990). Other curves have also been suggested for the distribution. Schubauer and Klebanoff (1955) fit an error function to their data; Abu-Ghannam and Shaw (1980) suggest

$$\gamma = 1 - \exp -b\xi^3. \tag{4}$$

With suitable fitting procedures all these expressions provide reasonable approximations to the data, (4) being perhaps slightly less satisfactory (Fraser, Milne, and Gandiner 1988; cf.

their Figures 3, 4, and 5). The advantage of (2), however, is its direct relation to spot theory, and the natural way in which extensions can be derived for more complex situations based on information on spot behavior. Thus, using a generalized intermittency distribution based on the hypothesis of concentrated breakdown (Narasimha 1985), one can deduce the appropriate distributions in pipe flow (Pantulu 1962, Narasimha 1985) and in flow past axisymmetric bodies (Narasimha 1984).

In using (2), it is necessary to take some care in determining λ, and in particular x_t. Sometimes x_t is taken as the point were $\gamma = 0.01$ (or some other suitably small number). This procedure is particularly misleading, because (Narasimha 1985): (a) measurement of small values of γ tends to be inaccurate, (b) breakdown to spots does not actually occur entirely at x_t but in a narrow belt across it, and (c) spot growth at low Reynolds numbers, especially near the point of birth, may not be linear (Schubauer and Klebanoff 1955). For all these reasons small departures from (2) are sometimes noticed near x_t. A procedure that avoids these problems (Narasimha 1957, 1985) is to plot $F(\gamma) = [-\ln(1-\gamma)]^{\frac{1}{2}}$ vs. x, and extrapolate from the best linear fit for $F(\gamma)$ to the point $\gamma = 0$. Failure to analyze results on this "$F(\gamma), t$ basis", as Walker and Gostelow (1989) call it, has sometimes been responsible for unjustified conclusions (as illustrated by Narasimha and Dey 1985). The value of following this procedure in understanding the transition zone in adverse pressure gradients has been emphasized by Walker and Gostelow (1990) and Gostelow and Walker (1990).

A special case of the generalized intermittency distribution for the transition zone on an arbitrary surface (Narasimha 1985) is a one-dimensional version of (2),

$$\gamma = 1 - \exp[-1.1\xi] \tag{5}$$

discovered first for flow in pipes (Pantulu 1962). However, the distribution (5) is applicable whenever the spot can grow only in one dimension (say streamwise), its "width" being constrained by geometry in other directions. Thus, (3) is also applicable in the later stages of the transition zone on an axisymmetric body, after a spot has wrapped itself around the body and becomes a sleeve (Rao 1974, Narasimha 1984); or when a turbulent slab extending across the whole body takes the place of a turbulent spot (Pfeil and Herbst 1979). Equation (5) is therefore best seen as the "1-D analogue" of the 2-D law (2).

There are many interesting situations where there is a "subtransition" from the 2-D to the 1-D law in the same flow. This is easily understood on a circular cylinder with axis aligned to the flow: a 2-D law near x_t changes to a 1-D law sufficiently far downstream (Narasimha 1984). A similar situation occurs in the study of the phenomenon of wake-induced transition, currently of great interest in turbomachinery (where the wakes of passing rotor blades periodically trip stator boundary layers: Dong and Cumpsty 1990a,b, Addison and Hodson 1990a,b): Mayle and Dullenkopf (1990) have provided an appropriate combination of (2) and (5) relevant to the problem.

There are other kinds of subtransition as well: e.g., in pressure gradient flows (Narasimha 1984), where $F(\gamma)$ shows a kink, reflecting presumably a relatively rapid change in spot spread or propagation characteristics near the corresponding station. Such changes in spread rate have been noticed in experiments on a single spot (Narasimha et al. 1984); similarly a change in propagation velocities is noticeable in the measurements of Dong and Cumpsty (1990b). The existence of a subtransition in some of the flows studied by Blair and Werle

(1981) was suspected by Dey and Narasimha (1988) on the basis of the observed mean-flow parameters; this has since been confirmed by Blair (private communication) through direct measurements of the intermittency.

