# UPPER STAGE IN-FLIGHT RETARGETING TO ENHANCE GEOSYNCHRONOUS SATELLITE OPERATIONS 

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#### Abstract

Real time utilization of propellant reserves that are not needed is available with the implementation of the in-flight retargeting capability for the Centaur Upper Stage. Application to a performance critical, geosynchronous mission is discussed. The operational duration of the satellite may be increased by selectively choosing the appropriate final orbit injection conditions. During ascent Centaur evaluates the amount of propellant excess available and adjusts the final orbit target to consume the excess. Typical satellite mission requirements are introduced to illustrate the mission analysis process to determine the pre-flight nominal target and the in-flight retarget function.


[^0]Satellite operational characteristics may be enhanced by innovative targeting of the upper stage. Until now expendable launch systems did not have very sophisticated and adaptive processes for targeting to their desired final orbits. Flight software programs loaded on board contain fixed orbits the vehicle must fly to. Hundreds of pounds of launch vehicle capability may be set aside as reserves to ensure a high probability of a successful mission in the event of dispersed conditions. If the launch system provides nominal or above nominal performance this additional capability is not used in any way to benefit the spacecraft mission objective. There is no flexibility in the flight algorithms to improve beyond the nominally targeted mission.

An in-flight retargeting capability has been developed for the General Dynamics Centaur Upper Stage which can utilize excess propellant reserves to achieve an enhanced final orbit, i.e. an orbit which upgrades the performance index of the satellite. The performance index is typically related to the operational lifetime of a satellite. Centaur detects how well the lower stages have performed and releases the appropriate amount of reserves for use. When the final orbit of the upper stage is a geosynchronous transfer orbit, the retargeting algorithm employed is strictly and solely to reduce the orbit's inclination. The spacecraft performs the injection into its operational orbit, at geosynchronous altitude.

When an upper stage is used to deliver the spacecraft into its operational geosynchronous orbit a more elegant retargeting strategy is desired, one which will increase the spacecraft's lifetime. The operational lifetime of a satellite is often dependent on the amount of spacecraft propellants required to trim out any injection errors and to maintain desired orbital characteristics. For example, the satellite's mission may require it to spend its time on station at the low inclinations. The oscillatory motion of a geosynchronous orbit plane is manifested in the propagation of its inclination and RAAN (Right Ascension of the Ascending Node) periodically about the values of 7.55 degrees inclination and 0.0 degrees RAAN. The period is approximately fifty to fifty-five years for injection inclinations less than 10 degrees. The period and the variation of the elements are a function of the initial injection inclination and RAAN.

We shall investigate a sample mission that requires the satellite to maximize its dwell time at inclinations below 6 degrees. However, the nominal performance of the launch vehicle delivers it to 10 degrees inclination. Mission analysis will identify the optimal combinations of targeted/injection inclinations and RAANs which will enhance spacecraft objectives and satisfy constraints based on the expected propellants available on the upper stage.

## RETARGETING STRATEGY

The Centaur is a high energy, liquid upper stage with a three burn capability. After a sub-orbital separation from the booster lower stage the first burn inserts Centaur into a low altitude park orbit. For typical geosynchronous orbit missions, the second burn provides insertion into a geosynchronous transfer orbit. The final Centaur burn circularizes the orbit at geosynchronus altitude and completes the inclination change. The objective of this capability is to evaluate actual booster performance and re-target the Centaur second and/or third burn in accordance with mission specifications. The Centaur first burn is not altered, allowing the nominal park orbit to be achieved. The intent is only to compensate for booster phase dispersions, not upper stage dispersions. This decision was made partly based on range safety implications. Other constraining factors considered in the design were: 1) simplicity, 2) minimal flight computer memory use, 3) credibility, 4) reliability, and 5) accuracy.

