THE ACCURACY OF DYNAMIC ATTITUDE PROPAGATION*

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ABSTRACT

Propagating attitude by integrating Euler's equation for rigid body motion has long been suggested for the Earth Radiation Budget Satellite (ERBS) but until now has not been implemented. Because of limited Sun visibility, propagation is necessary for yaw determination. With the deterioration of the gyros, dynamic propagation has become more attractive. Angular rates are derived from integrating Euler's equation with a stepsize of 1 second, using torques computed from telemetered control system data. The environmental torque model was quite basic. It included gravity gradient and unshadowed aerodynamic torques. Knowledge of control torques is critical to the accuracy of dynamic modeling. Due to their coarseness and sparsity, control actuator telemetry were smoothed before integration.

The dynamic model was incorporated into existing ERBS attitude determination software. Modeled rates were then used for attitude propagation in the standard ERBS fine-attitude algorithm. In spite of the simplicity of the approach, the dynamically propagated attitude matched the attitude propagated with good gyros well for roll and yaw but diverged up to 3 degrees for pitch because of the very low resolution in pitch momentum wheel telemetry. When control anomalies significantly perturb the nominal attitude, the effect of telemetry granularity is reduced and the dynamically propagated attitudes are accurate on all three axes.

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INTRODUCTION

This paper is based on work performed in the Flight Dynamics Division by the Earth Radiation Budget Satellite (ERBS) attitude support team. It describes the development and evaluation of a model of the ERBS attitude dynamics, based on direct integration of Euler's equation of motion, using telemetered control system information and estimated disturbance torques. Knowledge of the ERBS dynamics is important for accurately propagating the attitude over an entire orbit between occurrences of Sun data. The ERBS fine attitude determination system (FADS) was designed to use onboard inertial reference unit (IRU) measurements of spacecraft motion as a model for attitude motion. With the deterioration of the IRUs, an alternative method for deriving spacecraft body angular rates is necessary to utilize the existing FADS software and retain the fine attitude determination capability. Comparison of FADS attitudes propagated using rates from this alternative method to those using valid IRU rates provides a good assessment of the accuracy of the dynamic model.

The dynamic model presented here exploits available telemetered attitude control system data for computation of the control torques. However, since the control system engineering data were intended for spacecraft health and safety monitoring and trend analysis, the data have coarse digitization and are less frequent than is necessary for an accurate dynamics determination. Extensive preprocessing of the data is, therefore, required to approximate real control actuator behavior. The low quality of the telemetered control system data dominates the effects on solution accuracy of unmodeled disturbance torques, numerical integration error, and uncertainties in spacecraft mass properties.

The dynamic model is integrated into the Data Adjuster subsystem of the ERBS Attitude Determination System (ADS) and takes advantage of existing subroutines for data processing. This method of implementing the model is extremely efficient by eliminating redundant software development and allows the autonomy of the ADS to be preserved. Previous attempts at dynamic modeling have coupled the solution of the dynamics with complex state estimators, which solved for unmodeled parameters and propagated the attitude in one algorithm (References 1 through 3). In this work, every effort was made to keep both the mathematical detail and the solution method as simple as possible. The model demonstrates the feasibility of applying a simplified dynamics determination to real spacecraft data.

ERBS OVERVIEW

The ERBS was launched in October 1984. It is in a 600-kilometer, near circular orbit of 57-degree (deg) inclination. The ERBS is an angular momentum biased spacecraft, with attitude referenced to a geodetic coordinate system; yaw is defined about the local unit nadir vector (+z body axis); pitch is defined about negative orbit normal (+y body axis); and roll is defined about the remaining right-handed orthogonal unit vector. Pitch is controlled to within 1 deg of geodetic null with a y-axis angular momentum wheel using an analog control loop. Roll is controlled to within 1 deg of null by a pair of y-axis mounted electromagnets. Yaw is controlled to within 2 deg of null through roll/yaw kinematic coupling with electromagnets and active yaw angular momentum control by a pair of differentially driven ITHACO Scanwheels, mounted in the y-z plane. The Infrared (IR) Scanwheels, which measure geodetic pitch and roll, also serve as the sole input to the pitch and roll/yaw control loops. Figure 1 illustrates the ERBS body coordinate system and attitude actuator hardware. Table 1 summarizes the ERBS orbit and attitude characteristics. The remaining components of the Magnetic Control System (MCS) include a three-axis Schoenstedt magnetometer for electromagnet control input and attitude determination, one roll axis and one yaw axis electromagnet dipole torque rods for pitch momentum management, and the associated control electronics. Besides the Scanwheels and the magnetometer, the remaining attitude determination sensors are a pair of ADCOLE two-axis fine Sun sensors and two redundant three-axis Northrop IRUs (Reference 4).

