

NASA Technical Memorandum 103244

# A Methodology for Evaluating the Reliability and Risk of Structures Under Complex Service Environments

Michael C. Shiao  
*Sverdrup Technology, Inc.*  
*Lewis Research Center Group*  
*Brook Park, Ohio*

and

Christos C. Chamis  
*National Aeronautics and Space Administration*  
*Lewis Research Center*  
*Cleveland, Ohio*

Prepared for the  
31st Structures, Structural Dynamics and Materials Conference  
cosponsored by the AIAA, ASME, ASCE, AHS, and ACS  
Long Beach, California, April 2-4, 1990



(NASA-TM-103244) A METHODOLOGY FOR  
EVALUATING THE RELIABILITY AND RISK OF  
STRUCTURES UNDER COMPLEX SERVICE  
ENVIRONMENTS (NASA) 17 0

N91-17415

CSCL 20K

Unclas

63/39

0317638

A METHODOLOGY FOR EVALUATING THE RELIABILITY AND RISK OF  
STRUCTURES UNDER COMPLEX SERVICE ENVIRONMENTS

Michael C. Shiao  
Sverdrup Technology, Inc.  
Lewis Research Center Group  
Brook Park, Ohio 44142

and

Christos C. Chamis  
National Aeronautics and Space Administration  
Lewis Research Center  
Cleveland, Ohio 44135

SUMMARY

Aerospace components are often subjected to complex service environments such as random excitations and random temperature conditions. In addition, small variations in the material properties or geometry may significantly affect the structural performance and durability of aerospace components. Therefore, a methodology was developed to determine structural reliability and to assess the associated risk due to various uncertainties in the design variables. The methodology consists of a probabilistic structural analysis by a special computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress), a generic probabilistic material property model (multifactor, interaction equation), and a probabilistic fatigue analysis. The methodology is versatile and is equally applicable to structures operating at high-temperature and/or cryogenic environments. The relationship between the probability of crack initiation and fatigue cycles is developed. The probabilistic crack propagation at a given risk level is studied. The most probable fracture path at a given cycle is determined. Those results serve as a guideline for component certification and for setting inspection criteria. The methodology is demonstrated to be capable of formally evaluating structural reliability and risk - including uncertainties in material properties, structural parameters, and loading conditions. It is described in detail with an application to the space shuttle main engine (SSME) blade.

INTRODUCTION

Probabilistic structural analysis methods (PSAM) (ref. 1) have been developed to analyze the effects of fluctuating loads, variable material properties, uncertainties in analytical models and geometry, and other factors - especially for high-performance structures such as the space shuttle main engine (SSME) turbopump blades. In the deterministic approach, the uncertainties in the responses are not quantified, and the actual safety margin remains unknown. Risk is calculated after extensive service experience. However, probabilistic structural analysis provides a rational alternative method to quantify uncertainties in structural performance and durability. NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) is a probabilistic structural analysis computer code developed under the PSAM program, which integrates finite element methods and reliability algorithms (refs. 2 and 3). This code can predict the

scatter of structural response variables such as stress, displacement, natural frequencies, and buckling loads, due to structural and environmental uncertainties. These predictions are subsequently compared with their probable failure modes to assess the risk of component fracture. Probable failure modes are defined for different structures and their respective service environments. For example, failure events such as stress greater than strength, displacements exceeding the maximum displacements allowed or avoidance of resonance are often used for the reliability assessment. Probability of occurrence of those failure events can be determined once the probability distributions of the requisite structural response variables are calculated with NESSUS.

In a reliability and risk analysis, all suspected sources of uncertainties must be taken into account to keep the probability of failure in service environments within an acceptable range. Structural reliability and risk obtained by a formal probabilistic methodology can be useful in evaluating the traditional design and in setting quality control requirements, inspection criteria, and criteria for retirement for cause. This analysis can also identify candidate materials and design concepts in the absence of a technology base. This paper extends the previous work (ref. 4), which studied only the probabilistic crack initiation. Current work also includes probabilistic crack propagation and probabilistic fracture paths analysis.

## CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS

In a probabilistic structural analysis, the primitive (random) variables that affect the structural behavior have to be identified. These variables, which include temperature, material properties, structural geometry, and loading conditions, should be described by their respective probability distributions. A structural analysis performed by NESSUS with the predetermined probability distributions of all the primitive (random) variables will produce corresponding scatter (uncertainties) in the structural responses such as displacement, stress, and natural frequencies. This concept is illustrated by figure 1.

## STRUCTURE OF NESSUS

NESSUS consists of three major modules: NESSUS/PRE, NESSUS/FEM, and NESSUS/FPI. NESSUS/PRE, a pre-processor, prepares the statistical data needed for the probabilistic structural analysis. It allows the user to describe the uncertainties in the structural parameters (primitive random variables) at nodal points of a finite element mesh. The uncertainties in these parameters are specified over this mesh by defining the mean value and standard deviation of the random variable at each point, together with an appropriate form of correlation. Correlated random variables are then decomposed into a set of uncorrelated vectors by a modal analysis. In a strongly correlated random field, there are significantly fewer dominant random variables in the set of uncorrelated vectors than in a weakly correlated random field. Thus, the computational time required for the analysis is reduced significantly.

NESSUS/FEM, a finite element code used for the structural analysis, generates a database containing all the response information corresponding to a small variation of each independent random variable. The algorithm used in

NESSUS/FPI requires an explicit response function in order to perform a reliability analysis. In complicated structural analysis problems, response can only be available implicitly through a finite element model. To overcome this difficulty, the response function is expressed parametrically with this database.

NESSUS/FPI (Fast Probability Integrator), an advanced reliability module (ref. 3), extracts the database generated by NESSUS/FEM to develop a response or a performance model in terms of uncorrelated random variables. The probabilistic structural response is calculated from the performance model. The probability of exceeding a given response value is estimated by a reliability method, which treats the problem as a constrained minimization. This step is called a point probability estimation. The cumulative distribution function is generated by running FPI at several response values. One alternative for generating the distribution function for any given response is to conduct a direct Monte Carlo simulation study. However, in general, it is very costly. NESSUS/FPI not only produces an accurate probability distribution, but also requires less computing time than a Monte Carlo simulation, especially in low-probability regions.

