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How Much Redundancy?–Some Cost Considerations, Including Examples for Spacecraft Systems

Ronald C. Suich
California State University
Fullerton, California

and

Richard L. Patterson
Lewis Research Center
Cleveland, Ohio

NASA

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by

Ronald C. Suich

California State University

Fullerton, CA 92634

and

Richard L. Patterson

NASA Lewis Research Center

Cleveland, Ohio 44135

ABSTRACT

How much redundancy should be built into a subsystem such as a space power subsystem? How does a reliability or design engineer choose between a power subsystem with .990 reliability and a more costly subsystem with .995 reliability? How does the engineer designing a power subsystem for a satellite decide between one power subsystem and a more reliable but heavier power subsystem?

High reliability is not necessarily an end in itself. High reliability may be desirable in order to reduce the statistically expected loss due to a subsystem failure. However, this may not be the wisest use of funds since the expected loss due to subsystem failure is not the only cost involved. The subsystem itself may be very costly. We cannot consider either the cost of the subsystem or the expected loss due to subsystem failure separately. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected loss due to subsystem failure.

We consider a specific type of redundant system, called a k-out-of-n: G subsystem. Such a subsystem has n modules, of which k are required to be good for the subsystem to be good. We discuss five models which can be applied in the design of a power subsystem to select the unique redundancy method which will minimize the total of the cost of the power subsystem plus the expected loss due to the power subsystem failure. A BASIC computer program is available.

1. INTRODUCTION

Although much has been written about the computation of reliability, little guidance is given to the reliability or design engineer who is asked to build redundancy into a subsystem. How does the engineer choose between a subsystem with .990 reliability and a more costly subsystem with .995 reliability? How does the engineer designing a subsystem for a rocket decide between one subsystem and a more reliable but heavier subsystem?

To answer these questions the engineer needs to consider not only the cost of the subsystem but also the losses that would occur if the subsystem fails. These losses are weighed by the probability of their occurrence to yield the expected loss.

1.1 EXPECTED VALUE

Since much of the paper is founded upon the idea of expected value or expected loss, we'll review some of the fundamental uses of this concept in decision-making applications. Suppose that you may choose between actions A and B. In this example, action A always results in a $1000 return to you. Then A has a value of $1000 and we can say that the expected value of A, E(A), is $1000. Action B, on the other hand, results in a return to you of either $500, outcome B_1, or $1500, outcome B_2. This return is a random variable which depends upon circumstances beyond your control. The choices which you face are outlined in the box.
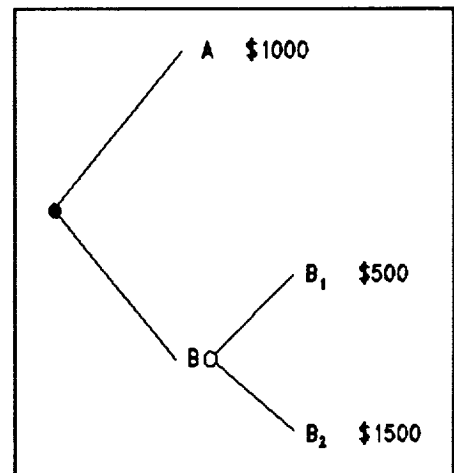
If B_1 and B_2 are equally likely, i.e., $Pr(B_1) = Pr(B_2) = .5$ (where Pr means "probability of"), then $E(B) = \$500 \times Pr(B_1) + \$1500 \times Pr(B_2) = \$500(.5) + \$1500(.5) = \$1000$. If you use expected value as your criterion, then you would be indifferent as to choice A or B, since both have an expected value of $1000. Also note that, although B has an expected value of $1000, you never receive $1000. Half of the time you receive $500 and half of the time you receive $1500. There are circumstances where you would not wish to use expected value as your criterion. Suppose that you had borrowed $1000 from a loan shark

and needed to be sure of receiving $1000 to repay your debt. A is the obvious choice.

Now suppose that the probabilities of B_1 and B_2 are .4 and .6, respectively. Then E(B) = $500(.4) + $1500(.6) = $1100. If you use expected value as your criterion, you would choose B over A. Again, in unusual circumstances, such as the need to repay $1000, you might choose A over B, even though A has the lower expected value. For these types of circumstances, we say that the certain return of $1000 has a higher expected utility to you than the _expected utility_ associated with an expected value of $1100, where the return can be either $500 or $1500.

Suppose instead that action A results in a loss of $1000 while action B can result in a loss of either $500 or $1500. We could, in a manner similar to that above, analyze actions A and B in terms of their expected losses. Our objective would be to minimize expected loss.

Throughout the remainder of this paper we will use expected value or expected loss as our criterion. For unusual circumstances, the procedures outlined in this paper can be applied using expected utility rather than expected value. For a more detailed discussion of utility, see [1].

