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Active Vibration Control Techniques
for
Flexible Space Structures
Final Report for the Period
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Executive Summary

This report describes the research performed in the period between April 1989 and September 1990, under the NASA Johnson Space Center Grant NAG 9-347 to Texas A&M University. Parts III and IV of the report describe the research results and findings of the two major tasks of the project. Namely, the report details two proposed control system design techniques for active vibration control in flexible space structures. Control issues relevant only to flexible-body dynamics are addressed, whereas no attempt has been made in this study to integrate the flexible and rigid-body spacecraft dynamics. Both of the proposed approaches revealed encouraging results, however, further investigation of the interaction of the flexible and rigid-body dynamics is warranted.

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Part I

Project Description

Chapter 1

Overview of the Project

In Part II of this report a brief summary of the structural model used in this research is presented, along with a statement of the structural control problem under investigation. Part III of the report presents the design of an active control law for the rejection of persistent disturbances in large space structures. The control system design approach is based on a deterministic model of the disturbances, with a Model-Based-Compensator (MBC) structure, optimizing the magnitude of the disturbance that the structure can tolerate without violating certain predetermined constraints. In addition to closed-loop stability, the explicit treatment of state, control and control rate constraints, such as structural displacement, control actuator effort, and compensator response time guarantees that the final design will exhibit desired performance characteristics. The technique is used for the vibration damping of a simple two bay truss structure which is subjected to persistent disturbances, such as shuttle docking. Preliminary results indicate that the proposed control system can reject considerable

persistent disturbances by utilizing most of the available control, while limiting the structural displacements to within desired tolerances. Further work, however, for incorporating additional design criteria, such as compensator robustness to be traded-off against performance specifications, is warranted.

In Part IV of the report a dynamic compensator based on the H_∞ -optimization of the sensitivity transfer function matrix is used to actively reject the persistent disturbances of a representative model of a flexible space structure. A variational approach is used to formulate a general state space solution to the multi-input, multi-output (MIMO) H_∞ -optimal control problem, without eliminating the feedforward terms. This allows use of the H_∞ -optimal synthesis algorithm on state-space models of structures which result from model order reduction. Disturbances encountered in flexible space structures, such as shuttle docking, are the primary interest of this study. Both the high-mode and the reduced-order models of a cantilevered two-bay truss are developed and are used to demonstrate the application of the H_∞ -optimal approach to flexible space structures. A computer algorithm which iteratively searches for an H_∞ -optimal control law is presented, with some adjustable parameters for matching additional design specifications. This study shows that the proposed H_∞ -optimal control law has good disturbance rejection capabilities for a wide class of persistent disturbances encountered in flexible space structures. Further studies, however, on the trade-offs of compensator robustness and the achieved disturbance rejection are warranted.

Part II

Flexible Structure Dynamics and Structural Control

Chapter 2

Introduction

Future National Aeronautics and Space Administration (NASA) and Department of Defense (DOD) space missions will require the availability and use of active disturbance rejection controllers in order to fulfill the increasingly sophisticated and demanding spacecraft performance requirements. One such requirement is the accurate pointing and tracking of the payload systems, solar concentrators, etc. Such subsystems, however, will be subjected to vibrations because of persistent external disturbances and because of structural disturbances resulting from multi-body flexible/flexible or rigid/flexible interactions.

If structural damage is to be prevented, the amplitude of structural vibrations must be controlled to within some prespecified limits. Traditionally, passive damping has been the primary avenue by which structural vibrations have been suppressed, with active damping given a secondary role. Even though it is widely believed that a combined design, optimizing the passive and active damping of a structure, will probably result in a

superior design, it is common practice to assume some amount of passive structural damping and proceed with the investigation of active damping techniques.

As the freedom of the designer increases because of the use of active control techniques, in addition to the traditional problem of vibration damping issues such as disturbance rejection, pointing and tracking are some of the problems that could be simultaneously addressed. In designing active control laws, the designer makes use of mathematical representations of a structure, usually derived from detailed finite element models. The accuracy of these models is extremely important and their limitations must be well understood by the designer and the user of the resulting active controllers. Accurate structural models may imply more effective controllers, however, as the accuracy of these models increases, so does the computational burden required to design an active control law. It is common practice to represent the infinite dimensional structural models in terms of finite dimensional approximations. Furthermore, selective retention of the most important system modes results in further simplifications which approximately represent the actual structure response up to a certain frequency. Therefore, satisfaction of certain stability and performance robustness criteria is of utmost importance when designing active control laws for flexible structures.

There is a vast literature dealing with the design of active control laws for large space structures which addresses issues such as vibration suppression, pointing and tracking. Atluri and Amos [1] present an excellent overview of the state-of-the-art in large space structure dynamics and

control as of 1988, while Craig [2] has a more recent literature survey on structural system modeling, identification and analysis. The majority of the active control techniques reported in the literature are based on the stochastic representation of external disturbances and the resulting control laws are products of the stochastic linear quadratic regulator problem. The next chapter presents a brief description of the modeling approach used in the mathematical representation of the structure under consideration, which is representative of a wide class of structures under consideration for future space missions. In addition, in the final paragraph of this chapter, the objectives of the control problem addressed in this study are defined.

Chapter 3

Modeling of a Flexible Structure and Control

Problem Definition

The dynamics of a structure are accurately represented by one or more partial differential equations, which are usually discretized using the finite element technique. This results in a finite dimensional representation of a structure suitable for digital computer simulations. In obtaining this high accuracy, however, finite element modeling results in very high order models which are virtually useless for control system design. This is primarily because of the extreme computational resources required and also because of the increased numerical inaccuracies introduced when dealing with very large scale matrix computations, such as solutions to the algebraic Riccati equation.

The cantilevered two-bay truss shown in Figure 1 is used to illustrate

the proposed nonlinear optimization approach for control system design. Such a truss could be considered part of a larger structure, e.g. the Space Station Freedom, and used in analyzing the pointing performance, for example, of a payload instrument. The position sensors on the truss are located at points 1,2,3 and 4. The control forces required to damp the truss vibrations are co-located with the sensors, and are denoted by $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$, respectively. An eight-mode (sixteenth order) model and a two-mode (fourth order) reduced order model is considered, based on the work presented in reference [3]. The truss material has an assumed weight density of $0.1 \frac{lb_f}{in^3}$ and modulus of elasticity 10^7 psi. The cross-sectional area of the structural members and their lumped masses are given in reference [3]. The maximum force that could be applied by any one of the force actuators along the Y-axis is limited to $\pm 100 lb_f$. The first eight modes of the truss considered in this study have modal frequencies: 3.1416, 10.3857, 22.7040, 29.5437, 31.1894, 32.8702, 55.8220, and $58.7790 \frac{rad}{sec}$, respectively. The governing equations of motion for the truss model can be written as:

$$m\ddot{\mathbf{r}}(t) + c\dot{\mathbf{r}}(t) + \mathbf{k}\mathbf{r}(t) = \mathbf{b}\mathbf{u}(t) + \mathbf{g}d(t) \quad (3.1)$$

where $\mathbf{r}(t)$, $\mathbf{u}(t)$ and $d(t)$ denote the truss physical coordinates, the control forces and a scalar external persistent disturbance, respectively. The mass \mathbf{m} , damping \mathbf{c} , and stiffness \mathbf{k} are (8×8) -dimensional matrices, whereas the control input distribution matrix \mathbf{b} and the disturbance distribution matrix \mathbf{g} are (8×4) and (8×1) -dimensional, respectively. A state space representation of the structure is given as follows:

$$\dot{\tilde{\mathbf{x}}}(t) = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}\mathbf{u}(t) + \tilde{\mathbf{G}}d(t) \quad (3.2)$$

$$\mathbf{y}(t) = \tilde{\mathbf{C}}\tilde{\mathbf{x}}(t)$$

where,

$$\begin{aligned} \tilde{\mathbf{A}} &= \begin{bmatrix} -\mathbf{m}^{-1}\mathbf{c} & -\mathbf{m}^{-1}\mathbf{k} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{B}} = \begin{bmatrix} -\mathbf{m}^{-1}\mathbf{b} \\ \mathbf{0} \end{bmatrix}, \\ \tilde{\mathbf{G}} &= \begin{bmatrix} -\mathbf{m}^{-1}\mathbf{g} \\ \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{C}} = [\mathbf{0} \quad \mathbf{b}^T], \quad \tilde{\mathbf{x}}(t) = \begin{bmatrix} \dot{\mathbf{r}}(t) \\ \mathbf{r}(t) \end{bmatrix} \end{aligned} \quad (3.3)$$

and where $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$, $\tilde{\mathbf{G}}$, and $\tilde{\mathbf{C}}$ are constant matrices with dimensions (16×16) , (16×4) , (16×1) and (4×16) , respectively, and where $\mathbf{y}(t)$ is the measured output vector. Numerical data for the above matrices are given in Table 3.1