5. Spot Formation Rates

It can be shown (Dhawan and Narasimha 1958) that a consequence of (2) is that the spot formation rate is given in terms of the transition zone length by the relation

$$n = 0.41U/\sigma\lambda^2. \tag{6}$$

For modelling the zone, it would be useful if n (or equivalently λ) could be determined in terms of the other parameters in the problem. Narasimha and Dey (1986) have reviewed earlier attempts at prescribing n. It has been argued (Narasimha 1984) that the appropriate non-dimensional parameter is $N = n\sigma\theta_t^3/\nu$, where θ_t is the momentum thickness at x_t; if N is constant n scales with boundary layer thickness and a viscous diffusion time, and it is implied that $Re_\lambda \sim Re_{x_t}^{3/4}$, a relation which is close to an earlier proposal by Dhawan and Narasimha (1958) and represents the available data just as well. Narasimha and Dey (1986) have examined all available data in detail, and inferred the likely variations of N with free stream turbulence level and Mach number. They find that the data suggests that N settles down to a constant value of about 0.7×10^{-3} in what has been identified as the turbulence-driven transition regime (Narasimha 1985); the evidence indicates that N is higher when residual non-turbulent disturbances drive transition and will be facility-dependent. Dey and Narasimha (1996) have similarly derived values of N in favorable pressure gradient flows. Gostelow (1989), analyzing the extensive experimental data reported by Walker and Gostelow (1990) on transitional boundary layers in adverse pressure gradients, has found that the use of the $F(\gamma), t$ basis provides not only a viable basis for analyzing the intermittency distributions but also for deriving N. A summary of the effect of pressure gradient on N, taken from Narasimha and Dey (1989), is given in Figure 3. The use of N to specify zone-lengths has also been found useful by Addison (1989).

It should perhaps be noted that in the models only the product $n\sigma$ is important (being sufficient to determine λ, see (6)), and not the factors separately; it is for this reason that N contains σ as well. The only attempt at determining the value of σ appears to be that of Narasimha (1978), who found by integration of the Schubauer-Klebanoff (1955) data that σ varies from about 0.25 near the wall to 0.29 away from it. (Some idealized spot shapes were also proposed for these values of σ.)

6. Prediction of Onset

This remains a major unsolved problem, of course. To predict x_t, therefore, the best one can do at present is to adopt some more or less empirical approach.

Various correlations for the prediction of onset have been proposed: Michel (1951), Dunham (1972), Singh (1974), and Abu-Ghannam and Shaw (1980), among others. Govindarajan and Narasimha (1990) have recently made a critical comparison of these methods and proposed a new one of their own, which differs from the previous ones in making an allowance

for residual non-turbulent disturbances that will in general vary from one facility to another. There is also a method proposed by Granville (1953) which takes into account the effect of an imposed pressure gradient on the flow.

Among the most widely used current techniques is the so-called e^n method. The rationale for this method is that the linear-instability regime in the boundary layer is generally rate-controlling, so that the total amplification in the regime should correlate with transition. It has been suggested that in atmospheric flight the value of n tends to be around 10 (Bushnell et al. 1988). In general, n should be a function of the disturbance level, as higher disturbances will require less amplification to reach critical levels. Proposals for such relations have also been made (Mack 1977, Arnal et al. 1984), although these have not yet considered the effect of residual non-turbulent disturbances.

As has been emphasized by Morkovin on various occasions (e.g., Morkovin 1989), transition depends on a variety of parameters, and the whole question of the receptivity of the boundary layer to external disturbances needs still to be satisfactorily tackled. Much more progress on such issues is required before onset prediction can be put on a more rational basis.

7. Assessment of Some Current Models

With the current wave of interest in transition, the first systematic assessments of various available models are beginning to be made. It is certain that the need for more careful experiments will quickly be felt (if it has not already been), especially those in pressure gradients with a fairly thoroughly documented disturbance environment.