#### PROBABILISTIC MATERIAL PROPERTIES MODEL

A generic material-behavior model (multifactor interaction equations) (ref. 5) is used to evaluate the scatter in material properties for structures subjected to high temperatures and high cyclic loading. The fundamental assumption for this model is that the material behavior can be simulated by primitive (random) variables. The general form of this model is shown in equation (1):

$$M_P = M_{PO} \left( \frac{T_F - T}{T_F - T_0} \right)^n \left( \frac{S_F - \sigma}{S_F - \sigma_0} \right)^p \left( \frac{\log N_{MF} - \log N_M}{\log N_{MF} - \log N_{MO}} \right)^q \quad (1)$$

where  $M_P$  is the material property at current temperature after the application of fatigue cycles;  $M_{PO}$  is the reference material property at reference temperature  $T_0$  (usually room temperature), reference stress  $\sigma_0$ , and reference fatigue cycles  $N_{MO}$ ;  $T_F$ ,  $S_F$ , and  $N_{MF}$  are the final temperature, final material strength, and final fatigue cycle, respectively;  $T$  is current nodal temperature;  $\sigma$  is the respective current nodal stress; and  $N_M$  is the number of fatigue cycles to be considered in the fracture analysis. The exponents  $n$ ,  $p$ , and  $q$  are determined from available experiment data or can be estimated from the anticipated material behavior due to the particular primitive random variable.

#### APPLICATION TO SPECIFIC EXAMPLE

The methodology described previously was applied to an SSME turbine blade (fig. 2) that was subjected to complex mechanical and thermal loads (ref. 6). Mechanical loads consisted of the centrifugal force induced by rotational speed and the differential pressure over the airfoil. Since it was difficult to maintain a constant rotational speed, the centrifugal force had to be considered as a random variable, and differential pressure was also random because

of pressure fluctuation. In addition, thermal loads were random because of combustion irregularities that caused a random temperature distribution in the blade. Uncertainties in the blade geometry arose during the manufacturing process. The scatter in material properties was caused by nonuniformities in the material. The SSME turbine blade was modeled by 40 four-node shell elements with 55 nodal points. For this example, seven random fields were considered as listed in table I.

### COMPUTATIONAL PROCEDURE

The material properties considered in this analysis were the Young's modulus, the thermal expansion coefficient, and the material strength. Their probability distributions were predicted by the probabilistic material property model (defined by eq. (1)) which is a function of its reference value, temperature, and fatigue cycles. The statistics of the primitive random variables are listed in table II. Since the material is a function of stress and stress is a function of material property in an implicit way, an iterative procedure was necessary to obtain a convergent solution for stresses and material properties.

The iteration procedure was started by predicting the probability distributions of the material properties with equation (1). Only the reference values and the blade temperature were used at the beginning because the variation of the blade stress was unknown. A probabilistic structural analysis was performed with NESSUS, and the probability distributions of stresses were determined. Since the stress field is a random process in space, the joint probability distributions of the stresses for the entire blade were needed. They were calculated by the first-order, second-moment reliability method. Figure 3 shows a surface in a standardized, normally distributed probability space for the stress at node  $i$ ,  $\sigma_i$ , equal to a realization,  $y_i$ . One side of this surface represents the domain where  $\sigma_i < y_i$ . The other side is the domain where  $\sigma_i > y_i$ . The surface is defined by equation (2):

$$\sum_{k=1}^L \alpha_{ik} U_k - \beta_i = 0 \quad (2)$$

where  $L$  is the number of uncorrelated random variables,  $\alpha_{ik}$  is the  $k^{\text{th}}$  sensitivity factor from NESSUS/FPI,  $U_k$  is the  $k^{\text{th}}$  normalized, independent random variable, and  $\beta_i$  is the reliability index. The joint cumulative distribution function for stresses at nodes  $i$  and  $j$  are calculated by equation (3):

$$P(\sigma_i \leq y_i \text{ and } \sigma_j \leq y_j) = \Phi(-\beta_i)\Phi(-\beta_j) + \int_0^{\rho_{ij}} \phi(-\beta_i, -\beta_j; z) dz \quad (3)$$

where  $\rho_{ij}$  is defined by

$$\rho_{ij} = \sum_{k=1}^L \alpha_{ik} \alpha_{jk} \quad (4)$$

The generic probabilistic material property model is applied again at a given fatigue cycle with these newly estimated stress variations. The probability distributions of material properties are updated, and NESSUS is rerun.

The procedure is repeated until the probability distributions of the material properties and stress converge.

### PROBABILISTIC FATIGUE CRACK INITIATION

The SSME blade is assumed to be subjected to a given number of fatigue cycles that degrade the modulus, thermal expansion coefficient, and strength. Crack initiation occurs when the stress is greater than the strength. The critical points of the large displacement and high stress are depicted in figure 4. As shown in figure 5, the modulus at the root in the leading edge reduces significantly with fatigue degradation.

Changes in the probability density function of the tip displacement with and without fatigue degradation are shown in figure 6. As expected, the tip displacements increase with fatigue effect because the stiffness degrades and the blade becomes softer.

Material strength is calculated by the probabilistic material property model considering both temperature and fatigue degradation (fig. 7). Once the probability distributions of blade stress and strength are determined at this fatigue cycle, the probability that the stress is greater than the corresponding material strength is determined from equation (5):

$$P_f = \int_{-\infty}^{+\infty} \left( \int_{-\infty}^x f_S(s) ds \right) f_{\sigma}(x) dx \quad (5)$$

where  $f_{\sigma}$  is the probability density function (pdf) of effective stress calculated by the probabilistic structural analysis using NESSUS;  $f_S$  is the pdf of material strength simulated by the probabilistic material property model. The number of cycles is varied, and the procedure described previously is repeated to develop a risk-fatigue cycle curve for critical locations (fig. 8). This curve is useful for assessing the risk of structural fracture. For instance, at a given acceptable risk level, the number of fatigue cycles to initiate local fracture can be determined. With this information available, criteria can be set for quality control, inspection intervals and retirement for cause.

### RISK-COST ASSESSMENT

The risk-cost assessment evaluates the relationship between structural reliability and total cost to achieve this reliability. The total cost is the sum of the initial cost associated with the structure and a fraction of consequential cost. The fraction is weighted by the probability of failure. The initial cost is the cost for component service readiness. The consequential cost is the cost incurred due to structural fracture. This relationship is represented by equation (6):

$$C_t = C_i + P_f \times C_f \quad (6)$$

where  $C_t$  represents the total cost,  $C_i$  represents the initial cost,  $P_f$  is the probability of failure, and  $C_f$  is the consequential cost if failure does

occur. A total cost-fatigue cycle curve is shown in figure 9. If it is desired to inspect a structure before the total cost exceeds an acceptable level, the proper time for inspection can be determined.