Following extensive examination of the structural modes, it is concluded that the two-bay truss system is most controllable and observable from the first two modes [3]. Thus, the above eight-mode model can be reduced to a lower order model to facilitate the numerical computations. This can be done by first defining the transformation operator \mathbf{T} as follows:

$$\mathbf{r}(t) = \mathbf{T}\boldsymbol{\eta}(t) \quad (3.4)$$

where \mathbf{T} defines a coordinate transformation from the modal coordinates $\boldsymbol{\eta}(t)$ to the physical coordinates $\mathbf{r}(t)$, given by

$$\omega^2 \mathbf{m}\mathbf{T} = \mathbf{k}\mathbf{T} \quad (3.5)$$

Substitution of equation (3.4) into equations (3.2) leads to the following representation:

$$\begin{aligned} \dot{\hat{\mathbf{x}}}(t) &= \hat{\mathbf{A}}\hat{\mathbf{x}}(t) + \hat{\mathbf{B}}\mathbf{u}(t) + \hat{\mathbf{G}}d(t) \\ \mathbf{y}(t) &= \hat{\mathbf{C}}\hat{\mathbf{x}}(t) \end{aligned} \quad (3.6)$$

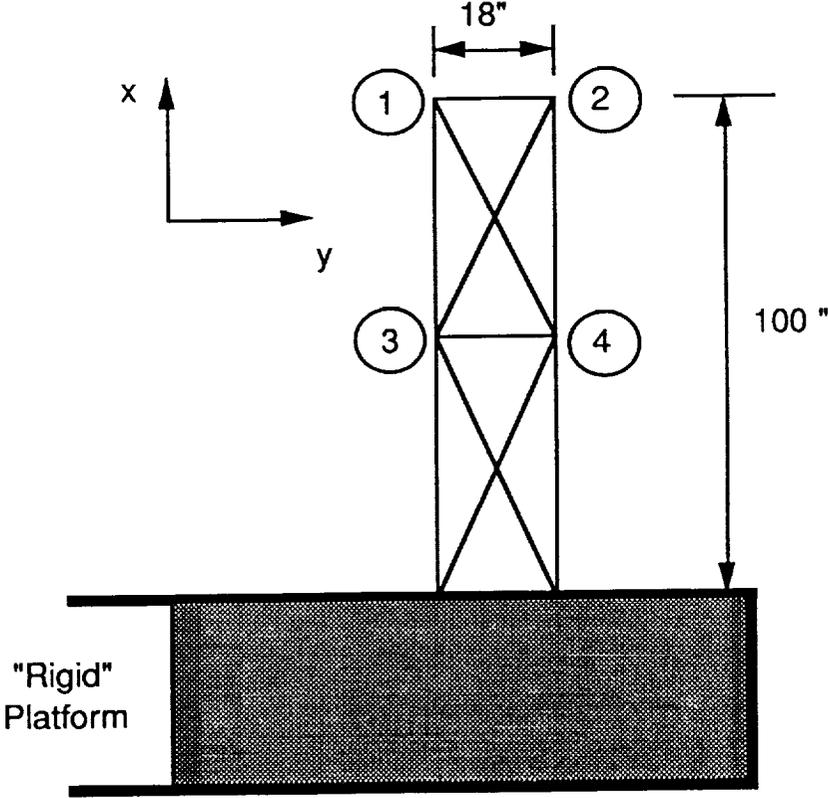


Figure 3.1: The AFWAL Two-Bay Truss Mounted on a Rigid Platform.

where

$$\begin{aligned}\hat{\mathbf{A}} &= \begin{bmatrix} -\mathbf{T}^{-1}\mathbf{m}^{-1}\mathbf{c}\mathbf{T} & -\mathbf{T}^{-1}\mathbf{m}^{-1}\mathbf{k}\mathbf{T} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} -\mathbf{T}^{-1}\mathbf{m}^{-1}\mathbf{b} \\ \mathbf{0} \end{bmatrix}, \\ \hat{\mathbf{G}} &= \begin{bmatrix} -\mathbf{T}^{-1}\mathbf{m}^{-1}\mathbf{g} \\ \mathbf{0} \end{bmatrix}, \quad \hat{\mathbf{C}} = [\mathbf{0} \quad \mathbf{b}^T\mathbf{T}], \quad \hat{\mathbf{x}}(t) = \begin{bmatrix} \dot{\boldsymbol{\eta}}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix}\end{aligned}\quad (3.7)$$

The system of equations (3.6) can now be written in the standard singularly perturbed form, as follows:

$$\begin{aligned}\begin{bmatrix} \dot{\mathbf{x}}_r(t) \\ \epsilon\dot{\mathbf{x}}_f(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_f(t) \end{bmatrix} + \begin{bmatrix} -\mathbf{B}_1 \\ -\mathbf{B}_2 \end{bmatrix} \mathbf{u}(t) + \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix} d(t), \\ \mathbf{y}(t) &= \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_f(t) \end{bmatrix}\end{aligned}\quad (3.8)$$

where $(\mathbf{A}_{11}, \mathbf{B}_1, \mathbf{G}_1, \mathbf{C}_1)$ are the system matrices corresponding to the first two modes and ϵ is a small perturbation number. By formally setting the perturbation parameter ϵ to zero, a two-mode reduced-order model can be obtained as follows:

$$\begin{aligned}\dot{\mathbf{x}}_r(t) &= \mathbf{A}_r \mathbf{x}_r(t) + \mathbf{B}_r \mathbf{u}(t) + \mathbf{G}_r d(t) \\ \mathbf{y}(t) &= \mathbf{C}_r \mathbf{x}_r(t) + \mathbf{D}_r \mathbf{u}(t) + \mathbf{E}_r d(t)\end{aligned}\quad (3.9)$$

where $\mathbf{x}_r(t)$ is the reduced-order model state vector, and

$$\begin{aligned}\mathbf{A}_r &= [\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21}], \quad \mathbf{B}_r = [-\mathbf{B}_1 + \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{B}_2] \\ \mathbf{C}_r &= [\mathbf{C}_1 - \mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{A}_{21}], \quad \mathbf{D}_r = [\mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{B}_2] \\ \mathbf{G}_r &= [\mathbf{G}_1 - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{G}_2], \quad \mathbf{E}_r = [-\mathbf{C}_2\mathbf{A}_{22}^{-1}\mathbf{G}_2]\end{aligned}\quad (3.10)$$

The reduced-order model state $\mathbf{x}_r(t)$, the input $\mathbf{u}(t)$ and the measured output $\mathbf{y}(t)$ are 4-dimensional vectors. The matrices \mathbf{A}_r , \mathbf{B}_r , \mathbf{C}_r , and \mathbf{D}_r are

(4×4) -dimensional, the vectors \mathbf{G}_r and \mathbf{E}_r are 4-dimensional, and the disturbance $d(t)$ is assumed scalar. Notice that the input, the measured output, and the disturbance vectors are preserved for both the eight-mode and the two-mode reduced-order models. The numerical values of the reduced-order model matrices are given in Table 3.2.

To enable a flexible space structure, such as the truss model presented in this section, to perform a desired mission, the encountered structural vibrations must be effectively controlled, among others. The main cause of such structural vibrations is the ever present extraneous disturbances and the inability of the various passive control mechanisms to minimize their effects on the structure. Additionally, for most space structures such external disturbances are usually neither precisely known nor resemble white Gaussian noise. Rather, they belong to the class of finite energy signals. The control objective for this work is to reject the maximum possible magnitude of the persistent disturbance $d(t)$ and to damp the resulting vibrations within the shortest possible time period, though time-optimal response is not an explicit design specification of the proposed optimization. Among the performance criteria satisfied by the resulting control law are the maximum controller bandwidth, the maximum control effort used, and the maximum deformation of the structure. Other than closed-loop stability, however, the single most important requirement that the control law must satisfy is the ability to perform the desired tasks using only the available control effort and without exceeding the maximum deformation limits at certain locations on the structure.

TABLE 3.1 (cont.)

Columns 9 thru 16

-3.4554	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-3.1165	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0098	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	-0.1079	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-0.5156	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	-1.0806	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.9729	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.8729
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

B (16×4 matrix) =

-0.0418	0.0418	-0.0018	0.0018
0.0260	2.0260	0.0098	0.0098
0.5135	0.5135	0.1748	0.1748

TABLE 3.1 (cont.)

