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# CONSTANT PROPELLANT USE RENDEZVOUS SCENARIO ACROSS A LAUNCH WINDOW FOR REFUELING MISSIONS\*

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## ABSTRACT

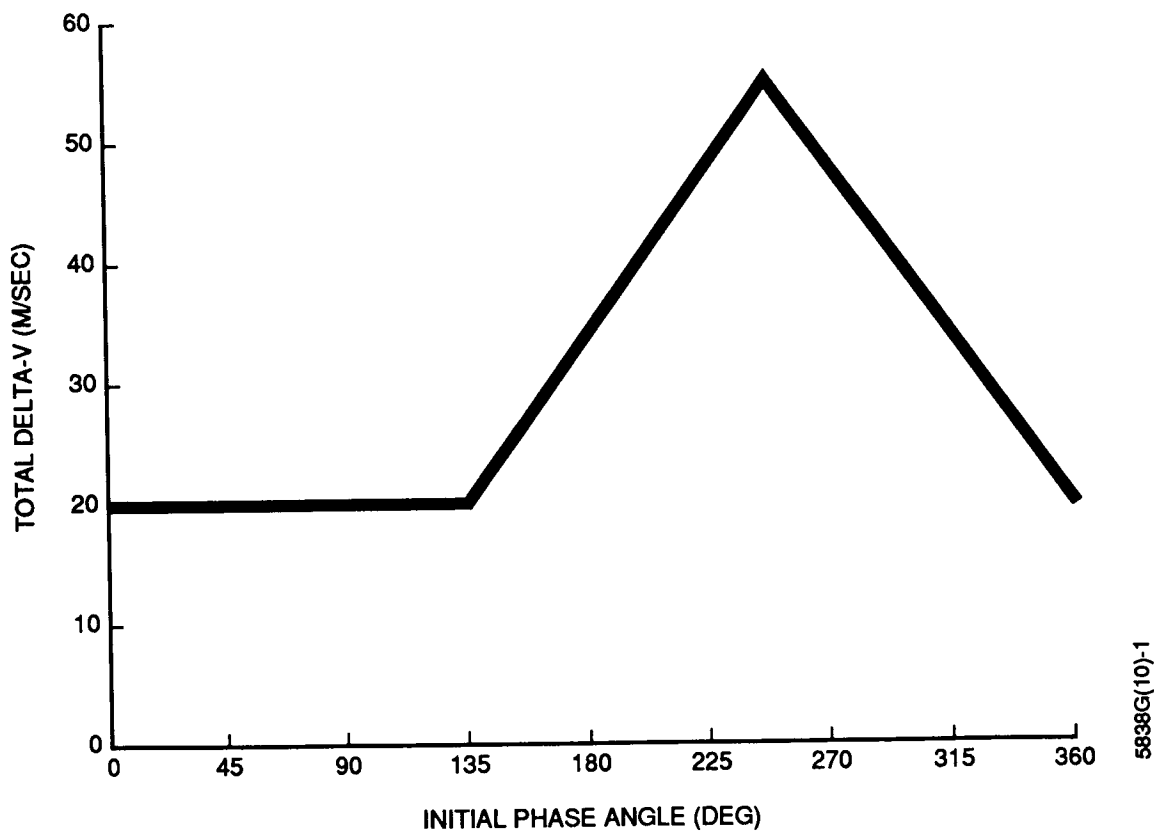
This study investigates active rendezvous of an unmanned spacecraft with the Space Transportation System (STS) Shuttle for refueling missions. The paper first presents the operational constraints facing both the maneuvering spacecraft and the Shuttle during a rendezvous sequence. For example, the user spacecraft must arrive in the generic Shuttle control box at a specified time after Shuttle launch. In addition, the spacecraft must be able to initiate the transfer sequence from any point in its orbit. The standard four-burn rendezvous sequence, consisting of two Hohmann transfers and an intermediate phasing orbit, is presented as a low-energy solution for rendezvous and retrieval missions. However, for refueling missions, the Shuttle must completely refuel the spacecraft and return to Earth with no excess fuel. This additional constraint is not satisfied by the standard four-burn sequence. Therefore, a variation of the four-burn rendezvous, the constant delta-V ( $\Delta V$ ) scenario, has been developed to satisfy the added requirement.

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# 1. INTRODUCTION

This report presents the results of an investigation into analysis and mission-planning techniques for unmanned user spacecraft involved in active rendezvous with the Space Transportation System (STS) Shuttle for refueling purposes. The requirements for an active rendezvous are (1) the maneuver sequence must possess a 360-degree phasing capability (i.e., the two spacecraft must rendezvous from any initial orientation) and (2) the rendezvous must be completed in a fixed amount of time. A standard four-burn rendezvous sequence has been developed for retrieval missions. In this sequence, the amount of fuel used during the rendezvous varies with the initial angular phasing between the two spacecraft, as shown in Figure 1 for a 3-day rendezvous. For refueling missions, an additional rendezvous requirement is that maneuver planning and analysis for premission planning must determine the exact amount of fuel the user spacecraft will expend during the rendezvous sequence. This allows the Shuttle to transport only the fuel necessary to refuel the spacecraft's tanks. However, since the initial phase angle varies during a Shuttle launch window, the delta-V ( $\Delta V$ ) required during the rendezvous is not fixed. Therefore, the standard four-burn sequence is not appropriate for a refueling mission. Consequently, a constant  $\Delta V$  rendezvous scenario was developed to meet all requirements.



**Figure 1. Total  $\Delta V$  for a 3-Day Rendezvous, Standard Four-Burn Sequence**

Section 2 presents background information on the derivation of the standard four-burn sequence from the standard Shuttle rendezvous policies. This sequence, which consists of

two Hohmann transfers and a coast period in a phase orbit, is shown to minimize fuel costs for any set of initial conditions. Section 3 then discusses the concepts of the constant  $\Delta V$  scenario and explains how it is derived from the standard four-burn sequence. Section 4 details the equations used to solve the constant  $\Delta V$  case and presents the results. Section 5 presents a summary of the conclusions reached in the report.

## **2. BACKGROUND: STANDARD FOUR-BURN RENDEZVOUS SEQUENCE**

This section presents an overview of the rendezvous sequence designed for retrieval missions. This is essential, since the retrieval sequence is the basis for the constant  $\Delta V$  case. The section first presents the requirements imposed by the Shuttle on an actively rendezvousing user spacecraft. Then, the four-burn rendezvous sequence is derived as the optimum sequence that satisfies all the Shuttle requirements while minimizing  $\Delta V$  requirements.