ERBS ADS OVERVIEW

The ERBS Attitude Determination System computes single-frame coarse attitudes to within 5 deg, fine attitudes to within 0.25 deg, attitude rates to within 0.005 deg/second, and monitors control system



Figure 1. ERBS Attitude Hardware and Body Coordinate System

performance (Reference 5). Attitude telemetry data are unpacked and converted to engineering units in the Telemetry Processor. The Data Adjuster subsystem applies sensor biases and misalignment corrections, generates reference vectors from ephemerides, and smooths raw data on option. The Coarse Attitude Determination System (CADS) computes single-frame attitudes using the Quaternion Estimator (QUEST) algorithm. QUEST requires no dynamics information for attitude propagation and has inherent accuracy much greater than the specified accuracy of 5 deg when accurate observations (Sun data) are available. The FADS uses IRU measurements as a mechanical motion model for attitude propagation to model sensor data. It then applies a batch least-squares optimization of observed to modeled sensor data to estimate a state vector of epochal attitude angles and gyro calibration parameters. The FADS is very sensitive to the quality of the input rate data.

Table 1. ERBS Attitude and Orbit Characteristics

ORBIT:	
SEMIMAJOR AXIS: 6891 km	
INCLINATION: 57 deg	
ECCENTRICITY: 0.0014 (NEAR-FROZEN ORBIT)	
ATTITUDE PARAMETERS:	
ANGULAR MOMENTUM BIASED, EARTH ORIENTED, 1 REVOLUTION PER ORBIT	
NOMINAL GEODETIC PITCH AND ROLL = 0.0 deg	
NOMINAL YAW = 0.0 OR 180.0 deg FOR SOLAR ARRAY ILLUMINATION	
SPACECRAFT PRINCIPAL MOMENTS OF INERTIA: l _x = 2860 kg-m ² l _y = 4590 kg-m ² l _z = 3040 kg-m ²	
ATTITUDE SENSORS:	
TWO ADCOLE FINE SUN SENSORS 64x64 deg 0.004 deg (i.s.b.) @ 1 Hz	
TWO ITHACO SCANWHEEL IR SENSORS 0.025 deg (I.s.b.) @ 1 Hz	
ONE SCHOENSTEDT THREE-AXIS FLUXGATE MAGNETOMETER 4.68 mg (I.s.b.) @ 1/8 Hz	
TWO IRUS WITH THREE NORTHROP RATE GYROS 0.001 deg/sec (I.S.b.) @ 1 Hz	
MAGNETIC CONTROL SYSTEM ACTUATORS:	
ONE PITCH MOMENTUM WHEEL 50 rpm (I.s.b.) @ 1/8 Hz, I MW = 0.0881/kg-m ²	
TWO ITHACO SCANWHEELS 12 rpm (I.s.b.) @ 1/8 Hz, I _{SW} = 0.0271 kg-m ²	
ONE ROLL AXIS AND ONE YAW AXIS, 50-AMPERE TURN METER SQUARED (ATm ²) MAGNETIC DIPOLE TORQUE RODS FOR PITCH MOMENTUM CONTROL 4.7 ATm ² (I.s.b.) @ 1/16 Hz	-
TWO PITCH AXIS 50 ATm ² DIPOLE TORQUE RODS FOR ROLL CONTROL 9.4 ATm ² (1.s.b.) @ 1/16 Hz	58400(