The re-targeting process is divided into several segments. The first segment is the in-flight detection phase; some detection criteria are imposed to determine if the vehicle is within the envelope of acceptable nominal performance. The predicted time-to-go to Main

Engine Cut-Off (MECO) calculated by the flight computer for the first Centaur burn is used to evaluate the performance of the lower stage(s). It is based on the state vector at booster/Centaur separation. The trajectory is integrated forward, assuming a nominal Centaur acceleration, until the nominal park orbit is achieved. Steering laws are incorporated into the integration process so that the time-to-go prediction is a realistic indication of the actual burn duration under these conditions. The use of nominal Centaur acceleration precludes a low thrust (long burn time) Centaur phase from influencing the re-targeting decision. Before activating the detection criterion, it is required that time-to-go converge such that the current time-to-go is within some tolerance of the value from each of the two previous guidance computation cycles, i.e.:

$$
\left|\mathrm{T}_{4 i}-\mathrm{T}_{41-1}+\Delta \mathrm{t}\right|<\varepsilon
$$

and

$$
\begin{align*}
& \qquad 1 \mathrm{~T}_{41}-\mathrm{T}_{4 \mathrm{i}-2}+2 \Delta t \mathrm{I}<\varepsilon  \tag{1}\\
& \Delta \mathrm{t}=\text { elapsed time between successive guidance computation cycles } \\
& \mathrm{T}_{4 \mathrm{i}}=\text { time-to-go to MECO at the ith guidance cycle } \\
& \varepsilon=\text { convergence tolerance, set at } 1 \text { sec. }
\end{align*}
$$

When both of these are satisfied, the difference between the in-flight predicted burn duration and the nominal burn duration (loaded pre-flight) is computed:

$$
\begin{equation*}
\Delta \mathrm{T}_{\mathrm{B}}=\mathrm{T}_{\mathrm{B}-\mathrm{NOM}}-\left(\mathrm{T}-\mathrm{T}_{\text {MES } 1}+\mathrm{T}_{4 \mathrm{i}}\right) \tag{2}
\end{equation*}
$$

$\mathrm{T}_{\mathrm{B}-\mathrm{NOM}}=$ nominal Centaur first burn duration
$\mathrm{T}_{\text {MES } 1}=$ Main Engine Start-1 (MES1) time from go-inertial $T=$ time from go-inertial

The re-targeting sequence proceeds only if the absolute difference is above some threshold. Additionally, some time limit will be set after which actual Centaur accelerations are admitted (for proper steering corrections) and the re-targeting option is disengaged. Since the predicted time-to-go is a standard guidance calculation there is minimal impact to the flight software and computer memory. The long successful history of Centaur flights has proven the reliability and accuracy of the computations. In particular the NASA Atlas/Centaur- 53 HEAO-C mission had similar detection software loaded on board.

A performance index is needed to quantitatively relate the performance excess (or shortfall) to revised final orbit parameters. At General Dynamics Space Systems Division the conventional measurement of performance is the Propellant Excess (PE). This is the total burnable propellants remaining in the final stage after the last main engine burn thrust decay. For this study it is baselined that the nominal mission results in a PE equal to zero. Having computed the difference in burn time from (2), the change in the amount of propellants at MECO1 (due to off-nominal performance of the booster stage) is now calculated based on a nominal mass flow rate:

$$
\begin{equation*}
\Delta \mathrm{W}_{\mathrm{P} 1}=\Delta \mathrm{T}_{\mathrm{B}} \cdot \dot{\mathrm{w}} \tag{3}
\end{equation*}
$$

The change in performance which is supplied by the subsequent Centaur burns can be measured by the variation in propellant excess ( $\triangle \mathrm{P} \mathrm{PE}$ ). The partial derivative based on the mass fraction across the nominal second and third burns is used to calculate $\triangle \mathrm{PE}$ :

The final phase of the algorithm re-targets the Centaur burns to an adjusted final orbit which will improve spacecraft capability. The variation of the targeted final orbit inclination and RAAN as a function of available PE is examined in the next sections. This functional relationship will be loaded into the flight computer. Pre-flight analysis determines the function constants for the set of desired final orbit parameters. The algorithm is structured to be able to deploy different functions dependent on the value of the PE. A complete description of the algorithm can be found in Reference 1.