### **2.1 STS SHUTTLE RENDEZVOUS REQUIREMENTS**

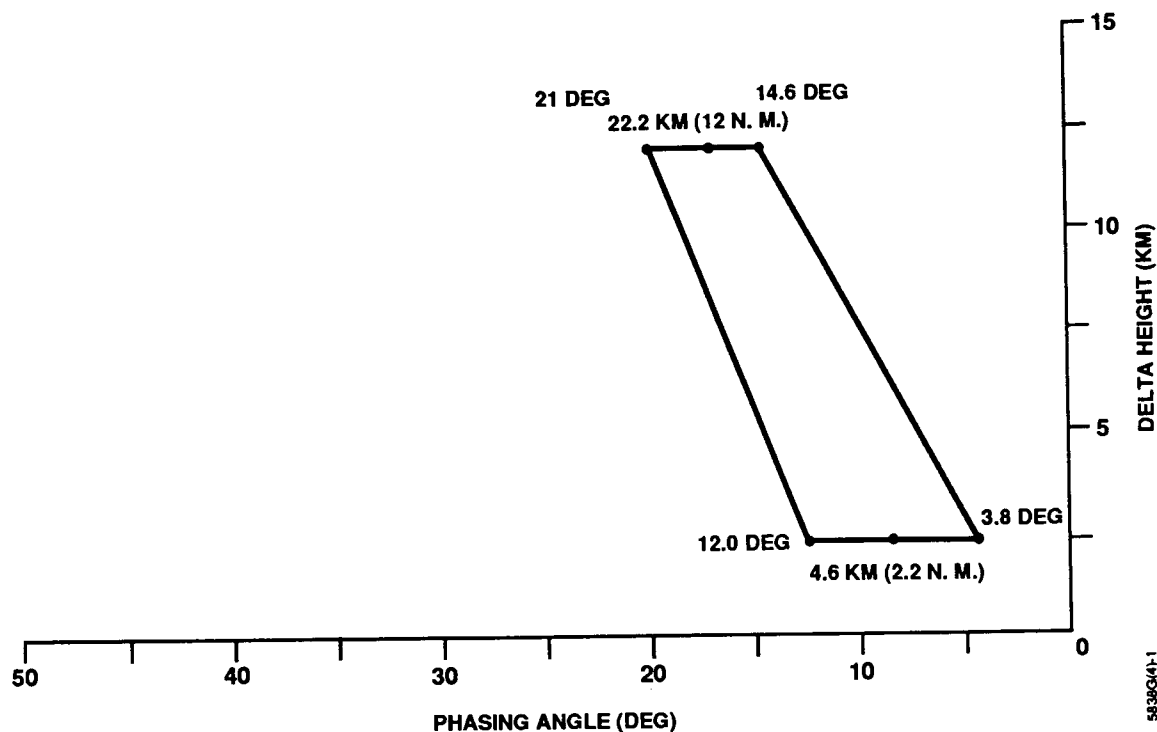
The rendezvous sequence is initiated when mission controllers at Johnson Space Center (JSC) issue the "Go for descent" declaration. This is done after the Shuttle has achieved orbit and a systems check has determined that the rendezvous sequence may proceed. Nominally, this occurs at 5 hours mission-elapsed time (MET), or 5 hours after launch.

Upon receiving the "Go for descent" declaration, the unmanned user spacecraft (chase spacecraft) must complete its rendezvous with the Shuttle (target spacecraft) at a predetermined time, currently given as 53 hours MET. JSC refers to this rendezvous completion time as the Control Box Start Time (CBST). The rendezvous is considered complete when the maneuvering spacecraft has achieved the Shuttle control box (Figure 2) and has ceased all translational maneuvering. As illustrated, the control box is a region above and ahead of the Shuttle with its origin at the Shuttle. The horizontal component measures angular separation along the Shuttle orbit, while the vertical component measures radial distance from the Shuttle.

Upon achieving the CBST at the completion of the rendezvous, the user spacecraft must satisfy a semimajor axis and eccentricity requirement limiting the difference in apogee and perigee altitudes to 14.8 kilometers (km). In addition, a maximum angular separation of 0.03 degree (deg) in the orbital planes of the spacecraft is required. The user spacecraft must be capable of absorbing up to approximately 0.1 deg of launch dispersion error in the orbit plane of the Shuttle. Finally, the user spacecraft must be capable of handling Shuttle launch slips of up to 1 hour. This, combined with the possibility of 24-hour Shuttle launch delays, requires that the user spacecraft be capable of completing rendezvous with the Shuttle from any initial orientation (or phasing) with the Shuttle. Stated differently, the user spacecraft must possess a 360-deg phasing capability with the Shuttle.

### **2.2 THE STANDARD FOUR-BURN RENDEZVOUS SEQUENCE**

This section describes the four-burn rendezvous sequence, which is well suited to the operational environment. This method satisfies all the Shuttle requirements while



**Figure 2. STS Shuttle Generic Control Box; Orbit Normal Out of Page**

minimizing  $\Delta V$ . The section begins with a discussion of the characteristics of the Hohmann transfer and proceeds to describe a rendezvous sequence consisting of a series of Hohmann transfers with an intermediate phase orbit. The rendezvous technique does not require any specific orbital conditions. However, to simplify the current discussion, it is assumed that the user spacecraft begins in a higher orbit than the Shuttle.

A Hohmann transfer is well known as the optimum maneuver sequence for transferring between two circular coplanar orbits. The first burn of such a maneuver places the chase spacecraft in an elliptic transfer orbit with perigee at the same altitude as the target orbit. The second burn occurs 180 deg after the first and makes the transfer orbit circular, leaving the chase spacecraft in the same orbit as the target vehicle.

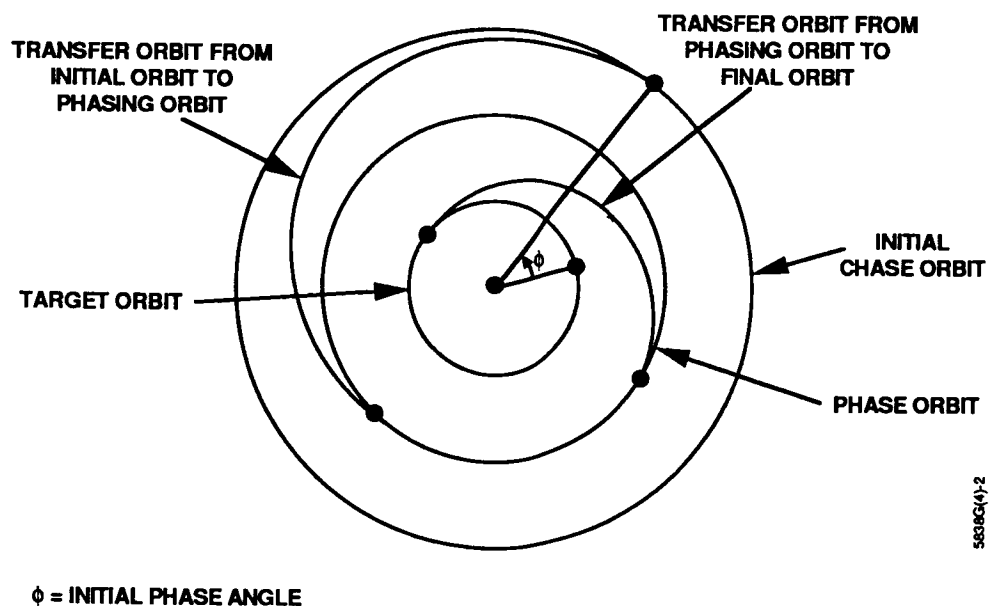
If the chase and target orbits are not coplanar, a plane change must be done at some point in the maneuver sequence. This could be accomplished by executing the entire plane change in either the initial or the final orbit, independently of the altitude change to be performed. However, the transfer  $\Delta V$  is optimized by a simultaneous execution of the plane-change and orbital-change maneuvers. Efficiency is further improved by distributing the plane changes between the two burns. In the examples discussed in this paper, however, rendezvous will be assumed to be coplanar. For a more detailed discussion on the Hohmann transfer as it pertains to rendezvous, see References 1 and 2.