The most extensive comparisons with published experimental data have come from Dey and Narasimha (1988, 1990b). Their model computes both laminar and turbulent boundary layers by appropriate integral schemes (Thwaites 1949 modified by Dey and Narasimha 1990a, and Green et al. 1973, respectively), and combines the two solutions linearly for the velocity in the proportions $(1-\gamma):\gamma$. Onset location is estimated from the correlation of Govindarajan and Narasimha (1989), and extent from similar correlations for the parameter N (Dey and Narasimha 1990c), as a function of free stream turbulence and pressure gradient. The scheme is modular, and enables any of the components (e.g., onset prediction) to be replaced by a better procedure should one become available or be necessary. Subtransitions (Narasimha 1984) are explicitly allowed for. Comparisons were provided with the experimental data of Narasimha et al. (1984), Abu-Ghannam and Shaw (1980), and Blair and Werle (1981); by and large they found reasonably good agreement. The model has been recently updated by Govindarajan (1990).

Two other assessments have been recently published. Abid (1990) introduced the intermittency models of Arnal (1984), Chen and Thyson (1971) and Dhawan and Narasimha (1958) into the algebraic scheme of Cebeci and Smith (1974), taking the fully turbulent eddy viscosity to be reduced by the factor γ in the transition zone, as in (1); he also made calculations using the scheme of McDonald and Fish (1973). Comparisons were made with the experiments of Blair and Werle (1980, 1981) in zero and favorable pressure gradients. Abid found that all models showed reasonable agreement with experiment in zero pressure gradient and low free stream turbulence. For accelerated flows, the transition zone lengths were not captured properly, but it must be recalled that the intermittency functions used

by Abid did not allow for the subtransitions known to occur in such flows and included in the model of Dey and Narasimha (1988, 1990b). The McDonald-Fish model was found to overestimate the effects of acceleration.

Dinavahi (1990) compares the predictions of two "transition functions" (as he calls them) coupled to the Baldwin-Lomax (1978) model for fully turbulent flow. The transition function is the factor that multiplies the Reynolds stress from the turbulence model to allow for the transition zone (the viscous stress being always included in full). One of the transition functions was the intermittency distribution (2), again without allowance for subtransitions. The second was the proposal of Arnal (1986), which exhibits an overshoot before settling down to unity further downstream. (This overshoot appears necessary in algebraic models in order to capture the observed peak values in such parameters as wall stress or heat transfer towards the end of the transition zone.) The author conjectures that the reasons are to be found in (a) the absence of a well-defined effective origin at x_t for the emerging asymptotic boundary layer in such algebraic models, and (b) inadequate accounting of the Reynolds stress contribution arising from the fluctuation *between* laminar and turbulent flow (Narasimha 1990). Comparisons were made with only two sets of experimental data. Both methods were found to do quite well in the Schubauer-Klebanoff (1955) 2-D incompressible flow, but in supersonic flow (past a cone of half-angle 10 deg. at a Mach number of 6, Stainback et al. 1972) there were slight deviations towards the beginning and end of the transition zone. No comparisons were made with pressure-gradient data.

The linear-combination method has worked well in 2D incompressible attached flows, and I see no reason why it should not do equally well at higher speeds. It has, however, an inherent weakness in strong adverse-pressure gradient flows, in a situation where a purely laminar boundary layer would separate but a transitional one would not. Intermittency-weighted Reynolds stress models appear inherently less limited in this respect, but they miss the overshoot in surface parameters that is so characteristic of the transition zone and is in many applications an important feature to capture. A separate overshooting transition function as discussed above would solve this problem, but the direct connection with the spot theory of intermittency is then lost. Clearly, further improvements in modelling the transition zone are required.

Direct solutions of the Navier-Stokes equations for transitional flow have recently been reviewed by Kleiser and Zang (1990). Considerable progress has been made in the past decade with numerical simulations, but no solutions are yet available for the transition zone with natural emergence of turbulent spots. A strong effort at developing such solutions seems highly desirable.