-0.1583	-0.1583	0.5078	0.5078
0.0977	-0.0977	0.1631	-0.1631
-0.2299	0.2299	-0.4576	0.4576
0.0957	0.0957	-0.1004	-0.1004
-0.4842	0.4842	0.2503	-0.2503
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

G (16 × 1 matrix) =

0.8351

0.0173

0.0591

-0.1251

TABLE 3.1 (cont.)

Columns 9 thru 16

0.0541	-0.0337	-0.6645	0.2048	-0.1264	0.2974	-0.1239	0.6266
-0.0541	-0.0337	-0.6645	0.2048	0.1264	-0.2974	-0.1239	-0.6266
0.0023	-0.0127	-0.2261	-0.6571	-0.2111	0.5921	0.1299	-0.3238
-0.0023	-0.0127	-0.2261	-0.6571	0.2111	-0.5921	0.1299	0.3238

D (4×4 matrix) =

0. 0. 0. 0.

0. 0. 0. 0.

0. 0. 0. 0.

0. 0. 0. 0.

E (4×1 matrix) =

0.

0.

0.

0.

TABLE 3.2 The System Matrices of the 2-Mode Structural Model

Ar (4×4 matrix) =

-0.0314	0.0000	-9.8697	0.0000
0.0000	-0.1039	0.0000	-107.8628
1.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000

Br (4×4 matrix) =

-0.3412	-0.3412	-0.1161	-0.1161
0.1040	0.1040	-0.3337	-0.3337
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

Gr (4×1 matrix) =

-0.0393
0.0822
0.0000
0.0000

TABLE 3.2 (cont.)

Cr = (4 × 4 matrix) =

0.0000	0.0000	1.0000	-0.3117
0.0000	0.0000	1.0000	-0.3117
0.0000	0.0000	0.3402	1.0000
0.0000	0.0000	0.3402	1.0000

Dr (4 × 4 matrix) =

1.0D-03 *

0.4480	-0.4230	-0.0263	0.0010
-0.4230	0.4480	0.0010	-0.0263
-0.0263	0.0010	0.4239	-0.3970
0.0010	-0.0263	-0.3970	0.4239

Er (4 × 1 matrix) =

1.0D-03 *

0.1331
0.0011
0.1097
-0.2509

References

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Part III

A Nonlinear Optimization Approach for Disturbance Rejection in Flexible Space Structures

Chapter 4

Introduction

There is a vast literature dealing with the design of active control laws for large space structures which addresses issues such as vibration suppression, pointing and tracking. Atluri and Amos [2] present an excellent overview of the state-of-the-art in large space structure dynamics and control as of 1988, while Craig [4] has a more recent literature survey on structural system modeling, identification and analysis. The majority of the active control techniques reported in the literature are based on the stochastic representation of external disturbances and the resulting control laws are products of the stochastic linear quadratic regulator problem. The purpose of this study is to introduce an alternate compensator design approach for rejection of persistent external disturbances, based on a deterministic uncertainty model. An advantage of the proposed approach, called the Set-Theoretic (ST) control system design method, is the ability to impose and guarantee the satisfaction of explicit time-domain constraints on the system states, control and control rates. Considering the serious consequences that

could result from saturating actuators and the resulting pointing errors, it is highly desirable to consider an approach that explicitly treats such compensator design specifications.

Part III of this report is organized as follows. Chapter 5 presents an overview of the ST control system design method, from the theoretical development to the numerical algorithm. Chapter 6 presents the results of the application of the active control system design technique to the flexible structure presented in chapter 3. Finally, chapter 7 gives a brief summary of the main results and accomplishments, along with some concluding remarks.

Chapter 5

The Set-Theoretic Design Method

5.1 The Unknown-but-Bounded Disturbance Model and Set-Theoretic Design

The foundation of the control system design method proposed for use in rejecting persistent disturbances in flexible structures, is based on the unknown-but-bounded (ubb) disturbance model [8]. This is an uncertainty modeling technique which differs from both the stochastic (or Bayesian) in that no statistics of the uncertainty are assumed, and the less familiar completely unknown (or Fisher) approach in that there is some information that is known about the identity of the uncertainty, namely its boundedness. The main objective of the ST control strategy, which models disturbances as ubb processes, is to keep the system states in a “Target Tube”,

a prespecified sequence of bounded sets, using control from the bounded control sets at only the available control rate, in the presence of these ubb input disturbances which take values that are elements of a bounded set. Therefore, in this control scheme primary emphasis is placed upon satisfying the state, control and control rate constraints. In view of the linearity of the assumed dynamic system, once these “hard” constraints are satisfied, stability of the closed-loop system is guaranteed.

The ST approach has first appeared in the literature in the late 60’s, as a deterministic tool for state estimation [8]. In the seventies, the method underwent some major theoretical developments, and it was not until the early 80’s that it was used as a controller design tool [10]. However, most of the developments utilized static controller structures, with dynamic compensation and frequency domain design specifications not considered. Additionally, several different forms of ST control appeared in the literature, utilizing different approximations to convex sets, such as ellipsoids, boxes, etc.. The development in this paper uses the ellipsoidal approximation to convex sets.

The main advantages of the Ellipsoidal ST control system design method are:

- (1) The state, control and control rate constraints that are placed upon the dynamic system, because of safety or deteriorating performance concerns, are treated explicitly, guaranteeing that the final design satisfies these constraints;
- (2) The ubb disturbance model is of great practical significance since measuring the stochastics of a disturbance, especially for the case of

large-scale dynamic systems such as space structures, may be a very difficult task; and

- (3) In addition to persistent system disturbances, such as shuttle docking, it is also possible to model uncertainties such as system parameter variations and unmodeled dynamics using the ubb uncertainty model. Some issues related to robustness to parameter variations and to unmodeled dynamics are currently under investigation.

Some of the disadvantages of the ST approach are:

- (1) Limitation to only linear or linearized models of dynamic systems, a problem common to most of the currently available systematic control synthesis methods; and
- (2) The conservative nature of the results, because of the “worst case” assumption inherent in the ST algorithm, which in some instances may be desirable for robustness purposes. Results can be made less conservative by extensive numerical simulations, however, no systematic analytical method exists, yet.

5.2 The Set-Theoretic Design Method

The following development requires some familiarity with set-theory and the set-theoretic representation of ellipsoids. The unfamiliar reader is encouraged to consult the appendices of reference [8] for the relevant background material.

5.2.1 Control Problem Statement

Let a linear, time-invariant dynamic system, such as the reduced-order structural model of equations (3.9), be given in state-space form by the following continuous time model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}d(t) \quad (5.1)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{E}d(t) \quad (5.2)$$

where $[\mathbf{A}, \mathbf{B}]$ is a stabilizable and $[\mathbf{C}, \mathbf{A}]$ is a detectable pair, $\mathbf{x}(t)$ is the n -dimensional state vector, $\mathbf{y}(t)$ is the m -dimensional measured output vector, $\mathbf{u}(t)$ is the r -dimensional input control vector, and where the external disturbance, $d(t)$, is assumed scalar. The matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} , and the vectors \mathbf{G} and \mathbf{E} are of appropriate dimensions. The disturbance model used throughout this study is the ubb, allowing the following representation:

$$|d(t)| \leq \sqrt{Q} \quad (5.3)$$

at all times t , where \sqrt{Q} is the bound of the disturbance. In the frequency domain, the above system has the following representation:

$$\mathbf{y}(s) = \mathbf{G}_p(s)\mathbf{u}(s) + \mathbf{G}_d(s)d(s) \quad (5.4)$$

where

$$\mathbf{G}_p(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (5.5)$$

and

$$\mathbf{G}_d(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{G} + \mathbf{E} \quad (5.6)$$

In addition, the following p -dimensional constrained output vector is defined, as follows:

$$\mathbf{y}_c(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{H}\mathbf{u}(t) + \mathbf{L}d(t) \quad (5.7)$$

where the matrices \mathbf{F} and \mathbf{H} , and the vector \mathbf{L} is of appropriate dimensions. The significance of the constrained output vector will become clearer during the development of the control synthesis technique, however, it suffices to mention that the constrained output vector is not necessarily composed of measured variables.