### PROBABILISTIC FRACTURE PATHS ANALYSIS

Often it is important to know whether a structure can still operate safely or will fracture (cleavage or ductile) when a crack has just initiated. This can be probabilistically quantified by determining the most probable fracture initiation site, its respective probably path, and the corresponding structural integrity degradation during these processes.

The probable fracture path is determined as follows:  $D^0$  represents the structure in its undamaged state (no pre-existing cracks), and  $D^{ijk}$  represents a damaged state in which fracture has occurred at nodes  $i$ ,  $j$ , and  $k$  consecutively. Let  $B_l^{ijk}$  denote the event where node  $l$  fractures next when the structure is in the damaged state  $D^{ijk}$ . Damaged state is defined as the terminal state (the structure collapses) when the probability of occurrence of event  $B_l^{ijk}$  is equal to 1.0 and the strain energy has a sudden jump. The probability of reaching the terminal state is the probability that events  $B_i^0$ ,  $B_j^i$ ,  $B_k^{ij}$ , and  $B_l^{ijk}$  all occur. There are many probable fracture paths; only the two most probable paths are shown in figure 10: paths 1 and 2. The mean strain energy along path 2 is shown in figure 11. For path 2, the first fracture location is at node 10 with a probability of crack initiation equal to 0.10. At damaged state  $D^{10}$ , the probability to advance the crack to node 9 is equal to 0.0002. When both nodes 10 and 9 are fractured, the probability that node 14 will fracture next is equal to 1.0. However, the change of mean strain energy is small from  $D^{10,9}$  to  $D^{10,9,14}$ . When the crack extends to the node 18, the mean strain energy has a sudden jump. It indicates that the global structural failure is imminent. Therefore,  $D^{10,9,14}$  is the terminal state. The probability of path 2 is determined by equation (7):

$$P(\text{path 2 occurs}) = P(B_{10}^0 \cap B_9^{10} \cap B_{14}^{10,9}) \quad (7)$$

where  $\cap$  denotes the intersection of probability events. Similarly, the probability of occurrence of path 1 is

$$P(\text{path 1 occurs}) = P(B_{15}^0 \cap B_{14}^{15} \cap B_{18}^{15,14}) \quad (8)$$

The probability of path 2 occurring is 20 times greater than that of path 1 occurring. Therefore, path 2 is the only significant fracture path in this study. It is not necessarily true for some other problems. The natural frequencies decrease along this fracture path as shown in figure 12. When the crack was just formed, only the first and third modes were affected. When the damaged state is at  $D^{10,9,14}$ , significant frequency reduction is observed.

### PROBABILISTIC CRACK PROPAGATION

Structural components that either contain cracks or develop cracks early in their lives may still carry the service loads safely. Practical limitations in manufacture, inspection, and use of many structural components also prohibit

complete elimination of flaws. In some cases, where initial flaws prevail, slowly propagating cracks occupy a significant portion of the usable component life time. Therefore, the determination of fatigue crack propagation is an essential part of setting guidelines for inspection intervals and retirement for cause. Inspections should be made to detect and repair the defects prior to the conditions for imminent global fracture. The fundamental failure mechanism by which a crack propagates is that the load effect is greater than the material resistance at the current structural conditions. At a given number of cycles, the material resistance is exceeded by the cyclic load effect, and a new fracture mechanism is formed. The crack is either initiated or advanced, and a reliability assessment of fatigue life is needed to provide a basis to ensure a safe operation.

The probabilistic structural life can be divided into two parts: crack initiation life and crack propagation life at a given risk level. The risk level is based on the cost to upgrade the material and on the cost for the consequences of structural failure. During the crack initiation life (fig. 13), the probability of crack initiation at node 10 is lower than the probability for acceptable risk until the number of cycles reaches  $10^{4.6}$ . A crack is assumed to occur at this time. A probabilistic structural analysis is performed for this damaged structure at this fatigue cycle. The location (node 9) with highest probability of failure is detected and the structure undergoes crack propagation. As shown in figure 14, the probability of forming a new failure mechanism is smaller than that for the acceptable risk at the beginning of the crack propagation life. The structure is considered to be safe and can resist additional cycles. Again, when the number of cycles increase, the material properties such as modulus, strength, and fracture toughness degrade according to equation (1). At  $10^{5.6}$  cycles, the acceptable risk level is reached, and the crack advances from node 10 to node 9. The crack propagation rate is determined as

$$\frac{da}{dN} = \frac{a(9,10)}{10^{5.6} - 10^{4.6}} = 1.6 \times 10^{-6} \text{ in./cycle} \quad (9)$$

where  $N$  is the number of cycles and  $a(9,10)$  is the distance between nodes 9 and 10. The procedure is repeated with increasing cycles until risk rapidly violates the acceptable bound (fig. 15). Prior to this (say, at  $10^5$  cycles), the structure must be retired for cause (global structure fracture). Inspection intervals can be set during the crack propagation process to assure safety. For example, if the acceptable probability of failure is set at  $10^{-3}$ , the structure should be inspected right after  $10^5$  cycles (fig. 14).

#### SUMMARY OF RESULTS

A methodology for evaluating reliability and risk for aerospace components has been developed. It consists of a probabilistic structural analysis by NESSUS, a generic probabilistic material property model, and a probabilistic fatigue analysis. The methodology is versatile and equally applicable to hot and cold structures where data is difficult to obtain. The relationship between risk associated with local crack initiation (or total cost) and fatigue cycle were developed. The most probable fracture path at given cycles was found. The probabilistic fatigue life at a given risk level was studied. This information provides guidelines for setting inspection intervals, retiring components for cause, and certifying components.