8. Future Work

Further progress in transition zone modelling requires several careful experimental programs. First of all, the behavior of turbulent spots when subjected to such influences as pressure gradient, distortion, curvature, three-dimensionality, compressibility, etc., needs to be investigated more extensively. Parameters of interest will include shape, velocities of propagation, conditional statistics, and flow structure. Experiments are also needed in two-dimensional flows with pressure gradient, both favorable and adverse, with a disturbance environment that is well understood and carefully controlled. In particular, data on flows

with separation bubbles are badly needed in turbomachinery applications. Very little has been done on three-dimensional transition zones. Significantly better models are unlikely to emerge without the benefit of all this experimental work, although certain improvements can be envisaged on current models and will undoubtedly appear as a result of work on hand.

In numerical simulation or direct solution of the Navier-Stokes equations, a study of the processes of generation and propagation of turbulent spots has just begun: this task should surely be pursued vigorously.

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Table 1. A brief summary of transition-zone models.

Authors	Type	Remarks
Dhawan and Narasimha (1958)	Linear combination	Combination of laminar and turbulent velocities in proportions determined by the intermittency. Requires onset (x_t) extent of zone, model for fully turbulent flow. Constant pressure. Simple.
Chen and Thyson (1971)	Linear combination	For axisymmetric flows. Special intermittency model, correlation for length. Limited validation.
Lakshminarayana (1976)	Linear combination	As in Dhawan and Narasimha. Integral method for axisymmetric body and high speed flows.
Arnal (1986)	Linear combination	Integral method. Linear combination for shape factor and skin-friction. Intermittency in terms of momentum thickness, not related to spot theory.
Fraser and Milne (1986)	Linear combination	Velocity and skin-friction as in Dhawan and Narasimha. Intermittency is error-function. Extent in terms of standard deviation of intermittency. Integral method.
Fraser et al. (1988)	Linear combination	Extenstion of Fraser and Milne, but different correlation for zone-length. Good agreement with data on turbine blades.
Dey and Narasimha (1989a, 1990)	Linear combination	Extenstion of Dhawan and Narasimha. Extent from new spot formation rate parameter. Integral method. High favorable pressure gradient data also predicted.
Harris (1971)	Algebraic	Eddy viscosity and thermal diffusivity. Intermittency of Narasimha. Requires extent. Compressible plane and axisymmetric flows.
Kuhn (1971)	Algebraic	Eddy viscosity. Method of integral relations for high speed flows. Intermittency distribution of Narasimha (1957).
Adams (1972)	Algebraic	Eddy viscosity. Intermittency distribution of Narasimha (1957) takes extent = $x_t f$ 2.96.
Cebeci and Smith (1974)	Algebraic	Eddy viscosity. Intermittency distribution of Chen and Thyson (1971). Predicts x_t.

Authors	Type	Remarks
Gaugler (1985)	Algebraic	Eddy viscosity, based on STAN5 code. Intermittency distribution of Abu-Ghannam and Shaw (1980). Onset and extent adjusted to obtain agreement with experimental data.
Michel et al. (1985)	Algebraic	Intermittency in terms of momentum thickness, exceeds 1 for ensuring agreement with data.
Krishnamoorthy (1986)	Algebraic	Extension of Patankar-Spalding (1970) for predicting heat transfer rates on turbine blades and nozzle guide vanes. Intermittency distribution of Narasimha (1957) x_t and extent from measurements. Effect of large free stream turbulence by addition to eddy viscosity, shows good agreement with experiments.
Krishnamoorthy et al. (1987)	Algebraic	Extension of Krishnamoorthy (1986) with onset momentum thickness Reynolds number = 160. Dhawan-Narasimha correlation for extent extended to pressure gradients.
McDonald and Fish (1973)	Differential	Integral form of a turbulent kinetic energy equation. Source terms in governing equation through which free stream turbulence triggers transition.
Blair and Werle (1980, 1981)	Differential	Extension of McDonald and Fish (1973) and McDonald and Kreskovsky (1974). Zero pressure gradient heat transfer generally predicted well (but not for the flow at free stream turbulence level = 0.25), less satisfactory for pressure gradient flows.
Wilcox (1981)	Differential	Stability related closure model. Tested for constant-pressure flows at low free stream turbulence levels.
Arad et al. (1982, 1983)	Differential	Modified two-equation model of Ng (1971). Requires adjustment of numerical constants.
Vancoillie (1984)	Differential	Based on $K-\varepsilon$ model. Conditional averages of all quantities require intermittency, which is taken as that of Narasimha (1957). Good agreement with data considered.