1.2 BALANCING TWO COSTS

Suppose that failure of a subsystem results in a loss of, say, c_1 dollars. c_1 includes all losses incurred by subsystem failure (but does not include the cost of the subsystem itself). A highly reliable subsystem will incur this loss infrequently while a less reliable subsystem will incur this loss with greater frequency. Since this loss occurs only when the subsystem fails, we need to consider the expected loss due to subsystem failure. Additionally, the main system itself (in which the subsystem is placed) also may fail due to causes other than failure of the subsystem in question. Let r be the reliability of the main system for other than failure of the subsystem. This expected loss due to subsystem failure is given by

$$E\{\text{loss due to subsystem failure}\} = rc_1 \, Pr\{\text{subsystem failure}\}.$$

We can minimize this expected loss by building a subsystem with an extremely low probability of failure, i.e., a subsystem with extremely high reliability. This is the rational for building extremely reliable subsystems. High reliability is not necessarily an end in itself. High reliability may be desirable in order to reduce the expected loss due to subsystem failure. However, this subsystem may be very costly and

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not the wisest use of funds since the expected loss due to subsystem failure is not the only cost involved. A less reliable subsystem would, of course, result in a higher expected loss due to subsystem failure but may be less costly to build. In this situation it is not clear that we should build the most reliable subsystem possible since this will minimize only the expected loss due to subsystem failure but does not consider the cost of the building the subsystem. We cannot consider either the cost of the subsystem or the expected loss due to subsystem failure separately. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected loss due to subsystem failure. The total quantity to be minimized is given by

C = cost of the subsystem + E{loss due to subsystem failure}.

In minimizing C we see that we are balancing the cost of the subsystem and the expected loss. An inexpensive subsystem with low reliability may result in a very high E{loss} due to subsystem failure while an expensive subsystem itself will be costly.

1.3 EXAMPLE

As an example, suppose that we have four possible subsystems under consideration. Subsystem A, which costs one unit (hundred of thousands of dollars) has a .1 probability of subsystem failure. Subsystem B, with a cost of two units, has a .05 probability of subsystem failure. Subsystem C, with a cost of four units, has a .025 probability of subsystem failure while subsystem D, with a cost of ten units, has a .01 probability of subsystem failure. The more reliable subsystems have higher costs. Without further information and analysis, there is no clear "best" subsystem and the choice is often based upon the amount budgeted for the subsystem.

In reality the subsystem is but a component of a larger system. The best subsystem for this larger system will depend upon both r, the reliability of this larger system and upon c_1, the loss due to failure of the subsystem. We'll consider examples of four different situations. In each situation assume that the reliability of the system (for other than failure of the subsystem in question) is .9, i.e. r = .9. For situation 1 let c_1 = 10. For situations 2, 3 and 4 let c_1 = 50, 100 and 1000 respectively. The four situations are listed in table 1 in order of increasing loss due to subsystem failure.

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Table 1

Comparison of the costs and E{loss} for four subsystems in each of four situations

situation	subsystem	cost of subsystem	r	c_1	Pr(subsystem failure)	E(loss)	C
1	→ A	1	.9	10	.1	.9	1.9
1	B	2	.9	10	.05	.45	2.45
1	C	4	.9	10	.025	.225	4.225
1	D	10	.9	10	.01	.09	10.09
2	A	1	.9	50	.1	4.5	5.5
2	→ B	2	.9	50	.05	2.25	4.25
2	C	4	.9	50	.025	1.125	5.125
2	D	10	.9	50	.01	.45	10.45
3	A	1	.9	100	.1	9	10
3	B	2	.9	100	.05	4.5	6.5
3	→ C	4	.9	100	.025	2.25	6.25
3	D	10	.9	100	.01	.9	10.9
4	A	1	.9	1000	.1	90	91
4	B	2	.9	1000	.05	45	47
4	C	4	.9	1000	.025	22.5	26.5
4	→ D	10	.9	1000	.01	9	19

In situation 1 subsystem A is optimal (denoted by the →) since its value of C = cost of the subsystem + E{loss} = 1 + (.9)(10)(.1) = 1 + .9 = 1.9 is lowest among the four subsystems. Although subsystem A has the highest E{loss} due to subsystem failure, its lower cost of building the subsystem results in the lowest total for C. In situation 4 subsystem D, with the highest cost of building the subsystem, has the lowest C because of its low E{loss}. Since c_1 = 1000, greater weight is given to the lower Pr{subsystem failure} which results in a relatively low E{loss} due to subsystem failure. In general, for higher values of c_1, i.e., for higher losses due to failure of the subsystem, a more reliable subsystem is required to minimize C. Therefore, there is no best overall subsystem, but rather a best

<u>subsystem for use in a particular system</u>.

1.4 K OUT-OF-N:G SUBSYSTEM

In this paper we will direct our attention to a specific type of subsystem, called a k-out-of-n:G subsystem. Such a subsystem has n modules, of which k are required to be good for the subsystem to be good. Selection of the different values of n and k results in different subsystems, each with different costs and reliabilities.

As an example consider the situation where the engineer has a certain power requirement. He may meet this requirement by having one large power module, two smaller modules, etc. The number of modules required is called k. For example, the engineer may decide that k = 4. This means that each module is 1/4 of the full required power. Therefore, the subsystem must have 4 or more modules for the full required power. The number of modules used in the subsystem is called n. For example, an n = 6 and k = 4 subsystem would have 6 modules each of 1/4 th power and thus would have the output capability of 1.5 times the required power. The engineer is free to choose n and k and may choose them to minimize C.

In the remainder of this paper we will be presenting five models, which cover different situations and circumstances and are generally presented in order of increasing complexity.

1.5 ASSUMPTIONS AND NOTATION

<u>ASSUMPTIONS</u>

In this paper we will assume perfect switching devices (if needed) of negligible cost and independence of the modules of the subsystem.