APPENDIX - SYMBOLS

$a(9,10)$	distance between nodes 9 and 10
$B_l^{ijk}$	event where node $l$ fractures next when the structure is in the damaged state $D_{ijk}$
$C_f$	consequential cost if failure does occur
$C_i$	initial cost
$C_t$	total cost
$D_{ijk}$	structure damaged at nodes $i, j, k$
$D^0$	structure in undamaged state
$f_s$	probability density function of material strength simulated by the material property model
$f_\sigma$	probability density function of effective stress calculated by the probabilistic structural analysis using NESSUS
$L$	number of uncorrelated random variables
$M_p$	material property at current temperature after the application of fatigue cycles
$M_{p0}$	reference material property at reference temperature $T_0$
$N$	number of mechanical cycles
$N_M$	number of fatigue cycles considered in the fracture analysis
$N_{MF}$	final fatigue cycle
$N_{M0}$	reference fatigue cycles
$P$	probability
$P_f$	probability of failure
$S$	material strength
$S_F$	final material strength
$T$	current temperature
$T_F$	final temperature
$T_0$	reference temperature (usually room temperature)
$U_k$	$k^{\text{th}}$ normalized independent random variable

$y_i$  realization  
 $\alpha_{ik}$   $k^{\text{th}}$  sensitivity factor from NESSUS/FPI  
 $\beta_i$  reliability index (fig. 3)  
 $\rho_{ij}$  correlation coefficient  
 $\sigma$  respective current point stress  
 $\sigma_0$  reference stress  
 $\Phi$  normal cumulative distribution function  
 $\phi$  normal probability density function

Subscripts:

E modulus  
 F final  
 f failure  
 i,j,k nodes i, j, and k  
 S material strength  
 t total  
 $\nu$  Poisson's ratio  
 $\sigma$  effective stress  
 0 reference condition

## REFERENCES

1. Chamis, C.C.: Probabilistic Structural Analysis Methods for Space Propulsion System Components. Presented at 3rd Space System Technology AIAA Conference, San Diego, CA, June 1986.
2. Lind, N.C.; Krenk, D.; and Madsen, H.O.: Methods of Structural Safety, Prentice-Hall, 1986.
3. Wu, Y.-T.: Demonstration of a New, Fast Probability Integration Method for Reliability Analysis, Advances in Aerospace Structural Analysis; Proceedings of the Winter Annual Meeting, edited by O.H. Burnside and C.H. Parr, ASME, 1985, pp., 63-73.
4. Shiao, M.C.; and Chamis, C.C.: Probability of Failure and Risk Assessment of Propulsion Structural Components. The 1989 JANNAP Propulsion Meeting, Vol. 1, edited by D.S. Eggleston and K.L. Strange, Johns Hopkins Univ., 1989, pp. 135-162.
5. Boyce, L.; and Chamis, C.C.: Probabilistic Constitutive Relationships for Cycle Material Strength Models. Structures, Structural Dynamics and Materials Conference, 29th, Part 3, AIAA, Washington, DC, 1988, pp. 1299-1306.
6. Probabilistic Structural Analysis for Select Space Propulsion System Structural Components. Annual Report by Southwest Research Institute, Rocketdyne, University of Arizona, Marc Analysis Corporation and Columbia University for NASA Lewis Research Center, Contract NAS3-24389, 1986.

TABLE I. - RANDOM INPUT DATA

Random fields	Number of dependent random variables	Mean	Standard deviation (or coefficient of variation)	Correlation length
X coordinate	55	Deterministic coordinate	0.01 in.	5.0
Y coordinate	↓	Deterministic coordinate	0.01 in.	5.0
Z coordinate		Deterministic coordinate	0.01 in.	5.0
Temperature		Steady-state temperature	60 °F	3.0
Young's modulus	40	23 000 000 psi	(0.10)	3.0
Pressure	36	Steady-state pressure	(0.20)	0
Rotation speed	1	40 000 rpm	(0.01)	(a)
Poisson's ratio	40	$0.45 \times 10^{-5}$	$0.45 \times 10^{-6}$	3.0

<sup>a</sup>Not applicable.

TABLE II. - PRIMITIVE VARIABLE PROBABILITY DISTRIBUTIONS FOR PROBABILISTIC MATERIAL PROPERTY MODEL

Variable <sup>a</sup>	Distribution type	Mean	Standard deviation
T <sub>F</sub> , °F	Normal	2750	51.4
T <sub>0</sub> , °F	Normal	68	2.04
S <sub>F</sub> , ksi	Normal	212.0	10.6
σ <sub>0</sub>	Constant	0	0
N <sub>M</sub> F	Lognormal	10 <sup>8</sup>	5×10
N <sub>M</sub> O	Lognormal	10 <sup>3</sup>	50
n <sub>E</sub>	Normal	0.25	0.0075
p <sub>E</sub>		0.25	
q <sub>E</sub>		0.25	
n <sub>ν</sub>		-0.25	
p <sub>ν</sub>		0.25	
q <sub>ν</sub>			
n <sub>S</sub>			
p <sub>S</sub>			
q <sub>S</sub>			

<sup>a</sup>The variables n, p, and q are the exponents in equation (1); E is the Young's modulus; ν is the Poisson's ratio; S is the strength; S<sub>F</sub> is the final material strength; and T, σ, and N<sub>M</sub> are the temperature, stress, and number of fatigue cycles at reference 0 or final F conditions.

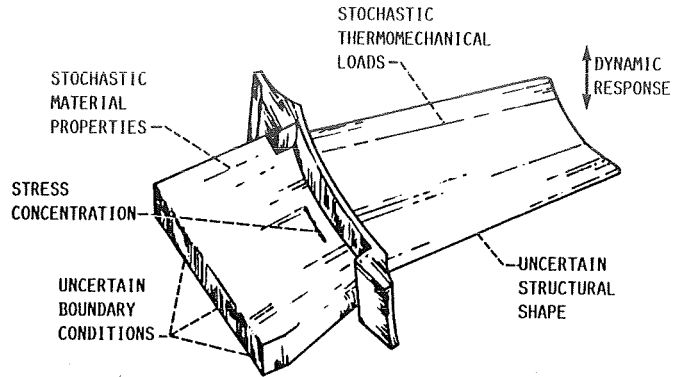


FIGURE 2. - UNCERTAINTIES IN PROBABILISTIC STRUCTURE ANALYSIS.