Authors	Type	Remarks
Wang et al. (1985)	Differential	Based on $K-\varepsilon$ model; sensitive to boundary conditions for K,ε for airfoil cascade. Discrepancy noted in transitional and turbulent regions on suction surfaces of turbine blades.
Krishnamoorthy et al. (1987)	Differential	$K-\varepsilon$ model of Jones and Launder with change in a constant. Tested for nozzle guide vane data. Underpredictions near trailing edge attributed to separation.

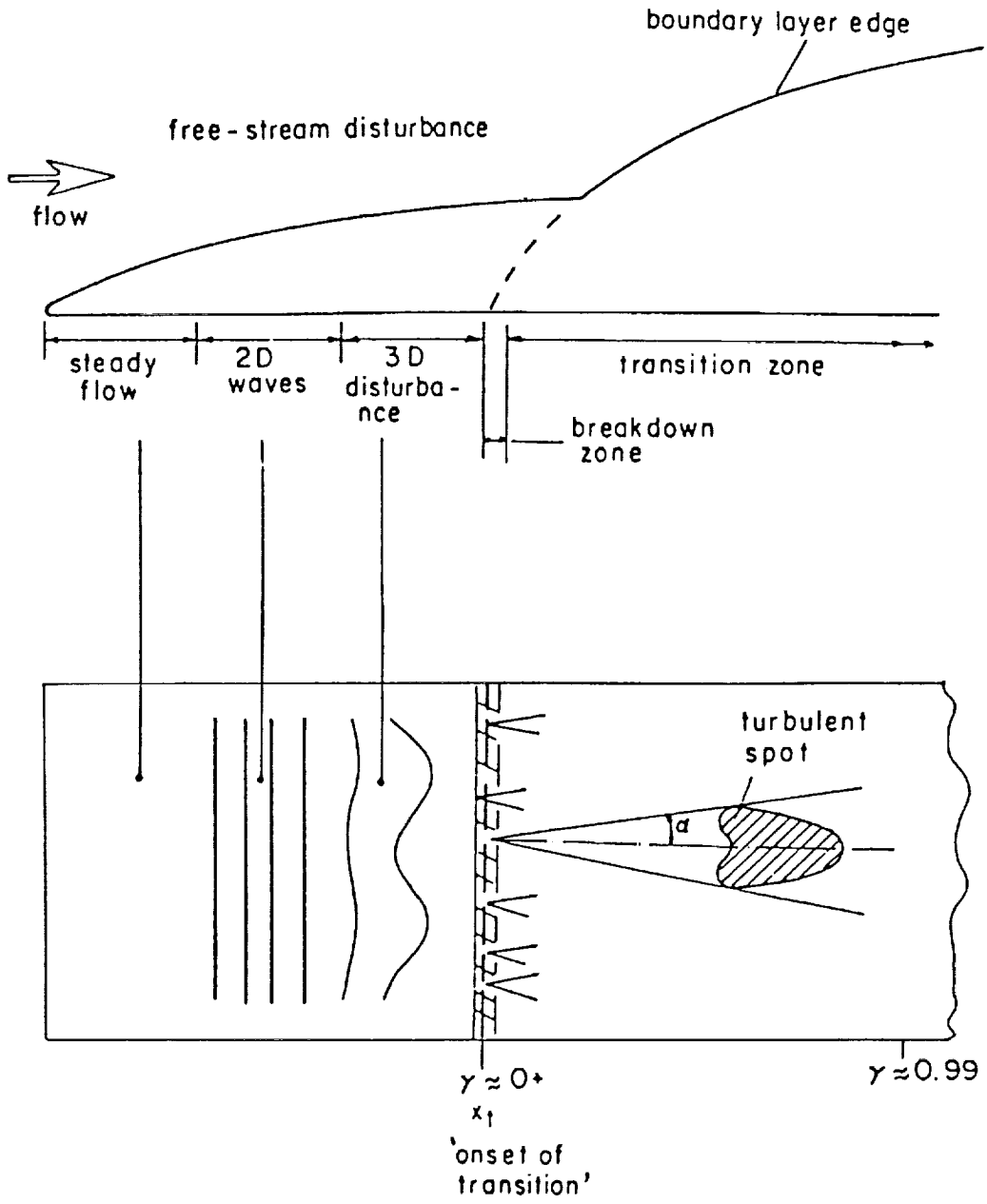

