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PASSIVE DAMPING CONCEPTS FOR TUBULAR BEAMS WITH PARTIAL ROTATIONAL AND TRANSLATIONAL END RESTRAINTS

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NOMENCLATURE

C	Coefficient of damping
C_c	Critical damping factor
D	Outer diameter of tubular member
d	Inner diameter of member
Δ_o	Initial center deflection
E	Young's modulus for member material
f	Natural frequency of member vibration
η	Efficiency
h	Panel length
I	Moment of inertia for member
K_t	Translational stiffness of end springs
K_r	Rotational stiffness of member ends
L	Length of member
M	Number of panels
M_s	Mass of system
M_d	Mass of dampers
t	Time
ρ	Mass per unit length
W	Transverse displacement of member
Q	Suspended weight
Z	Distance along member

NOMENCLATURE - Cont'd

Ω	Frequency of forcing function
Δt	Time increment
ζ	Damping ratio for member with dampers
ζ_o	Damping ratio for member with no dampers
δ	Logarithmic decrement
Δ_D	Member dynamic displacement with dampers
Δ_s	Member static displacement with no dampers
μ_0	Dynamic amplication factor with no dampers
μ	Dynamic amplification factor with dampers
A_r	Amplitude reduction index

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1. INTRODUCTION

1.1 Background and Previous Work

Various types of designs and configurations are being considered for the construction of a future permanent space station by NASA. One proposed configuration consists of a three-dimensional space frame with slender tubular members. The tubular members as well as the space structure as a whole are susceptible to unwanted vibrations induced by various types of perturbations. The overall shape of the space structure and the slenderness of its members with semi-rigid connections may result into a dynamic response characterized by low-frequency small-amplitude vibrations. The future large space structures will have as performance objectives short settling times and relatively fast response requirements or combinations thereof.

A structure can be damped using either active or passive control. Active control uses electronic sensors and an external source of energy to reduce the vibrations to admissible levels. Passive systems reduce vibrations by energy dissipation or by temporarily storing it. Damping may also be obtained by blending both passive and active control techniques. A considerable amount of research has been conducted on active techniques as compared to that on passive control. However, passive control is gaining increasing popularity due to its stability, high reliability, relatively low cost and little or no power requirements. A brief overview of the representative works in this field is given below.

Rogers and Richards (1) presented an overview of the passive and active control of space structures program. They noted that although it is possible in principle to achieve structural vibration control through purely active means, experience with complex structures has shown that the realities of the prototype system inaccuracies and sensor/actuator dynamics frequently combine to produce substandard performance. A more desirable approach would be to apply passive damping techniques to reduce the active control burden.

Chen and Wada (2) carried out a theoretical and experimental study of passive damping techniques in truss-type structures with emphasis on viscoelastic damping. Thin walled aluminum tubes with concentric constraining members were studied. The use of constraining members was found to enhance the damping.

Balas and Shephard (3) reviewed passive control of structural deformations in deployable reflectors. Using finite element codes, they were able to identify the dynamic characteristics and determine the optimal locations for the placement of passive damping elements to decrease the structural deformations of the reflector structure.

Gehling, Harcrow and Morosow (4) studied the benefits of passive damping as applied to active control of large space structures. Passive dampers were incorporated into a representative large space structure and their effect on candidate active control laws was investigated. Viscous dampers were implemented in time simulations with direct velocity feedback. They found that the use of discrete passive dampers in the approach to vibration suppression led to a reduction in demands placed upon active control system.

Atluri and Odonoghue (5) conducted an investigation into the active control of transient dynamic response in large space structures that are modeled as equivalent continua, with emphasis on the effects of initial stresses on the controllability of transverse dynamic response. A singular solution approach was used to derive a fully coupled set of nodal equations of motion which included non-proportional passive damping to develop a feedback control law. They presented examples involving the suppression of structure transient dynamic response vibration by means of an arbitrary number of control force actuators.

Crawley, Sarver and Mohr (6) carried out an experimental investigation of the damping of metallic and fibrous composite materials taking into account analytical modelling and experimental verification of frictional damping schemes. To demonstrate how material and frictional damping can be combined, they carried out a simple structural optimization, indicating the potential for significant savings in mass by addition of frictional dampers.

Razzaq, Volland, Bush and Mikulas (7) presented an experimental and theoretical study of the natural vibration of passively damped and vertically aligned tubular steel member with partial rotational end restraints. A damping concept consisting of a string-mass assembly was explored in addition to the inherent structural damping. Lead shots were used as masses. Natural vibration tests were conducted in the absence of an external axial force. A three-lead shot configuration was found to provide considerably greater damping than a single lead shot. Three types of analysis were presented and were in excellent agreement with test results.

Razzaq and Ekhelikar (8) studied a vertically aligned steel member with partial rotational end restraints. Experiments were conducted to evaluate the effectiveness of string-mass, polyethylene tubing, and chain damping concepts. The damping ratio was found to increase with the number of lead shots. The polyethylene tubing however, did not provide any significant damping. The amount of chain damping was found to increase with the length of the chain.

Razzaq and Bassam (9) investigated a horizontally aligned tubular aluminum member with partial end rotational restraints. Both natural and forced vibration tests were conducted. Four damping concepts, namely, copper brushes, wool swabs, nylon brush, and silly putty in chamber were investigated in various configurations. Experiments were also conducted on a grillage assembly consisting of two members with partial end rotational restraints at the member outer ends and spring supports at the member junction. A harmonic forcing function applied at the midspan of one member while the deflection-time response was monitored at the midspan of the other member. The dampers were provided only in the member being monitored for the deflection-time response. The silly putty in chamber dampers provided the maximum damping under the free vibration conditions. For the grillage assembly, the wool swab dampers were found to provide the maximum damping under both free and forced-free vibrations.

Razzaq and Mykins (10) conducted an experimental and a theoretical investigation of the wool swab damper, the copper brush, and the silly putty in chamber damping concepts for a single tubular member under free and forced-free vibrations. The silly putty in chamber concept proved to be the most efficient one whereas the copper brush concept provided the highest damping ratio under natural vibration. The silly putty in chamber dampers were further investigated theoretically under forced-free vibration and were found to be more effective at or near resonance frequency.

Research conducted to date on single tubular members has been on members with rotational end springs only, that is, in the absence of any translational end springs. This research report presents the outcome of an experimental and theoretical investigation of seven different damping devices for a tubular aluminum member with both rotational and translational end springs.

1.2 Problem Definition

Figure 1 shows a schematic of the experimental setup. It consists of a tubular member with a length L, an outer diameter D and an inner diameter d. The member is supported at both ends by identical springs of translational stiffness K_t and rotational stiffness K_r.

The problem is to investigate the effectiveness of the following passive damping concepts under natural and forced-free vibrations:

1. Wool Swab Dampers
2. Copper Brush Dampers
3. Silly Putty in Chamber Dampers
4. Water-Filled Balloon Dampers
5. Egg-Filled Balloon Dampers
6. Water-Filled Plastic Egg Dampers
7. Oil-Filled Plastic Egg Dampers

The natural vibrations are induced by suddenly releasing a constant static load suspended by a string at member midspan. The forced-free vibrations are induced by means of a mechanical vibrator which is disengaged after a certain period of time. The damping efficiency of each of the above concepts is determined experimentally from the deflection-time response graphs.

1.3 Objectives and Scope

The main objectives of this study are:

1. Identification of potential passive damping concepts for slender tubular structural members with rotational and translational end springs under natural and forced-free vibrations.
2. Evaluation of damping efficiencies of the various damping concepts mentioned in Section 1.2.
3. Evaluation of the suitability of a theoretical finite-difference analysis by comparison to the experimental results for the case of natural vibrations.

Only member flexural and translation motion is considered. The natural vibration study is conducted on the seven damping concepts and for only one specific initial deflection. The most suitable of the seven dampers is further investigated under forced-free vibrations. In addition only one set of end springs is used for all of the experiments.

1.4 Assumptions and Conditions

The following assumptions and conditions are adopted in this study:

1. The deflections are small,
2. The material of the member is linearly elastic,
3. Vibrations are planar,
4. Damping forces are opposite and proportional to the velocity,
5. The member is tested under 1-g environment,
6. The effect of secondary induced forces is negligible,
7. The end springs are linearly elastic.

2. NATURAL VIBRATION ANALYSIS

2.1 Governing Equations

Figure 1 shows the test setup for natural vibration. The tubular member of length L has end springs of translational stiffness K_t lb/in and rotational stiffness K_r lb-in/radian respectively. The material of the beam is elastic. The governing equilibrium equation is (Reference 11):

$$EI \frac{\partial^4 W}{\partial Z^4} + \rho \frac{\partial^2 W}{\partial t^2} + C \frac{\partial W}{\partial t} = 0 \tag{1}$$

where:

W = Beam lateral displacement

EI = Beam flexural rigidity

ρ = Mass per unit length

C = Damping coefficient

The initial and boundary conditions for this problem are as follows:

$$W(0,0) = \frac{Q}{2K_t} \tag{2}$$

$$W(L,0) = \frac{Q}{2K_t} \tag{3}$$

$$EI \frac{\partial^2 W}{\partial X^2}(0,t) = K_r \frac{\partial W}{\partial X}(0,t) \tag{4}$$

$$EI \frac{\partial^2 W}{\partial X^2}(L, t) = -K_r \frac{\partial W}{\partial X}(L, t) \tag{5}$$

Equations 2 and 3 are the initial conditions whereas equations 4 and 5 represent the natural boundary conditions dependent upon flexural stiffness of the beam ends and the rotational springs.

2.2 Finite-Difference Solution

Employing second order central finite-difference expressions (Reference 7), the governing equilibrium equation becomes:

$$\begin{aligned} &\frac{EI}{h^4}(W_{i-2,j} - 4W_{i-1,j} + 6W_{i,j} - 4W_{i+1,j} + W_{i+2,j}) + \\ &\frac{\rho}{(\Delta t)^2}(W_{i,j-1} - 2W_{i,j} + W_{i,j+1}) + \\ &\frac{C}{(\Delta t)}(-W_{i,j} + W_{i,j+1}) = 0 \end{aligned} \tag{6}$$

in which:

h = panel length along the longitudinal axis of the beam, and Δt is the time increment. The subscript i refers to the i^{th} panel point over the range $0 < x < L$, and the subscript j refers to the number of time increments such that the time at j is given by the following equation:

$$t_j = j(\Delta t) \qquad for \qquad j = 0, 1, 2, \ldots.. \tag{6a}$$

The boundary conditions represented by Equations 4 and 5 may similarly be represented as:

$$\left\{\frac{EI}{h} + \frac{K_r}{2}\right\} W_{-1,j} + \left\{\frac{EI}{h} - \frac{K_r}{2}\right\} W_{1,j} = 0 \tag{7}$$

$$\left\{\frac{EI}{h} - \frac{K_r}{2}\right\} W_{m-1,j} + \left\{\frac{EI}{h} + \frac{K_r}{2}\right\} W_{m+1,j} = 0 \tag{8}$$

Applying equation 6 at i = 1, 2, 3, ..., M+1 and imposing conditions 2, 3, 4, and 5 leads to the following matrix equation:

$$\{W_{i,j+1}\} = C_1 \, [T] \, \{W_{i,j}\} + C_2 \{W_{i,j-1}\} \tag{9}$$

where:

$$\begin{aligned} C_1 &= \frac{-1}{(B_3 + B_4)}, \\ C_2 &= B_3 \, C_1, \\ B_3 &= \frac{\rho}{\Delta t^2}, \\ B_4 &= \frac{c}{\Delta t}. \end{aligned} \tag{10}$$

and [T] is a symmetric coefficient matrix of the order (M+1) in which:

$$\begin{aligned} T[1,1] &= T[M+1, M+1] = -\frac{4EI}{h^4} \\ T[2,2] &= T[M,M] = 6B_1 - 2B_2 - B_4 - B_1 \times B_5 \\ T[k,k+1] &= -4B_1 \quad for \quad k = 1,2,\ldots,M \\ T[k,k+2] &= B_1 \quad for \quad k = 1,2,\ldots,M-1 \\ T[k,k] &= 6B_1 - 2B_2 - 2B_3 - B_4 \quad for \quad k = 2,3,\ldots,M \end{aligned} \tag{11}$$

Due to symmetry:

$$T[I,J] = T[J,I] \tag{11a}$$

and

$$B_1 = \frac{EI}{h^4} \; ,$$

$$B_5 = \frac{\dfrac{EI}{h} - \dfrac{K_r}{2}}{\dfrac{EI}{h} + \dfrac{K_r}{2}} \; , \tag{12}$$

$$B_6 = \frac{-(\Delta t)^2}{2h^4} \frac{EI}{\rho} \; .$$

The remaining terms in [T] are equal to zero. Equation 9 is used to predict the lateral beam deflections $W_{i,j+1}$ for j = 3, 4, 5, ... if $W_{i,J}$ and $W_{i,J-1}$ are known. The term $W_{i,1}$ may be determined as follows:

$$W_{i,1} = \frac{\Delta_0}{(1 + 2B_8)} \left\{ Sin\left(\frac{\pi Z}{L}\right) + B_8\left(1 - cos\left(2\frac{\pi Z}{L}\right)\right)\right\} + \frac{Q}{2k_t} \tag{13}$$

in which:

$$B_8 = \frac{K_r L}{4\pi EI} \tag{13a}$$

and Z is the longitudinal distance along the member axis. Similarly, $W_{i,2}$ is determined as follows (Reference 7):

$$W_{i,j} = B_6 W_{i-2,1} - 4 B_6 W_{i-1,1} + (6 B_6 + 1) W_{i,1} - 4 B_6 W_{i-1,1} + B_6 W_{i+2,1} \; . \tag{14}$$

The rotational stiffness of the ends is determined from the member mid-span deflection due to a known static load. Using elastic theory, the midspan vertical deflection of a beam of length L due to a static load Q at Z = L/2 is:

from which:

$$\Delta_0 = \frac{QL^3}{96\,EI}\left\{2 - \frac{\frac{3}{2}}{\left(1 + \frac{2EI}{K_r L}\right)}\right\} \qquad (15)$$

$$K_r = \frac{4QL^3EI - 192(EI)^2\Delta_0}{(96\Delta_0 LEI - 0.5QL^4)} \qquad (16)$$

Equation 16 is used to evaluate K_r in this study.

3. EXPERIMENTAL INVESTIGATION

3.1 Test Setups

3.1.1 For Natural Vibration

The experimental setup is shown in Figure 1. The specimen is a long slender tubular member of length 251 in. with outer and inner diameters of 2 in. and 1.78 in respectively. The setup consists of a proximity probe and a deflection-time plotter. The proximity probe which is connected to a deflection-time plotter is placed at member midspan to record the midspan deflections. The tubular member is connected to a prototype multi-joint connection through a sleeve as shown in Figure 2. To the joint is connected to a pivot by means of a 5/8" screw. The pivot is such that it fits tightly into the hollow of the compression spring of stiffness 7 lb/in. The spring is secured to the web of a channel bracket which in turn is secured to the top of the column as shown in Figures 3a and 3b. Steel plates 12" by 7/8" by 1/8", are cantilevered from the vertical webs of the channel bracket and attached to the pivot which in turn acts as a pivot between the plates. Thus the cantilever steel plates and the compression springs contribute to the translational stiffness of the ends which is such that the ends vibrate with low frequency small amplitude vibrations. The member is horizontally aligned with gravity forces acting in the plane of vibration. The effective length of the member with the joints attached is 259 in. and is the length used in the theoretical analysis.

3.1.2 For Forced-Free Vibrations

The setup for the member forced-free vibrations tests is shown in Figures 4a and 4b. This consists of a proximity probe as shown in Figure 5, and the deflection time plotter. The forced vibrations are induced by a mechanical vibrator (Model 203-25-DC) with an oscillator (Model TPO-250). The vibrator applies a forcing function of the type:

$$F(t) = F_0 \, Cos\,(\Omega t) \tag{17}$$

in which:

F_0 = 4.0 lb,

t = time, secs.,

Ω = the forcing function circular frequency, rads/sec.