MATHEMATICAL BACKGROUND

Euler's equation for rigid body rotation, including the contribution of internal wheel angular momenta, $\vec{h}(t)$, is written as

$$[I] \frac{d\vec{\omega}(t)}{dt} = \vec{N}(t) - \frac{d\vec{h}(t)}{dt} - \vec{\omega}(t) \times \{[I] \vec{\omega}(t) + \vec{h}(t)\}$$
(1)

where $\vec{\omega}(t)$ is the body angular velocity vector, [I] is the body moment-of-inertia tensor, and $\vec{N}(t)$ is the sum total external torque acting on the body. This equation is solved for the angular velocity state, $\vec{\omega}(t)$ in body coordinates. For the ERBS, $\vec{h}(t)$ is the sum of the Scanwheel momenta $(\vec{h}_{SW_A} \text{ and } \vec{h}_{SW_B})$ and momentum wheel angular momentum (\vec{h}_{MW}) resolved in body coordinates:

$$\vec{h}(t) = \vec{h}_{MW} + \vec{h}_{SW_A} + \vec{h}_{SW_B}$$
 (2)

The Scanwheels are differentially driven ($\omega_{SW_A} + \omega_{SW_B} = \text{constant}$) so that they produce no y-axis component of torque. The momentum wheel exerts very strong control over the pitch axis. The dh(t)/dt term in Equation (1) is approximated by the difference in wheel speed over the sampling interval.

The sum total external torque, $\vec{N}(t)$ in Equation (1), is approximated as the contributions from nonwheel control torques (magnetic dipole torques, \vec{N}_{MAG}) and environmental torques (gravity gradient, \vec{N}_{GG} , and aerodynamic, \vec{N}_{AERO})

$$\vec{N}(t) = \vec{N}_{MAG} + \vec{N}_{GG} + \vec{N}_{AERO}$$
(3)

Solar radiation pressure is not included; it is expected to be one order of magnitude less than gravity gradient or aerodynamic forces, and its variability and attitude dependence over an orbit would unnecessarily complicate the environmental torque model. Also excluded is the residual dipole moment torque that is easily modeled as a constant torque bias, but it was never determined for the ERBS mission (Reference 6). The disturbance torques arising from science instrument activity are also neglected, and an estimate of their magnitude is not available. Both the residual dipole moment and the science instrument torques are assumed to be small. The magnitude of the gravity gradient torque is a maximum of 1 x 10^{-4} N-m in roll for a 1-deg offset from null attitude and two orders of magnitude less in pitch and yaw. The aerodynamic torque is important only for pitch and can be as high as 1×10^{-4} N-m.

It should be noted that the gravity gradient and aerodynamic torques are low frequency phenomena that are observable only over long timespans (on the order of one orbit), while the magnetic dipole and wheel control torques are high frequency components of the spacecraft dynamics.

Equation (1) is integrated as a system of three coupled equations for each component of $\vec{\omega}(t)$ using a second-order Runge-Kutta method, with a fixed stepsize corresponding to the data discretization interval of 1 sec. The stepsize is made as small as possible to keep the frequency response of the modeled torques high relative to the actual dynamical response of the spacecraft. The coarseness of the telemetry data and uncertainty in disturbance torques and mass properties preclude the need for a higher order numerical integration method.

The integration for each component of $\vec{\omega}(t)$ is begun at the nominal values of zero for roll and yaw, and the one revolution per orbit rate of 0.062 deg/sec for pitch. It is possible to start the integration from an anomalous state relative to the orbital frame only if a priori estimates of the body rates are available.

ERBS TELEMETRY DATA

The data required to compute the state of the ERBS dynamics consist of Scanwheel and momentum wheel speeds, magnetic dipole moments, sensed magnetic field, and IR scanner pitch and roll angles for use in the gravity gradient and aerodynamic torque expressions. In addition, Sun sensor data are used for attitude determination, and the IRU rates provide the reference for evaluation of the dynamic model.

Reconstruction of the ERBS attitude dynamics from control system telemetry is complicated by the coarseness and sparsity of the telemetry data from the principal control actuators. Ideally, the telemetered data frequency should be high relative to the characteristic frequency of the actuators. The resolution in magnitude should also be high relative to typical trends in the data. The pitch momentum wheel telemetry satisfies neither of these criteria.