As only the measured output vector is available for feedback, a dynamic compensator is required for use in the feedback loop. The dynamic compensator structure used in this development is a Model Based Compensator (MBC), with the following state-space representation [9]:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{K}_1\mathbf{v}(t) \quad (5.8)$$

$$\mathbf{v}(t) = -\mathbf{e}(t) - \mathbf{C}\mathbf{z}(t) - \mathbf{D}\mathbf{u}(t) \quad (5.9)$$

$$\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{y}(t) \quad (5.10)$$

$$\mathbf{u}(t) = -\mathbf{K}_2\mathbf{z}(t) \quad (5.11)$$

where the n -dimensional vector $\mathbf{z}(t)$ is the compensator state, the m -dimensional error vector $\mathbf{e}(t)$ is the compensator input and the control vector $\mathbf{u}(t)$ is the compensator output. The matrices \mathbf{K}_1 and \mathbf{K}_2 are called the filter and control gain matrices, respectively, and they are of appropriate dimensions. As we are dealing only with regulation, the reference signal $\mathbf{r}(t)$ is zero. The compensator transfer function matrix, from the error $\mathbf{e}(t)$ to the control signal $\mathbf{u}(t)$, is given by the following expression:

$$\mathbf{u}(s) = \mathbf{K}(s)\mathbf{e}(s) \quad (5.12)$$

where

$$\mathbf{K}(s) = \mathbf{K}_2(s\mathbf{I} - \mathbf{A} + \mathbf{BK}_2 + \mathbf{K}_1\mathbf{C} - \mathbf{K}_1\mathbf{DK}_2)^{-1}\mathbf{K}_1 \quad (5.13)$$

Combining equations (5.1), (5.2), (5.7), and (5.8) through (5.11), the following closed-loop state-space representation is obtained:

$$\dot{\mathbf{X}}(t) = \mathbf{A}_{cl}\mathbf{X}(t) + \mathbf{G}_{cl}d(t) \quad (5.14)$$

$$\mathbf{y}(t) = \mathbf{C}_{cl}\mathbf{X}(t) + \mathbf{E}_{cl}d(t) \quad (5.15)$$

$$\mathbf{y}_c(t) = \mathbf{F}_{cl}\mathbf{X}(t) + \mathbf{L}_{cl}d(t) \quad (5.16)$$

whereas the compensator output, that is the plant input vector, is expressed as follows:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{X}(t) \quad (5.17)$$

and where,

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{z}(t) \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{0} & -\mathbf{K}_2 \end{bmatrix}, \quad (5.18)$$

$$\mathbf{A}_{cl} = \begin{bmatrix} \mathbf{A} & -\mathbf{BK}_2 \\ \mathbf{K}_1\mathbf{C} & \mathbf{A} - \mathbf{BK}_2 - \mathbf{K}_1\mathbf{C} \end{bmatrix}, \quad \mathbf{G}_{cl} = \begin{bmatrix} \mathbf{G} \\ \mathbf{0} \end{bmatrix}, \quad (5.19)$$

$$\mathbf{C}_{cl} = \begin{bmatrix} \mathbf{C} & -\mathbf{DK}_2 \end{bmatrix}, \quad \mathbf{E}_{cl} = \mathbf{E} \quad (5.20)$$

$$\mathbf{F}_{cl} = \begin{bmatrix} \mathbf{F} & -\mathbf{HK}_2 \end{bmatrix}, \quad \mathbf{L}_{cl} = \mathbf{L} \quad (5.21)$$

The closed-loop state vector $\mathbf{X}(t)$ is $(2n)$ -dimensional, and the matrices \mathbf{A}_{cl} , \mathbf{C}_{cl} , and \mathbf{F}_{cl} , and the vectors \mathbf{G}_{cl} , \mathbf{E}_{cl} , and \mathbf{L}_{cl} are of appropriate dimensions. The transfer function matrix of the closed-loop system can now be written as follows:

$$\mathbf{y}(s) = \mathbf{G}_{cl}(s)d(s) \quad (5.22)$$

where

$$\mathbf{G}_{cl}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})^{-1}\mathbf{G}_{cl} + \mathbf{E}_{cl} \quad (5.23)$$

In terms of the open-loop transfer function matrices, $\mathbf{G}_{cl}(s)$ can be expressed as:

$$\mathbf{G}_{cl}(s) = [\mathbf{I} + \mathbf{G}_p(s)\mathbf{K}(s)]^{-1} \mathbf{G}_d(s) \quad (5.24)$$

The forward loop transfer function matrix, $\mathbf{G}_d(s)$, offers no degrees of freedom to the designer, as it is completely determined by the open-loop system dynamics. Therefore, if it is desired to keep the maximum amplification of the closed-loop transfer function, $\mathbf{G}_{cl}(s)$, as low as possible up to a certain frequency, thus reducing the effect of the disturbance $d(s)$ on the regulated output $\mathbf{y}(s)$, it is necessary to design a compensator $\mathbf{K}(s)$ such that the minimum singular value of the forward loop transfer function matrix, $\mathbf{G}_p(s)\mathbf{K}(s)$, is as large as possible in the desired frequency range. This is a form of the sensitivity minimization problem, expressed in the frequency domain, which in the following sections is formulated in the time domain.

As mentioned in earlier paragraphs, one of the main features of the ST design method is the ability to guarantee satisfaction of certain “hard” (time-domain) constraints on the various system variables. Therefore, it is required that the closed-loop system described by equations (5.14) through (5.16), satisfies the following constraints at all times:

- (1) The constrained output vector $\mathbf{y}_c(t)$ is required to take values that are bounded at all times. This constraint can be quantitatively expressed as follows:

$$|y_{ci}(t) - y_{c0i}| \leq \sqrt{S_i}; \quad i = 1, 2, \dots, p \quad (5.25)$$

at all times t , where $S_i = y_{cimax}^2$; y_{c0i} ($i = 1, 2, \dots, p$) are known elements specifying the center, \mathbf{y}_{c0} , of the output vector $\mathbf{y}_c(t)$. The variables y_{cimax} ($i = 1, 2, \dots, p$) are the pre-specified bounds on the amplitudes of the relevant output perturbations, with respect to the center value \mathbf{y}_{c0} , defining the p -dimensional vector \mathbf{y}_{cmax} .

- (2) The control vector $\mathbf{u}(t)$, is required to take values that are bounded at all times. This constraint can be quantitatively expressed as follows:

$$|u_j(t) - u_{0j}| \leq \sqrt{T_j}; \quad j = 1, 2, \dots, r \quad (5.26)$$

at all times t , where $T_j = u_{jmax}^2$; u_{0j} ($j = 1, 2, \dots, r$) are known elements specifying the center, \mathbf{u}_0 , of the control vector $\mathbf{u}(t)$. The variables u_{jmax} ($j = 1, 2, \dots, r$) are the pre-specified bounds on the amplitudes of the relevant control input perturbations, with respect to the center value \mathbf{u}_0 , defining the r -dimensional vector \mathbf{u}_{max} .

- (3) The control rate vector, $\dot{\mathbf{u}}(t)$, is also required to take bounded values, as follows:

$$|\dot{u}_j(t) - \dot{u}_{0j}| \leq \sqrt{R_j}; \quad j = 1, 2, \dots, r \quad (5.27)$$

at all times t , where $R_j = \dot{u}_{jmax}^2$; \dot{u}_{0j} ($j = 1, 2, \dots, r$) are known elements specifying the center, $\dot{\mathbf{u}}_0$, of the control rate vector $\dot{\mathbf{u}}(t)$. The variables \dot{u}_{jmax} ($j = 1, 2, \dots, r$) are the pre-specified bounds on the amplitudes of the relevant control rate perturbations, with respect to the center value $\dot{\mathbf{u}}_0$, defining the r -dimensional vector $\dot{\mathbf{u}}_{max}$.

Satisfaction of a combination of certain time-domain constraints, such as control and control rate constraints, can also give an estimate on the upper

bound of certain important frequency-domain quantities, such as controller bandwidth. The compensator bandwidth is not explicitly taken into account by the ST approach, however, appropriate choice of the control and control rate constraints can limit this important frequency-domain quantity, preventing undesirable compensators. A desired controller bandwidth can be precisely obtained only through iterative design.

The control problem under consideration can be stated as follows:

Design a compensator $K(s)$ for the dynamic system described by equations (5.1) and (5.2), which at all times uses only the available control, according to equation (5.26), at the bounded rate, given by equation (5.27). It is also required that the constrained system output, defined by equation (5.7), is kept within the desired bounds of equation (5.25), in the presence of an ubb input disturbance, $d(t)$, which is required to take values that are bounded by \sqrt{Q} .