The latter is controlled using the oscillator. The forcing function F(t) is transmitted from the vibrator to the tubular member through a vibrator connector as shown in Figure 6. The connector consists of three segments PQ, QR and RU hinged together by pins at Q and R. End R is connected to the vibrator. The U end is connected to the lower part of a metal hose clamp provided around the tubular member at midspan. Sections QR and RU can be disengaged at R by pulling out pin RS and the arm is then disengaged thus cutting off the forcing function and leaving the member to vibrate naturally.

3.2 Passive Damping Concepts

The string-mass assembly used for damping consists of small masses of energy absorbent material stringed together at equal spacings by a nylon line (sportfisher monofilament line manufactured by K-mart corporation, Troy, Michigan 48084, 8013.9, No. EPM-40, inventory control number 04528201391) having a tensile capacity of 40 lb. The masses are suspended inside the hollow member by securing the nylon line at both ends of the tubular member under nominal tension as shown Figures 7a and 7b.

The seven different types of passive dampers investigated are described herein.

3.2.1 Wool Swab Dampers

Figure 8 shows a wool swab damping device, having a length of 3 in. and weighing 7.23 grams. It is manufactured by Omark Industries , Onalaska, Wisconsin 54650 with a U.S patent control number 076683422187. It has a 0.75 in. long cylindrical aluminum piece attached to 2.125 in. of rolled (threaded) wire which forms the core around which the wool swab is attached, and is normally used to clean 12 gauge shotguns.

3.2.2 Copper Brush Dampers

Figure 9 shows a copper brush damper 0.8125 in. diameter, with a length of 3.125 in. and weighs 13.195 gms. It has the same structure and dimensions as the wool swab damper except that the rolled wire core now holds copper bristles. It is also used to clean 12 gauge shotguns and is manufactured by Omark Industries, Onalaska, Wisconsin 54650 with a U.S patent 41986 and inventory control number 07668341989.

3.2.3 Silly Putty in Chamber Dampers

This device is shown in Figure 10. It consists of a 0.75 in. diameter ball of silly putty placed inside hollow cylindrical PVC pipe chamber. Silly putty is an elastoplastic material manufactured by Binney and Smith Inc., PA 18042, inventory control number of 07166208006. The chamber is made from a 1.0 in. long piece of "Bristol Pipe" (PVC -1120, Schedule 40, ASTM-D-1785, nominal 1 in. pipe) having an original outer diameter of 1.058 in. and a wall thickness 0.15 in. To reduce the mass of the damper the inner diameter was increased by machining to 0.914 in. The weight is further reduced by drilling seven holes each 0.25 in. in diameter around its periphery. The silly putty balls are held inside the chamber by means of plastic wrap (Saran wrap) stretched over the ends and held in place by an adhesive tape so that the silly putty balls are free to bounce around inside the chamber. The total weight of the chamber is 8.438 gms.

3.2.4 Water-Filled Balloon Dampers

This consists a toy balloon half-filled with ordinary tap water and half-filled with air so that the inflated balloon is about 0.6 in. in diameter and weighs 8.423

gms. The air space is left so that the water is free to oscillate inside the balloon for better energy absorption.

3.2.5 Egg-Filled Balloon Dampers

This consists of the same balloons referred to in Section 3.2.4 but in this case they are half-filled with a beaten up egg and the remainder with air resulting in a diameter of 0.60 in. and weighing 8.28 gms. The Egg was used as a viscous fluid for energy absorption because motor oils were first used and were found to attack the balloons chemically.

3.2.6 Water-Filled Plastic Egg Dampers

This damper consists of hollow plastic egg shaped chamber (ellipsoidal) with a minor diameter of 1.5 in. and a major diameter of 2.0 in. The chamber weighs 5.9 gms. when empty. Water is injected into the chamber through a hole at one end so that it is half-filled giving it a weight of 17.16 gms. Enough air space is left in the chamber for the water to oscillate.

3.2.7 Oil-Filled Plastic Egg Dampers

This damper consists of the same plastic egg chambers referred to in Section 3.2.6 but instead they are injected with high performance gear oil, SAE 80W-80W-90, Part No. 831. The weight of this damper is 16.7 gms.

3.3 Experimental Procedure

3.3.1 For Natural Vibrations

Natural vibration is induced in the member by suspending a constant static load of 3.9 lbs at midspan, which causes an initial center deflection of 0.49 in., and then releasing the load suddenly by cutting the suspending string. Three test runs are conducted for each configuration and the deflection-time response recorded. The probe is then shifted to the ends and the experiment repeated to get the deflection-time response for the member ends. Configurations of 1, 2, 3, and 5 dampers are investigated for the wool swab dampers, copper brush dampers, silly putty in chamber dampers and water-filled balloon dampers. Only one configuration of two equally spaced dampers is investigated for the egg-filled balloons, water-filled plastic eggs and oil-filled plastic egg dampers. From the deflection-time response, the logarithmic decrement is determined from the first ten cycles which in turn is used to determine the damping ratio.

The logarithmic decrement δ is determined as follows (Reference 11):

$$\delta = \frac{1}{n} \log_e \frac{X_1}{X_n} \tag{18}$$

in which:

X_1 = amplitude of first cycle,

X_n = amplitude of n^{th} cycle (n=10).

The damping ratio ζ is determined as follows:

$$\zeta = \frac{\delta}{\sqrt{((2\pi)^2 + \delta^2)}} \tag{19}$$

Knowing the damping ratio, the coefficient of damping C is determined from the following expression:

$$C = \zeta C_c \tag{19a}$$

in which:

C = the coefficient of damping

$$C_c = 2\sqrt{EI\rho\lambda} \tag{19b}$$

The efficiency index, η of the dampers is calculated using:

$$\eta = \frac{(\zeta - \zeta_0)}{M_d} \tag{20}$$

in which:

ζ_0 = damping ratio of the member without any dampers,

ζ = damping ratio for the member with dampers,

M_d = total mass of the damper assembly.

The natural frequency of vibration for the member is obtained using the following formula:

$$f = \frac{1}{2\pi}\sqrt{\frac{192\,EI\,K_t\,(2\,EI + K_r\,L)}{M_s\,[96\,EI\,(2\,EI + K_r\,L) + 2\,K_t\,L^3\,(4\,EI + 0.5\,K_r\,L)]}} \tag{21}$$

where K_r is given by Equation 16, and K_t is defined in Section 2.1. The derivation of Equation 21 is given in Appendix A, wherein the system mass M_s is also defined.

3.3.2 For Forced-Free Vibration

Forced-free vibrations are conducted for 1, 2, 3, and 5 equally spaced water-filled balloon dampers. The vibrator connector is engaged into position by locking it to the member by means of a clamp. The vibrator is then activated, forcing the member to vibrate with a frequency controlled by the oscillator. When a steady vibration amplitude is reached the vibrator is disengaged and the member allowed to vibrate naturally. The vibrator employed for this tests allowed only a limited amount of travel which meant that the deflection of the member at the location of the vibrator was restricted to the maximum travel of the vibrator and this affected the results obtained. Nevertheless, forced-free vibration tests were conducted with forcing function frequencies of 2, 3, 4, 5 and 6 Hz. Base experiments are also conducted on the member with no dampers for the same frequencies. Three test runs were conducted at each frequency and an average value of ζ obtained. An investigation was also carried out to determine the resonance frequency of the member. Due to the limitations of the vibrator travel, the forcing function was applied at the one- third point of the member and the deflection readings taken at member midspan for frequencies ranging from 2 Hz to 6 Hz. To evaluate the performance of a damping concept the following "amplitude reduction factor," A_r , is utilized:

$$A_r = \frac{\mu_0 - \mu}{M_d}$$

in which μ_0 and μ are the member dynamic amplification factors for member with no dampers and with dampers, respectively.

3.4 Test Results and Discussion

3.4.1 For Member Natural Vibration

The test results for the natural vibration tests are presented in Tables 1-4 and Figures 11 and 12. Table 1 summarizes the damping ratios for the various damping concepts. Tables 2 and 3 present a summary of the average values of the efficiency index and the damping ratio. In summary:

1. The damping ratio of the empty member is found to be 6.4×10^{-3}.
2. The maximum average damping ratio of 8.05×10^{-3} is obtained by using 2 water-filled balloon dampers with a corresponding efficiency index of 17.3 as shown in Tables 3 and 4. However, the maximum efficiency index of 28.8 is obtained by using one wool swab damper at the center.
3. From Figure 11, which shows the efficiency index vs. number of dampers, it is observed that the peaks occur at one damper for all of the various damping concepts. This implies that a single damper at the center provides the greatest efficiency with respect to the mass, as compared to 2, 3, or 5 dampers. Note that the copper brush dampers provide a negative efficiency for 1, 2, and 3 dampers and only exhibit a positive efficiency for 5 dampers. Likewise, the wool swab dampers exhibit a negative efficiency at 5 dampers even though they give the highest efficiency of 28.8 for 1 damper.
4. The softer dampers, namely, the water-filled balloon dampers and the wool swab dampers perform better for 1, 2, and 3 dampers whereas the

stiffer dampers, i.e, the silly putty and copper brush dampers perform better for 4 and 5 damper configurations.

3.4.2 For Forced-Free Vibration

Forced-free vibrations are also conducted for 1, 2, 3, and 5 equally spaced water-filled balloon dampers at forcing function frequencies of 1, 2, 3, 4, 5, and 6 Hz. The results of this study are presented in Tables 5 and 6.

In Table 5, which shows the results of the free vibration after disengaging the forcing function, the ratio of the dynamic midspan amplitude, Δ_D, to the static deflection, Δ_s, is also given for the forcing function located at member midspan. It is observed that during the forced vibration, 2 and 5 water filled balloons lead to a reduction in the dynamic magnification factor at a forcing function frequency of 6 Hz. however, 3 and 5 balloon dampers cause an increase in the dynamic amplication factor at 6 Hz. A reduction in dynamic amplication factor is also obtained by 5, 1, 3, and 5 balloon dampers at frequencies of 3, 4, 2, 5 Hz, respectively. Figure 24 shows the experimental Δ-t relationship for 2 and 5 balloon dampers. However, the water-filled balloons are found to have a detrimental effect on the member natural vibration after disengaging the forcing function, leading to a negative efficiency indices as shown in Table 5. However, the dampers can lead to positive results during the forced vibration phase as shown by the positive amplitude reduction factors in Table 5.

3.5 Performance of Damping Concepts

The damping concepts investigated in this research are found to be beneficial for the member under natural vibration. The maximum efficiency index is of 28.8 is realized using one wool swab damper whereas it is 23.7 when two water-filled balloon dampers are used. In general, the various types of dampers worked well if only 1 or 2 of them were used, as evident from Figures 11 and 12.

The water-filled balloon dampers provide a negative efficiency index after disengaging the forcing function from the member. During the forced vibration phase, these dampers can provide effective member damping under certain conditions, as indicated by some of the positive amplitude reduction factors listed in Table 5.

4. TEST VERSUS THEORY FOR NATURAL VIBRATION

The algorithm for central finite-difference solution is formulated in Section 2.1. To ensure convergence and numerical stability a time increment, Δt, of 0.001 sec., and a panel length of L/12, are found to be sufficient. The algorithm is found to be quite sensitive to variations in the rotational stiffness, K_r. Figure 21 shows a sample comparison between the theoretical and the experimental Δ-t plots for the member natural vibration with no dampers. The theoretical amplitudes are found to be within 10% of the experimental values. The frequency from the finite-difference solution, from the formula in Equation 21, and from the experiment is found to be 3.0 Hz, 3.12 Hz, and 2.83 Hz, respectively.

5. CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusions

A. Under Natural Vibration

1. A single damper at member midspan provides the maximum damping efficiency compared to 2, 3, or 5 dampers.
2. If 1 or 2 dampers are used, those made of softer material, namely wool swab and water-filled balloon, perform better than those with harder material, that is, copper brush and silly putty in chamber.
3. Two water-filled balloons provide the maximum damping ratio.

B. Under Forced-Free Vibration

1. The water-filled balloons give negative damping efficiencies upon disengagement of the forcing function from the member.
2. Under certain conditions, the water-filled balloons can result into positive amplitude reduction factors.

From the above conclusions it is evident that passive damping provides a possible approach to structural vibration reduction.

5.2 Future Research

More research needs to be conducted on passive damping concepts. The member support system should be modified considerably in order to achieve relatively low-stiffness translational boundary conditions. Furthermore, theoretical analysis should also be formulated for the forced vibration case.

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TABLES

TABLE 1. Damping Ratios from National Vibrations for the Various Dampers

Damper Type	Number of Devices	ζ_1 (x 10^{-3})	ζ_2 (x 10^{-3})	ζ_3 (x 10^{-3})	ζ_{AVG} (x 10^{-3})
None	0	6.53	6.32	6.35	6.40
Water-Filled Balloons	1	7.60	7.46	7.56	7.54
	2	8.28	8.15	7.94	8.05
	3	6.55	6.75	6.59	6.61
	5	7.28	7.15	7.24	7.24
Copper Brush	1	5.92	5.99	5.96	5.96
	2	6.14	6.05	5.98	6.08
	3	6.83	6.86	7.06	6.92
	5	7.75	7.78	7.09	7.75
Silly Putty in Chamber	1	7.19	6.98	7.09	7.05
	2	6.95	6.68	6.68	6.75
	3	7.87	7.89	7.16	7.64
	5	7.71	7.79	7.53	7.69
Wool Swab	1	7.66	7.51	7.61	7.59
	2	7.71	7.08	6.90	7.23
	3	5.38	5.23	5.41	5.34
	5	6.31	6.39	6.20	6.30
Oil-Filled Plastic Eggs	2	6.86	6.87	6.69	6.83
Water-filled Plastic Eggs	2	7.77	7.35	7.23	7.53
Egg-Filled Balloons	2	7.02	6.78	6.88	6.93

TABLE 2. Efficiency Indices for Various Damping Devices (x 10^{-3})

Number of Dampers	Type of Passive Dampers						
	Wool Swabs	**Silly Putty in Chambers**	**Copper Brushes**	**Water-Filled Balloons**	**Water-Filled Plastic Eggs**	**Oil-Filled Plastic Eggs**	**Egg-Filled Balloons**
1	28.8	11.2	-5.8	23.7	---	---	---
2	10.0	3.5	-0.9	17.3	5.8	2.3	5.6
3	-1.5	-.2	2.3	1.5	---	---	---
5	-4.8	5.4	3.2	3.5	---	---	---

TABLE 3. Damping Ratios for Various Damping Devices (x 10^{-3})

Number of Dampers	Type of Passive Damper						
	Wool Swabs	Silly Putty in Chamber	Cooper Brush	Water-Filled Balloons	Water-Filled Plastic Eggs	Oil-Filled Plastic Eggs	Egg-Filled Balloons
1	7.54	7.05	5.96	7.54	---	---	---
2	7.23	6.75	6.08	8.05*	7.53	6.83	6.93
3	5.34	7.64	6.92	6.61	---	---	---
5	6.30	7.69	7.75	7.24	---	---	---

* Maximum damping ratio.