The ERBS pitch axis control is dominated by the momentum wheel. Momentum wheel telemetry conversion is nonlinear and the least significant bit at nominal speed (approximately 2500 revolutions per minute (rpm)) is approximately 50 rpm over an 8-sec interval. This corresponds to a resolvable change in wheel angular momentum of about 0.46 N-m-sec over 8 sec. Since the resolution in onboard deliverable momentum wheel torque is 4×10^{-4} N-m (Reference 4), observability of the momentum wheel control torque in the downlink telemetry is very poor and limits accurate modeling of the pitch-axis dynamics. The maximum deliverable torque of the momentum wheel is 0.054 N-m so that more than one time interval is necessary to resolve even the maximum rate of change in wheel speed.

Yaw axis control is dominated by the differentially driven Scanwheels. Resolution in the Scanwheel speed is much better, about 12 rpm at nominal speed (approximately 2000 rpm). Because the Scanwheel

moments of inertia are smaller than the momentum wheel moment and because the component of Scanwheel yaw control is small (sine 10 deg), observability of the yaw axis control torque from telemetry is good.

Magnetic dipoles are used for roll axis control and for pitch angular momentum management. Although the resolution in the magnitude of the dipole moments is good, the period of the telemetered data is 16 seconds. Due to the relatively weak contribution of the dipoles to the ERBS dynamics (maximum 4 x 10^{-3} N-m in roll), the lack of time resolution in the dipole telemetry is less important. Also, the digitization of the magnetometer data contributes a smaller error to the dipole torques.

DATA CONDITIONING TECHNIQUES

To accurately represent the ERBS dynamics, the control system telemetry data require extensive preprocessing. Interpolation between low-frequency data points is necessary for the control torques to be evaluated at each integration point. As a benefit of a properly chosen smoothing or interpolation method, increased accuracy may be synthesized from coarse data. The behavior of different smoothing and interpolation techniques varies with data characteristics.

Incorporated in the ERBS ADS is a polynomial smoothing algorithm that fits a Chebyshev polynomial of user-specified order to a set of data (Reference 4). The smoother provides interpolation between low-frequency data points. For the Scanwheel data, the Chebyshev polynomial smoothing algorithm may be tuned to approximate actual Scanwheel activity. The fit is limited by array size to a maximum of 600 sec through one cycle of the Data Adjuster. Problems can arise at the endpoints of high order fits. Since a high-order fit is necessary to match peak-to-peak trends, a modification to the algorithm allows extrapolated beginning and endpoint divergences to be replaced with the constant values from the first and last raw data points, respectively. Figure 2 illustrates an 18th-order Chebyshev fit to raw Scanwheel data.

The coarseness of the raw momentum wheel data, resembling a series of step discontinuities about a mean, exacerbates the representation of realistic wheel behavior using a simple smoother. The raw momentum wheel data of Figure 3 most likely results from transitions near a telemetry digitization boundary. An 18th-order Chebyshev fit to the data is represented by the smooth line. A high-order polynomial follows the step discontinuities too closely while causing boundary matching problems for multiple Data Adjuster cycles. For a low-order polynomial, this problem is reduced, but detail is lost. Trends are represented by two or more steps in the same direction. Matching the step values exactly considerably overestimates the change in momentum wheel speed and drives unrealistic momentum into the model of pitch axis rates.

A more realistic fit to the momentum wheel data is obtained by preprocessing the raw data in a simple running average algorithm to reduce discontinuities. A Chebyshev polynomial is then fit to the averaged data to interpolate between the 8-sec data points. The polynomial fit also permits additional tuning to mimic real wheel behavior. Figure 4 illustrates averaged and smoothed momentum wheel data. Problems still exist in extracting actual wheel behavior from the coarse data.

Good results are achieved using the Chebyshev fit for the low-frequency magnetometer data because the telemetry digitization is small relative to the magnitudes of the trends in the data. Figure 5 illustrates a 12th-order fit to magnetometer data. Typical magnetic dipole moment telemetry is represented by the points in Figure 6. Polynomial smoothing of dipole data is inappropriate because of the large discontinuities. The dipole moments are interpreted as constant valued between 16-sec data points. This assumption is made because the chance of the subcommutator interrogating the dipole at a particular value is proportional to the amount of time the dipole moment maintains that value. The solid line in Figure 6 represents the interpolated dipole moment values. Error imparted to the dynamic model by the low frequency of dipole telemetry is acceptable because of the small magnitude of the dipole control torques.