Several optimization problems can be formulated using the above control problem statement. In most physical applications, however, the bounds on the available control action and the rate at which the control effort is available, as well as the desired bounds on certain critical system outputs (or states) are a priori known, as part of the controller design specifications. Therefore, a meaningful variable to optimize is the maximum tolerable disturbance magnitude, Q . Even though there are certain other aspects of a controller that would have to be traded-off against the optimum value of Q , such as the controller robustness to unmodeled dynamics and to system parameter variations, this study does explicitly address these aspects of control system design, because such an optimization problem becomes

multiobjective in nature.

Assuming the special compensator structure expressed by equations (5.8) through (5.11), the optimal persistent disturbance rejection problem can be formulated as a nonlinear constrained optimization problem, with the subject of the optimization being the bound Q of the input disturbance amplitude [7]. The control problem, which is not limited to the controller class represented by the MBC structure, can be reformulated as follows:

Find the filter and control gain matrices \mathbf{K}_1 and \mathbf{K}_2 that maximize the disturbance bound \sqrt{Q} , subject to the inequality constraints expressed by equations (5.25), (5.26), and (5.27), and the dynamic constraint expressed by the closed-loop system equation (5.14).

5.2.2 The Set-Theoretic Synthesis Problem

One of the main features of the ST approach is the representation of dynamic system states by convex sets which, for mathematical simplification, are further approximated by ellipsoids. In order to arrive at the governing equation for the time propagation of the state vector in terms of its ellipsoidal set representation, the set of *reachable states*, $\Omega_{\mathbf{X}}(t)$, must be determined [8]. It can be shown that, for a dynamic system described by equations (5.1) and (5.2) with the controller structure of equations (5.8) through (5.11), or equivalently for the closed-loop dynamic system described by equations (5.14) and (5.15), with the initial condition:

$$\mathbf{X}(0) \in \Omega_{\mathbf{X}}(0) = \{\mathbf{X} : (\mathbf{X} - \mathbf{X}_0)^T \Psi^{-1} (\mathbf{X} - \mathbf{X}_0) \leq 1\} \quad (5.28)$$

where \mathbf{X}_0 is the center of $\Omega_{\mathbf{X}}(t)$, and Ψ is a $(2n \times 2n)$ -dimensional positive definite matrix defining the ellipsoidal set $\Omega_{\mathbf{X}}(0)$ of the possible initial states, there exists a $(2n \times 2n)$ -dimensional matrix $\Gamma(t)$ which, if positive semidefinite, describes the ellipsoids bounding the sets of reachable states at all times. That is, the reachable states $\mathbf{X}(t)$ are contained, at time t , within the ellipsoidal set given by

$$\mathbf{X}(t) \in \Omega_{\mathbf{X}}(t) = \{\mathbf{X} : (\mathbf{X} - \mathbf{X}_0)^T \Gamma^{-1}(\mathbf{X} - \mathbf{X}_0) \leq 1\} \quad (5.29)$$

Furthermore, it can be shown that the matrix $\Gamma(t)$ satisfies the following dynamic equation [8]:

$$\dot{\Gamma}(t) = \mathbf{A}_{cl}\Gamma(t) + \Gamma(t)\mathbf{A}_{cl}^T + \beta(t)\Gamma(t) + \frac{\mathbf{G}_{cl}Q\mathbf{G}_{cl}^T}{\beta(t)}; \quad \Gamma(0) = \Psi, \quad \beta(t) > 0 \quad (5.30)$$

where $\beta(t)$ is a free parameter introduced in the approximation of an arbitrary convex set by a bounding ellipsoidal set. The positivity of the free parameter $\beta(t)$ signifies that a non-empty convex set cannot be approximated by an ellipsoid of equal volume that contains it.

Furthermore, for a time-invariant dynamic system with constant β , if the closed-loop system matrix \mathbf{A}_{cl} is stable then equation (5.30) has a steady-state solution Γ_s . At steady-state, equation (5.30) reduces to the following well-known Lyapunov equation:

$$\{\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I}\}\Gamma_s + \Gamma_s\{\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I}\}^T + \frac{\mathbf{G}_{cl}Q\mathbf{G}_{cl}^T}{\beta} = \mathbf{0} \quad (5.31)$$

In addition to the approximate representation of general convex sets by ellipsoids, use of the steady-state solution of equation (5.30) is the second approximation introduced in the ST control synthesis. This results in

conservative estimates of the system state bounds, and therefore on conservative estimates of the disturbance rejection capability of the closed-loop system. However, evaluation of transient bounds will require more complex solution algorithms and more importantly, a solution to equation (5.30) can not be guaranteed. Equation (5.31) has now replaced the dynamic constraint (5.14) of the nonlinear optimization problem.

In order to satisfy the control constraints imposed upon the closed-loop system and expressed by equation (5.26), the following sufficient condition can be proven [8]:

$$\mathbf{K}_j \Gamma \mathbf{K}_j^T \leq T_j; \quad j = 1, 2, \dots, r \quad (5.32)$$

where \mathbf{K}_j is the j th row vector of the gain matrix \mathbf{K} defined by equation (5.18).

In order to derive an equivalent inequality for the constrained output and control rate vectors, the following $(p+r)$ -dimensional augmented vector is defined:

$$\mathbf{Y}_c(t) = \begin{bmatrix} \mathbf{y}_c(t) \\ \dot{\mathbf{u}}(t) \end{bmatrix} \quad (5.33)$$

The constrained output now becomes as follows:

$$\mathbf{Y}_c(t) = \tilde{\mathbf{H}}\mathbf{X}(t) + \tilde{\mathbf{E}}d(t) \quad (5.34)$$

where

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{F}_{cl} \\ \mathbf{K}\mathbf{A}_{cl} \end{bmatrix}; \quad \tilde{\mathbf{E}} = \begin{bmatrix} \mathbf{L}_{cl} \\ \mathbf{K}\mathbf{G}_{cl} \end{bmatrix} \quad (5.35)$$

It can also be shown that in order to guarantee satisfaction of the constrained output and control rate constraints, the following inequality is a

sufficient condition:

$$\tilde{E}_i Q \tilde{E}_i^T + \tilde{H}_i \Gamma \tilde{H}_i^T + 2 \left\{ \tilde{E}_i Q \tilde{E}_i^T \tilde{H}_i \Gamma \tilde{H}_i^T \right\}^{\frac{1}{2}} \leq \tilde{S}_i; \quad i = 1, 2, \dots, (p+r) \quad (5.36)$$

where \tilde{E}_i is the i th element of the vector $\tilde{\mathbf{E}}$, \tilde{H}_i is the i th row vector of the matrix $\tilde{\mathbf{H}}$, and

$$\tilde{\mathbf{S}} = \begin{bmatrix} \mathbf{S} \\ \mathbf{R} \end{bmatrix} \quad (5.37)$$

Having substituted the time-domain constraints (5.25), (5.26) and (5.27) with the sufficient conditions (5.32) and (5.36), and the dynamic constraint (5.14) with the governing equation (5.31), the following nonlinear constrained optimization problem can be formulated:

Find the gain matrices \mathbf{K}_1 and \mathbf{K}_2 that maximize Q , subject to the constraints (5.31), (5.32), and (5.36), as well as the free parameter constraint $\beta > 0$ and the ellipsoidal representation constraint $\Gamma_s \geq 0$.

5.2.3 Solution of the Set-Theoretic Synthesis Problem

As stated above, the ST synthesis problem can be solved using nonlinear constrained optimization techniques. However, this imposes certain difficulties in finding realizable solutions that meet all of the problem constraints. Additionally, it has been the experience of the authors that nonlinear constrained optimization algorithms require excessive computational effort. Therefore, the following observations are made which help transform the ST synthesis problem to a nonlinear unconstrained optimization problem, allowing use of available software from the nonlinear programming

literature.