<u>NOTATION</u>

n	number of modules in the subsystem
k	minimum number of modules which must be good for the subsystem to be good
r	reliability of the system for other than failure of the subsystem
c_1	loss due to failure of the subsystem
c_2	loss due to subsystem output at v_c

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c_3 cost of a one module subsystem capable of full output

c_4 cost of a module in a k-out-of-n:G subsystem when k is fixed

c_5 cost of launching a one module subsystem capable of full power

g(k)   function which relates cost of subsystem to the number of modules in the subsystem

w(k)   function which relates cost of launching the subsystem to the number of modules in the subsystem

v_c fraction of subsystem output necessary so that the mission is not a failure

p   probability that a module is good

q   probability that a module fails or 1-p

C   the total of the cost of the subsystem plus the expected loss due to subsystem failure

λ failure rate of a module

T_0 mission time

2. MODEL 1

Suppose that the modules are independent and all have common probability p of being good and common probability of failure q = 1-p. Let X count the number of good modules. Then

$$Pr\{subsystem\ failure\} = Pr\{X < k\} = \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}$$

and E{loss due to subsystem failure} = rc_1 Pr{subsystem failure}

$$= rc_1 Pr\{X < k\} = rc_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}. \tag{1}$$

First consider a simple situation where k is fixed. Here we are free to choose only n. Then n-k will be the redundancy or number of spares in the subsystem. If each module costs c_4 then the cost of subsystem = nc_4. Using this with (1) we obtain

C = cost of subsystem + E{loss due to subsystem failure}

= nc_4 + rc_1 Pr{subsystem failure} =

$$= nc_4 + rc_1 Pr(subsystem\ failure) = nc_4 + rc_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}. \quad (2)$$

We wish to find the n which minimizes C (Note that n-k is the number of spares which will minimize C).

As an example, consider the situation where k = 1, i.e. only one module is required to be operational for the subsystem to be operational. Suppose that the reliability of this single module is .95, i.e., p = .95. Let the reliability of the system other than for failure of the subsystem be .9, i.e. r = .9. Suppose that the cost of one module is 1, i.e., c_4 = 1. We'll again consider the four situations with c_1 = 10, 50, 100 and 1000 and present the results in table 2.

Table 2

An example of model 1 (with k fixed at 1)
with c_4 = 1 and p = .95.

	cost of subsystem	r	c_1	Pr(subsystem failure)	E(loss)	C
→ n = 1	1	.9	10	.05	.45	1.45
n = 2	2	.9	10	.0025	.0225	2.0225
n = 3	3	.9	10	.000125	.001125	3.001125
n = 1	1	.9	50	.05	2.25	3.25
→ n = 2	2	.9	50	.0025	.1125	2.1125
n = 3	3	.9	50	.000125	.005625	3.005625
n = 1	1	.9	100	.05	4.5	5.5
→ n = 2	2	.9	100	.0025	.225	2.225
n = 3	3	.9	100	.000125	.01125	3.01125
n = 1	1	.9	1000	.05	45	46
n = 2	2	.9	1000	.0025	2.25	4.25
→ n = 3	3	.9	1000	.000125	.1125	3.1125

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With c_1 = 10 the subsystem with the lowest C is the n = 1 subsystem, a subsystem with no spares. This subsystem has a reliability of 1 - Pr{subsystem failure} = 1 - .05 = .95. With c_1 = 50 the n = 2 subsystem, with one spare and a subsystem reliability of .9975, becomes the best. Here the increase in c_1 from 10 to 50 favors subsystems with higher reliability. At c_1 = 1000, the best subsystem, with C = 3.1125, has n = 3, a subsystem with two spares (Note that a subsystem with n = 4 could not be best in this situation since the cost of building this subsystem is four and therefore C will exceed four). Obviously, the greater the loss due to subsystem failure, the greater the number of spares required to minimize C. This same principle holds for subsystems with k of 2 or more.

3. MODEL 2

Suppose in model 1 that we are also free to choose k in our subsystem. Let c_3 be the cost of a subsystem of one module. Further suppose that the cost of a subsystem with exactly k modules is c_3 g(k). Here g(k) is the factor which measures the (generally) increased cost of building a subsystem consisting of k smaller modules rather than one large module. If g(k) = 1 for all k, then a subsystem of k modules costs the same as a subsystem consisting of a single module. Any g(k) may be used. For example, if a subsystem of 2 smaller modules costs 4 times as much as a single module subsystem then g(2) = 4. Therefore this subsystem would cost c_3 g(k) = c_3 g(k) = $4c_3$. If a subsystem of 3 smaller modules costs 7 times as much as a single module subsystem then g(3) =7. Other values for g(k) may be defined in a similar manner. Therefore g(1) = 1, g(2) = 4, g(3) = 7, etc. We also assume that each module in the subsystem costs c_3g(k)/k, which is 1/k th of the total cost for k modules. Since we have a total of n modules in the subsystem, then the cost of the subsystem = nc_3g(k)/k. Using this with (1) we obtain

C = cost of subsystem + E{loss due to subsystem failure}

$$= nc_3 g(k)/k + rc_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}. \quad (3)$$

For any particular situation with given values of c_1, c_3, r, p and g(k) we select the n and k to

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minimize C in (3). The n and k thus selected will be the optimal subsystem. A BASIC program can be used to search for the n and the k. See section 8 of this paper for more information.