## PROPAGATION OF GEOSYNCHRONOUS ORBITS

Earth oblateness. gravitational attractions of the Sun and Moon, and solar radiation pressure affect the long-term orbital variations of geosynchronous satellites. Algorithms that propagate the oscillation of the orbit characteristics have been developed to estimate the stationkeeping requirements for satellites. An understanding of the time history of the satellite orbit precession provides a strategy to adjust the injection targeting for Centaur.

A semi-analytical method which predicts inclination and RAAN given the initial inclination and RAAN values has been developed by Allan and Cook (Ref. 2). This procedure is based on analysis which used an averaged disturbing function in the equations of motion. Expression by vectorial elements (in the satellite orbit frame) reduced the resultant Lagrange equations to a compact and tractable form. Chao and Baker (Ref. 3) generated a program that numerically integrates the equations. Comparison of the semi-analytical approach with the numerical method (Ref. 4) yielded favorable results for several ten-year inclination and RAAN histories. Also noteworthy is that for synchronous, circular orbits an approximate solution to the Lagrange equations yields a "stable" orbit which is at 7.55 degrees inclination and 0 degrees RAAN.
Allan and Cook provide the expression for the period of oscillation of the orbit under these influences by inverting and integrating the equation of motion along one axis:

$$
\begin{equation*}
T=4 \cdot\left(\lambda_{3}-\lambda_{2}\right)^{-1 / 2} \cdot\left(\lambda_{0}-\lambda_{1}\right)^{-1 / 2} \cdot K(k) \tag{5}
\end{equation*}
$$

where $\lambda_{i}$ are eigenvalues of the set of "Euler" equations and $\lambda_{0}$ is an integration constant $\mathrm{K}(\mathrm{k})$ is the complete elliptic integral of the first kind and

$$
\begin{equation*}
k=\left(\lambda_{2} /\left(\lambda_{3}-\lambda_{2}\right)\right)^{1 / 2} \cdot \tan \Phi \tag{6}
\end{equation*}
$$

$\Phi$ is the dihedral angle between the stable orbit and the orbit of interest

$$
\begin{aligned}
& \Phi=\operatorname{arc} \cos \left(\cos \alpha_{0} \cos i_{0}+\sin \alpha_{0} \sin i_{0} \cos \text { RAAN }_{0}\right) \\
& \alpha_{0}=7.55 \text { deg. }
\end{aligned}
$$

Figure 1 illustrates the variation in the period of precession of the orbit based on the initial inclination and RAAN. The variation of the precessional period is about one year at the low inclinations (less than 5 degrees) but may be up to seven years at inclinations around 25 degrees. Figure 2 presents the RAAN and inclination time history for three different injection states: inclination of 8 degrees and RAAN of 270 , 300 , and 330 degrees. Although the difference in period is only ten months among the


Figure 1 Variation of Precession Period with Inclination and RAAN


Polar axis $=$ RAAN
Radial axis $=$ Inclination

Figure 2 RAAN and Inclination History
three cases, the time spent at a particular inclination band varies significantly among them. For instance the central angle of the arc length of intersection between the 270 degree RAAN trace and the region of inclination less than 8 degrees is 91 degrees. From equation (7), a satellite drifting along this trajectory will experience a dwell time of 13.4 years at inclinations of 8 degrees or less.

> Dtime $=\mathrm{T} \cdot($ Central Angle/360)
> $\mathrm{T}=$ period of oscillation

Similarly, the 300 degree RAAN trajectory results in a dwell time of 15.9 years and the 330 degree RAAN trajectory yields a dwell time of 23.4 years.

## UPPER STAGE MISSION DESIGN

The nominal mission design for the Centaur results in targeting to final orbit parameters which can be achieved by the launch system with a 99.87 percent probability. To guarantee this confidence a certain amount of propellants (vehicle performance) is held in reserve to compensate for dispersions in flight. With Inflight Retargeting, part of this reserve is released and used to achieve a different orbit which enhances the satellite objectives.