As only a scalar ubb disturbance model has been used in developing the ST formulation, the matrix representing the set of reachable states, Γ , can be expressed in terms of the disturbance bound, Q , as follows:

$$\Gamma = Q\Theta \quad (5.38)$$

where Θ is a $(2n \times 2n)$ -dimensional matrix. This is true because any matrix, in this case Γ , can be expressed as the product of a scalar and another matrix, here Θ . Substitution of equation (5.38) into equation (5.31), results in the following simplified Lyapunov equation:

$$\{\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I}\}\Theta + \Theta\{\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I}\}^T + \frac{\mathbf{G}_{cl}\mathbf{G}_{cl}^T}{\beta} = \mathbf{0} \quad (5.39)$$

Additionally, substitution of equation (5.38) into equations (5.32) and (5.36), results in the following inequality constraints:

$$Q [\mathbf{K}_j \Theta \mathbf{K}_j^T] \leq T_j; \quad j = 1, 2, \dots, r \quad (5.40)$$

$$Q \left[\tilde{\mathbf{E}}_i \tilde{\mathbf{E}}_i^T + \tilde{\mathbf{H}}_i \Theta \tilde{\mathbf{H}}_i^T + 2 \left\{ \tilde{\mathbf{E}}_i \tilde{\mathbf{E}}_i^T \tilde{\mathbf{H}}_i \Theta \tilde{\mathbf{H}}_i^T \right\}^{\frac{1}{2}} \right] \leq \tilde{S}_i; \quad i = 1, 2, \dots, (p+r) \quad (5.41)$$

The special form of equations (5.40) and (5.41) can now be used to eliminate these inequality constraints as follows:

$$Q = \min \begin{cases} \frac{\tilde{S}_i}{\tilde{\mathbf{E}}_i \tilde{\mathbf{E}}_i^T + \tilde{\mathbf{H}}_i \Theta \tilde{\mathbf{H}}_i^T + 2\sqrt{\tilde{\mathbf{E}}_i \tilde{\mathbf{E}}_i^T \tilde{\mathbf{H}}_i \Theta \tilde{\mathbf{H}}_i^T}}; \\ \quad i = 1, 2, \dots, (p+r) \\ \frac{T_j}{(\mathbf{K}_j \Theta \mathbf{K}_j^T)}; \quad j = 1, 2, \dots, r \end{cases} \quad (5.42)$$

In order to satisfy the β parameter positivity constraint and in order to guarantee that a solution to equation (5.39) exists, the zero value is assigned

to Q as follows:

$$\begin{aligned} \text{If } \beta \leq 0 \text{ then } Q &= 0 \\ \text{If } (\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I}) \text{ unstable then } Q &= 0 \end{aligned} \quad (5.43)$$

5.3 Set-Theoretic Design as a Nonlinear Programming Problem

In the previous paragraphs the ST synthesis problem has been transformed to an unconstrained nonlinear optimization problem, by systematically removing all of the inequality and dynamic constraints. A solution to this optimization problem can be found via the ST synthesis algorithm as follows [7]:

- (1) Generate a feasible initial point $(\mathbf{K}_1, \mathbf{K}_2, \beta)$ for the algorithm, where a feasible point is defined as a triplet of $(\mathbf{K}_1, \mathbf{K}_2, \beta)$ for which the resulting closed-loop matrix $(\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I})$ is stable and the parameter β is positive;
- (2) For a given point $(\mathbf{K}_1, \mathbf{K}_2, \beta)$ solve equation (5.39) for Θ ;
- (3) Calculate the objective function Q from equation (5.42) and (5.43);
and
- (4) Search over the feasible points $(\mathbf{K}_1, \mathbf{K}_2, \beta)$ repeating steps (2) through (4) until the optimum Q is obtained.

Existence of a Solution: A sufficient condition for the existence of a solution to the ellipsoidal ST control synthesis problem with the MBC

structure is the stabilizability of the pair $[\mathbf{A}, \mathbf{B}]$ and the detectability of the pair $[\mathbf{C}, \mathbf{A}]$.

Proof: A more transparent form of the closed-loop system matrix, \mathbf{A}_{cl} , given in equation (5.19) can be obtained by applying the following change of variables [1]:

$$\mathbf{w}(t) = \mathbf{x}(t) - \mathbf{z}(t) \quad (5.44)$$

In view of equations (5.1) and (5.8), it follows that:

$$\dot{\mathbf{w}}(t) = (\mathbf{A} - \mathbf{K}_1\mathbf{C})\mathbf{w}(t) + (\mathbf{G} - \mathbf{K}_1\mathbf{E})d(t) \quad (5.45)$$

In addition, equation (5.1) can be rewritten as:

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{BK}_2)\mathbf{x}(t) - \mathbf{BK}_2\mathbf{w}(t) + \mathbf{G}d(t) \quad (5.46)$$

The combination of equations (5.45) and (5.46), reveals the following form for the closed-loop system matrix \mathbf{A}_{cl} :

$$\mathbf{A}_{cl} = \begin{bmatrix} \mathbf{A} - \mathbf{BK}_2 & +\mathbf{BK}_2 \\ \mathbf{0} & \mathbf{A} - \mathbf{K}_1\mathbf{C} \end{bmatrix} \quad (5.47)$$

Now, it is true that

$$\begin{aligned} \det(\lambda\mathbf{I} - \mathbf{A}_{cl}) &= \det \begin{bmatrix} \lambda\mathbf{I} - \mathbf{A} + \mathbf{BK}_2 & -\mathbf{BK}_2 \\ \mathbf{0} & \lambda\mathbf{I} - \mathbf{A} + \mathbf{K}_1\mathbf{C} \end{bmatrix} \\ &= \det(\lambda\mathbf{I} - \mathbf{A} + \mathbf{BK}_2)\det(\lambda\mathbf{I} - \mathbf{A} + \mathbf{K}_1\mathbf{C}) \end{aligned} \quad (5.48)$$

The above determinant equality states that if the two sub-systems $\mathbf{A} - \mathbf{BK}_2$ and $\mathbf{A} - \mathbf{K}_1\mathbf{C}$ are stable (i.e. all of their eigenvalues have strictly negative real parts), then the closed-loop system matrix \mathbf{A}_{cl} is also stable. However,

it is well-known fact of linear system theory that for a stabilizable pair $[\mathbf{A}, \mathbf{B}]$, there is at least one matrix \mathbf{K}_2 for which $(\mathbf{A} - \mathbf{BK}_2)$ is stable. Similarly, for a detectable pair $[\mathbf{C}, \mathbf{A}]$, there is at least one matrix \mathbf{K}_1 for which $(\mathbf{A} - \mathbf{K}_1\mathbf{C})$ is stable. This implies that there is at least one pair $(\mathbf{K}_1, \mathbf{K}_2)$ for which the system with closed-loop system matrix \mathbf{A}_{cl} is stable. In other words

$$\text{Max} \{ \text{Re } \lambda(\mathbf{A}_{cl}) \} \leq 0 \quad (5.49)$$

However,

$$\text{Max} \left\{ \text{Re } \lambda(\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I}) \right\} \leq \text{Max} \{ \text{Re } \lambda(\mathbf{A}_{cl}) \} + \frac{1}{2}\beta \quad (5.50)$$

Therefore, with a pair $(\mathbf{K}_1, \mathbf{K}_2)$ that stabilizes \mathbf{A}_{cl} , a positive β can always be found for which the system $(\mathbf{A}_{cl} + \frac{1}{2}\beta\mathbf{I})$ is stable. Consequently, there is at least one point $(\mathbf{K}_1, \mathbf{K}_2, \beta)$, for which the Lyapunov equation (5.39) (and equation (5.31)) yields a unique solution $\Theta > 0$ (and $\Gamma_s > 0$). This condition guarantees the existence of at least one positive value of Q , as defined by equation (5.42). The maximum value of Q can therefore be found by an appropriate optimization procedure.

There are numerous issues that relate to the implementation of the previously described algorithm. These include numerical issues in solving the various equations and obtaining feasible starting points for the optimization, especially for a high order system. Additionally, the possibility of obtaining a local instead of a global minimum for the above nonlinear programming problem exists. However, use of sophisticated optimization algorithms, such as simulated annealing, has considerably reduced the number of iterations required for proper convergence while lowering the possibility of obtaining a local minimum for Q .

The ST control system synthesis algorithm can be summarized by simply listing the guaranteed properties of the final linear design. For a given system described in state space form by equations (5.1) and (5.2), a persistent external disturbance modeled according to equation (5.3), and for a set of predefined output, control and control rate constraints, the set-theoretic synthesis step is guaranteed to satisfy the following design specifications:

- (1) Stability of the closed-loop system;
- (2) Satisfaction of the constrained output, control and control rate constraints that have initially been placed upon the dynamic system. In addition, indirect satisfaction of compensator bandwidth constraints through the appropriate choice of the control bound to control rate bound ratio; and
- (3) Ability to reject the maximum possible disturbance magnitude, regardless of its spectral composition. However, bandwidth constraints limit this capability to within a certain frequency range.

5.4 Set-Theoretic Analysis of Control Systems

An additional unique feature of the ST approach is the ability to analyze and evaluate the performance of a linear control system which is designed using this or any other control synthesis algorithm based on a deterministic disturbance model. The ST analysis, for the specific case of a MBC, can be stated as follows:

Given a dynamic system, as described by equations (5.14) through (5.16), find the corresponding maximum ubb input disturbance amplitude that the dynamic system can tolerate, and the corresponding possible constrained output, control and control rate excursions.