RESULTS

Two criteria are used to assess the accuracy of the modeled dynamics: comparison of the modeled rates to valid IRU rates from the same data span and comparison of FADS attitudes propagated by modeled



Figure 2. Raw Scanwheel Speed Data (Points) and 18th-Order Chebyshev Polynomial Fit (Smooth Line)

rates with FADS attitudes propagated by valid IRU rates. For the evaluation, telemetry data were chosen from periods when all three IRU axes were functioning and gyro noise was minimal. Full orbit timespans of data were processed with the model. Figure 7 shows the results of one run with the modeled rates represented by the smooth line together with raw IRU telemetry. The IRU data are considered an absolute reference. Root mean square noise in the IRU telemetry for this period is about three digitization steps (0.003 deg/sec).

Agreement between the modeled and measured rates is good for roll and yaw, in most cases within 0.002 deg/sec of the mean of the IRU signal, but poor for pitch. Although some trends are seen in the modeled data, much of the pitch dynamics information is not resolvable in the large momentum wheel telemetry digitization. The constant offset between the modeled and measured rates in Figure 7 is due to the arbitrary integration starting point and may be corrected with an appropriate initial value corresponding to the constant of integration. The FADS is able to estimate these values as rate biases.

The most significant test of this method of dynamic modeling is its ability to accurately propagate attitudes over a full orbit. The FADS was used to produce one-orbit attitudes using modeled rates in place of measured IRU rates. Figures 8 through 13 show the FADS roll, pitch, and yaw derived from modeled rates of Figure 7 together with the reference FADS angles derived from IRU measured rates. In this example, only the epochal angles and rate biases were included in the estimated state. The modeled roll and yaw match the reference to within 0.2 deg. Pitch, however, diverges up to 3 deg.

To demonstrate the effect of environmental torques on the model, the data of Figure 7 were reprocessed with no environmental torques included. Figure 14 shows the results of this run. No change in the roll or yaw rates was observed, but lack of the constant component of the aerodynamic torque causes the pitch axis rate to gradually diverge. Although it is difficult to resolve the environmental torques because of the





Figure 3. Raw Momentum Wheel Speed Data (Points) and 18th-Order Chebyshev Polynomial Fit (Smooth Line)

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Figure 5. Twelfth-Order Chebyshev Polynomial Fit (Smooth Line) to Raw Magnetometer Data (Points)





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Figure 7. Comparison of Modeled Rates (Smooth Line), Including Environmental Torques, To Measured IRU Rates for One Orbit



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Figure 9. FADS Reference Roll Using IRU Data







Figure 11. FADS Reference Pitch Using IRU Data

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Figure 13. FADS Reference Yaw Using IRU Data



Figure 14. Comparison of Modeled Rates (Smooth Line) Excluding Environmental Torque Models to Measured IRU Rates

coarseness of the control telemetry, their effect on the actual spacecraft dynamics is diminished due to gyroscopic stiffness from the strong angular momentum bias of the ERBS.

ATTITUDE ANOMALIES

Beginning November 27,1989, the ERBS was configured under dual scanner MCS control during a period when the field of view trace of one Scanwheel viewed the Sun during sunrise and sunset events. The Sun pulse resulted in control anomalies of about 2 deg on all three axes. During this period, the pitch axis gyro was failed, and the roll axis gyro was severely degraded, precluding use of the FADS for reference attitudes to evaluate the dynamic model. The CADS was used for reference attitudes but gave accurate solutions only when Sun data were available following sunrise. Figures 15 through 20 show CADS angles computed with Sun, magnetometer, and uncorrupted IR Scanwheel data with corresponding data from modeled rates through a Sun interference occurrence. Agreement between the CADS attitudes and modeled FADS attitudes is within 0.2 deg even for pitch. The large change in momentum wheel speed reduces the effect of telemetry digitization and allows the pitch dynamics to be more accurately determined. Figure 21 shows the raw momentum wheel data from this period. The Sun interference occurrence demonstrates the accuracy of the dynamic model when large trends in control system activity make control torques more observable.