If two spacecraft are to rendezvous using a Hohmann transfer, the correct angular separation, or phasing, must exist between the spacecraft at the initiation of the transfer. This

angle is referred to as the Hohmann phase angle (HPA). The relative periods of the two orbits determine the value of the HPA.

The synodic period represents the length of time required for spacecraft in different orbits to return to the same orientation with respect to each other. This is the time between successive occurrences of the HPA. If the synodic period is greater than the amount of time allotted for a particular rendezvous scenario, the required HPA may not be achievable for all initial orientations. For a 2-day rendezvous, the synodic period is longer than the rendezvous duration if the initial user spacecraft altitude is less than 145 km above the nominal Shuttle altitude of 315 km. For a spacecraft such as the Gamma Ray Observatory (GRO), which is nominally only 35 km above the Shuttle at the start of the rendezvous sequence, additional measures must be taken.

The required 360-deg phasing capability can be achieved while maintaining the  $\Delta V$  advantages inherent in the Hohmann transfer by employing a series of Hohmann transfers. Such a sequence, the four-burn rendezvous sequence (Figure 3), consists of two Hohmann transfers. The first transfer places the chase spacecraft in an intermediate orbit called the phase orbit. The second transfer maneuvers the chase vehicle to the target spacecraft. The phase orbit is computed such that the HPA between the phase and target orbits is reached at the time of the final transfer. By varying the altitude of the phase orbit, the user spacecraft can achieve rendezvous with the Shuttle from any initial relative orientation.



**Figure 3. Four-Burn Transfer Scenario**

The concept of linking in- and out-of-plane corrections to save  $\Delta V$  is as applicable to the four-burn scenario as it is to the case of a direct Hohmann transfer. To combine plane

changes and altitude changes, each of the four burns must occur along the relative node defined by the intersection of the user spacecraft and Shuttle orbit planes at the termination of the rendezvous sequence. However, to simplify the cases examined in this paper, rendezvous will be assumed to be coplanar.

To apply the four-burn sequence, it is necessary to accurately compute the semimajor axis of the phase orbit, given a set of initial conditions. This is done using the following equation:

$$0 = \left[ \left[ \frac{\mu}{a_t^3} \right]^{1/2} - \left[ \frac{\mu}{a_p^3} \right]^{1/2} \right] T - \phi - 2\pi - \frac{\pi}{\sqrt{8}} \left[ \left[ \frac{a_p + a_c}{a_p} \right]^{3/2} + \left[ \frac{a_p + a_t}{a_p} \right]^{3/2} \right] \quad (2-1)$$

where

- $\mu$  = Earth's gravitational constant (398,600.64 km<sup>3</sup>/sec<sup>2</sup>)
- $a_t$  = target spacecraft semimajor axis
- $a_c$  = chase spacecraft semimajor axis
- $a_p$  = phase orbit semimajor axis
- $\phi$  = initial phase angle
- $T$  = rendezvous duration

Equation (2-1) is solved iteratively until a value for  $a_p$  is found that makes the right-hand side of the equation arbitrarily close to zero.

Figure 4 shows phase orbit altitude as a function of phase angle,  $\phi$ , for a 3-day transfer from 350 to 315 km. The figure demonstrates that two phase orbit solutions exist for each initial phase angle: one above the target spacecraft and the other below. The solid portions of the curves show the phase orbit solutions having the lower  $\Delta V$  requirement for each specific initial phase angle. The crossover point from the upper to the lower solution occurs when both solutions require equivalent  $\Delta V$  expenditure.

Further examination of variations in phase orbit altitude with rendezvous time and initial spacecraft altitudes suggests several noteworthy trends. The phase orbit semimajor axis is essentially a linear function of phase angle, with the upper and lower solutions being nearly parallel. Furthermore, the y-intercept of the upper phase orbit altitude/phase angle function is the target spacecraft semimajor axis, and its slope varies inversely with  $T$ , the rendezvous duration. With these relationships in mind, it is possible to write the following three analytical equations, which accurately predict the phase orbit altitudes and the

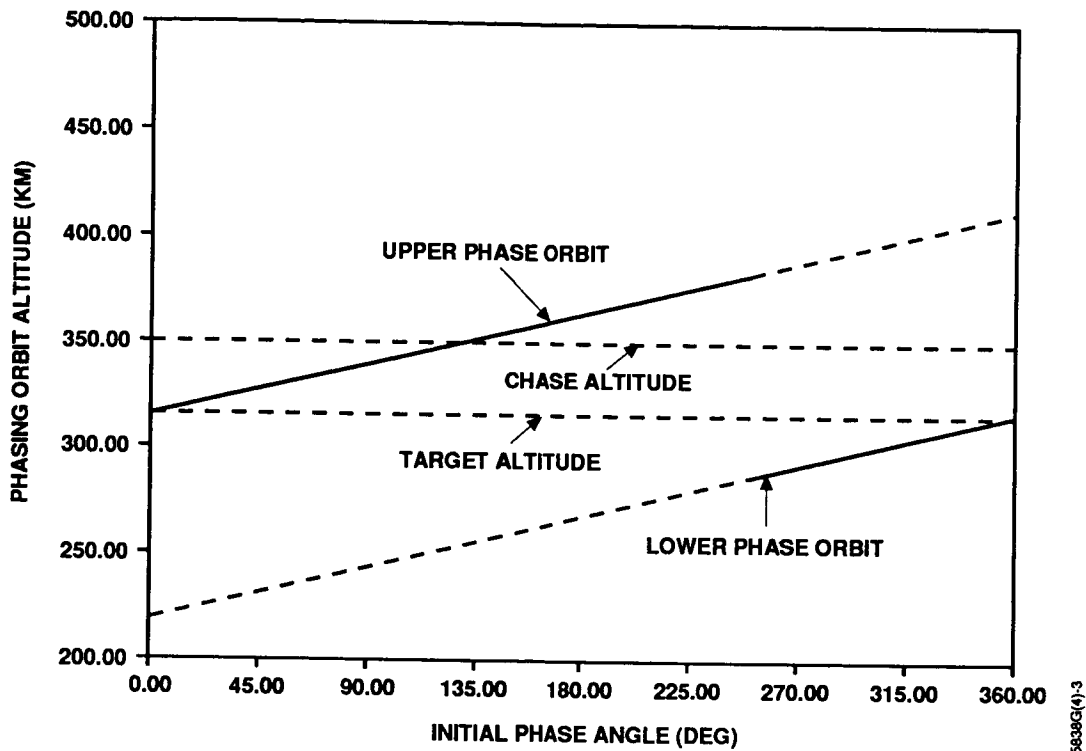


Figure 4. Phase Orbit Altitude as a Function of Initial Spacecraft Phase Angle for a 3-Day, 350- to 315-km Scenario

crossover point over the ranges of Shuttle altitudes (300 to 350 km), user spacecraft altitudes (300 to 500 km), and rendezvous durations (2 to 5 days) under consideration:

$$a_{pu}(\phi) = \frac{k_u}{T} \phi + a_t \quad (2-2)$$

$$a_{pl}(\phi) = \frac{k_l}{T} \phi + \left[ a_t - 2\pi \frac{k_l}{T} \right] \quad (2-3)$$

$$\phi_c = \frac{T}{k_l + k_u} \left[ a_c - a_t + \frac{2\pi k_l}{T} \right] \quad (2-4)$$

where

- $a_{pu}$  = semimajor axis of the upper phase orbit
- $a_{pl}$  = semimajor axis of the lower phase orbit
- $\phi_c$  = phase angle at which crossover occurs
- $k_u, k_l$  = constants