Figure 1. A schematic picture of various stages in the transition from laminar to turbulent flow in a flat plate. α and γ respectively denote the spot spread angle and intermittency. The route shown here, one among many that are possible, seems to be relevant when external disturbances are low. The instability stages may be bypassed when the environment is highly disturbed.

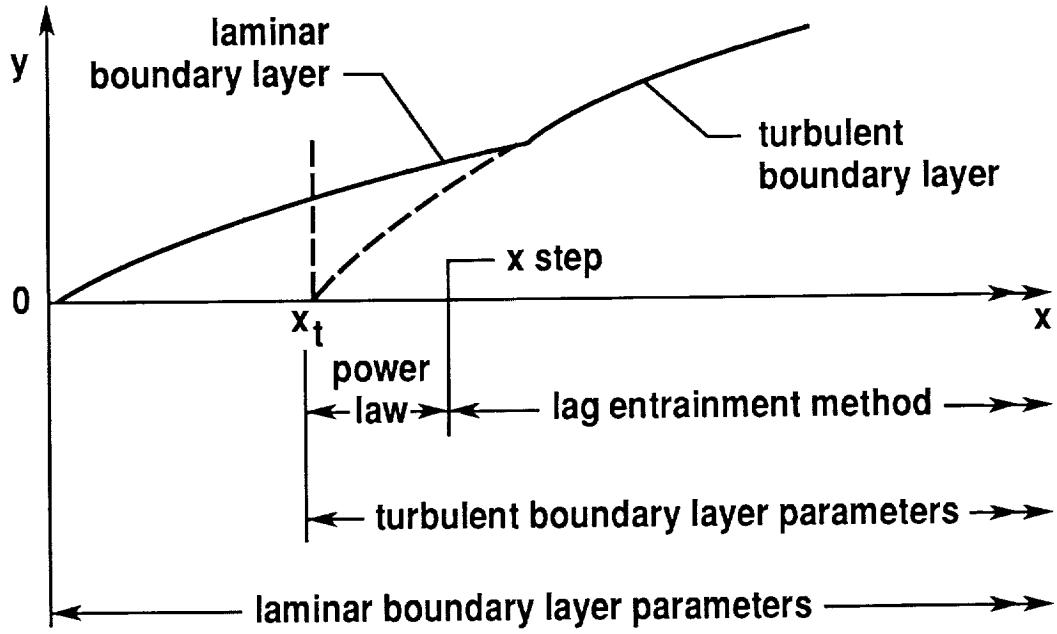

Figure 2. Various computational domains adopted in the linear-combination type integral model of Dey and Narasimha (1988, 1990b).