Consider the example of a space electrical power subsystem. A rough rule of thumb says that the cost of smaller modules for a space electrical power subsystem is proportional to the electrical power raised to the .7. Thus, for this example $g(k) = k(1/k)^7$. Therefore, a subsystem consisting of a single module capable of full power would cost $c_3 g(1) = c_3 1(1/1)^7 = 1.0c_3$, a subsystem consisting of 2 modules, each of 1/2 power, would cost $c_3 g(2) = c_3 2(1/2)^7 = 1.23c_3$ to build, etc. An n = 3 and k = 2 subsystem, i.e., one having 3 modules each of 1/2 power, would cost nc_3 $g(k)/k = 3 \times 1.23c_3/2 = 1.85c_3$ to build.

Suppose that the loss due to subsystem failure, c_1, is 1000 (hundreds of thousands of dollars). Let the reliability of the system for other than failure of the subsystem be .99, i.e. r = .99. Suppose that the cost of building a single module capable of full power is 2 (hundreds of thousands of dollars), i.e. c_3 = 2. Suppose each module has a reliability of .95. For illustration in table 3 we'll compute the components of equation (3) along with the reliability of the subsystem for various values of n and k (where k is the number of modules required for full power).

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Table 3

An example of model 2, for various
subsystems for r = .99, c_1 = 1000, c_3 = 2 and p = .95

n	k	Cost of Subsystem	E{loss}	C	Reliability of Subsystem
1	1	2	49.5	51.5	.95
2	1	4	2.475	6.475	.9975
2	2	2.462	96.525	98.9873	.9025
3	1	6	.1238	6.1238	.999875
3	2	3.6934	7.1775	10.8709	.99275
3	3	2.7808	141.1988	143.9796	.857375
4	1	8	.0062	8.0062	.9999937
→ 4	2	4.9246	.4764	5.4010	.9995188
4	3	3.7077	13.8786	17.5863	.9859812
4	4	3.0314	183.6389	186.6703	.8145062
5	1	10	.0003	10.0003	.9999997
5	2	6.1557	.0297	6.1854	.99997
5	3	4.6346	1.1465	5.7812	.99884
5	4	3.7893	22.3666	26.1559	.97741
5	5	3.2413	223.9569	227.1982	.77378
6	1	12	.00002	12.00002	.999999+
6	2	7.3869	.0018	7.3887	.9999982
6	3	5.5616	.0086	5.6471	.9999136
6	4	4.5471	2.2075	6.7547	.9977701
6	5	3.8896	32.4461	36.3357	.9672261
6	6	3.4235	262.2591	265.6826	.7350918

A look at table 3 shows that the simple one module power subsystem (n = 1 and k = 1) has the lowest cost, 2, of building the subsystem. However, the high expected loss due to subsystem failure results in a high overall value for C of 51.5. The subsystem of n = 2 and k = 1 consists of two modules, only one of which is required for full power. Although this subsystem is more expensive to build than the n = 1 and k = 1 subsystem, its higher subsystem reliability results in a lower E{loss} due to subsystem failure and a lower total C. The n = 2 and k = 2 subsystem has a lower reliability (.9025)

because this subsystem requires that both modules be operational for full power. This lower reliability results in a higher $E\{loss\}$ and therefore a higher C. A look over the entire table shows that the minimum value of C occurs at $n = 4$ and $k = 2$ with a subsystem reliability of .99952. This subsystem consists of four modules, of which two are required for full power. One can also see from table 3 that there are a number of near optimal subsystems ($n = 4$ and $k = 2$, $n = 5$ and $k = 3$, $n = 6$ and $k = 3$) all of which have approximately the same values for C. Choices among these may also be made based upon other criteria. The n and k which minimize C are given for various values of c_3 and p in table 4.

Table 4

Optimum n and k (for model 2) for $r = .99$
and $c_1 = 1000$ which yield minimum C

c_3		$p = .80$	$p = .90$	$p = .95$	$p = .99$	$p = .999$
	n,k	16,7	10,5	7,4	2,1	3,2
1	Cost of Subsystem	4.10	3.24	2.65	2.00	1.85
	Rel. of Subsystem	.99975	.99985	.99981	.9990	.99999
	C	4.34	3.39	2.84	2.09	1.85
	n,k	12,5	8,4	4,2	3,2	1,1
2	Cost of Subsystem	7.78	6.06	4.92	3.69	2.00
	Rel. of Subsystem	.99942	.99957	.99952	.99971	.99900
	C	8.35	6.49	5.40	3.99	2.99
	n,k	8,3	6,3	2,1	3,2	1,1
5	Cost of Subsystem	18.54	13.90	10.00	9.23	5.00
	Rel. of Subsystem	.99877	.99873	.99750	.99970	.99900
	C	19.76	15.16	12.48	9.53	5.99

The values for $n = 4$ and $k = 2$ are the same as those given in table 3 where $c_3 = 2$ and $p = .95$. Also note that, for a given p, that C is minimized for smaller values of n and k for the more costly subsystems, e.g., for $p = .8$, the subsystem $n = 16$ and $k = 7$ minimizes C for $c_3 = 1$ while $n = 12$ and $k = 5$ minimizes C for $c_3 = 2$. For a given c_3, less redundancy is required for more reliable modules. This same pattern holds for different cost and loss functions. Although this pattern is intuitive, an advantage of this method is that it provides an exact solution along with its expected losses.