CONCLUSIONS

It has been demonstrated for the ERBS that dynamic modeling for attitude determination is feasible when control system data of sufficient frequency and resolution are available. Insufficient knowledge of control torques limits the accuracy of the modeled attitude dynamics. Poor resolution in the ERBS momentum wheel downlink telemetry results in poor pitch rate determination. It was found that improved granularity of the momentum wheel data was more important than increased time resolution, since the telemetry frequency provides sufficient input to the model relative to the timescale of the onboard control processes. When a significant perturbation to the nominal state occurs, the effect of telemetry resolution is reduced. The dynamic processes on all three axes are then observable in downlink telemetry, and the model performs well.

The dependence of the modeled dynamics on accurate control system information dominates the effect of environmental torques on the system. Better telemetry resolution is required to assess the accuracy of the disturbance torque models. The high momentum bias of the ERBS may diminish the effect of environmental torques on the actual dynamics, however. More detailed environmental torque models may be necessary for zero angular momentum spacecraft using this method.

One recommendation for further improvement in the model is to increase resolution of the momentum wheel control torques. Efforts to derive actual wheel behavior by preprocessing momentum wheel telemetry data resulted in limited accuracy. An alternative method would extract the commanded momentum wheel speed from a model of the onboard MCS pitch control law based on IR Scanwheel pitch angle input. Resolution in the Scanwheel fine pitch angle is 0.025 deg, which corresponds to a step change in pitch axis angular momentum of 0.25 N-m/sec, or a momentum wheel step of 27 rpm over the 8-sec interval. Since the Scanwheel pitch angle results from the sum total torques acting on the spacecraft pitch axis, this method would attribute all pitch motion to momentum wheel control. This would preclude estimation of the y-axis disturbance torques but could cause problems with magnetic dipole coupling in pitch axis control.

Another method for improving momentum wheel speed resolution would derive the analog tachometer input to the onboard analog to digital converter from the digitized telemetry data. A least-squares fit would be applied to the raw data, subject to known constraints on the control system. The success of this method would depend on the uniqueness of the analog input function.







Figure 16. FADS Roll Angle During Sun Interference Anomaly Using Modeled Rates



Figure 17. CADS Pitch Angle During Sun Interference Anomaly



Figure 18. FADS Pitch Angle During Sun Interference Anomaly Using Modeled Rates









Integration of the dynamic model in the ERBS Attitude Determination System is very efficient. This approach exploits many capabilities of the existing ground support software, such as telemetry processing, data smoothing, graphic displays, and the fine attitude determination algorithm. For future work, incorporating this simple dynamic model into a state estimation algorithm using attitude sensor data would allow additional parameters, such as unmodeled torque biases, to be estimated. Improvement in the accuracy of attitudes propagated in this way could be expected.



Figure 21. Raw Momentum Wheel Speed Data During Sun Interference Anomaly

REFERENCES

- 1. E. J. Lefferts and F. L. Markley, "Dynamic Modeling for Attitude Determination," paper presented at AIAA Guidance and Control Conference, San Diego, California, August 16-18, 1976
- 2. Computer Sciences Corporation, CSC/TM-76/6235, Nimbus-G Attitude Determination Feasibility Study Utilizing the Attitude Dynamics Generator (ADGEN), F. L. Markley and J. W. Wood, December 1976
- 3. --, CSC/TM-77/6115, Evaluation of Wheel Rate Data for LANDSAT Attitude Modeling, J. Fein, April 1977
- 4. --, CSC/SD-82/6013, Earth Radiation Budget Satellite (ERBS) Attitude Ground Support System Functional Specifications and Requirements, G. Nair et al., September 1982
- 5. --, CSC/SD-84/6004UD1, Earth Radiation Budget Satellite (ERBS) Attitude Determination System (ADS) System Description and User's Guide, Volume 1, R. Strang, December 1985
- 6. --, CSC/TM-84/6071, Earth Radiation Budget Satellite (ERBS) Attitude Analysis and Support Plan, M. Phenneger and J. Rowe, August 1984