The solution to this problem can be obtained by implementing the following algorithm:

- (1) Form the matrices \mathbf{A}_{cl} , \mathbf{G}_{cl} , $\tilde{\mathbf{H}}$, and $\tilde{\mathbf{E}}$, and the vectors $\tilde{\mathbf{S}}$ and \mathbf{T} ;
- (2) Solve equation (5.39) for Θ ;
- (3) Calculate Q from equation (5.42); and
- (4) Calculate the left-hand-sides of equation (5.40) and (5.41).

This problem is of practical significance, because its solution yields a good reflection on the capability and performance of a given ST control system, without resorting to numerical simulations. However, as mentioned in earlier paragraphs, the estimates of the calculated bounds will be somewhat conservative as a result of the approximations involved in solving the ST problem.

Chapter 6

Computer Simulation Results

The cantilevered two-bay truss, the AFWAL structure, a model of which is presented in section II is assumed to be mounted on a relatively rigid base, which may be representing another structure [5]. Persistent disturbances are assumed to be acting directly upon the two-bay truss structure. Such disturbances may be representative of a shuttle docking or of other interactions of the structure with the environment. The structure is allowed to move only along the Y -axis and its physical displacement is measured by four sensors. The structure is well represented by its first eight flexible modes, which are assumed to be equally damped with 0.5% passive damping.

The eight-mode model is reduced in order to obtain a lower-order model for control system design. The first two modes of the structure, with frequencies 3.14 and $10.39 \frac{rad}{sec}$, are retained [5], the reduced-order model is augmented with four integrators at the plant input (one for each input channel), and the resulting four input, eight state, four output system is

used for design purposes. The augmented plant model is given by:

$$\mathbf{G}_a(s) = \mathbf{G}_p(s) \frac{\mathbf{I}}{s} \quad (6.1)$$

Figure 6.1 depicts the block diagram of the feedback loop under consideration. Since the model used to design the active control law will be inaccurate at frequencies approximately above $10 \frac{\text{rad}}{\text{sec}}$, it is required that the controller bandwidth is about $10 \frac{\text{rad}}{\text{sec}}$.

The control problem for the above structure is completely determined if the design specifications are explicitly expressed, as follows: It is desired to reject the maximum possible disturbance magnitude, while keeping the maximum deformation of the points 1, 2, 3, and 4 to within 0.1 *in.* of the resting position of the structure. This is to be accomplished using only the available control power of a maximum 100 *lb_f*, at the available control rate of a maximum $1000 \frac{\text{lb}_f}{\text{sec}}$. The combination of the above control and control rate limitations, impose a minimum compensator response time on the order of 0.1 *sec*, which in turn implies a maximum compensator crossover frequency of $10 \frac{\text{rad}}{\text{sec}}$. The control and output constraints considered result from the physical limitations of the actuators and the desire to limit the structure deformation at both the tip and at the mid-point.

All of the simulations for the AFWAL structure were performed using the eight mode model, even though control system design was performed using the reduced-order two mode approximation. The open-loop response of the structure to a persistent disturbances is shown in Figure 6.2. This figure depicts the displacement of the truss tip (point 1 in Figure 1), when an external pulse force of 50 *lb_f* is applied for 5 *secs*, starting at $t = 1 \text{ sec}$. The maximum structure displacement is about 0.4 *in.* and after the dis-

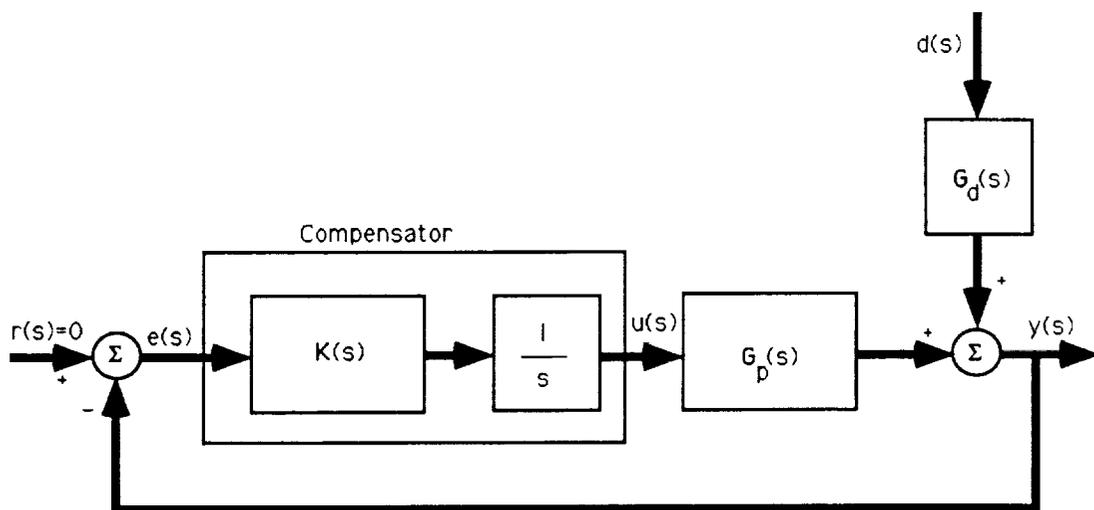


Figure 6.1: Block Diagram of the Feedback Loop.

turbance is removed, the vibrations damp out at a fairly slow rate. It is desired to limit this structural displacement and to damp-out the resulting oscillations as rapidly as possible, within the limitations imposed by the compensator design specifications.

Application of the ST control design technique to the AFWAL structure resulted in an active control law that is expected to satisfy all of the design requirements if a disturbance of a maximum 50 lb_f amplitude and arbitrary shape is applied directly on the structure tip. Table 6.1 presents a summary of the ST design and of the resulting ST analysis. The table clearly shows that, in contrast to the transient simulations which follow, the structure displacement, the control and the control rate at point 1 (Figure 3.1) are binding. This discrepancy can be primarily attributed to the two approximations in the ST design method. These are:

- (1) The approximation of a convex set by an ellipsoid which contains it and,
- (2) The use of the steady-state bounds in the nonlinear optimization, instead of the transient bounds (steady-state solution of equation (5.30)).

Both of these approximations contribute towards increasing the conservative nature of the design, however, they are both required in order to transform the design problem into a relatively simpler nonlinear optimization problem.

Figures 6.3 and 6.4 depict the transient response simulation of the structure to a 10 sec pulse of 50 lb_f magnitude. Figure 6.3 shows the displace-