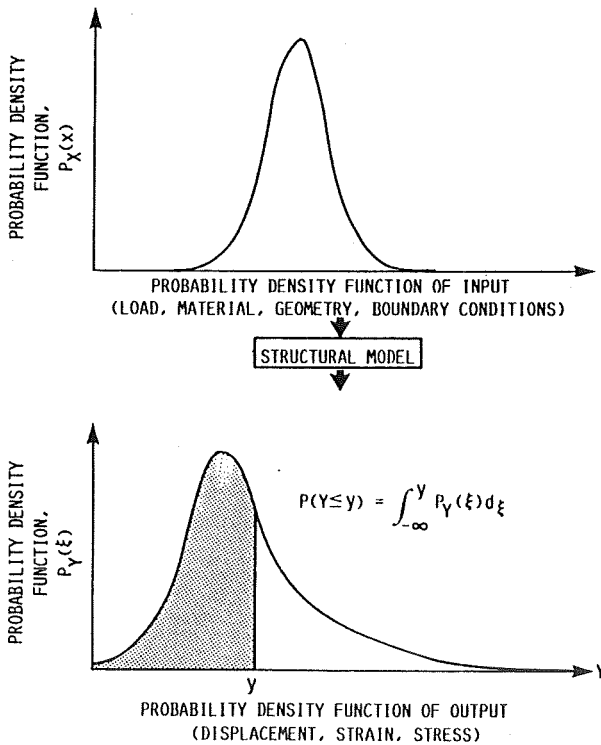


FIGURE 1. - CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS.

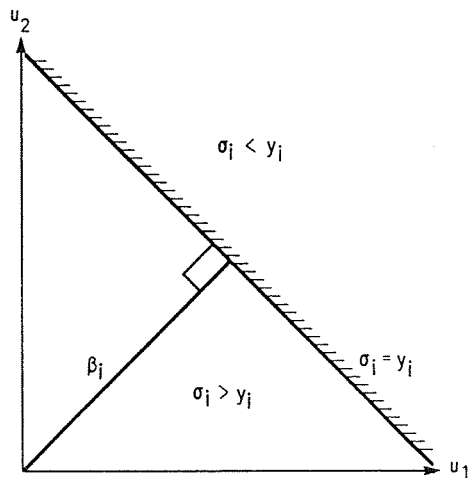


FIGURE 3. - LINEARIZED SURFACE FOR STRESS y, WHERE σ<sub>i</sub> IS THE STRESS AT NODE i, y<sub>i</sub> IS THE REAL VALUE, AND β<sub>i</sub> IS THE RELIABILITY INDEX.

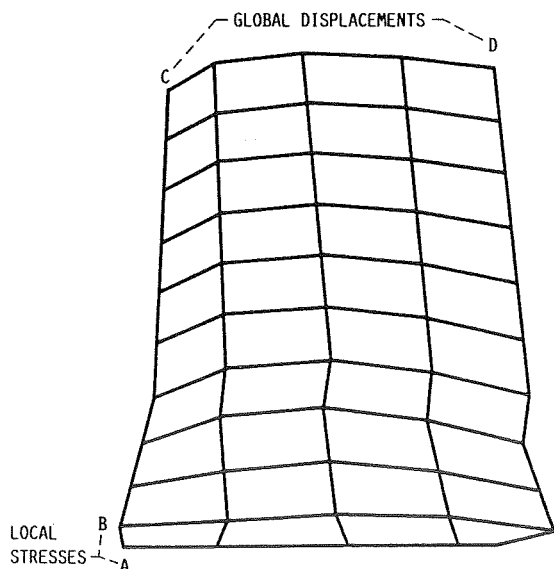


FIGURE 4. - SPACE SHUTTLE MAIN ENGINE (SSME) BLADE SHOWING LOCATIONS WHERE PROBABILISTIC STRUCTURAL RESPONSE WAS EVALUATED.

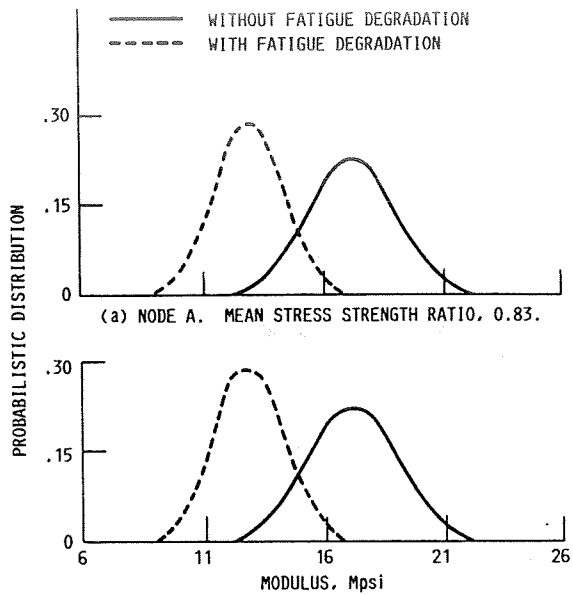
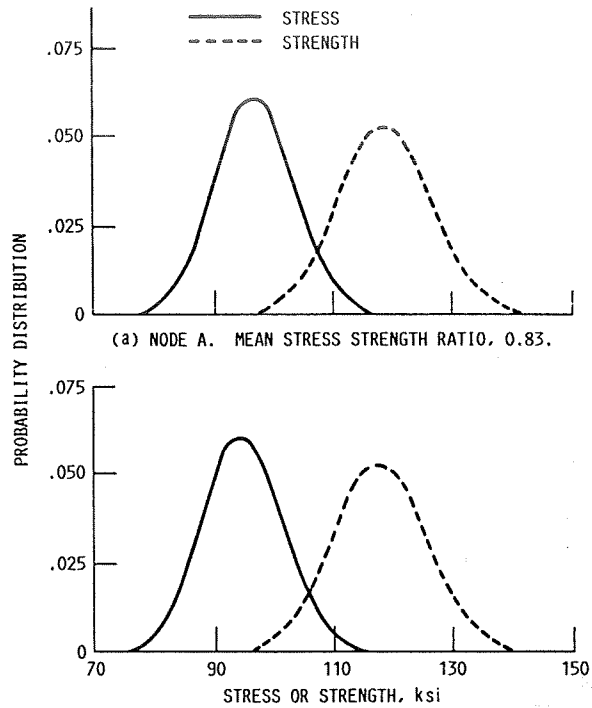


FIGURE 5. - PROBABILISTIC MODULUS SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL. (NODES A AND B SHOWN IN FIG. 4.)



(b) NODE B. MEAN STRESS STRENGTH RATIO, 0.82.

FIGURE 7. - PROBABILISTIC FATIGUE STRESS OR STRENGTH SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL. (NODES A AND B SHOWN IN FIG. 4.)