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We may use table 4 to find the n and k which minimize C. However, we may also use it to compare different subsystems. Suppose that we could build, or purchase from supplier 1, a one module subsystem capable of full power for $c_3 = 1$ with $p = .8$. Suppose further that supplier 2 could build a similar subsystem but with $p = .95$ for $c_3 = 2$. A third subsystem from supplier 3 has $c_3 = 5$ with $p = .999$. Here we will assume that more complicated subsystems will all increase by $g(k) = (1/k)^7$. To compare the 3 suppliers compare the optimal subsystems from each. The first entry in table 4 shows, for the first supplier with $c_3 = 1$ and $p = .8$, that the optimum solution ($n = 16$ and $k = 7$) is to build a subsystem of 16 modules, each 1/7 th of full power. The cost of building this subsystem is $nc_3 g(k)/k =$ $16 \times 1(1/7)^7 = 4.10$, while its reliability is .99975 with a total for C of 4.34. The optimum solution for supplier 2 (with $c_3 = 2$ and $p = .95$) is to build an $n = 4$ and $k = 2$ subsystem of 4 modules, each of 1/2 power. The cost of building such a subsystem is 4.92. It has a reliability of .99952 with C = 5.40. Since 4.34 < 5.40, choose the subsystem from supplier 1. Supplier 3 (with $c_3 = 5$ and $p = .999$) would not be chosen since his optimum solution ($n = 1$ and $k = 1$) has C = 5.99, which is higher than C = 4.34, the lowest C for supplier 1.

4. LAUNCHING COSTS OF THE SUBSYSTEM

There are circumstances when other cost factors are important and need to be considered. One such situation would be in space applications, where the weight of the subsystem is an important cost factor. The modification required here is to add the cost of launching the subsystem to C so that

C = cost of subsystem + cost of launching subsystem   (4)

+ E{loss due to subsystem failure}

If we are choosing between two subsystems of equal weight, then the same cost of launching the subsystem is contained in C for both subsystems and may therefore be dropped from further consideration. So the only time we need consider the launching costs is when the weights are different.

In the previous section we chose (with the use of table 4) a subsystem of $n = 16$, $k = 7$, $p = .80$ and C = 4.34 from supplier 1 over the subsystem from supplier 2 with $n = 3$, $k = 2$, $p = .95$ and C = 5.40. If these subsystems are used in a launch situation, we need to ask whether there are any

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differences in launch costs. Although the subsystem from supplier 2 has a higher C, it may be lighter and therefore have a lower launch cost. Therefore, we need to compare the minimum C's in (4) between the two suppliers to determine which supplier should be used. In order to minimize C in (4) we need a general cost of launching subsystems of different n's and k's. Let w(k), a function similar to the g(k) function, be the function which gives the (generally) increasing costs of launching more complex subsystems, i. e., subsystems having smaller modules. For example, if w(2) = 1.5, then a subsystem consisting of 2 smaller modules will cost 1.5 times as much to launch as the one module subsystem. If w(k) is specified then C becomes

$$C = nc_3 g(k)/k + nc_5 w(k)/k + rc_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}. \tag{5}$$

Consider the three suppliers mentioned previously, i.e., supplier 1 with modules of p = .80 for c_3 = 1, supplier 2 with p = .95 for c_3 = 2 and supplier 3 with p = .999 for c_3 = 5. Table 5 presents the optimum n and k for various values of c_5, the cost of launching a one module subsystem.

Table 5

Optimum n and k for model 2 for the example of
the three suppliers which yield minimum cost C

Supplier	c_3		$c_5 = 0.5$	$c_5 = 1.0$	$c_5 = 2.0$	$c_5 = 3.0$
1	1	n,k	14,7	12,5	10,4	10,4
		Rel. of Subsystem	.99962	.99942	.99914	.99914
		C	6.37	8.35	12.22	16.01
2	2	n,k	4,2	4,2	4,2	2,1
		Rel. of Subsystem	.99952	.99952	.99952	.99750
		C	6.63	7.86	10.33	12.48
3	5	n,k	1,1	1,1	1,1	1,1
		Rel. of Subsystem	.99900	.99900	.99900	.99900
		C	6.49	6.99	7.99	8.99

Supplier 1 should be used if c_5 = 0.5 since C is lowest for supplier 1. For c_5 = 1, 2 or 3, supplier 3

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should be chosen. Note that supplier 3 does not have the subsystem with the highest reliability. Throughout the remainder of this paper we will not explicitly consider the cost of launching the subsystem, since this cost may be included in the other models by inclusion of the term $nc_5 w(k)/k$ in the equation for C in each model.

5. MODEL 3

Now consider a more generalized loss function than that of model 2. Suppose that loss due to subsystem failure is given by figure 1, where v is the ratio of the actual output of the subsystem to the specification output.

If v drops below some critical value v_c, the loss is c_1, e.g., if the output falls so that v is below a critical fraction v_c, the mission is a complete failure. However, if v is at v_c, then the loss is only c_2. As v increases above v_c, this loss decreases until there is no loss at full output.
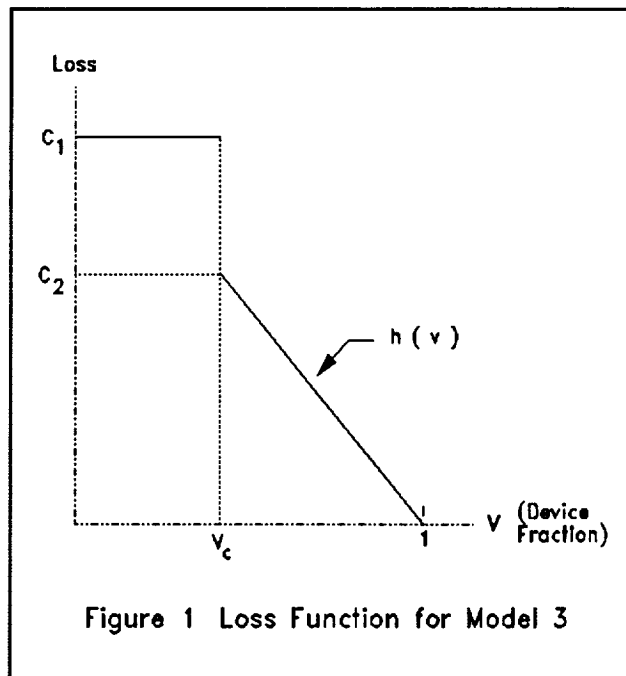