TABLE 4. Summary of Member Natural Vibration Test Results

Type of Damper	**Number of Dampers**	**Weight of Damping Assembly (gms)**	**Average Damping Ratio (x 10^{-3})**	**Efficiency Index (in/lb-sec^2)**
No Dampers	0	0.00	6.40	0.0
Copper Brush Dampers	1	13.20	5.96	-5.8
	2	26.39	6.08	-0.9
	3	39.60	6.92	2.3
		65.96	7.75	3.2
Wool Swab Dampers	1	7.23	7.54	28.8
	2	14.46	7.23	10.0
	3	21.69	5.34	-1.5
	5	36.15	6.30	-4.8
Silly Putty in Chamber Dampers	1	8.42	7.05	11.2
	2	16.88	6.75	3.5
	3	25.32	7.64	-9.2
	5	42.20	7.69	5.4
Water-Filled Balloon Dampers	1	8.42	7.54	23.7
	2	16.84	8.05	17.3
	3	25.26	6.61	1.5
	5	42.10	7.24	3.5
Oil-Filled Plastic Egg Dampers	2	16.56	6.93	5.6
Water-Filled Plastic Egg Dampers	2	34.32	7.53	5.8
Oil-Filled Plastic Egg Dampers	2	33.40	6.83	2.3

TABLE 5. Member Forced-Free Vibration Test Results
for Water-Filled Balloon Dampers

Number of Dampers	Forcing Function Frequency (Hz)	Δ_D/Δ_S	Average Damping Ratio (x 10^{-3})	Efficiency Index (in/lb-sec^2)	Amplitude Reduction Factor (in/lb-secs)
0	2	0.55	4.98	---	---
	3	0.51	4.59	---	---
	4	0.57	6.56	---	---
	5	0.51	6.96	---	---
	6	0.28	6.70	---	---
1	2	0.57	3.16	-37.7	-264.9
	3	0.53	4.13	-9.5	-264.9
	4	0.48	4.25	-47.9	1192.1
	5	0.41	6.25	-14.7	1324.5
	6	0.36	6.20	-10.4	-1059.6
2	2	0.53	4.16	-17.0	264.9
	3	0.51	5.13	-32.5	0.0
	4	0.57	4.10	-51.1	0.0
	5	0.53	6.22	-15.3	-264.9
	6	0.22	3.06	-37.9	794.7
3	2	0.55	2.25	-18.0	0.0
	3	0.51	3.38	-8.4	0.0
	4	0.53	3.58	-20.7	529.8
	5	0.51	4.94	-14.0	0.0
	6	0.48	5.34	-9.4	-2649.1
5	2	0.51	3.73	-8.6	529.8
	3	0.47	5.15	-2.3	529.8
	4	0.67	4.28	-9.5	-1324.5
	5	0.43	4.18	-11.5	1059.6
	6	0.22	4.02	-18.6	794.7

TABLE 6. Member Midpoint Amplitudes at Various Frequencies for the Empty Member With Forcing Function Located at One-Third Point

Frequency (Hz)	**Amplitude (in.)**
1.00	0.35
2.00	0.39
2.20	0.31
2.50	0.31
2.75	0.32
3.00	0.34
4.00	0.40
4.50	0.38
5.00	0.20
5.50	0.12
6.00	0.07
6.50	0.02
7.00	0.16
7.50	0.05
8.00	0.04
9.00	0.01
9.50	0.06
10.00	0.04

FIGURES

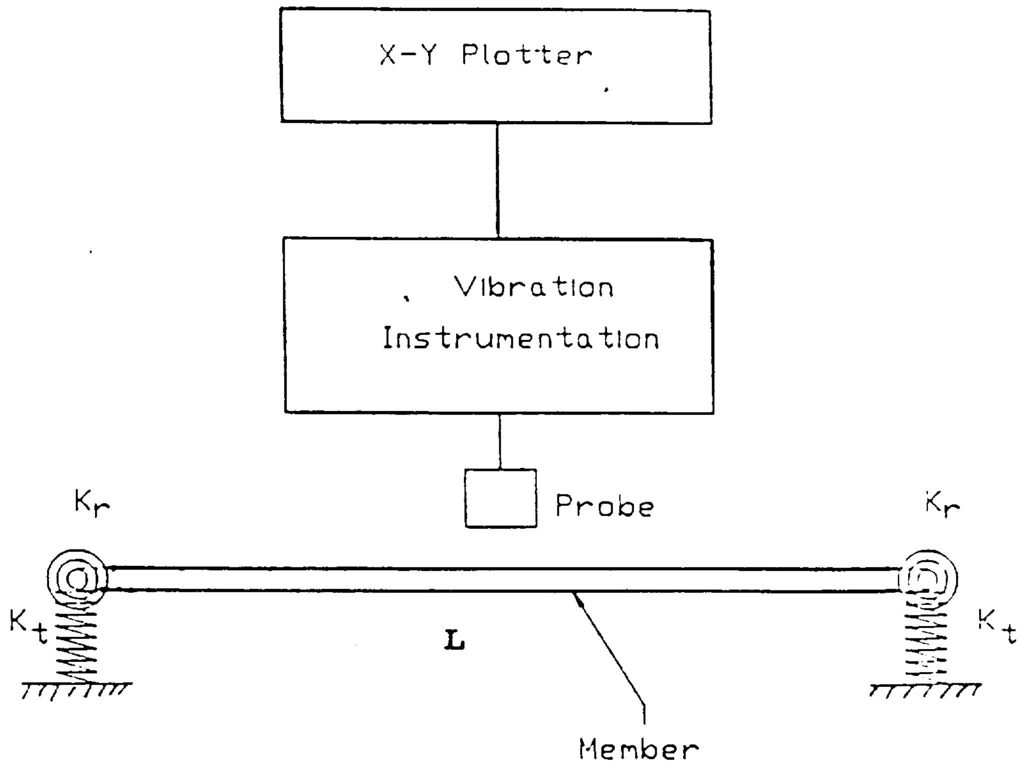


Figure 1. Test setup for member natural vibration

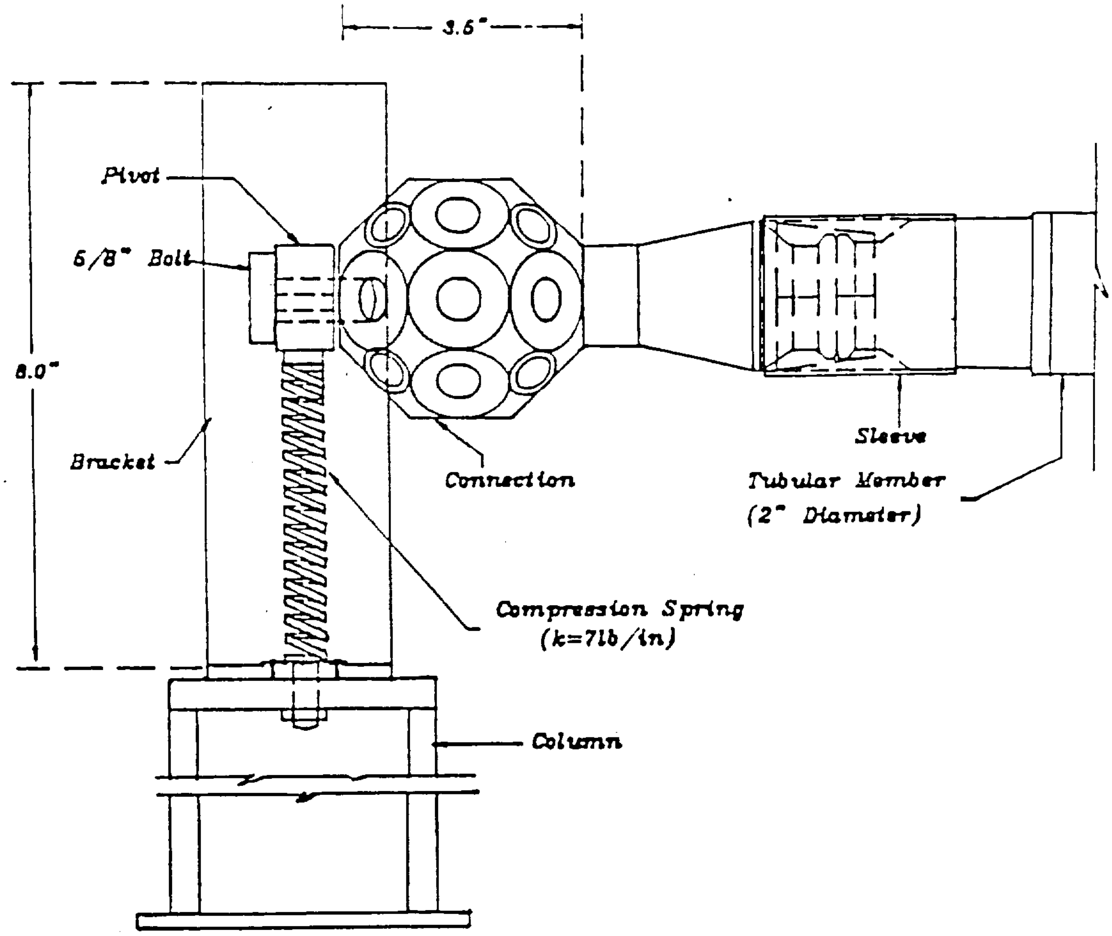
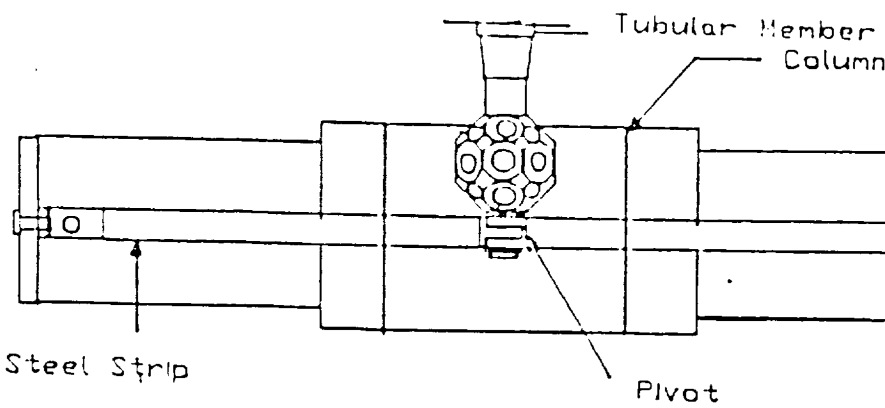


Figure 2. Details of member end connections (Section A-A of Figure 3a)