The expressions for  $k_u$  and  $k_l$  were derived by taking a Taylor series expansion of an expression for phase orbit altitude based on spacecraft angular rates and assuming only linear terms to be significant. Numerical analysis can be performed to demonstrate that, in agreement with the initial simplifying assumption of a linear relationship between phase orbit altitude and  $\phi$ ,  $k_u$  and  $k_l$  do remain essentially constant over the ranges under consideration. The derivation of  $k_l$  and  $k_u$  and the associated numerical analysis can be found in Reference 3.

### 3. CONCEPTS OF THE CONSTANT $\Delta V$ RENDEZVOUS

The purpose of this section is to introduce the constant  $\Delta V$  rendezvous scenario. A constant  $\Delta V$  rendezvous means that for the same two spacecraft and rendezvous duration, a rendezvous requires the same  $\Delta V$  for every initial angular orientation (phasing). Such a rendezvous is required if the Shuttle is to refuel a user spacecraft and return to Earth without excess fuel. Calculation of the fuel the user spacecraft requires during the rendezvous allows the Shuttle to transport only the fuel necessary to refuel the spacecraft's tanks.

In the standard four-burn rendezvous sequence, the fuel cost of the rendezvous varies with the initial phase angle of the two spacecraft and with the rendezvous duration. Since the possibility of launch slips and delays makes it impossible to predict the initial phase angle of the two spacecraft and the fuel cost of the rendezvous, a variation of the four-burn sequence must be developed that ensures that a constant  $\Delta V$  rendezvous occurs.

The  $\Delta V$  required for a four-burn sequence can be computed for specific GRO and Shuttle altitudes and rendezvous duration. For a low-Earth orbit, the  $\Delta V$  of a maneuver is a linear function of the altitude change. However, since the chase and target altitudes are fixed in the case of a four-burn scenario, the total change in altitude ( $\Delta A$ ) and  $\Delta V$  are functions of the altitude of the phase orbit. Figures 5 and 6 show the phase orbit altitudes and corresponding total  $\Delta V$  costs associated with all possible initial phase angles for a rendezvous occurring between 350 km and 315 km altitude. Two phase orbit solutions exist for each phasing, one above the target orbit and the other below the target orbit. Figure 5 shows the lower  $\Delta V$  cost solutions as solid lines. When the phase orbit lies between the initial and final orbits, the total altitude change and, therefore,  $\Delta V$  cost remain constant. For phase orbit altitudes above the initial chase orbit, the  $\Delta V$  has a maximum value when the lower phase orbit fuel cost equals the upper orbit cost. The lower solution is then employed, and the total altitude change and  $\Delta V$  costs decrease.



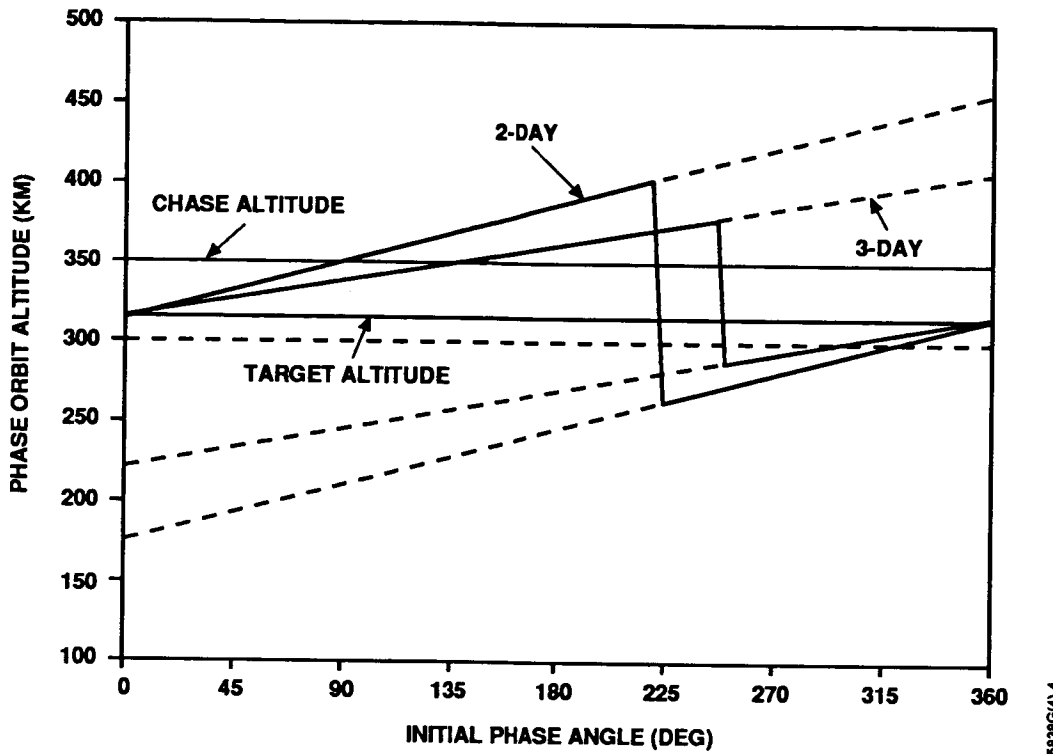
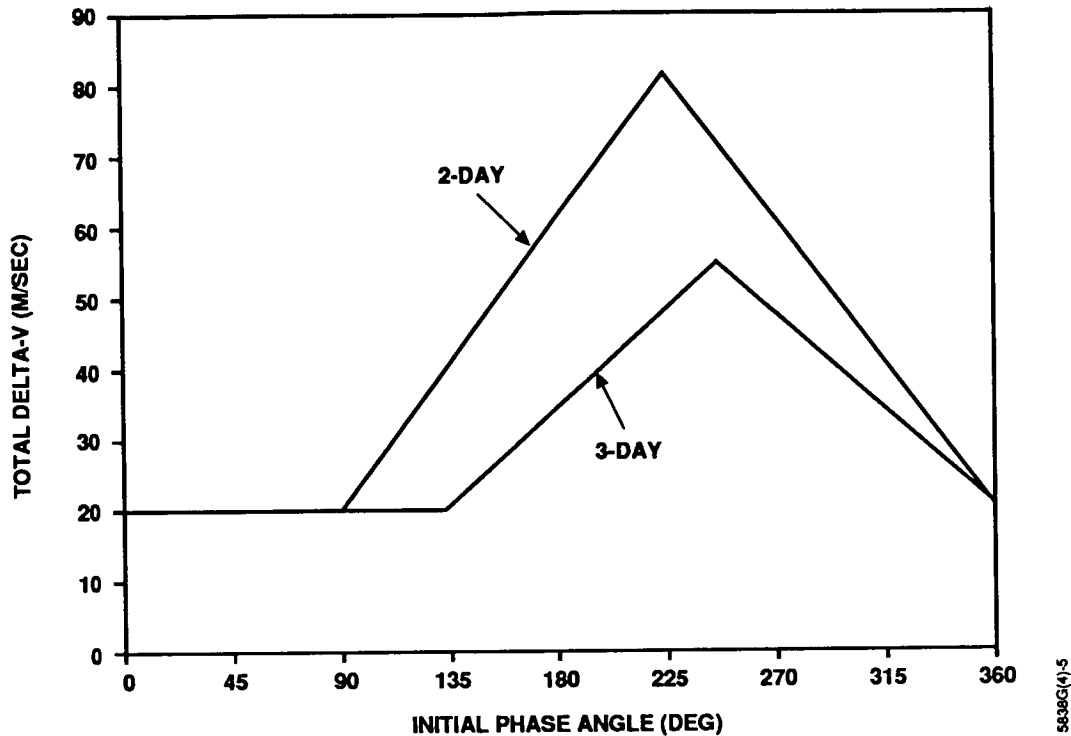