Figure 1 Loss Function for Model 3

Figure 1 shows h as a linear loss function but other loss functions, e.g., a decreasing multi-step function, might be appropriate. Note that, since we have identical modules, h takes on values only at $v = x/k$. If we let $h(v) = a - av$, for $v_c < v < 1$, where $a = c_2/(1-v_c)$, then

$$C = nc_3 g(k)/k + rc_1 \sum_{x=0}^{x<kv_c} \binom{n}{x} p^x q^{n-x} + r \sum_{x \geq kv_c}^{k-1} \binom{n}{x} p^x q^{n-x} (a - ax/k). \tag{6}$$

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Note that the third term on the rhs is expected loss due to partial failure of the subsystem. Again we can find, by means of the BASIC program described in section 8, the n and k which minimize C.

6. MODEL 4

Suppose that we have a situation similar to model 3 with $c_1 = c_2$ but now wish to consider time of the mission. If we assume that the individual elements fail exponentially with failure rate λ, then the probability of an element still operating successfully at time t is $\exp(-\lambda t)$. To find f(x,t), first let f(t|x) be the time at which the xth success occurs (the waiting time for the (n-x)th failure), given that n-x failures have occurred before mission time T_0. Then
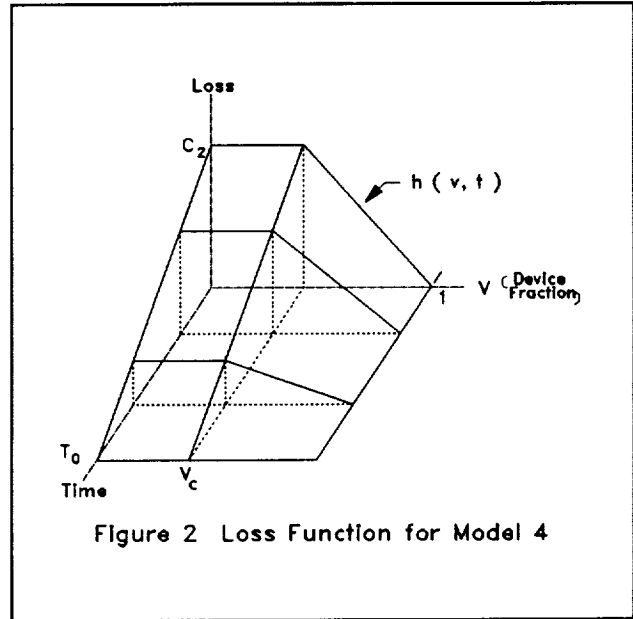

Figure 2   Loss Function for Model 4

$$f(t|x) = L(t|x) \bigg/ \int_0^{T_0} L(t|x)\,dt \qquad\qquad 0 < t < T_0$$

where $L(t|x) = \dfrac{n!}{x!(n-x-1)!}\,[\exp(-\lambda t)]^x\,\lambda\exp(-\lambda t)[1-\exp(-\lambda t)]^{n-x-1}$.

$\therefore\ f(x,t) = f(t|x)\ g(x)$

where $g(x) = \dbinom{n}{x}[\exp(-\lambda T_0)]^x\,[1-\exp(-\lambda T_0)]^{n-x} \qquad x=0,1,...,n.$

If the output fraction is at v_c at the start of the mission, our loss is c_2. We further assume that as the

output fraction increases above v_c, then this loss decreases until there is no loss at full output. With output at or above v_c, losses decrease with increasing time until there is no loss beyond mission time T_0. In figure 2 we assumed our loss function h(v,t), for any given v above v_c, will decrease linearly with time until no loss occurs above T_0. Additionally, for any given t, h(v,t) decreases as v increases above v_c.

Consider now a general loss function h(v,t) [not necessarily the one illustrated by figure 2]. Again, for a given t, h takes on values only for $v = x/k$. Now we have

$$C = nc_3 g \ (k)/k + r\sum_{x=0}^{n} \int_0^{T_0} h \ (x/k,t) \ f \ (x,t) \, dt. \tag{8}$$

Note that the last term on the rhs of (7) is the expected loss due to subsystem failure. If we let

$$h \ (x/k,t) = d \ (x/k)\sum_{j=0}^{m} b_j \ t^j \tag{9}$$

then, after integrating, (7) becomes

$$C = nc_3 g(k)/k + r\sum_{j=0}^{m} b_j \ [\ \sum_{x=0}^{k-1} J(x) d(x/k) \sum_{i=0}^{n-x-1} (-1)^{i+1}\binom{n-x-1}{i}$$

$$[\exp(-\lambda T_0(x+i+1))\sum_{s=0}^{j} (j! T_0^{j-s})/((j-s)!(\lambda(x+i+1))^{s+1}]-j! [\lambda(x+i+1)]^{-j-1}] \ ] \tag{10}$$

$$\text{where } J(x) = [g(x)]^{-1}\sum_{i=0}^{n-x-1} (-1)^i \binom{n-x-1}{i}[1-\exp(-\lambda T_0(x+i+1)](x+i+1)^{-1}$$

$$\text{with } g(x) = \binom{n}{x}[\exp(-\lambda T_0)]^x \ [1-\exp(-\lambda T_0)]^{n-x} \qquad x = 0,1,...,n.$$