Plan View



End View

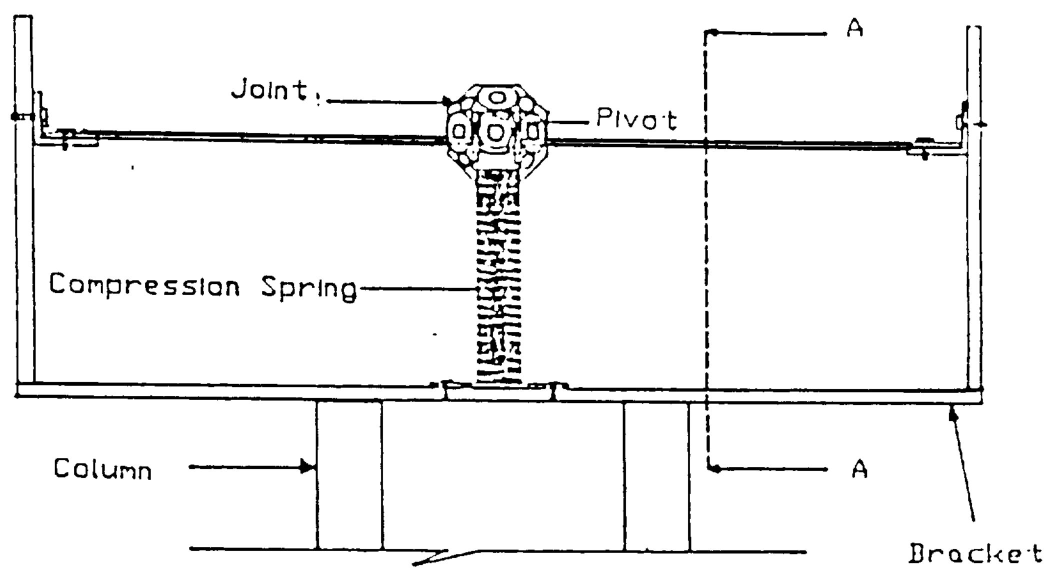


Figure 3a. Details of member end supports

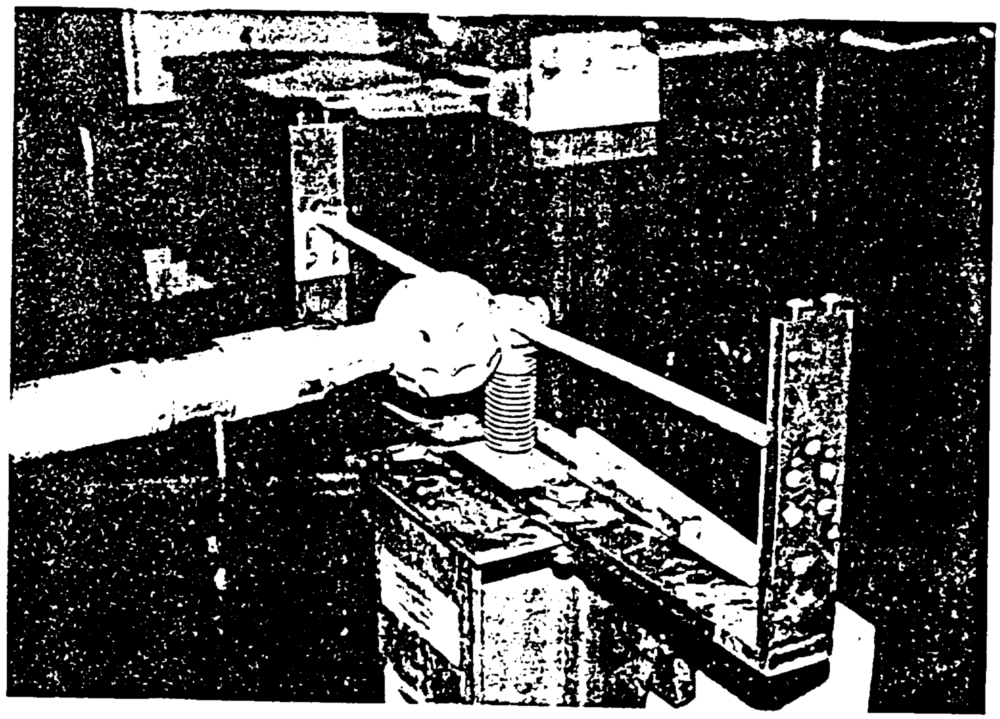


Figure 3b. End connection details

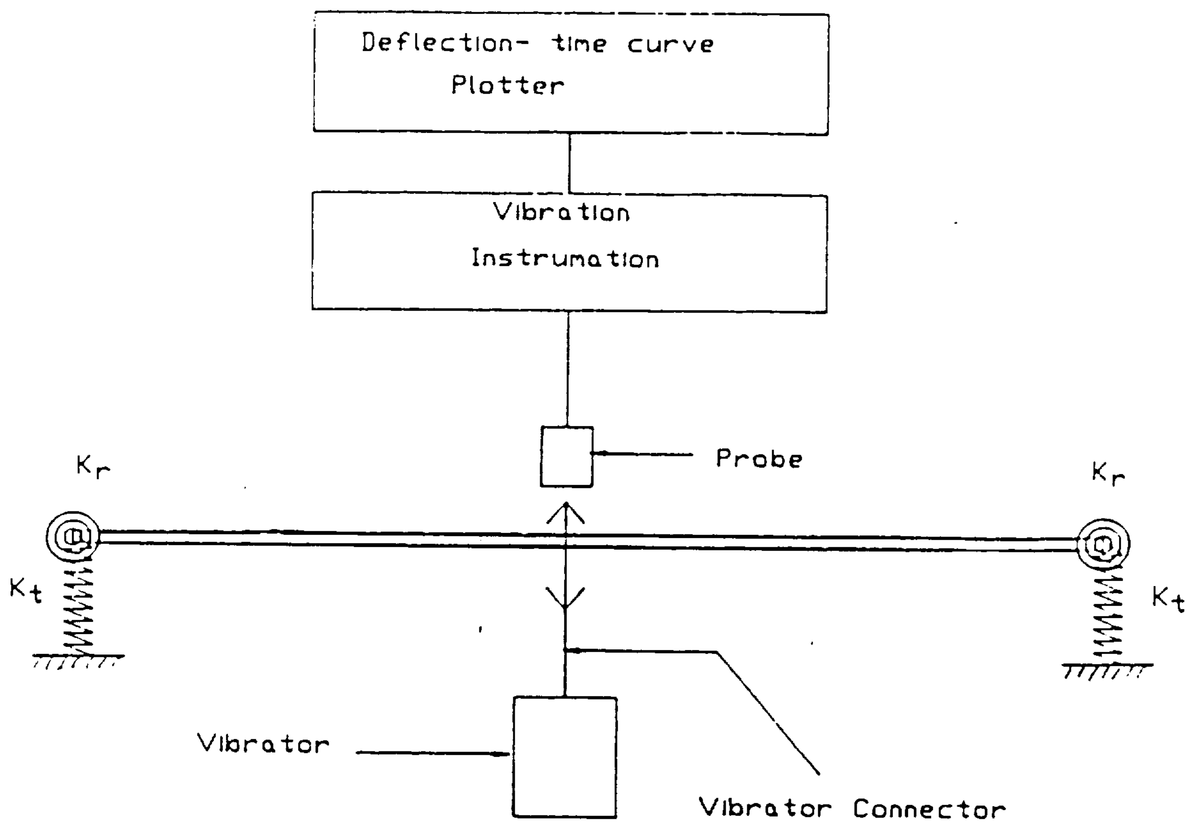


Figure 4a. Schematic of forced-free vibration test setup

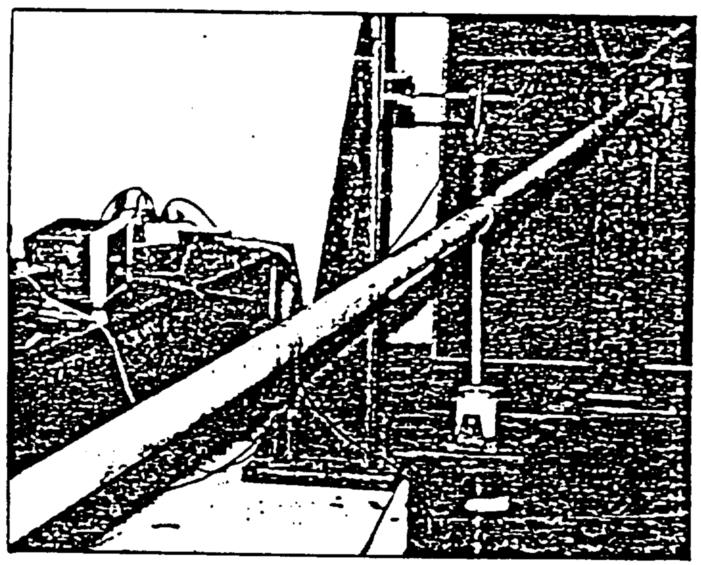


Figure 4b. Member forced-free vibration test setup

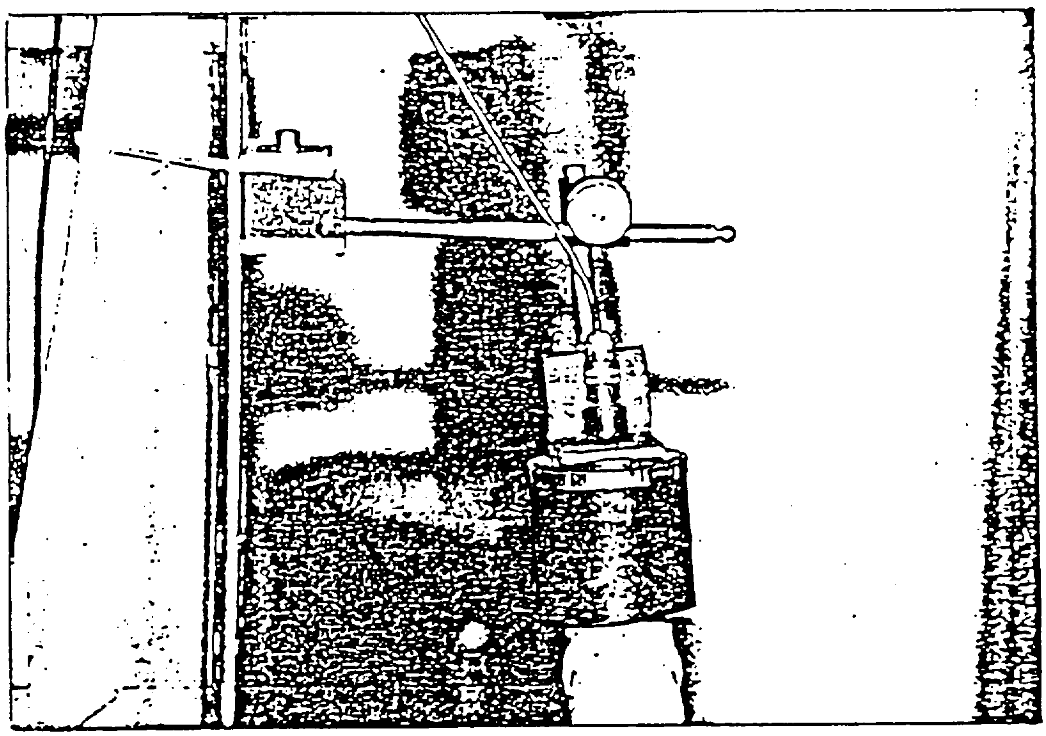


Figure 5. Proximity Probe

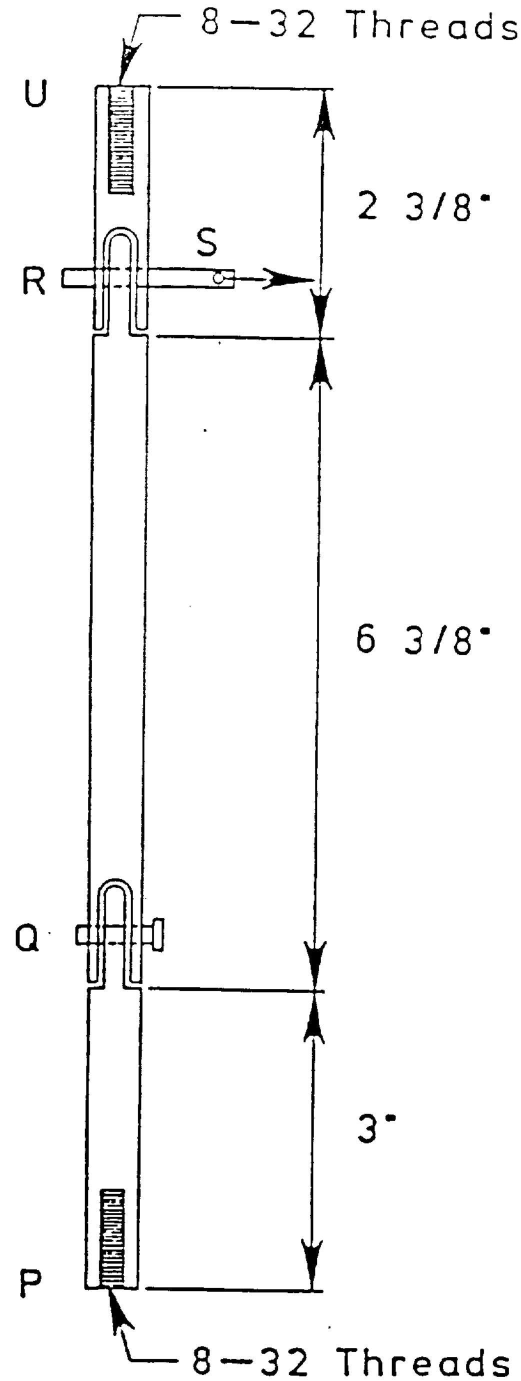
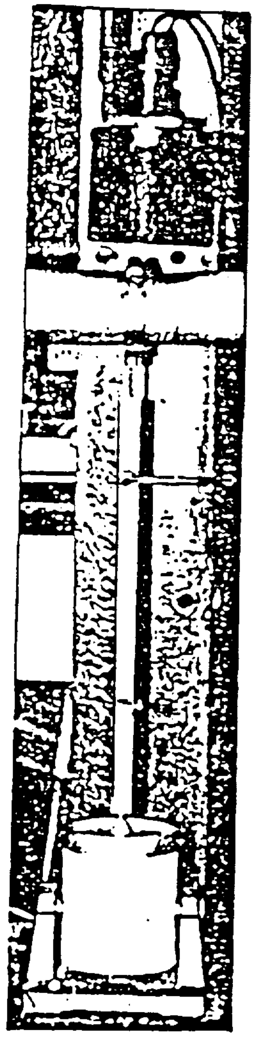
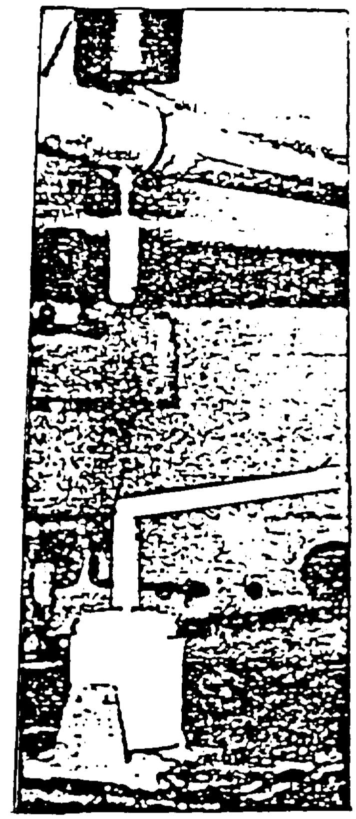