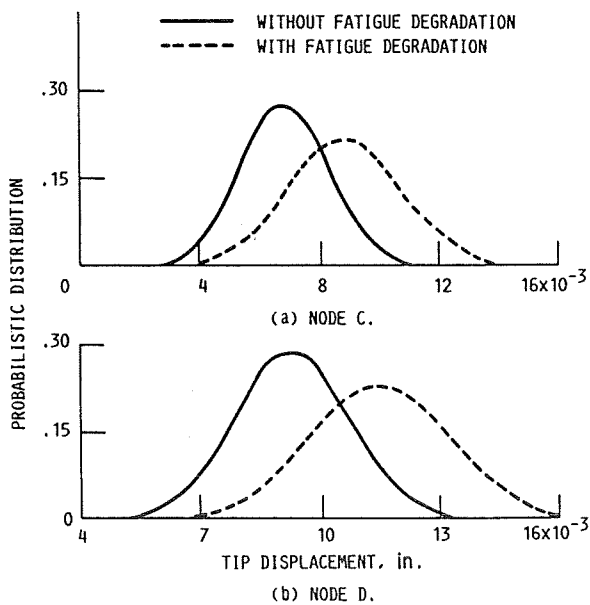


FIGURE 6. - PROBABILISTIC DISPLACEMENTS CALCULATED BY NESSUS. (NODES C AND D SHOWN IN FIG. 4.)

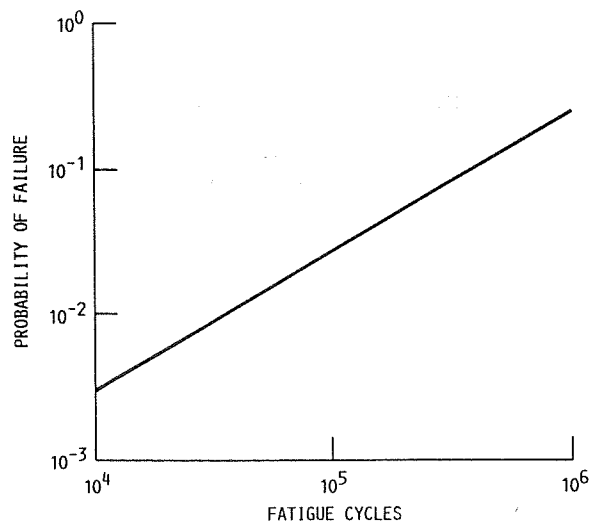


FIGURE 8. - PROBABILITY OF LOCAL FAILURE AT NODE A (FIG. 4) DUE TO FATIGUE CYCLES.

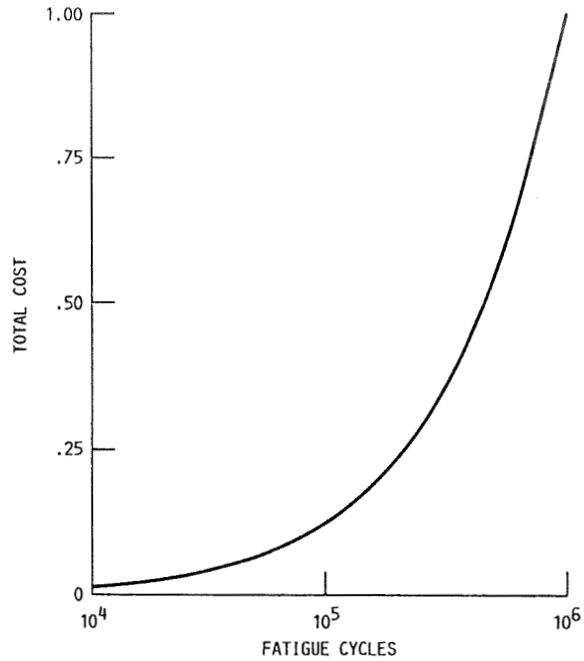


FIGURE 9. - RISK-COST ASSESSMENT.

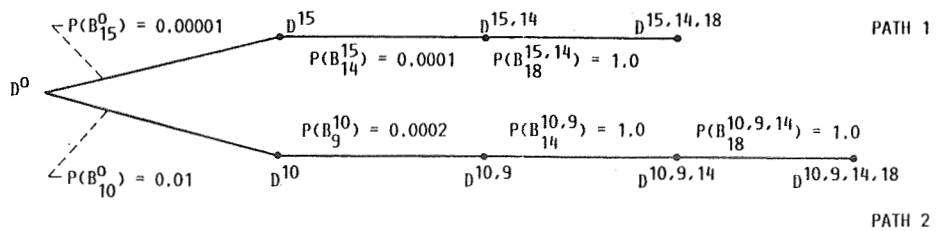
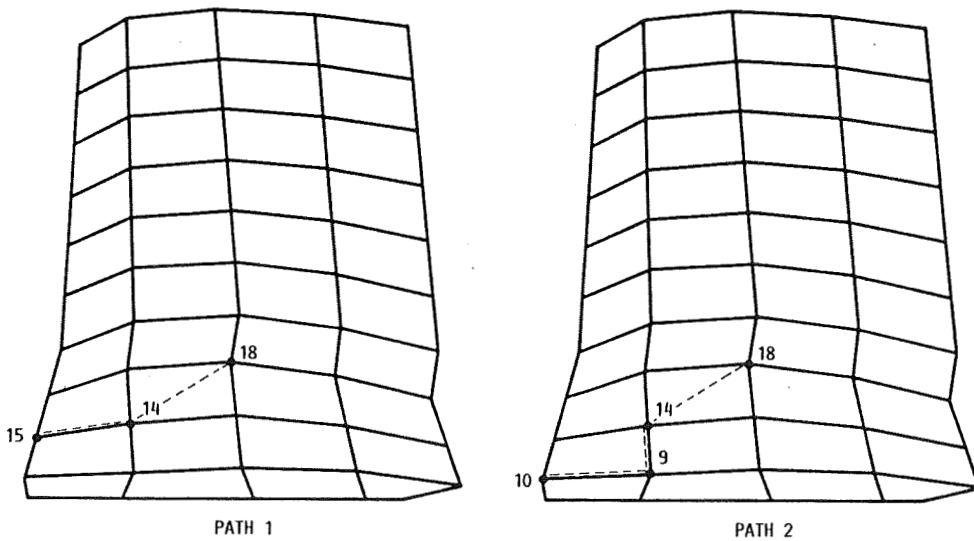


FIGURE 10. - MOST PROBABLE FRACTURE PATH CAUSED BY 100 000 CYCLES, WHERE D INDICATES DAMAGE STATES AT VARIOUS NODES AND P(B) INDICATES THE PROBABILITY THAT THE NEXT DAMAGE EVENT WILL OCCUR AT A PARTICULAR NODE. (SEE EQ. (7).)