Given particular mission requirements such as satellite weight, the Centaur is capable of achieving a geosynchronous orbit with a certain inclination for its nominal level of performance. As reported earlier, for that inclination there is a corresponding RAAN which maximizes the time spent at certain inclination bands. This selection is done pre-flight to generate the complete nominal mission final orbit target parameters which are loaded onboard the flight computer. By adjusting the time of launch each day, achieving the desired RAAN does not incur any performance penalty.

To determine the targeted RAAN simply requires solving for the intersection of two non-concentric circles. Referring to the coordinate system used in Figure 2 the polar equations of an inclination band and the precession trace are expressed:

$$
\begin{align*}
& r=1  \tag{9}\\
& \mathrm{r}^{2}-2 \cdot \mathrm{~s} \cdot \mathrm{r} \cdot \cos (\mathrm{RAAN})+\left(\mathrm{s}^{2}-\mathrm{P}^{2}\right)=0  \tag{10}\\
& 1=\text { inclination } \\
& \mathrm{s}=\text { inclination of "stable" point }=7.55 \mathrm{deg} \text {. } \\
& P=\text { radius of the precession trace based on } \\
& \mathrm{P}^{2}=\mathrm{i}_{0}^{2}-2 \cdot \mathrm{i}_{0} \cdot \mathrm{~s} \cdot \cos \left(\operatorname{RAAN}_{0}\right)+\mathrm{s}^{2} \tag{11}
\end{align*}
$$

Equation (10) is solved by substituting eq. (9) and (11):

$$
\begin{equation*}
\text { RAAN }=\arccos \left\{ \pm\left[\left(1^{2}+s^{2}-P^{2}\right) / 2 \cdot s \cdot 1\right]\right\} \tag{12}
\end{equation*}
$$

The two RAAN values indicate the RAAN along the precession trace where the satellite enters the inclination region of interest and the RAAN at which it exits. These angles provide the arc length of travel along the precession trajectory. The central angle, $\beta$, of this arc length is calculated by the law of cosines from the triangle shown in Figure 3.


Figure 3 Geometry of Intersection Central Angle

$$
\begin{equation*}
\beta=2 \arccos \left[\left(\mathrm{P}^{2}+\mathrm{s}^{2}-\mathrm{i}^{2}\right) /(2 \cdot \mathrm{~s} \cdot \mathrm{P})\right] \tag{13}
\end{equation*}
$$

Equation (8) is then used to calculate the dwell time.
Let us examine the application to a sample geosynchronous mission. The satellite requirements are: 1.) maximize the time spent at inclinations of 6 degrees or less with a minimum of 10 years and maximum of 12 years; 2.) assume no spacecraft $\Delta V$ has been allocated for out-of-plane injection usage; 3.) the total mission duration will not exceed 15 years. If the nominal launch vehicle can only deliver the spacecraft to 10 degrees inclination, this becomes a performance critical mission. A retargeting function, executed in-flight, may help meet the requirements by utilizing propellant reserves which are not needed.

In developing the retargeting strategy, the nominal target must first be established. Obviously the nominal targeted inclination is 10 degrees but the targeted RAAN is still to be determined. Since any RAAN may be achieved by selecting the appropriate time of day to launch without incurring a performance degradation, one can allow the satellite requirements to drive its selection. Figure 4 shows the dwell time, the drift time, and the total mission duration time for a range of RAANs at 10 degrees inclination. It can be seen that a RAAN of 290 degrees will satisfy the mission requirements: the dwell time at inclinations of 6 degrees and less is 10.5 years, the time to drift into that region is 4.5 years, and thus the total mission duration is 15.0 years. The nominal target is 10 degrees inclination and 290 degrees RAAN. The propellant reserves carried on board will ensure this orbit will be achieved even with three-sigma "low performance" launch system dispersions.

During flight Centaur will detect and evaluate the actual performance of the booster lower stage; if the vehicle is operating nominally, or at any level above the low threesigma threshold, there are propellant reserves which may be utilized. This propellant excess (PE) can be used to reduce the inclination from 10 degrees. Figure 5 illustrates the effect on the dwell time, drift time, and total mission duration by reducing the injection inclination (RAAN remains the same). For this mission the maximum dwell time of 12 years is satisfied at an injection inclination of 8.4 degrees. Additional PE can then be used to reduce the drift time of 2.8 years at the injection RAAN of 290 degrees and injection inclination of 8.4 degrees. To accomplish this and to maintain the requirement of a 12 year dwell time dictates adjusting the RAAN and inclination according to the precession path (see Figure 2). To determine the RAAN and inclination pairs along this path recall equation (10) and (11).