Figure 6a. Details of vibrator connection



(I) Vibrator connector in engaged position



(II) Disengaged position

Figure 6b. Vibrator connector in engaged and disengaged positions

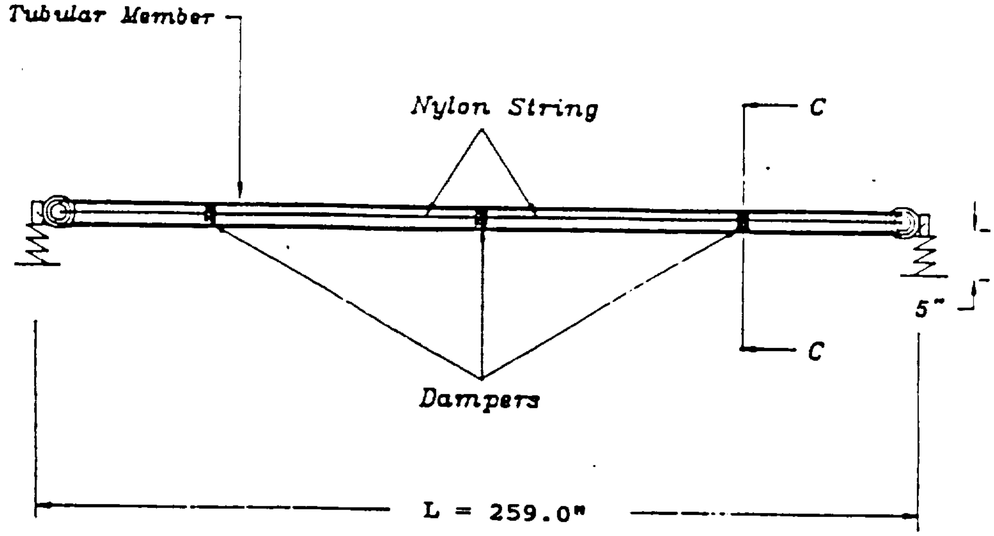


Figure 7a. Arrangement of passive dampers in tubular member

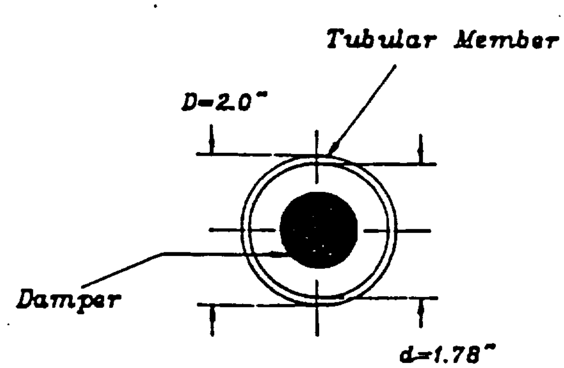


Figure 7b. Cross-sectional view of member at damper location

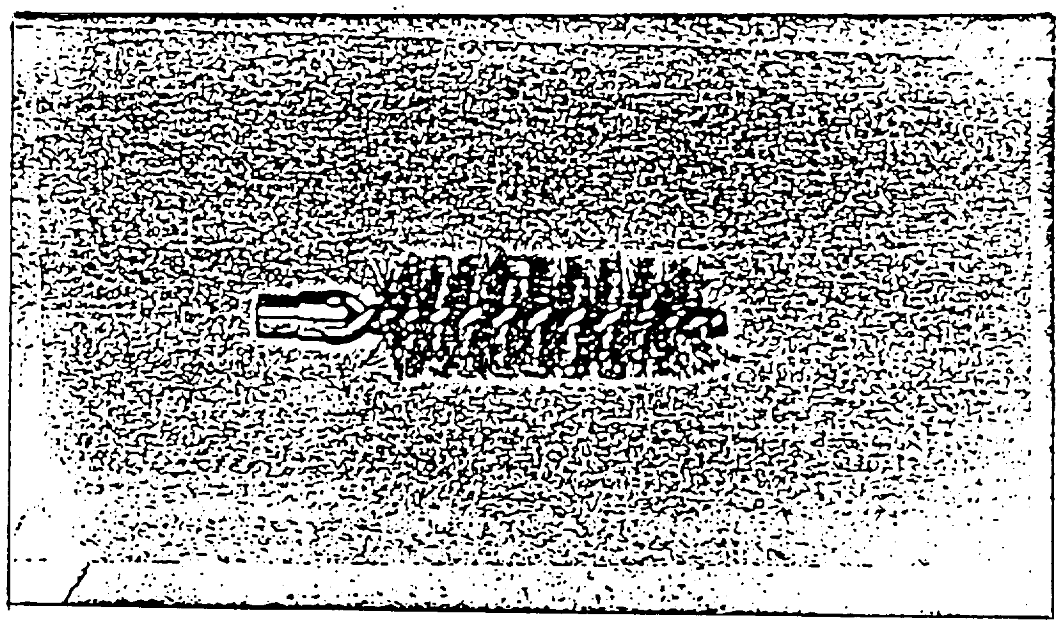


Figure 8. Wool swab damper

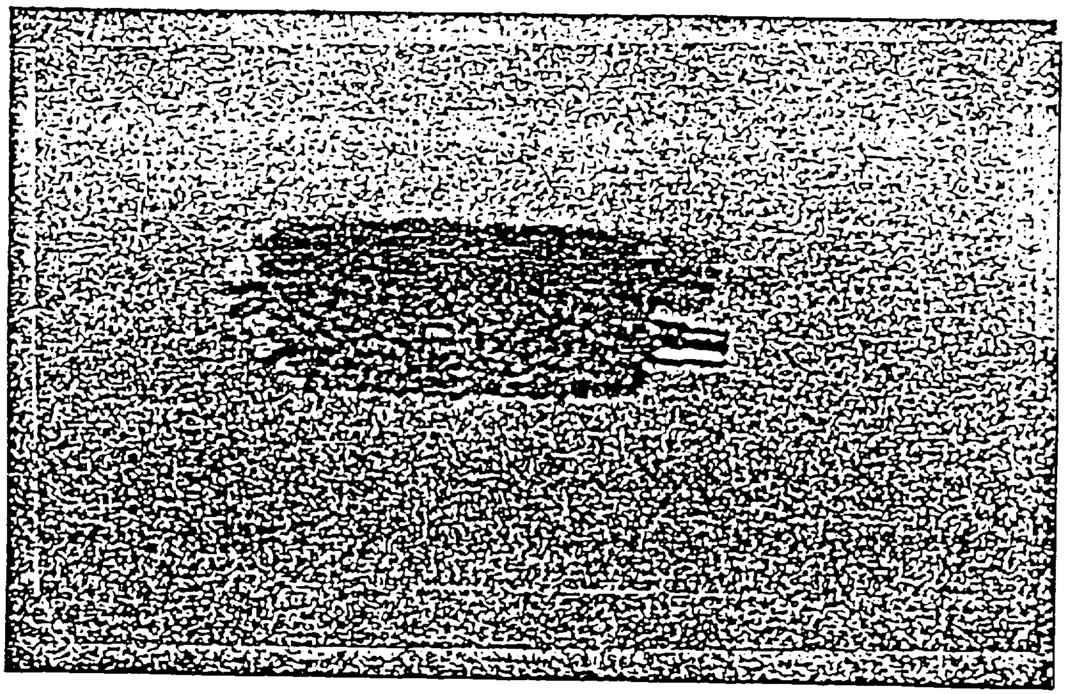


Figure 9. Copper brush damper

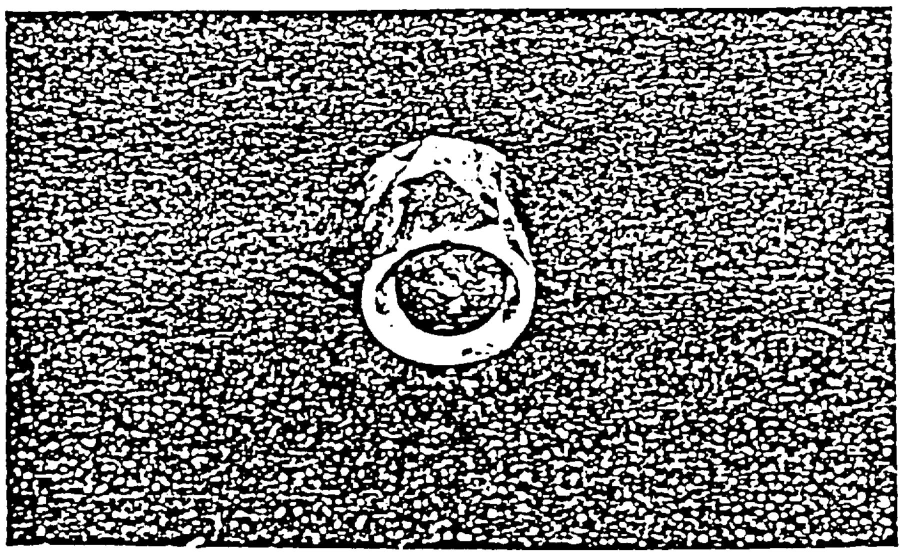


Figure 10. Silly putty in chamber damper

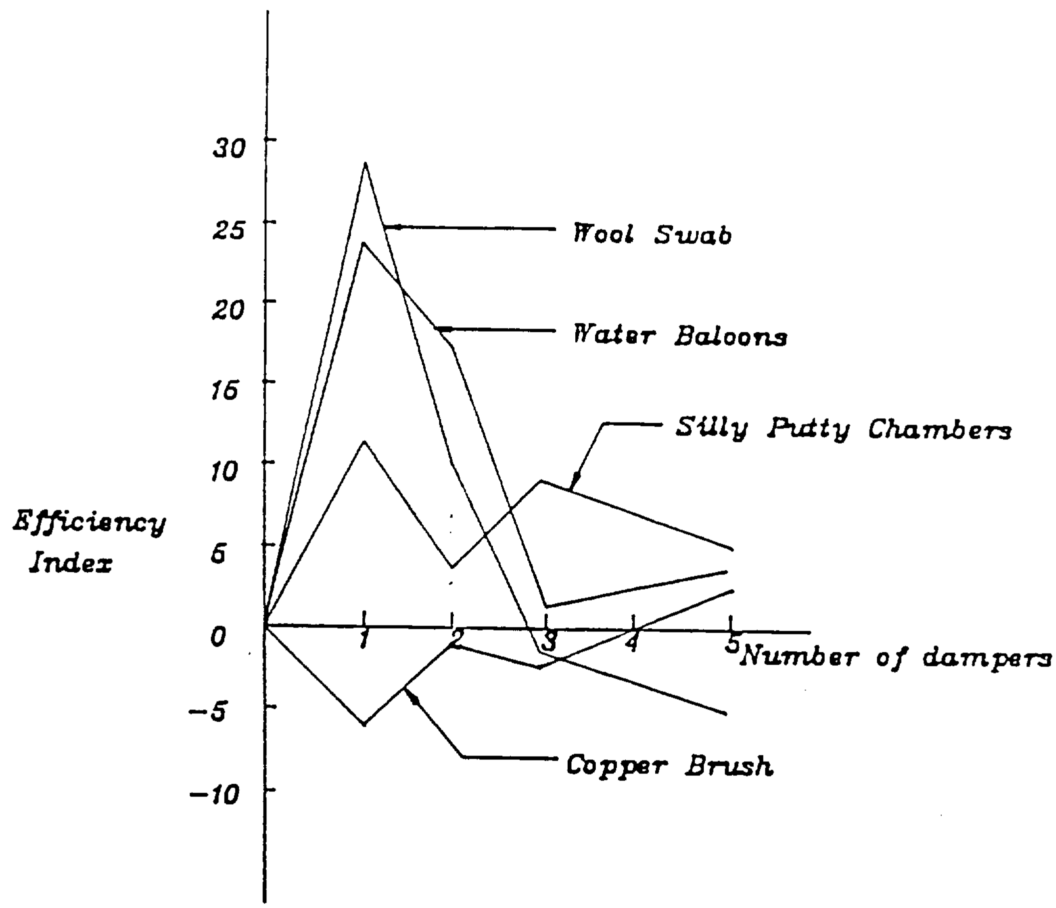


Figure 11. Efficiency index versus number of dampers