Figure 5. Phase Orbit Altitude as a Function of Initial Phase Angle (2- and 3-Day Rendezvous)

The standard four-burn scenario, composed of two Hohmann transfers and an intermediate coast period in a phase orbit, yields the optimum fuel cost solution for the rendezvous of two spacecraft. The phase orbit altitudes shown in Figure 5 result in the minimum altitude change  $\Delta A$  and, thus,  $\Delta V$  for each possible phasing for a rendezvous occurring between 350 and 315 km. For the 3-day curve in Figure 5, the 247-deg phase angle requires the largest total altitude change during the four-burn sequence. These phase orbit altitudes are 380 km or 285 km, respectively, and both altitudes result in a total  $\Delta V$  of 55 meters/second (m/sec). Since no method exists that can change the phase orbit altitude such that  $\Delta A$  decreases, a constant  $\Delta V$  rendezvous is not possible below 55 m/sec. For example, the graph in Figure 6 show that a constant  $\Delta V$  of 40 m/sec for a 3-day rendezvous is not possible for phase angles between 200 deg and 295 deg, since 40 m/sec is below the minimum cost profile. Instead, 55 m/sec would be the minimum constant  $\Delta V$  value possible. Therefore, in order to achieve a constant  $\Delta V$  of 55 m/sec, a variation of the standard four-burn sequence must be designed that requires all phase angles to use phase orbit altitudes of 380 km or 285 km. This would ensure a constant altitude change; thus, a constant  $\Delta V$  scenario would exist.

Altering the phase orbit altitude is accomplished by incorporating an initial coast period into the rendezvous before burn 1. As the spacecraft coast freely in their initial orbits, the phasing between the spacecraft changes due to the differences in the angular rates of



**Figure 6. Total  $\Delta V$  as a Function of Initial Phase Angle (2- and 3-Day Rendezvous)**

the spacecraft. In addition, an initial coast time reduces the effective rendezvous duration; e.g., a 3-day rendezvous duration with an initial coast period of 12 hours has only 2-1/2 days to execute the four-burn sequence. Both the change in phasing and the reduced time interval in which to execute the rendezvous burn sequence combine to move the phase orbit farther away from the chase and target orbits, which increases the total altitude change.

Figures 7 and 8 demonstrate the effects of an initial coast on phase orbit altitude and  $\Delta V$ . Initial coast durations of 12 and 24 hours are plotted along with the standard four-burn case (no initial coast). As the coast period increases, the slope of the phase orbit altitude plot increases, either raising the phase orbit farther above the initial orbit or lowering the phase orbit farther below the final orbit. As shown in Figure 8, the additional altitude change increases the rendezvous costs. For the 3-day constant  $\Delta V$  example, the phase orbit must be raised or lowered to altitudes of 380 km or 285 km, resulting in a constant  $\Delta V$  rendezvous of 55 m/sec. A 12-hour initial coast period will satisfy this constraint for phase angles of 225 and 285 deg, whereas a 24-hour coast is required for phasings of 205 and 325 deg. It is evident that for each initial phasing, a unique coast time is required that will enable the phase orbit to be altered to the constant  $\Delta V$  altitude.

Figure 9 illustrates the constant  $\Delta V$  technique for an initial phase angle of 180 deg. For a 3-day rendezvous, the target phase orbit altitude is 380 km, yielding a  $\Delta V$  of 55 m/sec.

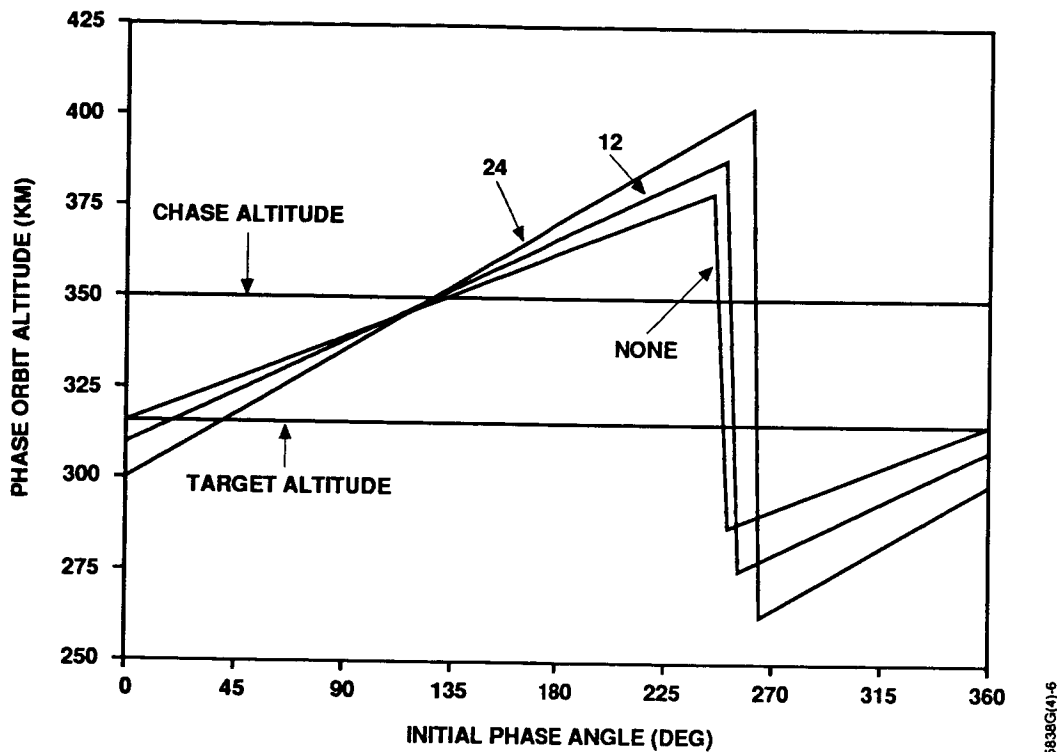


Figure 7. Phase Orbit Altitude as a Function of Initial Phase Angle (No Initial Coast Period and 12- and 24-Hour Initial Coast Periods; 3-Day Rendezvous)