We wish to find the n and k which minimize C. Minimizing C in (8) is appropriate for any loss function of the form (9). Let us use the loss function given in figure 2. For $0 \le x < kv_c$, d(x/k) = 1, m = 1, $b_0 = c_2$ and $b_1 = -c_2 T_0^{-1}$. For $kv_c \le x \le k-1$ we have d(x/k) = 1 - x/k, m = 1, $b_0 = a$ and $b_1 = -aT_0^{-1}$

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where $a = c_2(1-v_c)^{-1}$ with $0 < v_c < 1$. Using (9) we obtain

$$C = nc_3 g(k)/k$$

$$+ r \Bigg\{ c_2 \sum_{x=0}^{x<kv_c} \sum_{l=0}^{n-x-1} (-1)^{l+1}\binom{n-x-1}{l}[\lambda(x+l+1)]^{-1}[\exp(-\lambda T_0(x+l+1))-1]$$

$$- c_2 T_0^{-1} \sum_{x=0}^{x<kv_c} J(x) \sum_{l=0}^{n-x-1} (-1)^{l+1}\binom{n-x-1}{l}$$

$$\Big[\exp(-\lambda T_0(x+l+1))[T_0(\lambda(x+l+1))^{-1}+(\lambda(x+l+1))^{-2}]-(\lambda(x+l+1))^{-2}\Big]$$

$$+ a \sum_{x \geq kv_c}^{k-1} J(x)(1-x/k)\sum_{l=0}^{n-x-1} (-1)^{l+1}\binom{n-x-1}{l}[\lambda(x+l+1)]^{-1}[\exp(-\lambda T_0(x+l+1)-1]$$

$$- a T_0^{-1} \sum_{x \geq kv_c}^{k-1} J(x)(1-x/k)\sum_{l=0}^{n-x-1} (-1)^{l+1}\binom{n-x-1}{l}$$

$$\Big[\exp(-\lambda T_0(x+l+1))[T_0(\lambda(x+l+1))^{-1}+(\lambda(x+l+1))^{-2}]-(\lambda(x+l+1))^{-2}\Big] \Bigg\}$$

(11)

6.1 Model 4 Applications

Model 4 might reasonably be applied to non-recoverable systems which, at the end of their service life, have no intrinsic or salvage value or which are prohibitively expensive to recover. Examples include undersea sonar systems anchored in deep water, instrument/telemetry packages located in remote regions or communications satellites in geosynchronous orbit. For a geosynchronous communications satellite a number of subsystems could be chosen as an example. Let us examine the satellite power system which can be divided into smaller identical modules. We will use the rule of thumb which says that the cost of a space power subsystem is proportional to the electrical power raised to the .7, which gives $g(k) = k(1/k)^7$. Suppose that the mission life is 7 years and that the reliability of the satellite (exclusive of the power subsystem) over the mission life is .90. Because the satellite needs power for stationkeeping, computers and cooling, at least 10% of the specification power is needed for the satellite to survive. Therefore, v_c is 0.1. Suppose that the satellite generates $2 million per month revenue. In the event of satellite failure, a new satellite could be launched within two years at a cost of $115 million. Therefore $c_2 = 163$ (115 plus 48 in lost revenue). Here we will assume that revenue is roughly

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proportional to power, i.e., if a module of the power system fails, then one or more channels are no longer available (in other applications, failure of a module may result in increased load on the remaining modules, thereby increasing the failure rate.) Table 6 gives the n and k which minimizes C for various values of c_3 and different failure rates of a module of the power subsystem. As c_3 increases, both n and k decrease and higher values of λ (lower reliability) require greater redundancy to minimize C.

Table 6

Optimum n and k (for model 4) which yield
minimum cost C for the example Note: λ (10^{-6} /hour)

c_3		$\lambda = 15$	$\lambda = 10$	$\lambda = 2.5$
	n,k	10,2	10,3	4,2
	Cost of Subsystem	6.16	4.63	2.46
1	Rel. of Critical Subsystem	.993811	.999590	.999592
	Rel. of Full Subsystem	.952789	.969021	.989740
	C	7.35	5.43	2.90
	n,k	4,1	3,1	2,1
	Cost of Subsystem	20	15	10
5	Rel. of Critical Subsystem	.869185	.903686	.979800
	Rel. of Full Subsystem	.869185	.903686	.979800
	C	25.16	19.22	11.03
	n,k	3,1	2,1	1,1
	Cost of Subsystem	30	10	10
10	Rel. of Critical Subsystem	.782483	.789883	.857872
	Rel. of Full Subsystem	.782478	.789883	.857872
	C	40.38	31.90	20.69

7. MODEL 5

Suppose that we have a situation similar to model 4 but now assume a loss of c_1 if the output fraction from the subsystem is below v_c anytime during the life of the mission. An example of such a loss function is given by figure 3.