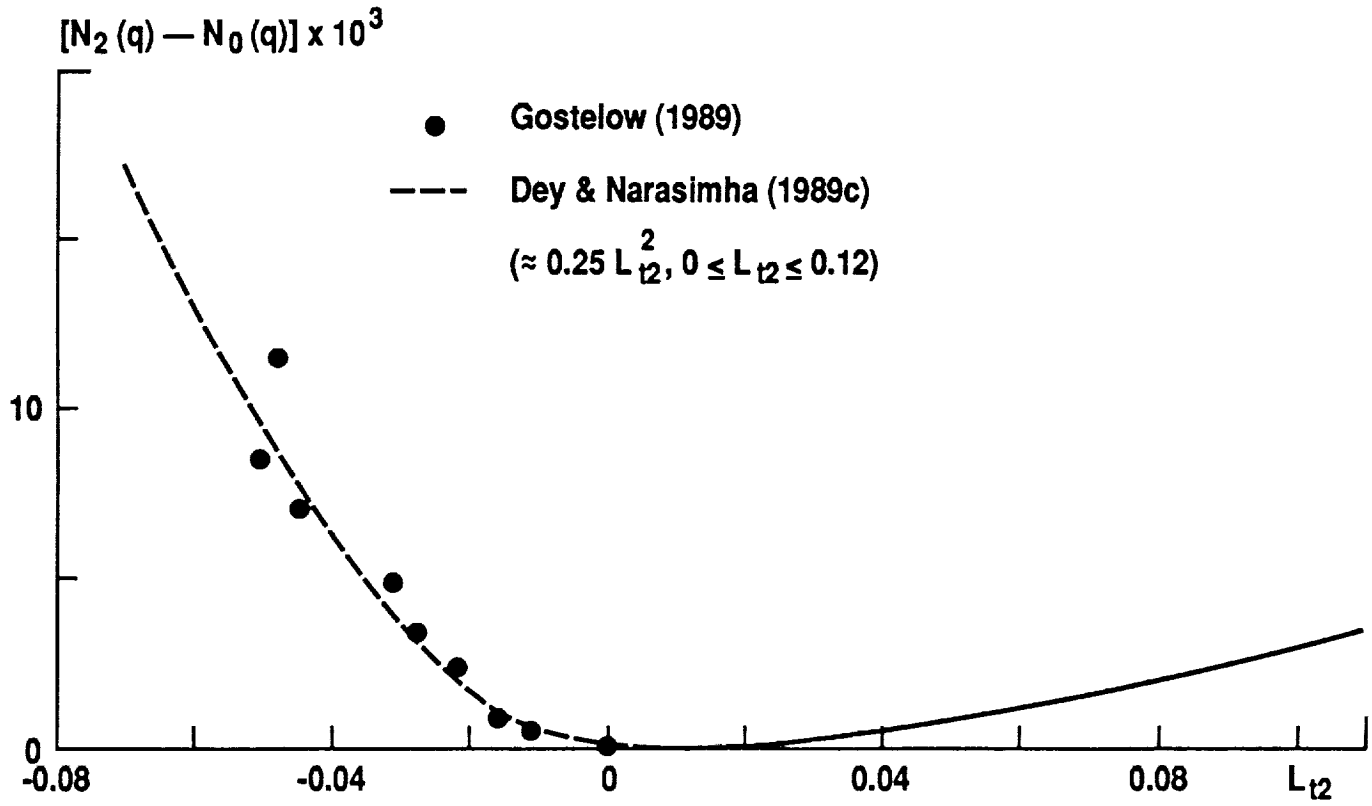

Figure 3. A preliminary proposal for the variation of the differential spot formation rate parameter N_2 at the location of the subtransition point, at any given value of the free-stream turbulence level q, as a function of the Thwaites pressure gradient parameter at subtransition (from Narasimha and Dey 1989).

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16. Abstract

Recent developments in the modelling of the transition zone in the boundary layer are reviewed (the zone being defined as extending from the station where intermittency begins to depart from zero to that where it is nearly unity). The value of using a new non-dimensional spot formation rate parameter, and the importance of allowing for so-called subtransitions within the transition zone, are both stressed. Models do reasonably well in constant pressure 2-dimensional flows, but in the presence of strong pressure gradients further improvements are needed. The linear combination approach works surprisingly well in most cases, but would not be so successful in situations where a purely laminar boundary layer would separate but a transitional one would not. Intermittency-weighted eddy viscosity methods do not predict peak surface parameters well without the introduction of an overshooting transition function whose connection with the spot theory of transition is obscure.

Suggestions are made for further work that now appears necessary for developing improved models of the transition zone.

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