If a standard four-burn rendezvous is employed, the phase orbit altitude required is 363 km, which requires a  $\Delta V$  of 34.5 m/sec. Therefore, an initial coast must be performed to increase the phase orbit altitude. Figure 9 shows the progression of the initial coast period for both phase angle and phase orbit altitude. After the spacecraft has coasted for 41 hours and 18 minutes, execution of a four-burn rendezvous sequence, given the new phase angle and remaining rendezvous time, will require a phase orbit altitude of 380 km. If such a procedure is executed at each initial phase angle, the  $\Delta V$  profile will be horizontal at 55 m/sec, as shown in Figure 10. Furthermore, all rendezvous requirements will be met and  $\Delta V$  expenditures will be minimum.

#### 4. COAST TIME EQUATIONS

A set of analytic formulas that describe the coast time necessary for a coplanar constant  $\Delta V$  rendezvous was derived based on linear approximations for computing the phase orbit altitude, and it was determined that the relation between  $\Delta V$  and altitude changes. The primary approximation is that  $\Delta V$  is linearly proportional to the total altitude change,

$$\Delta V = m(\Delta A) \quad (4-1)$$

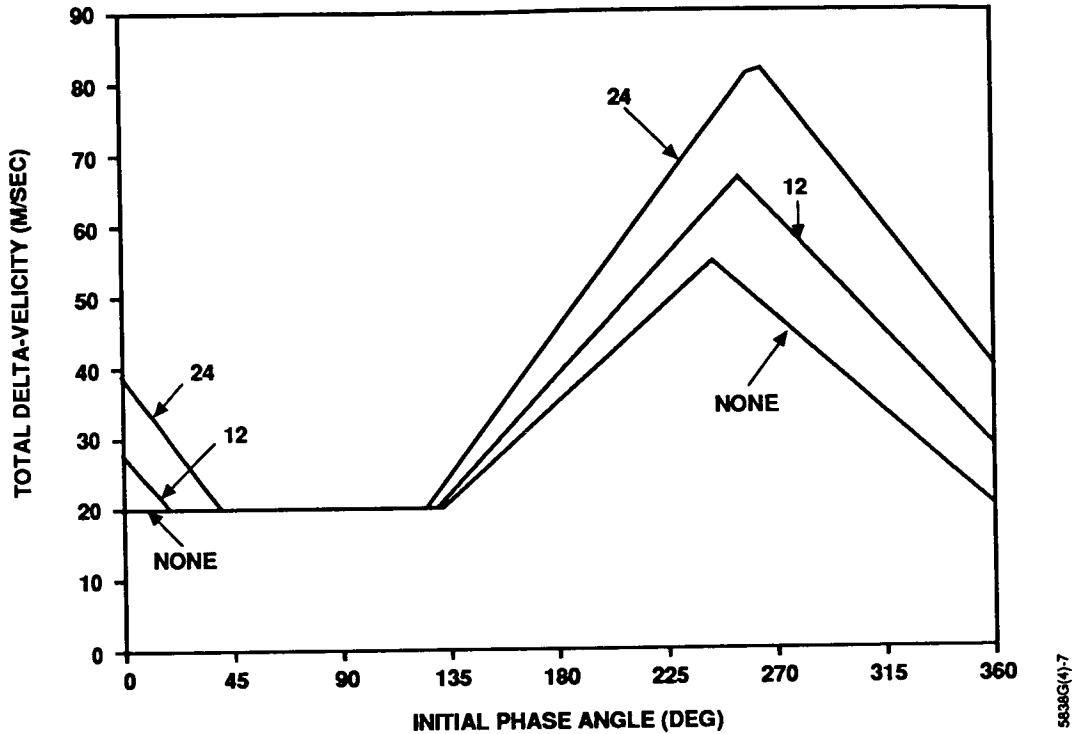


Figure 8. Total  $\Delta V$  as a Function of Initial Phase Angle (No Initial Coast Period and 12- and 24-Hour Initial Coast Periods; 3-Day Rendezvous)

The slope,  $m$ , can be determined by examining a single Hohmann transfer. First, the equation for the  $\Delta V$  of the transfer is modified to be a function of altitude change and final altitude. Then, the partial derivative of this function with respect to  $\Delta A$  results in an equation for the slope,  $m$ . This equation has been tested numerically, and the slope has been shown to be nearly constant (approximately  $0.0005 \text{ m/sec}^{-1}/\text{km}^{-1}$ ) for transfers below 500 km.

The total altitude change in a four-burn sequence is defined by the following equation:

$$\Delta A = |A_p - A_c| + |A_p - A_t| \quad (4-2)$$

where

- $A_c$  = initial chase (user spacecraft) orbit altitude
- $A_p$  = phase orbit altitude
- $A_t$  = target (final) orbit altitude