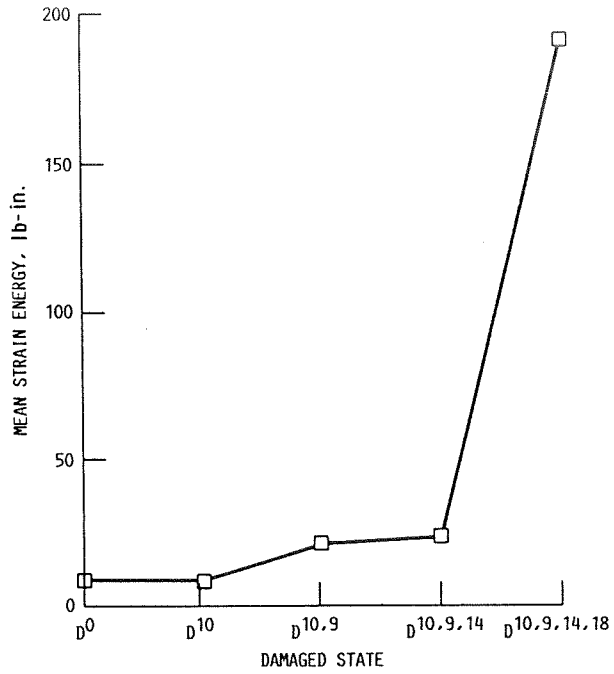


FIGURE 11. - CHANGE OF MEAN STRAIN ENERGY IN FRACTURE PATH 2, WHERE D INDICATES THAT STRUCTURE IS DAMAGED AT SPECIFIED NODES.

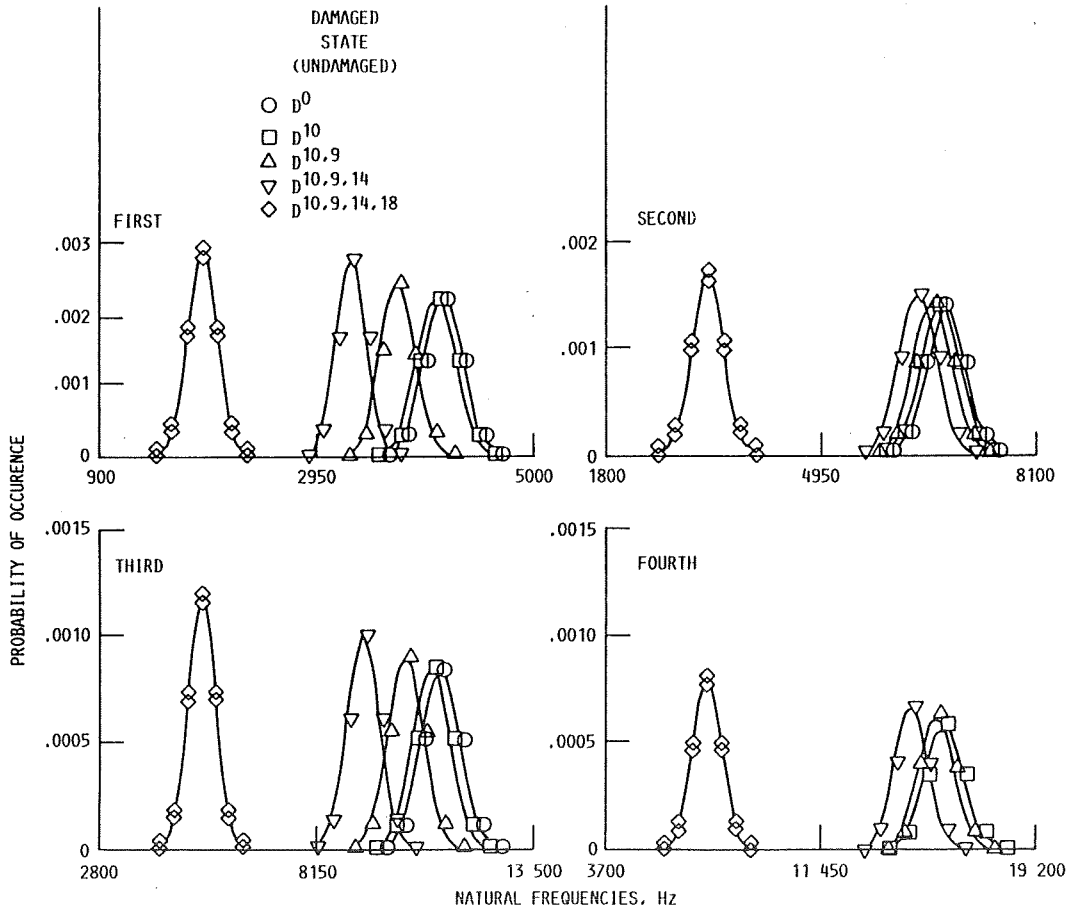


FIGURE 12. - CHANGE OF NATURAL FREQUENCIES IN FRACTURE PATH 2.

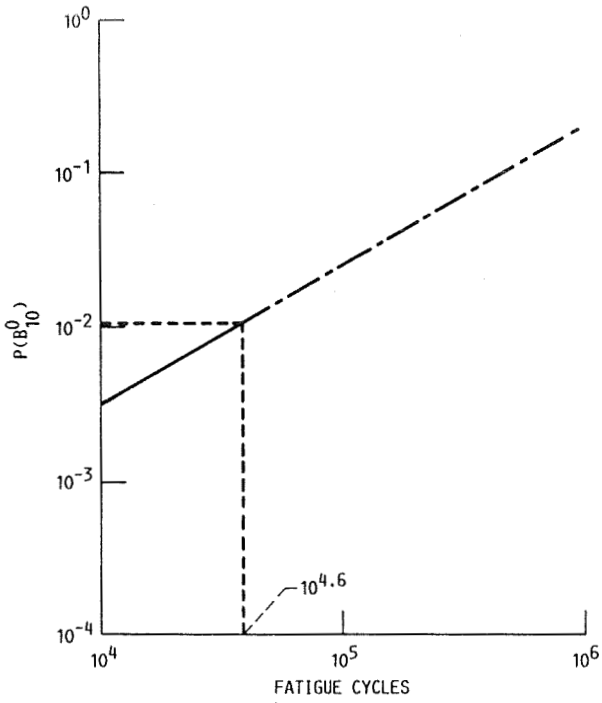


FIGURE 13. - PROBABILITY OF CRACK INITIATION AT NODE 10 AT DAMAGED STATE  $D^0$  (OR  $P(B_{10}^0)$ ).