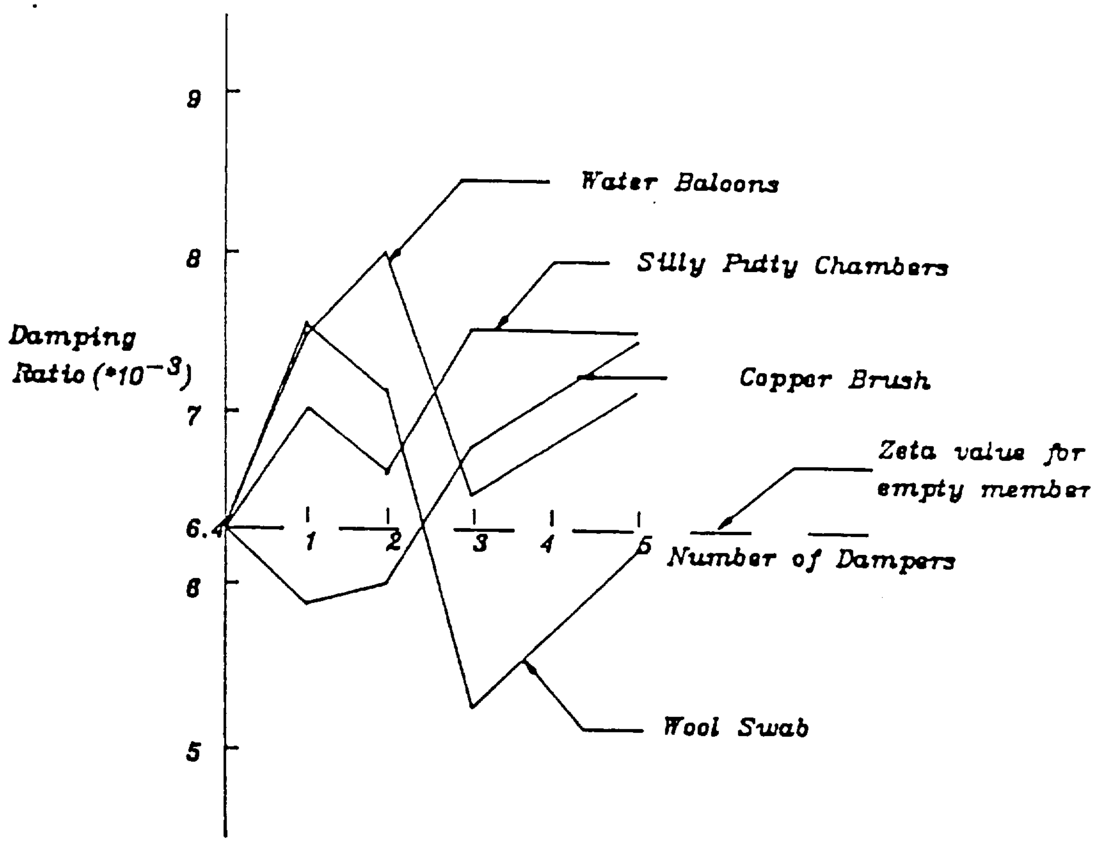


Figure 12. Damping ratios versus number of dampers

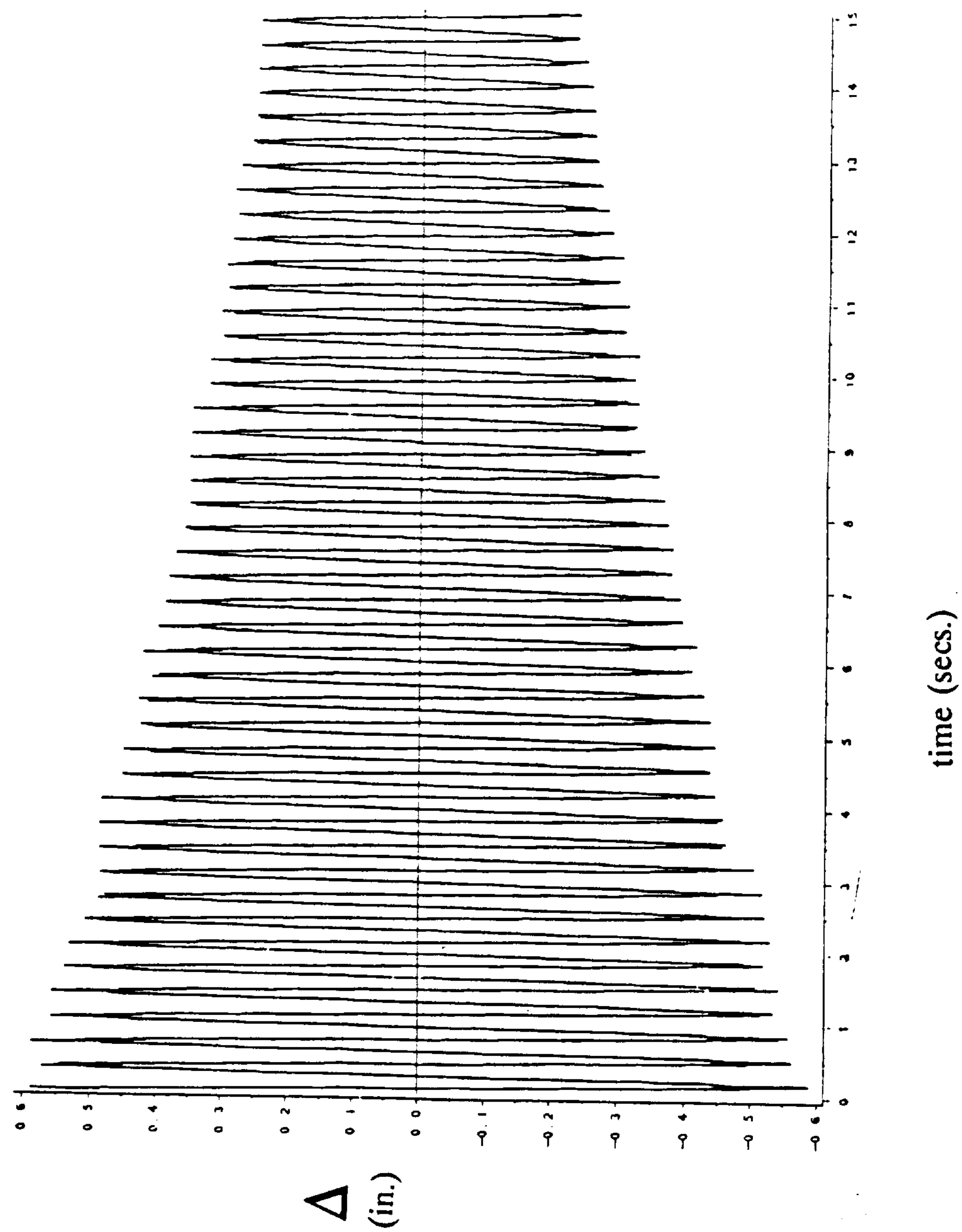


Figure 13. Theoretical Δ-t plot for member with no dampers

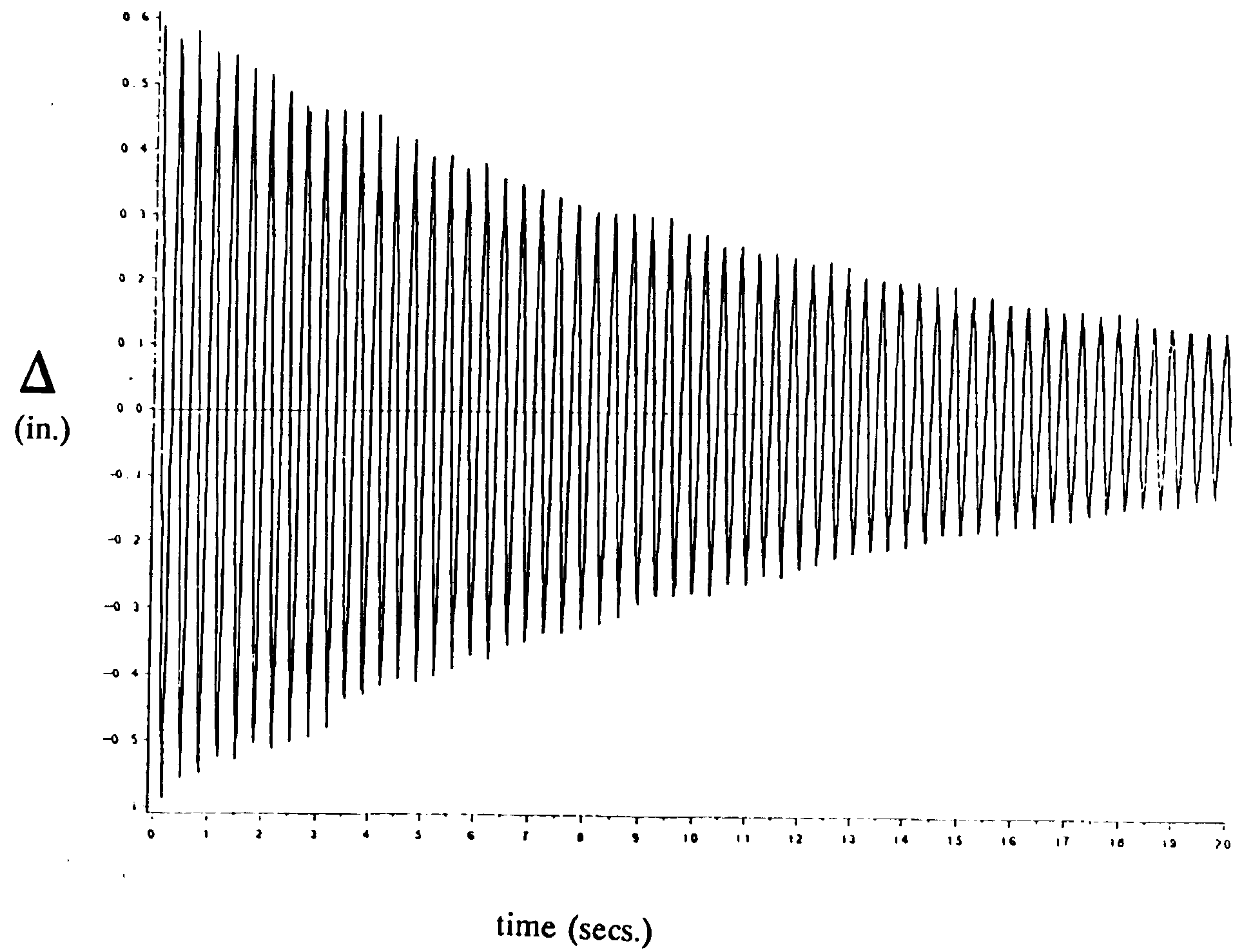


Figure 14. Theoretical Δ-t plot for member with two water-filled ballon dampers

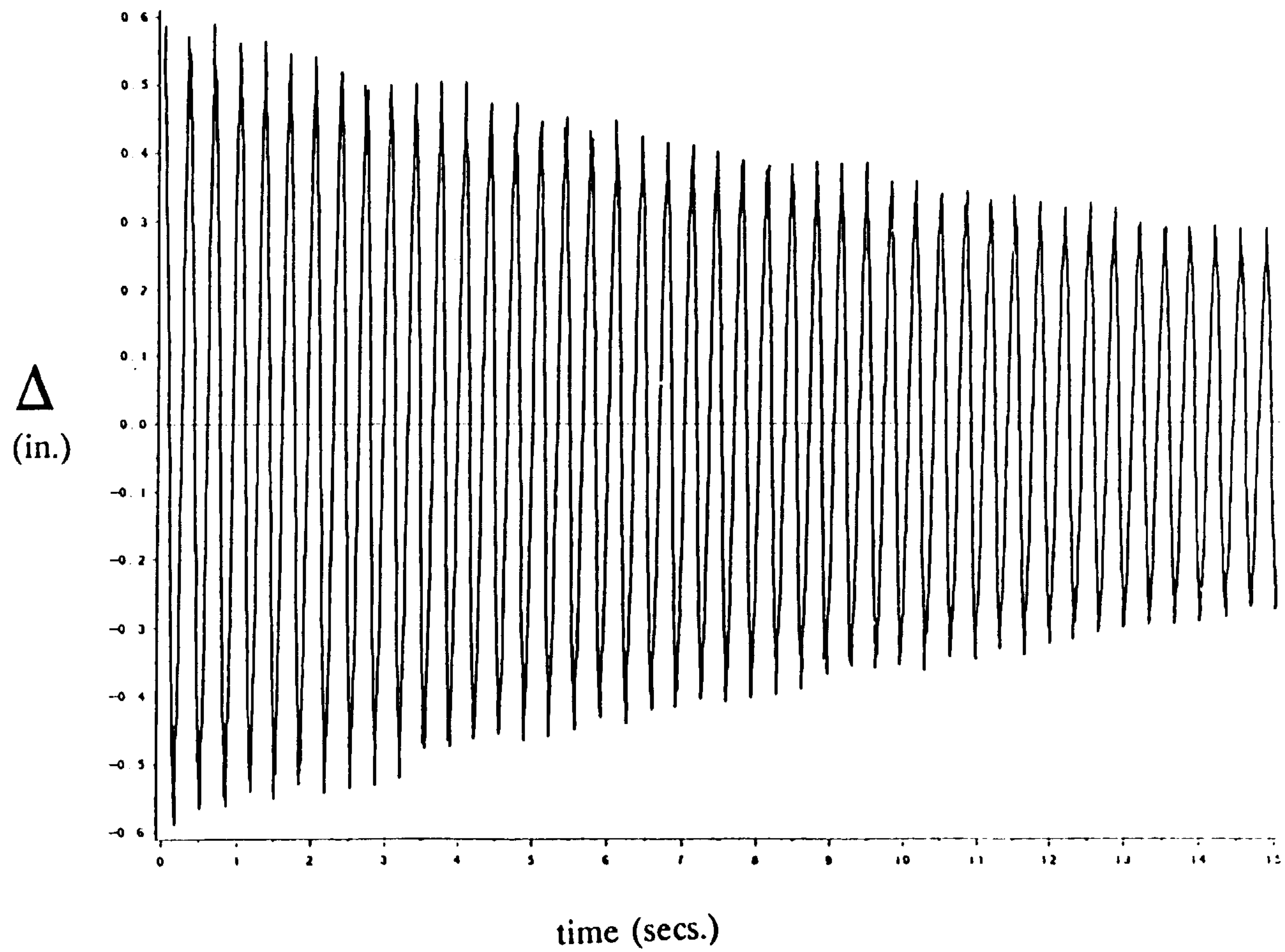


Figure 15. Theoretical Δ-t plot for member with three water-filled balloon dampers

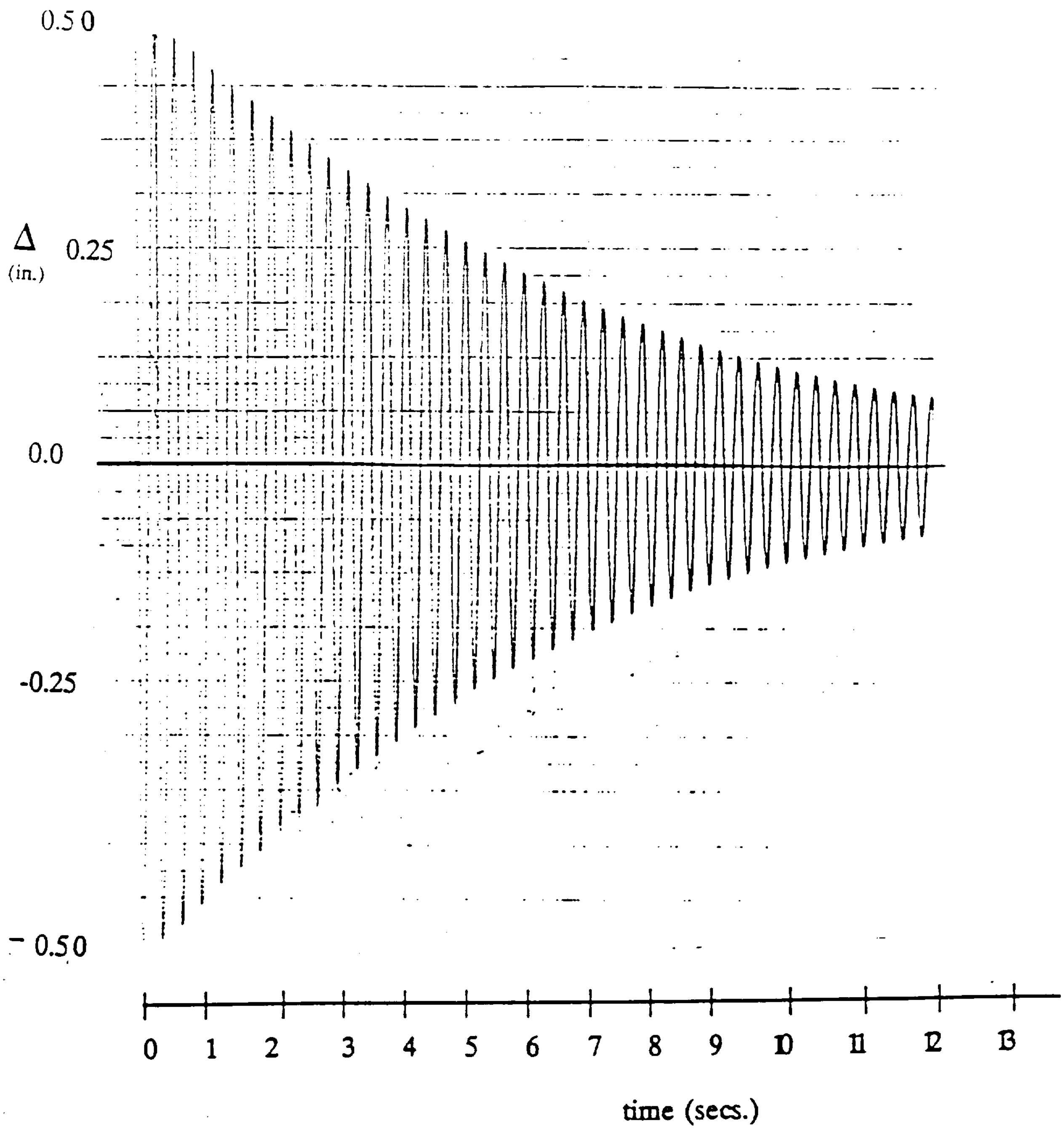


Figure 16. Experimental Δ-t relationship for member with two water-filled balloon dampers

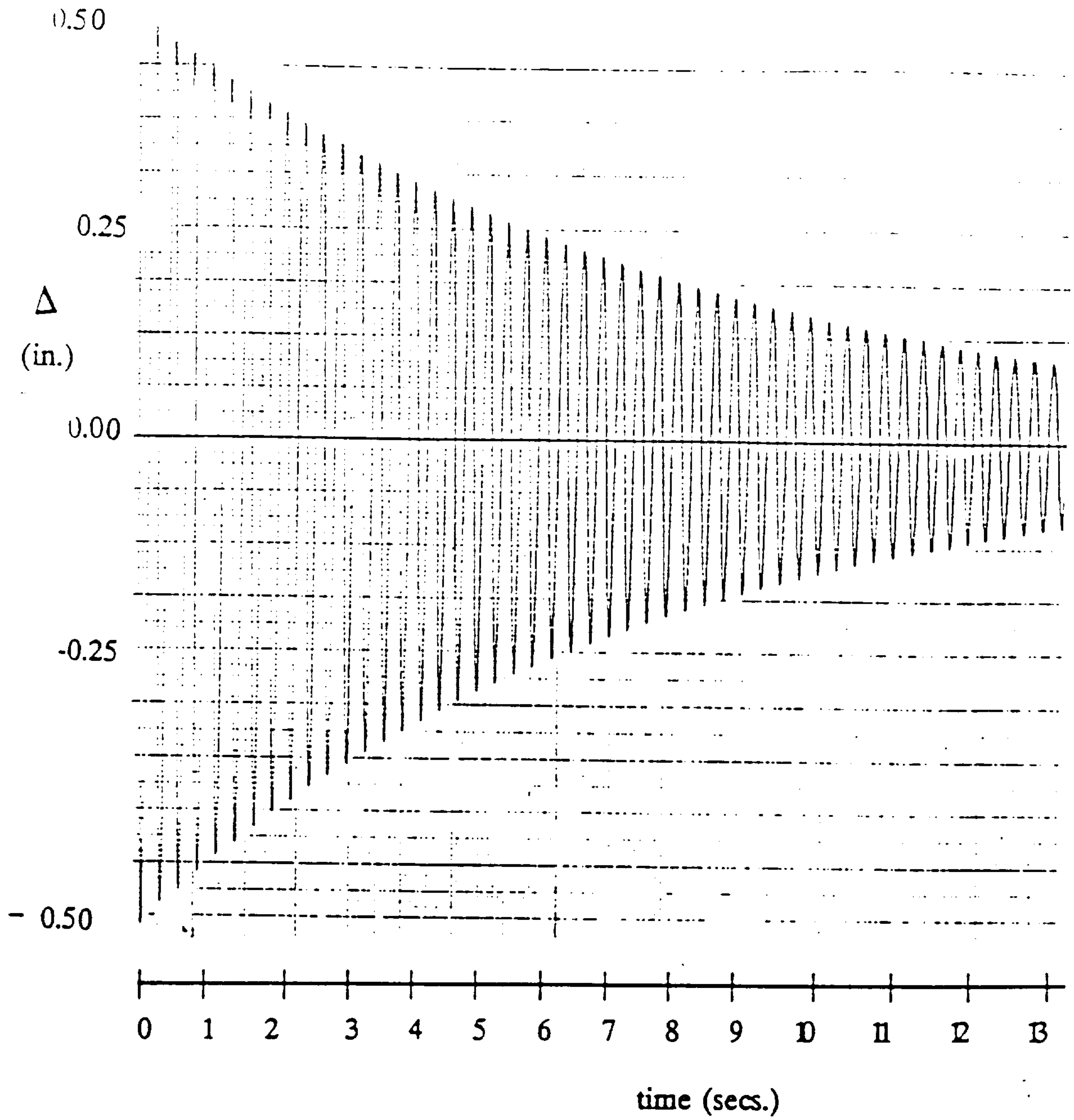


Figure 17. Experimental Δ-t relationship for member with one water-filled balloon damper