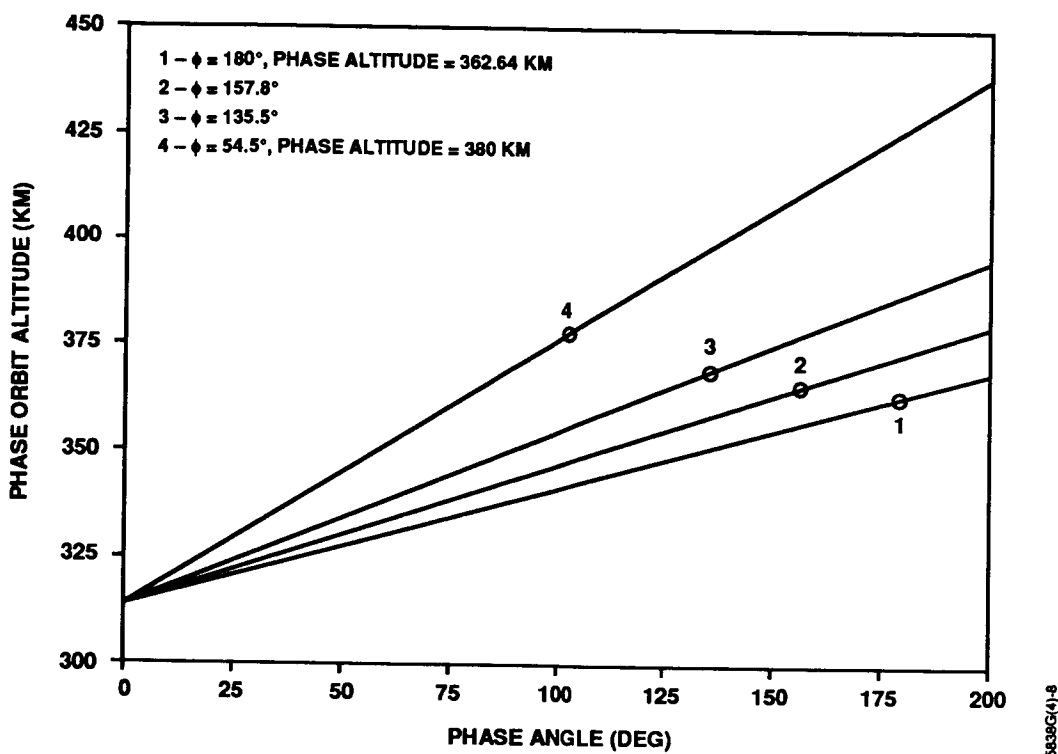


Figure 9. Changes in Phase Altitude and Angle as Initial Coast Period Increases

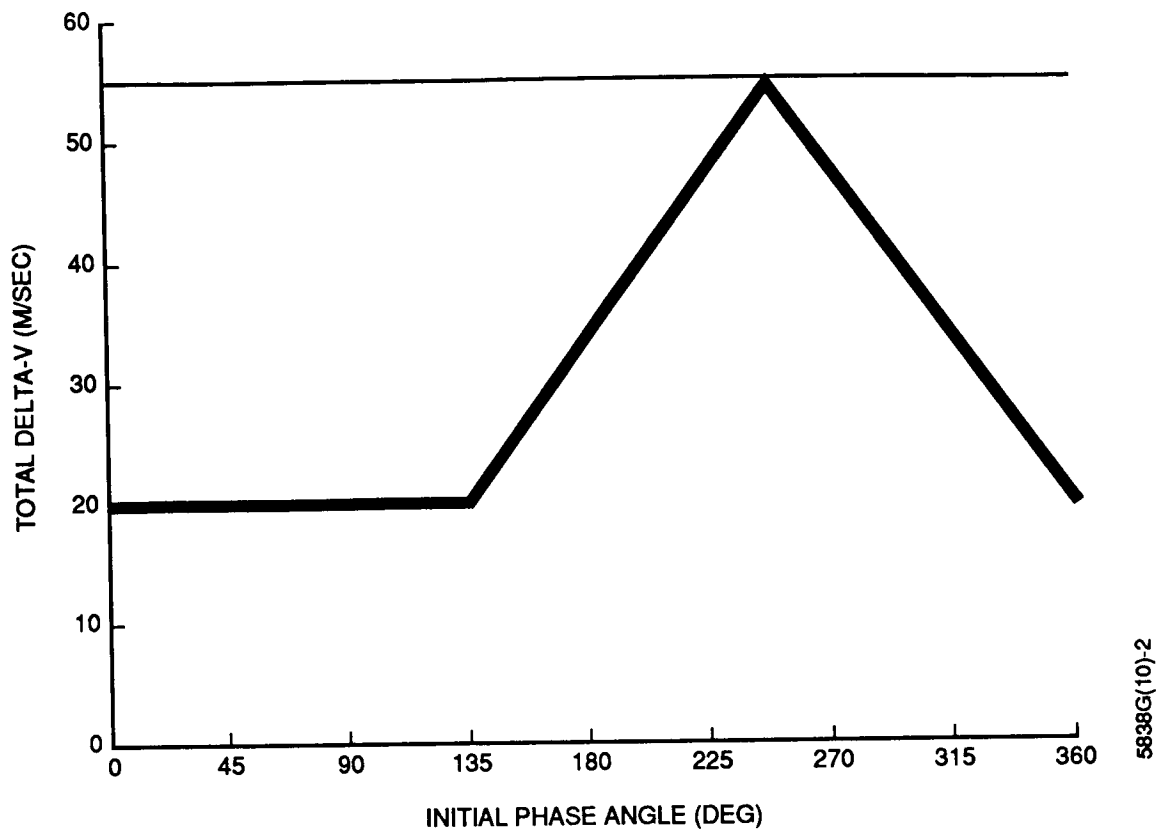
Absolute values are required, since the position of the phase orbit may lie between the chase and target orbits, above the chase orbit, or below the target orbit.

The phase orbit altitude can be computed from the analytic equations discussed in Section 2.2. These equations describe the upper and lower phase orbit altitudes for a standard four-burn rendezvous as functions of the rendezvous duration and the initial phase angle.

For a constant  $\Delta V$  rendezvous, the equations must be modified to include the initial coast period: The modified equations include the changes in the rendezvous time and the phase angle.

$$A_{pu} = k_u \frac{(w_c - w_i) t + \Phi_o}{(T - t)} + A_t \quad (4-3)$$

$$A_{pl} = k_l \frac{(w_c - w_i) t + \Phi_o}{(T - t)} + A_t - \frac{2\pi k_l}{(T - t)} \quad (4-4)$$



**Figure 10. Total  $\Delta V$  for a 3-Day Rendezvous, Standard Four-Burn Sequence Versus Constant  $\Delta V$  Sequence**

where

- $A_{pu}$  = phase orbit solution above the target orbit
- $A_{pl}$  = phase orbit solution below the target orbit
- $T$  = total initial rendezvous duration
- $t$  = initial coast time
- $w_c$  = angular rate of the chase vehicle in its initial orbit
- $w_t$  = angular rate of the target vehicle in its orbit
- $\Phi_0$  = initial phase angle
- $k_u$  = equation constant, 45.76 km day
- $k_l$  = equation constant, 44.18 km day

When Equations (4-3) and (4-4) are then substituted into Equation (4-2), three different solutions result for  $\Delta A$  as a function of phase angle, corresponding to the three possible positions of the phase orbit relative to the initial chase and target orbits. Specifically,

these solutions are (1) above the chase orbit, (2) below the target orbit, and (3) between the chase and target orbits. The equations for the total altitude change are as follows:

$$\Delta A_u = A_t - A_c + \frac{2 * k_u}{(T - t)} [(w_c - w_t) t + \Phi_o] \quad (4-5)$$

$$\Delta A_l = A_c - A_t - \frac{2 * k_l}{(T - t)} [(w_c - w_t) t + \Phi_o - 2\pi] \quad (4-6)$$

$$\Delta A_b = A_c - A_t \quad (4-7)$$

where

$\Delta A_u$  = total altitude change for a phase orbit above the chase orbit

$\Delta A_l$  = total altitude change for a phase orbit below the target orbit

$\Delta A_b$  = total altitude change for a phase orbit between the chase and target orbits

Multiplying the above equations by the slope constant,  $m$ , in Equation (4-1) yields equations for  $\Delta V$  as a function of initial coast time,  $t$ .

$$\Delta V_u = m_u \left[ A_t - A_c + \frac{2k_u[(w_c - w_t) t + \Phi_o]}{(T - t)} \right] \quad (4-8)$$

$$\Delta V_l = m_l \left[ A_c - A_t + \frac{2k_l[(w_c - w_t) t + \Phi_o - 2\pi]}{(T - t)} \right] \quad (4-9)$$

$$\Delta V_b = m_b (A_c - A_t) \quad (4-10)$$

Solving for  $t$  in these equations yields

$$t_u = \frac{[T (\Delta V/m - A_t + A_c) - 2k_u \Phi_o]}{[2k_u (w_c - w_t) + \Delta V/m - A_t + A_c]} \quad (4-11)$$

$$t_1 = \frac{[T (A_c - A_t - \Delta V/m) + 4\pi k_1 - 2k_1 \Phi_o]}{[2k_1 (w_c - w_t) + A_c - A_t - \Delta V/m]} \quad (4-12)$$

$$t_b = \text{undefined} \quad (4-13)$$

where  $\Delta V$  = maximum  $\Delta V$  case with no initial coast.