Using this loss function, for $x < kv_c$, $b_0 = c_2$ and $b_1 = 0$ and for $kv_c \leq x \leq k-1$, we have $d(x/k) = 1 - x/k$, $m = 1$, $b_0 = a$ and $b_1 = -aT_0^{-1}$ where $a = c_2 (1 - v_c)^{-1}$ with $0 < v_c < 1$. Using (9) we obtain

Figure 3   Loss Function for Model 5
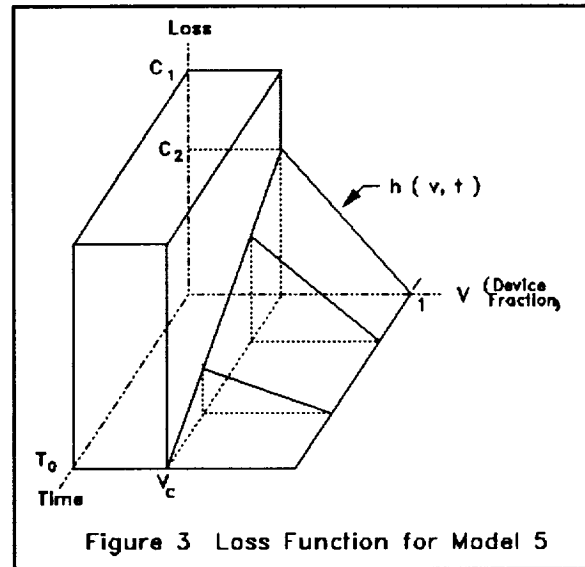
$$C = nc_3 g(k)/k + rc_1 \sum_{x=0}^{x < kv_c} \binom{n}{x} [\exp(-\lambda T_0)]^x [1-\exp(-\lambda T_0)]^{n-x}$$

$$+ r \left[a \sum_{x \geq kv_c}^{k-1} d(x)(1-x/k) \sum_{l=0}^{n-x-1} (-1)^{l+1} \binom{n-x-1}{l} \right] [\lambda(x+l+1)]^{-1}[\exp(-\lambda T_0(x+l+1)-1]$$

$$- aT_0 + \sum_{x \geq kv_c}^{k-1} d(x)(1-x/k) \sum_{l=0}^{n-x-1} (-1)^{l+1} \binom{n-x-1}{l}$$

$$\left[\exp(-\lambda T_0(x+l+1))[T_0(\lambda(x+l+1))^{-1}+(\lambda(x+l+1))^{-2}]-(\lambda(x+l+1))^{-2} \right] \,].$$

(12)

Model 5 could be applied to recoverable systems, systems which have inherent salvage value or manned systems. Examples include manned aircraft or spacecraft, recoverable undersea vehicles or spacecraft. Model 5 implies that if the output fraction of the subsystem falls below the critical value v_c, something catastrophic will occur, such as loss of the whole system or loss of life. With these systems, loss or significant degradation of a critical subsystem might cause loss of the craft and occupants.

8. BASIC PROGRAMS

The authors will provide, upon request, copies of Basic programs (Quickbasic 4.5) to both evaluate C and also to search for an n and k which minimize C. These programs are also appropriate for models 1, 2, 3, 4 and 5 (If you wish a copy on disk, please send a formatted 5 1/4" double-density floppy with your request). We also note that models 2, 3, 4 and 5 may be used when k is fixed by replacing $nc_3 g(k)/k$ by nc_4. Additionally, the cost of launching the subsystem may easily be considered merely by including this cost in C for the various models.

9. SUMMARY

Table 7 contains a summary of the five models which can be applied in a redundancy cost analysis.

Table 7

Redundancy Cost Models Considered in this Paper

Model 1   Simplest cost model. The subsystem consists of n modules, of which k are required for success of the mission. If less than k modules are good, a loss of c_1 occurs. In model 1, k is fixed. The g(k) cost function is also available to be used where increased redundancy brings in more (non-linear) cost. For spacecraft, launching costs may also be included in all the models.

Model 2   Same as model 1 except k may also vary.

Model 3   Model 3 expands on models 1 and 2. Linear (or other) loss functions are utilized. If less than k modules are good, some loss will occur but not necessarily the entire loss of c_1. The loss which occurs depends upon some critical output fraction v_c.

Model 4   Model 4 brings in the time domain to the loss function. Modules in the subsystem will fail exponentially with rate λ.

Model 5   Model 5 handles situations where output fraction below v_c causes a loss which is not time dependent, for example, to manned space missions where loss of a major portion of a critical subsystem may cause loss of life.

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10. REFERENCES

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15. Supplementary Notes

16. Abstract

How much redundancy should be built into a subsystem such as a space power subsystem? How does a reliability or design engineer choose between a power subsystem with 0.990 reliability and a more costly subsystem with 0.995 reliability? How does the engineer designing a power subsystem for a satellite decide between one power subsystem and a more reliable but heavier power subsystem? High reliability is not necessarily an end in itself. High reliability may be desirable in order to reduce the statistically expected loss due to a subsystem failure. However, this may not be the wisest use of funds since the expected loss due to subsystem failure is not the only cost involved. The subsystem itself may be very costly. We cannot consider either the cost of the subsystem or the expected loss due to subsystem failure separately. We therefore minimize the total of the *two costs*, i.e., the total of the cost of the subsystem plus the expected loss due to subsystem failure. We consider a specific type of redundant system, called a *k*-out-of-*n*: G subsystem. Such a subsystem has *n* modules, of which *k* are required to be good for the subsystem to be good. We discuss five models which can be applied in the design of a power subsystem to select the unique redundancy method which will minimize the total of the cost of the power subsystem plus the expected loss due to the power subsystem failure. A BASIC computer program is available.

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