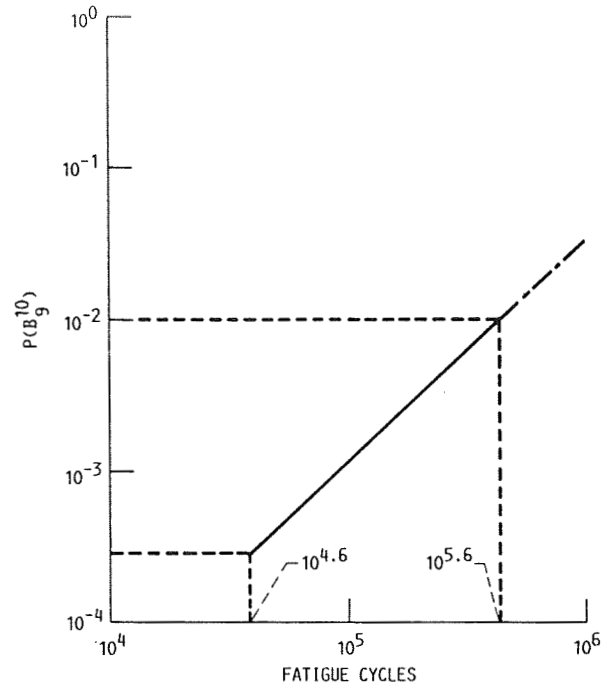


FIGURE 14. - PROBABILITY OF CRACK EXTENDED TO NODE 9 AT DAMAGED STATE  $D^{10}$  (OR  $P(B_9^{10})$ ).

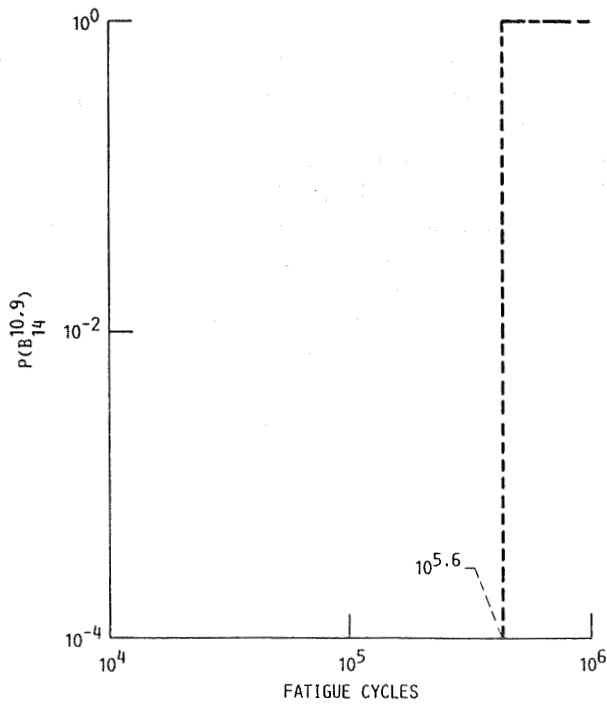


FIGURE 15. - PROBABILITY OF CRACK EXTENDED TO NODE 14 AT DAMAGED STATE  $D^{10,9}$  (OR  $P(B_{14}^{10,9})$ ).



1. Report No. NASA TM-103244		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle A Methodology for Evaluating the Reliability and Risk of Structures Under Complex Service Environments				5. Report Date	
				6. Performing Organization Code	
7. Author(s) Michael C. Shiao and Christos C. Chamis				8. Performing Organization Report No. E-5669	
				10. Work Unit No. 553-13-00	
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191				11. Contract or Grant No.	
				13. Type of Report and Period Covered Technical Memorandum	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001				14. Sponsoring Agency Code	
15. Supplementary Notes Prepared for the 31st Structures, Structural Dynamics and Materials Conference cosponsored by the AIAA, ASME, ASCE, AHS, and ACS, Long Beach, California, April 2-4, 1990. Michael C. Shiao, Sverdrup Technology, Inc., Lewis Research Center Group, 2001 Aerospace Parkway, Brook Park, Ohio 44142. Christos C. Chamis, NASA Lewis Research Center.					
16. Abstract Aerospace components are often subjected to complex service environments such as random excitations and random temperature conditions. In addition, small variations in the material properties or geometry may significantly affect the structural performance and durability of aerospace components. Therefore, a methodology was developed to determine structural reliability and to assess the associated risk due to various uncertainties in the design variables. The methodology consists of a probabilistic structural analysis by a special computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress), a generic probabilistic material property model (multifactor, interaction equation), and a probabilistic fatigue analysis. The methodology is versatile and is equally applicable to structures operating at high-temperature and/or cryogenic environments. The relationship between the probability of crack initiation and fatigue cycles is developed. The probabilistic crack propagation at a given risk level is studied. The most probable fracture path at a given cycle is determined. Those results serve as a guideline for component certification and for setting inspection criteria. The methodology is demonstrated to be capable of formally evaluating structural reliability and risk—including uncertainties in material properties, structural parameters, and loading conditions. It is described in detail with an application to the space shuttle main engine (SSME) blade.					
17. Key Words (Suggested by Author(s)) Probabilistic; Risk; Cost; Fatigue; Reliability; Random; Failure; Crack; Propagation; Initiation; Fracture			18. Distribution Statement Unclassified—Unlimited Subject Category 39		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of pages 16	22. Price* A03

National Aeronautics and  
Space Administration

Lewis Research Center  
Cleveland, Ohio 44135

Official Business  
Penalty for Private Use \$300

FOURTH CLASS MAIL

ADDRESS CORRECTION REQUESTED



Postage and Fees Paid  
National Aeronautics and  
Space Administration  
NASA 451

**NASA**

---