If the optimum solution (before adding the coast) for a given phase angle is the upper solution (phase orbit above the chase orbit), then Equation (4-11) describes the coast time required before burn 1 for a constant  $\Delta V$  rendezvous. If it is originally the lower solution (phase orbit below the target orbit), then Equation (4-12) should be used.

If the original phase orbit is between the initial and final orbits, the altitude change is constant. Therefore, since no equation exists as a function of  $t$  for the constant portion of the curve in Figure 6, no formula for an initial coast time can be extracted. This occurs because an initial coast will decrease the phase angle that still requires a phase orbit between the chase and target orbits, and the total altitude change remains the same. However, as the coast time increases, the phase angle reaches 0 deg, or 360 deg. At these angles, Equation (4-11) may be used to solve for the additional coast time required to achieve a constant  $\Delta V$ . In summary, the coast time required from the initial phase angle would equal the coast time from the initial phasing to a phasing of 0 deg plus the coast time generated from Equation (4-10) for a phase angle of 360 deg. A simpler approach to the problem is to apply Equation (4-11) and adjust the initial phase angle by adding 360 deg. From this angle on the lower solution, an initial coast time may be found directly. Therefore, the following equation solves for  $t$  as a function of  $\Phi$  for initial phase angles that require a phase orbit between the chase and target orbits:

$$t_1 = \frac{[T (A_c - A_t - \Delta V/m) + 4\pi k_1 - 2k_1 (\Phi_o + 2\pi)]}{[2k_1 (w_c - w_t) + A_c - A_t - \Delta V/m]} \quad (4-14)$$

Equations (4-11), (4-12), and (4-14) are the only equations needed to compute the coast time before burn 1 that will result in a constant  $\Delta V$  rendezvous.

Figure 11 presents the coast time solutions for a 2- and 3-day GRO/STS rendezvous. The maximum coast time equals the rendezvous duration and occurs when the low-energy phase orbit altitude (without an initial coast) equals the initial altitude. The minimum point on each curve equals zero. This occurs at the maximum  $\Delta V$  case in Figure 3, which is the constant  $\Delta V$  value chosen.

The coast time results from the above equations are presented in Figure 6 for 2- and 3-day rendezvous. These values were tested in rendezvous cases using the current rendezvous software, RENDEV. For the majority of the phase angles, the  $\Delta V$  costs for a 2-day



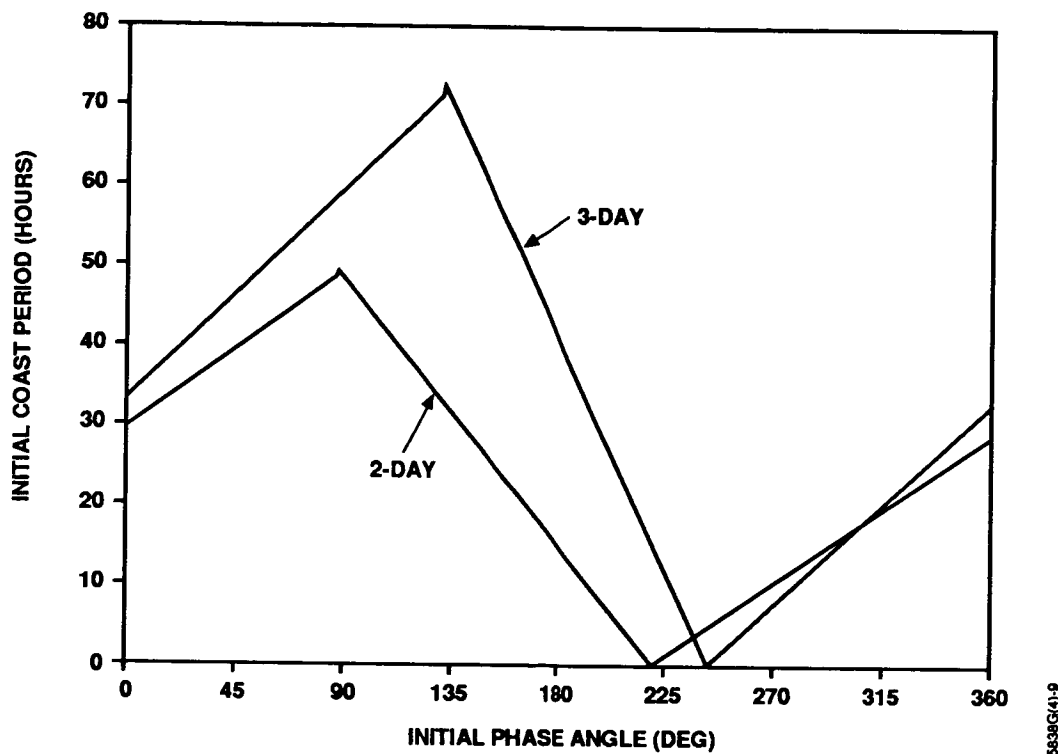


Figure 11. Initial Coast Period as a Function of Initial Phase Angle (2- and 3-Day Rendezvous)

rendezvous were within 6.5 percent of the maximum  $\Delta V$  value computed by RENDEV. For the 3-day case, errors were under 5 percent for most initial phase angles. These percentage errors can be attributed mainly to the phase orbit altitude approximation formulas. The limitation of these formulas is that they do not include the time and phasing changes in the transfer orbits. This approximation can offset the phase orbit altitude by as much as 2 km. Still, the overall coast time results are good approximations for most cases. The phase angles for which the coast times are not accurate occur when the coast time is within 12 hours of the total rendezvous duration. The accuracy of the linear approximations used in the derivations declines rapidly in the 12 hours or less available for the four-burn rendezvous sequence.

## 5. CONCLUSIONS

This paper has considered active rendezvous between a low-Earth-orbit user spacecraft and the STS Shuttle for refueling missions. A four-burn rendezvous sequence consisting of a series of Hohmann transfers, which was derived in a previous study, is presented as an optimal solution for rendezvous and retrieval missions. However, this sequence does not readily satisfy the mission constraints for refueling scenarios. Therefore, a variation of the standard four-burn sequence is derived as a method that satisfies all constraints for refueling missions while optimizing  $\Delta V$  costs.

The characteristics of the constant  $\Delta V$  rendezvous scenario are described in detail. In addition, a number of analytic equations are derived that solve for the initial coast period used in the rendezvous solution. These equations were tested with current software, RENDEV, which models the four-burn sequence after the initial coast period. The coast time equations were found to be good approximations for the majority of initial phase angles. However, for a small range of phasings, the solutions are not accurate, since the approximations made in the analytic equations for phase orbit altitudes are not valid. Therefore, accurate solutions for all phasings require iterative solutions.

## REFERENCES

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