Figure 4 Dwell Time, Drift Time, and Total Mission Duration Varaition with RAAN for inclination $=10 \mathrm{deg}$.


Figure 5 Dwell Time, Drift Time, and Total Mission Duration Variation with Inclination for RAAN $=290$ deg.

$$
\begin{align*}
& \mathrm{i}^{2}-2 \cdot \mathrm{~s} \cdot \mathrm{I} \cdot \cos (\mathrm{RAAN})+\left(\mathrm{s}^{2}-\mathrm{P}^{2}\right)=0  \tag{14}\\
& \mathrm{P}^{2}=\mathrm{i}_{0}^{2}-2 \cdot \mathrm{i}_{0} \cdot \mathrm{~s} \cdot \cos \left(\mathrm{RAAN}_{\mathrm{O}}\right)+\mathrm{s}^{2}  \tag{15}\\
& \mathrm{i}_{0}=8.4 \text { degrees } \\
& \quad \mathrm{RAAN}_{0}=290 \text { degrees } \\
& \mathrm{s}=\text { inclination of "stable" point }=7.55 \mathrm{deg} .
\end{align*}
$$

Table 1 contains the retargeted values of inclination and RAAN and the associated change in drift times.

Table 1 Reduction in Drift Time from Retargeted RAAN and Inclination

| RAAN (DEG) | INC (DEG) | DWELL TIME (YRS) | DRIFT TIME (YRS) |
| :---: | :---: | :---: | :---: |
| 290 | 8.4 | 12.0 | 2.77 |
| 289 | 8.22 | 12.0 | 2.56 |
| 288 | 8.04 | 12.0 | 2.56 |
| 287 | 7.87 | 12.0 | 2.35 2.14 |
| 286 | 7.69 | 12.0 | 2.14 1.94 |
| 285 | 7.52 | 12.0 | 1.74 |
| 284 | 7.35 | 12.0 | 1.74 1.54 |
| 283 | 7.18 | 12.0 | 1.54 |
| 282 | 7.01 | 12.0 | 1.34 |
| 281 | 6.84 | 12.0 | 1.15 |
| 280 | 6.69 | 12.0 | 0.96 0.78 |
| 279 | 6.53 | 12.0 | 0.60 |
| 278 | 6.37 | 12.0 | 0.42 |
| 277 | 6.21 | 12.0 | 0.24 |
| 276 | 6.07 | 12.0 | 0.07 |

To illustrate the final form of the retargeting function that would be loaded on the Centaur flight computer for this hypothetical mission we will use typical values of propellants required to change inclination and RAAN associated with a Centaur geosynchronous orbit.

$$
\begin{align*}
& \delta \mathrm{PE} / \delta \mathrm{i}=34.02 \mathrm{~kg}(75 \mathrm{lbs}) / \text { deg. inclination }  \tag{16}\\
& \delta \mathrm{PE} / \delta \mathrm{RAAN}=9.07 \mathrm{~kg}(20 \mathrm{lbs}) / \text { deg. RAAN } \tag{17}
\end{align*}
$$

The retargeted values of inclination and RAAN from the nominal pre-flight set as a function of the propellant excess (PE) as determined in flight is shown in Figure 6.


Figure 6 The Retargeting Function

## CONCLUSION

Upper stage in-flight retargeting may enhance geosynchronous satellite mission objectives by using excess propellant reserves to achieve different final orbit injection conditions. By analysis of the precessional motion at geosynchronous altitudes a strategy is formulated to adjust the final orbit injection inclination and RAAN which will meet dwell time, drift time, and total mission duration requirements. The variation in inclination and RAAN as a function of the additional propellants detected to be avallable during flight is generated for loading onto the flight computer.

## REFERENCES

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