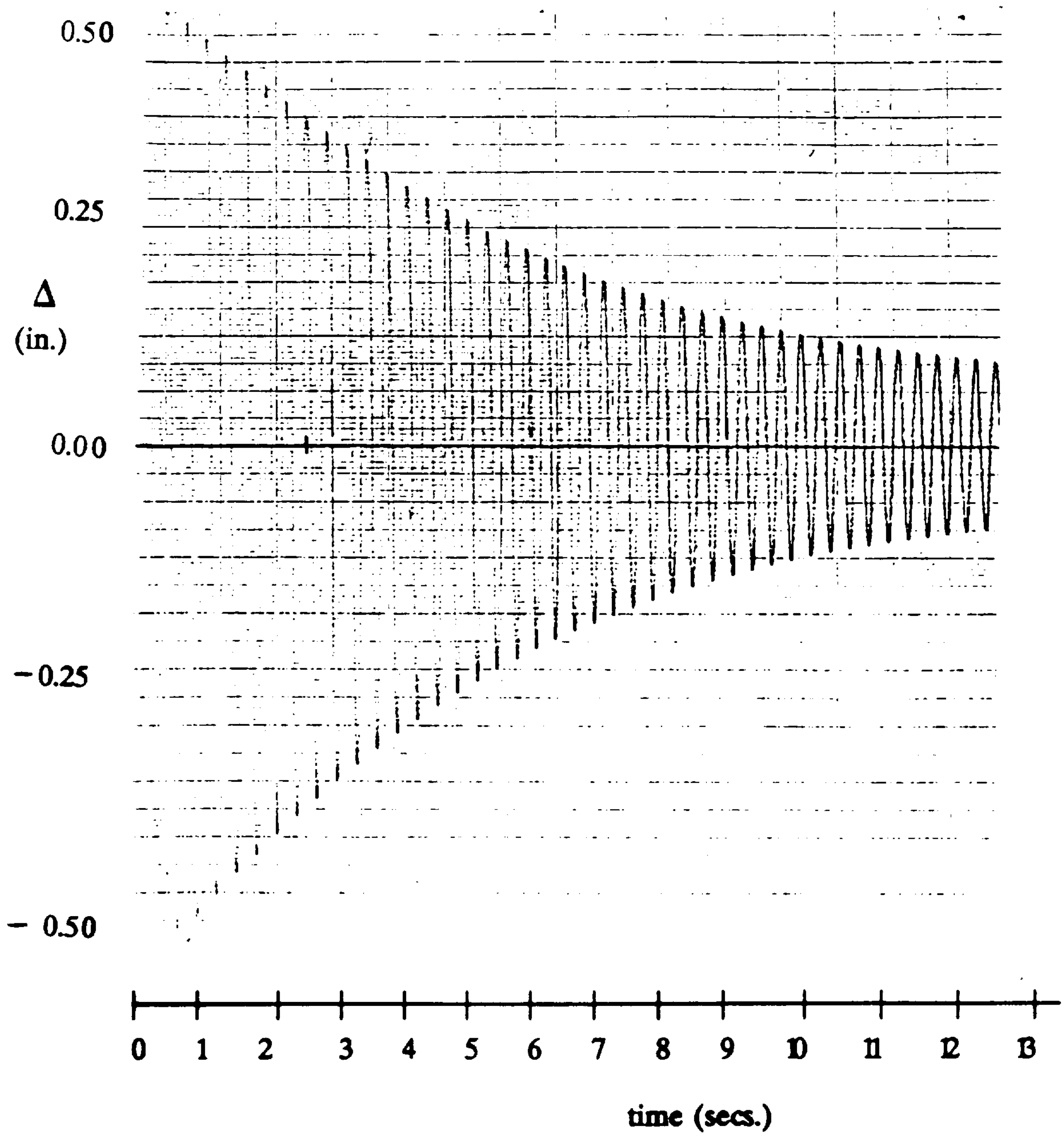


Figure 18. Experimental Δ-t relationship for member with no dampers

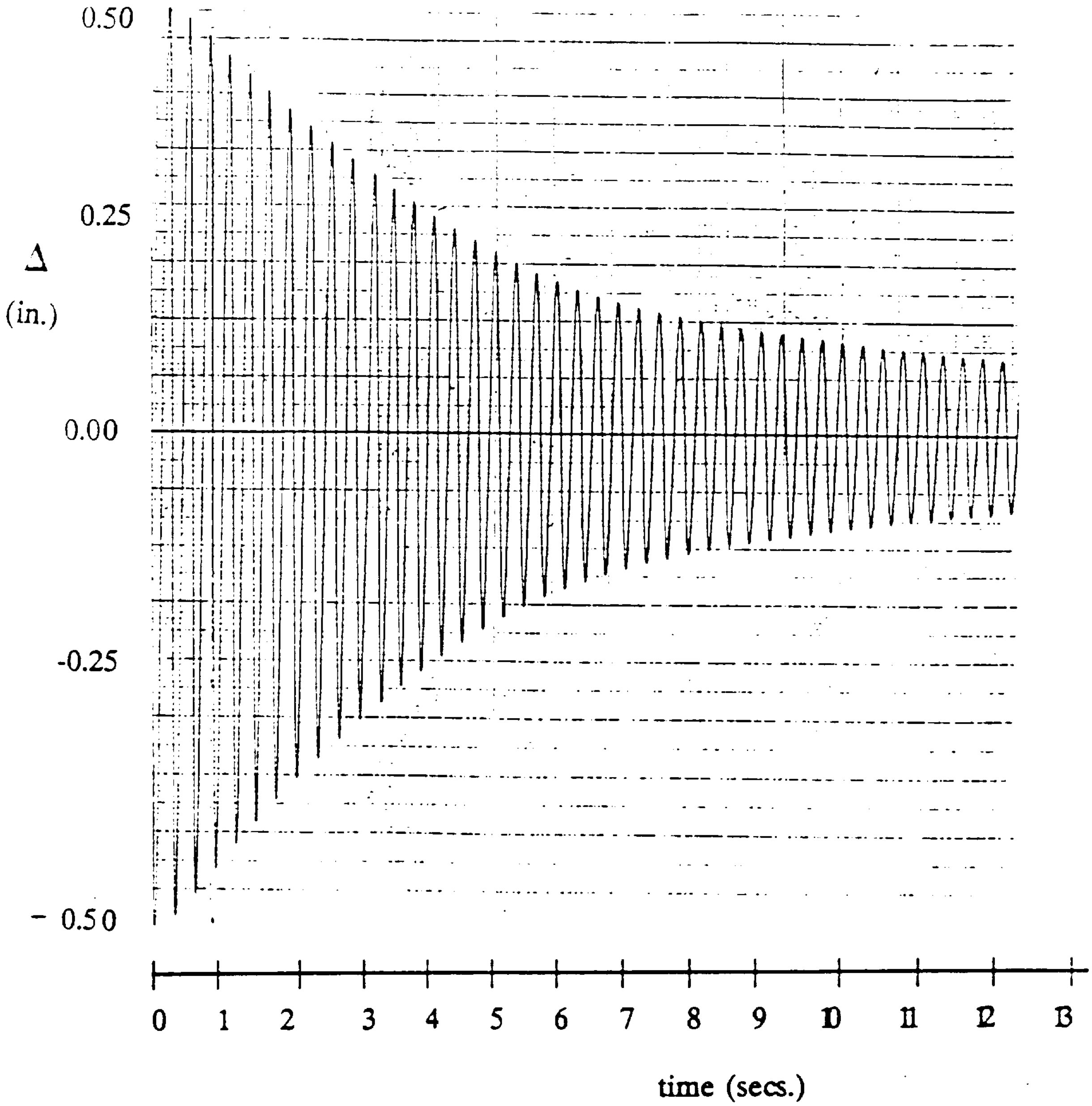


Figure 19. Experimental Δ-t relationship for member with one silly putty damper

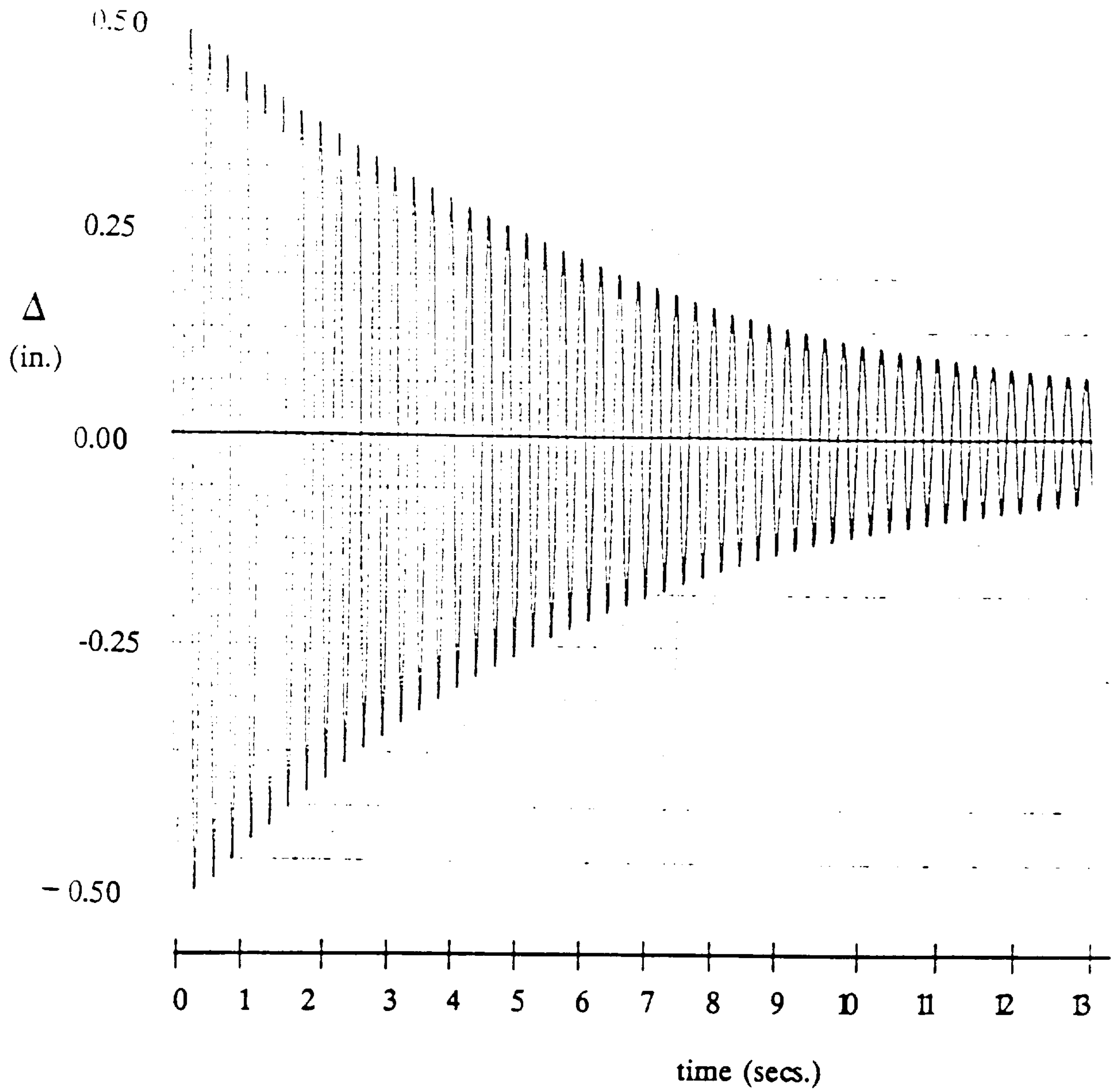


Figure 20. Experimental Δ-t relationship for member with two wool swab dampers

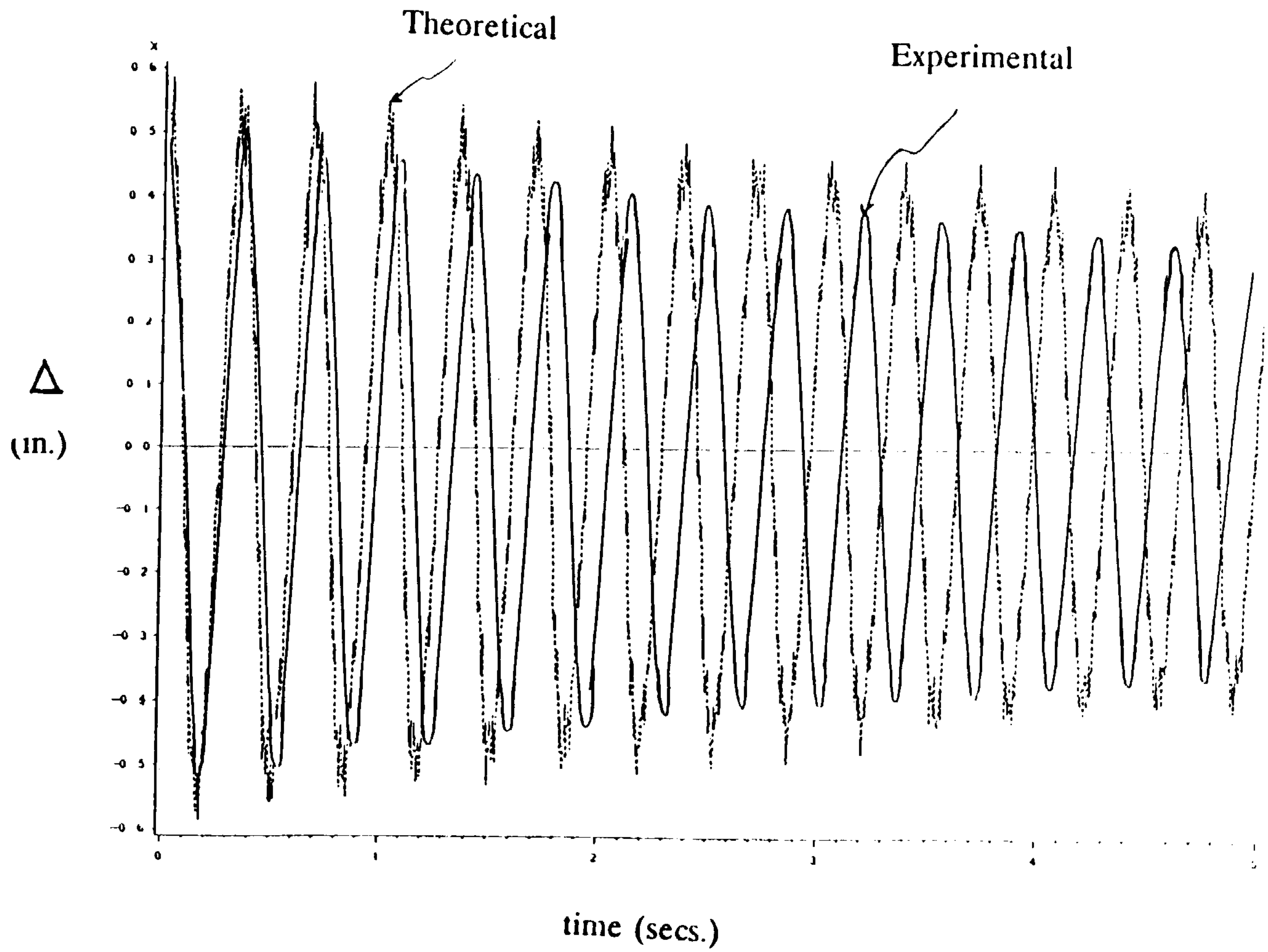
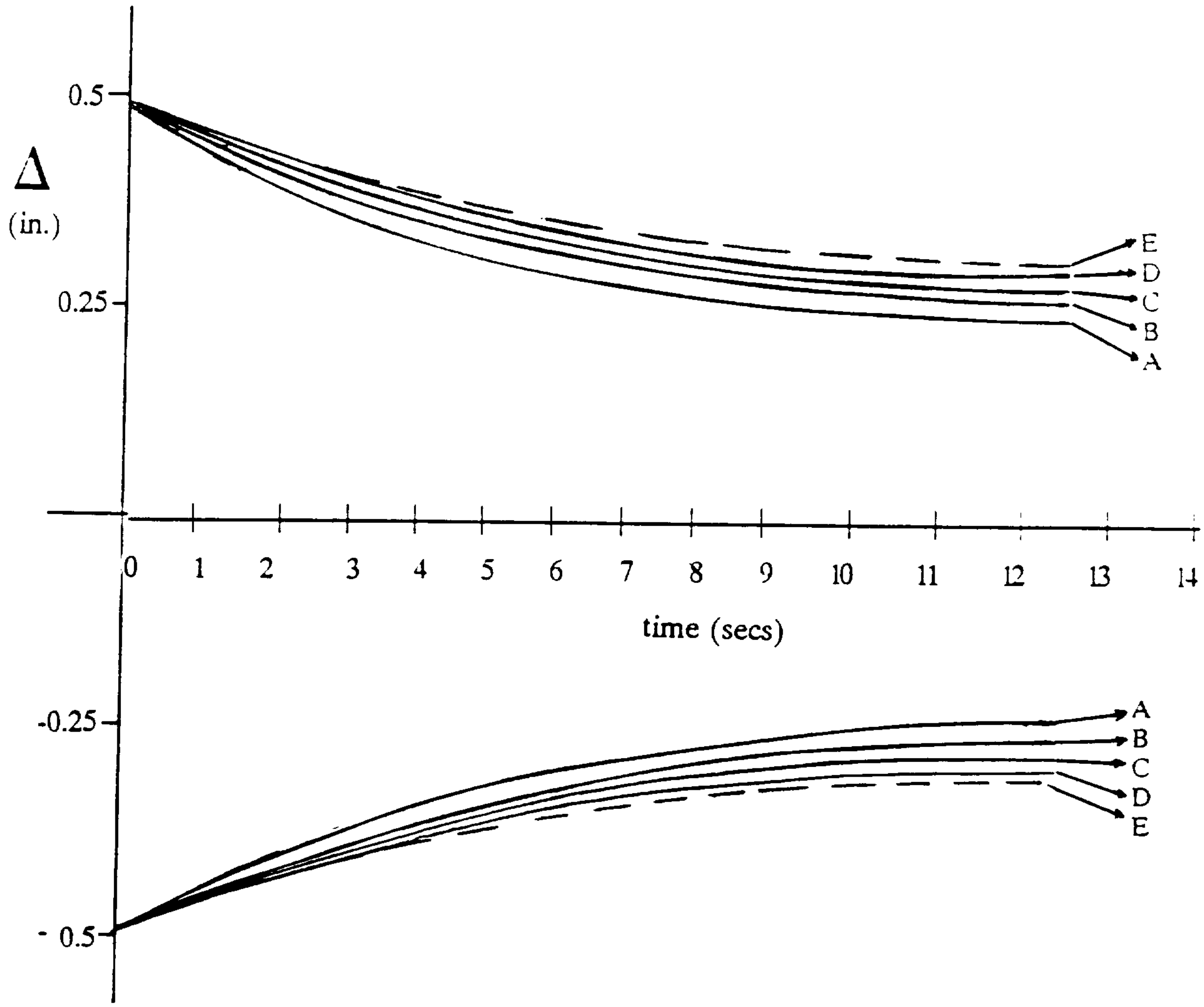
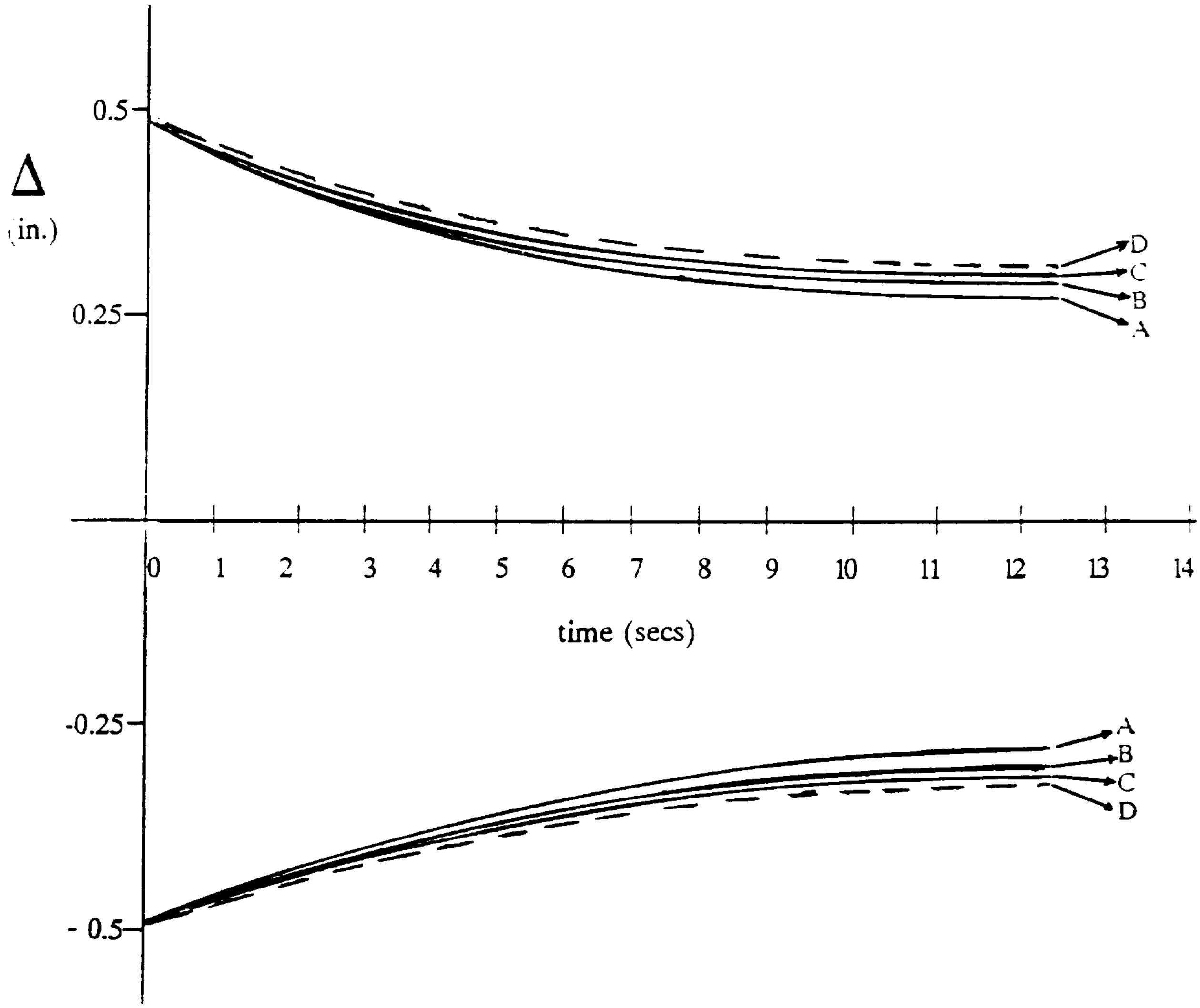


Figure 21. Theoretical versus experimetnal Δ-t relationship for member with no dampers



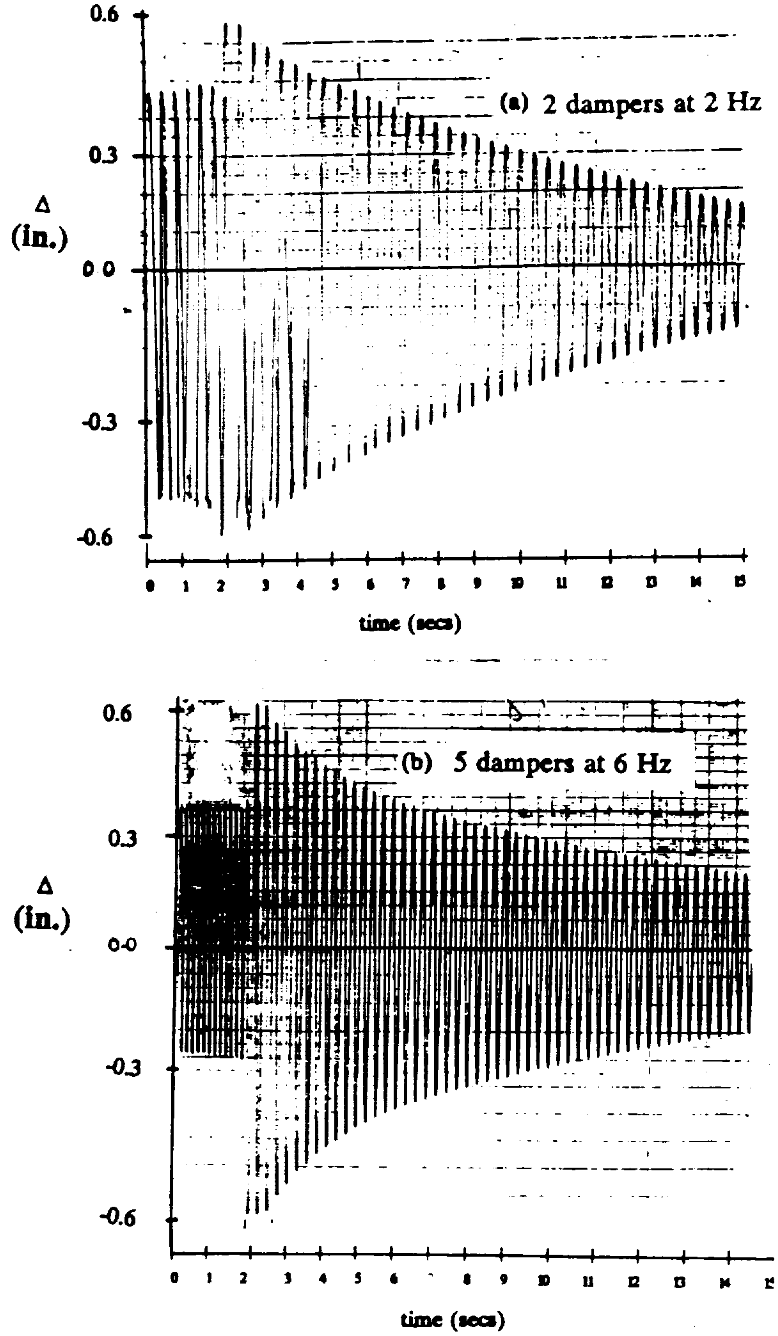
A Two water-filled balloons.
B Five copper brushes
C Five silly putty
D One wool swab
E Empty member

Figure 22. Δ-t envelopes for the best configuration of water-filled balloon, cooper brush, silly putty and wool swab dampers



A Two water-filled plastic eggs
B Two egg-filled balloons
C Two oil-filled plastic eggs
D Empty member

Figure 23. Δ-t envelopes for two equally spaced oil-filled plastic dampers, water-filled platsic eggs, and egg-filled balloon dampers



(b.) For frequency of 6 Hz.

Figure 24. Experimental Δ-t plots for member forced-free vibration for two and five water-filled balloons

APPENDIX

APPENDIX A

Derivation of Natural Frequency Formula

The deviation of the frequency formula given by Equation 21 is presented in the appendix.

Using Equation 15, the equivalent member midspan translational stiffness, Q/Δ_0, represented by K_{br} , is obtained as:

$$K_{br} = \frac{96\,EI}{L^3}\left(\frac{2\,EI + K_r\,L}{4\,EI + 0.5\,K_r\,L}\right) \qquad (A.1)$$

Since the stiffness of each of the end translational springs is K_t , the equivalent total system stiffness is found to be:

$$K_{eq} = \frac{192\,EI\,K_t\,(2\,EI + K_r\,L)}{96\,EI\,(2\,EI + K_r\,L) + 2\,K_t\,L^3\,(4\,EI + 0.5\,K_r\,L)} \qquad (A.2)$$

The natural frequency of the system (at member midspan) can be estimated using:

$$f = \frac{1}{2\,\pi}\sqrt{\frac{K_{eq}}{M_s}} \qquad (A.3)$$

where M_s is the combined mass of the beam, and the end connections excluding the translational springs. Equation 21 is obtained by substituting Equation A.2 into A.3.

APPENDIX B

Computer Program

This appendix presents the computer program used for the theoretical analysis in this research and a sample output. The program is based on the theoretical analysis given in Chapter 2. The input data is generated by the program and a sample output is given at the end.

PROGRAM TO EVALUATE DISPLACEMENT-TIME RELATIONSHIP FOR MEMBER WITH PARTIAL END TRANSLATIONAL AND ROTATIONAL RESTRAINTS.

```
T        THE COEFFICIENT MATRIX
KT       ROTATIONAL STIFFNESS
KR       TRANSLATIONAL STIFFNESS
E        YOUNG'S MODULUS FOR BEAM MATERIAL
MOI      MOMENT OF INERTIA
L        LENGTH OF MEMBER
YO       INITIAL MEMBER DEFLECTION DUE TO STATIC LOAD
YE       INITIAL MEMBER END DISPLACEMENT
WO       SUSPENDED MASS
RMASS    TOTAL MASS OF SYSTEM
H        LENGTH OF PANEL
M        NO OF PANELS
RHO      MASS PER UNIT LENGTH
C        COEFFICIENT OF DAKPING
DT       TIME INCREMENT
EI       MODULUS OF RIGIDITY OF MEMBER
```

```
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION DX(3),D2X(4),T(12,12),W(16,25000),
     >KR,KRN,KRD,KB,KT,E,MOI,L,YO,WO,RMASS,H,RHO,DT,AMB,C,
     >T(250),X(250),Y(250)
      DATA TMASS,E,MOI/27.0D0,10.0E+06,0.34500D0/
      DATA YO/0.4910/,PIE/3.14156264D0/
      DATA L/259.0D0/,WO/3.90/
                INDE=1
      ITERM=15000
      DELTA=0.00001
         CD=0.0053370D0
      M=12
      MID=(M+2)/2
      H=L/M
      GR=386.2
      RMASS=TMASS/GR
      L=L*1.0D0
      RHO=RMASS/L
      EI=MOI*E
C     DO 147 KC=200,1000,400
C            CD=KC*0.000010D0
        KT=20.0
       INDE=INDE+1
      YE=(WO/(2.0*KT))
      YOE=YO-YE
      II=100
           DO 1 IT=1,ITERM
            IF(IT.GT.1)GO TO 1718
      DT=II*DELTA
      KRN=(4.0D0*WO*EI)*L**3-192.0D0*YOE*(EI**2)
      KRD=(96.0D0*YOE*L*EI)-(0.5D0*WO*L**4)
      KR=KRN/KRD
      TOP=12.0D0*(PIE*EI)**2+80.0D0*EI*KR*L+12.0D0*(KR*L)**2
      BOT=12.0D0*(PIE*EI)**2+32.0D0*EI*KR*L+2.250D0*(KR*L)**2
      PI=(PIE/L)**2
289       FORMAT(2X,'FREQUENCY....OF...THE....SYSTEM....IS=',F6.3)
      EMB=KR*L/(KR*L+2.0D0*EI)
       FREQ=(1.0/(2.0D0*L))*SQRT((EI*PI/RHO)*(TOP/BOT))
C
```

```
      AMB=(48.0D0**2*EI*RHO)/(L**6)
      CRIT=2.0D0*SQRT(EI*RHO*(AMB))
C     CRIT=0.010D0
      C=(CRIT*CD)
           WRITE(6,889)FREQ
           WRITE(6,888)CD,RMASS,DT,KR,KT,WO,YE,C
              BB=-(4.0*(EI/H**4))
              B1=EI/H**4.0
              B3=RHO/DT**2
              B4=C/DT
              B5=(EI/H-KR/2.0D0)/(EI/H+KR/2.0D0)
              B6=-(DT**2*EI)/(2.0D0*H**4*RHO)
              B7=0.0D0
                C1=-(1/(B3+B4))
                C2=B3*C1
                 B8=(KR*L)/(4.0D0*PIE*EI)
        DO 22 I=1,M+1
         DO 62 J=1,M+1
            T(I,J)=0.0D0
62       CONTINUE
22       CONTINUE
         T(1,1)=-BB
         T(M+1,M+1)=BB
        T(2,2)=-(6.0D0*B1-2.0D0*B3-B4-B1*B5)
        T(M,M)=T(2,2)
             DO 77 IN=3,M-1
                T(IN,IN)=-(6.0D0*B1-2.0D0*B3-B4)
77           CONTINUE
             DO 79 IN=1,M
                 T(IN,IN+1)=4.0D0*B1
                 T(IN+1,IN)=T(IN,IN+1)
79            CONTINUE
              DO 86 IN=1,M-1
                 T(IN,IN+2)=-B1
                 T(IN+2,IN)=-B1
86            CONTINUE
                                                    DO 221 I=1,M+1
                                                    DO 621 J=1,M+1
                                                         T(I,J)=-T(I,J)
621                                                  CONTINUE
221                                                  CONTINUE
C        WRITE(6,367)
367    FORMAT(18X,'THE TAU MATRIX',10X,'THE TAU MATRIX'/)
C                WRITE(6,46)((T(IY,IZ),IY=1,M+1),IZ=1,M+1)
46          FORMAT(7(F8.3,3X))
1718              I1=IT-1
                  I2=IT-2
                 TIME=(IT-1)*DT
      DO 2 IX=1,M+1
C         IF(IT.LT.3.AND.IX.GT.MID)GO TO 213
          IF(IX.GT.MID)GO TO 213
          IF(IT.GT.1)GO TO 520
                         DX(1)=0.0D0
C                 Z=(PIE*(IX-1)*H)/L
                  Z=(IX-1)*H
       D2X(1)=(-1.0)*KT*W(1,1)/(RMASS)
             ZXC=(WO*L**3/(48*EI)*(Z/L))*(3.0*(1-EMB)+3.0*EMB*(Z/L)-3
     >.0*(Z/L)**2)
         W(IX,IT)=YE+ZXC
          GO TO 2
```

```
520      IF(IT.GT.2)GO TO 10
         IF(IX.GT.1)GO TO 587
        W(IX,IT)=W(1,1)+DT*DX(1)+(DT**2*D2X(1))
         XY= W(IX,IT)
       GO TO 2
587      IF(IX.EQ.2)GO TO 15
         IF(IX.GT.2)GO TO 4569
         IF(IX.EQ.3)GO TO 17
15         WWWWWW=2.0*W(1,1)-W(2,1)
C               WRITE(6,89)W(1,1),W(2,1),WWWWWW
89        FORMAT(3(F10.5,10X))
      W(IX,IT)=B6*W(2,1)+(B7-4.0D0*B6)*W(1,1)+(6.0D0*B6-2.0D0*B7+1)*W(
     >2,1)+(B7-4.0D0*B6)*W(3,1)+B6*W(4,1)
C     W(IX,IT)=B6*WWWWWW+(B7-4.0D0*B6)*W(1,1)+(6.0D0*B6-2.0D0*B7+1)*W(
C    >2,1)+(B7-4.0D0*B6)*W(3,1)+B6*W(4,1)
           GO TO 2
17       W(IX,IT)=B6*W(1,1)+(B7-4.0D0*B6)*W(2,1)+(6.0D0*B6-2.0D0*B7+1)*W
     >(3,1)+(B7-4.0D0*B6)*W(4,1)+B6*W(5,1)
           GO TO 2
4569  W(IX,IT)=B6*W(IX-2,I1)-4.*B6*W(IX-1,I1)+(6.*B6+1.)*W(IX,I1)-4.*B6
     >*W(IX+1,I1)+B6*W(IX+2,I1)
        GO TO 2
10          TIME=(IT-1)*DT
            W(IX,IT)=0.0D0
          DO 37 ID=1,M+1
                CNT=0.0
                 CNT=W(ID,IT-1)*T(IX,ID)*C1
                 W(IX,IT)=W(IX,IT)+CNT
37         CONTINUE
           W(IX,IT)=W(IX,IT)+C2*W(IX,IT-2)
             GO TO 2
213          W(IX,IT)=W(M+2-IX,IT)
2        CONTINUE
C               F(IT)=W(1,IT)*KT
111                        WRITE(6,54)TIME,W(MID,IT),INDE
C            IF(IT.GT.1.AND.QY.GE.0.0.OR.W(IX,IT).EQ.0.0)GO TO 1
C       WRITE(6,53)TIME,(W(IX,IT),IX=1,MID)
54    FORMAT(F12.6,5X,F30.15,5X,I4)
C53      FORMAT(F8.5,2X,7(F6.4,2X))
1         CONTINUE
C147        CONTINUE
            STOP
          END
```

FREQUENCY = 2.7039887
MASS OF SYSTEM IS = 0.06992
ZETA = 0.0053370
MASS OF SYSTEM IS = 0.069912
TIME. INCREM. = 0.001000
ROT. STIFN. = 1433.53636

TIME	DISPLACEMENT
0.000000	0.542146216070619
0.001000	0.538966503610973
0.002000	0.531046018378898
0.003000	0.522864931750910
0.004000	0.517922381770703
0.005000	0.516458619557011
0.006000	0.516054024257297
0.007000	0.513306907724214
0.008000	0.505385196684587
0.009000	0.492430812910998
0.010000	0.480241970810839
0.011000	0.479272441907707
0.012000	0.497753870909039
0.013000	0.533630945128433
0.014000	0.573163570884083
0.015000	0.598500352034303
0.016000	0.598538862221702
0.017000	0.575429508487454
0.018000	0.543103893303588
0.019000	0.518853922752098
0.020000	0.512331472641889
0.021000	0.518945087213722
0.022000	0.523626464507927
0.023000	0.513370853998041
0.024000	0.488224225156640
0.025000	0.460805231138725
0.026000	0.445207750982013
0.027000	0.445803134680174
0.028000	0.455094355727704
0.029000	0.460722470879810
0.030000	0.455123134328270
0.031000	0.441081854113309
0.032000	0.429547056001133
0.033000	0.430523321608627
0.034000	0.443319819639937
0.035000	0.454804191477768
0.036000	0.448606384643693
0.037000	0.417952346088904
0.038000	0.371022530641591
0.039000	0.325073501735181
0.040000	0.295491570851923
0.041000	0.288086280247661

etc.