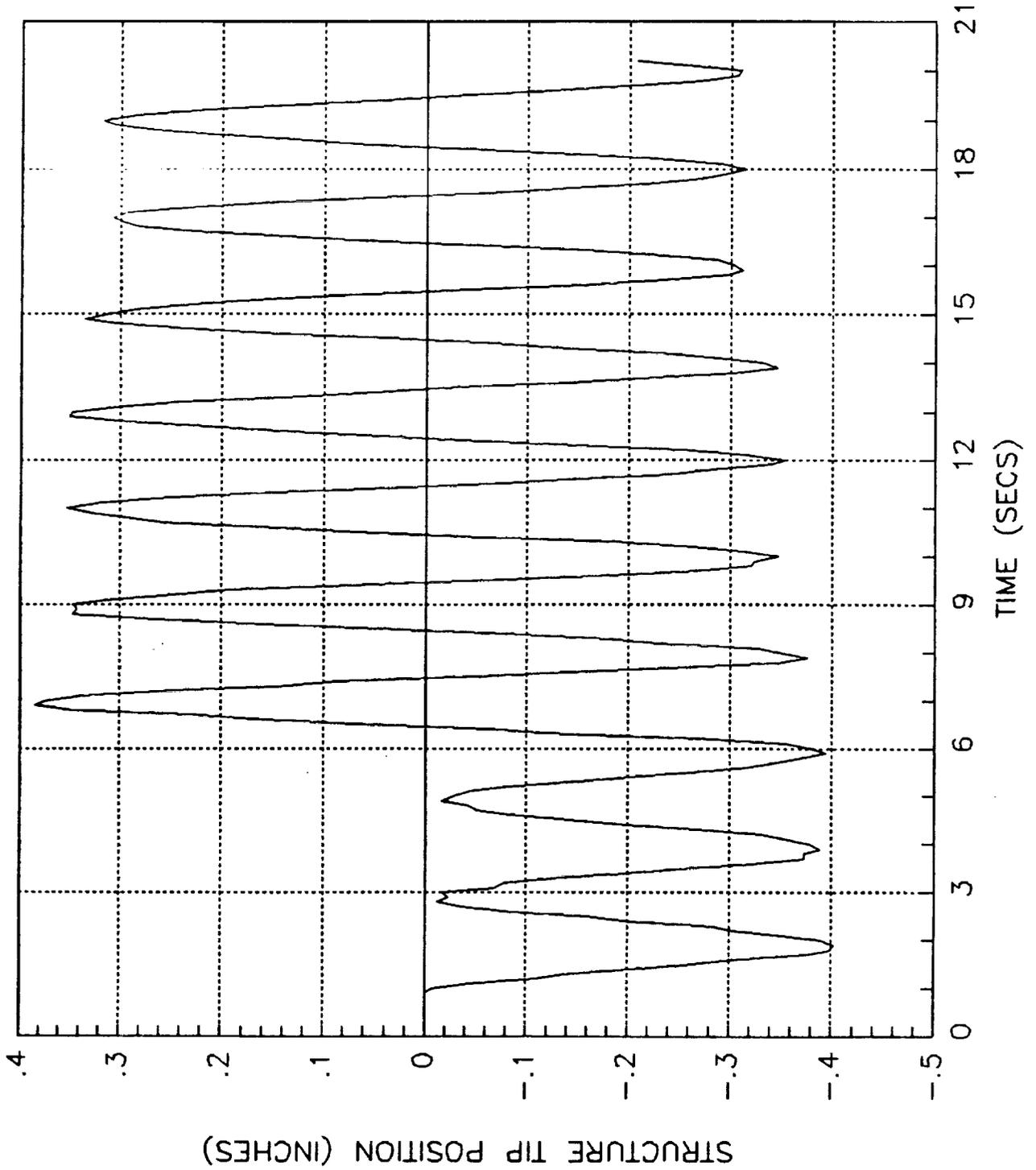


Figure 6.2: Open-Loop Response of the AFWAL Structure for a 5 second $50 lb_f$ pulse.

ment of the structure tip and Figure 6.4 the control effort applied to produce this displacement. The maximum displacement of the structure tip is about 0.03 *in.* and the resulting vibrations seem to damp out at a fairly fast rate.

Table 6.1. Set-Theoretic Design Summary

Max Ampl.	Q	50 <i>lb_f</i>
Free Parameter	β	1.88
Variable	Spec. Bound	Calc. Bound
Position at 1	0.1 <i>in.</i>	0.1 <i>in.</i>
Position at 2	0.1 <i>in.</i>	0.098 <i>in.</i>
Position at 3	0.1 <i>in.</i>	0.099 <i>in.</i>
Position at 4	0.1 <i>in.</i>	0.1 <i>in.</i>
Control effort at 1	100 <i>lb_f</i>	100 <i>lb_f</i>
Control effort at 2	100 <i>lb_f</i>	100 <i>lb_f</i>
Control effort at 3	100 <i>lb_f</i>	5.35 <i>lb_f</i>
Control effort at 4	100 <i>lb_f</i>	43.1 <i>lb_f</i>
Control rate at 1	1000 $\frac{lb_f}{sec}$	1000 $\frac{lb_f}{sec}$
Control rate at 2	1000 $\frac{lb_f}{sec}$	1000 $\frac{lb_f}{sec}$
Control rate at 3	1000 $\frac{lb_f}{sec}$	90.71 $\frac{lb_f}{sec}$
Control rate at 4	1000 $\frac{lb_f}{sec}$	250.5 $\frac{lb_f}{sec}$

Additionally, the control effort used to produce such response is less than the 100 *lb_f* required to saturate the actuators. It can be observed, however, that the structure tip maximum displacement is far from reaching

the bounds imposed upon the design, whereas the control and control rates are the binding constraints. The latter can be inferred by careful examination of the control effort slope in Figure 6.4. Additionally, this can be verified by the ST analysis presented in Table 4.1. Similarly the displacement of the truss mid-point has a maximum displacement which is well within the 0.1 *in.* bound required by the design.

In addition to the presented transient response simulations, Figure 6.5 depicts the magnitude plot of the forward loop transfer function matrix, $\mathbf{G}_a(s)\mathbf{K}(s)$, frequency response, for the two-mode structure. This figure indicates that the cross-over frequency of the maximum singular value is about $10 \frac{rad}{sec}$, implying compensator response time on the order of 0.1 *sec.* This can also be verified by the transient response simulation of the structure tip control effort, shown in Figure 5. The structure mid-point actuator responses are slower and require less control effort than the actuators at the structure tip.

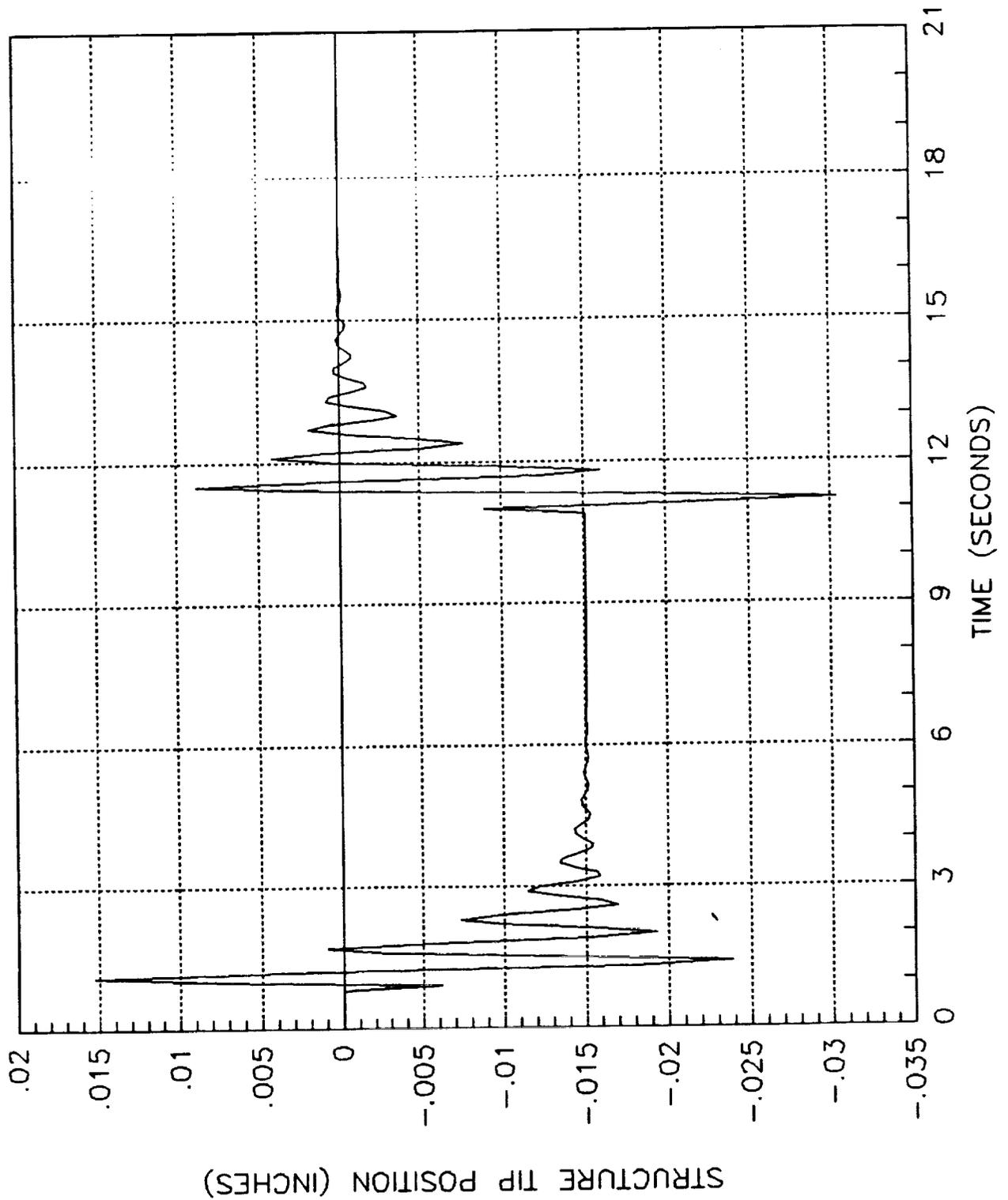


Figure 6.3: Tip Position Response of the AFWAL Structure to a 50 lb_f pulse (S-T Controller).

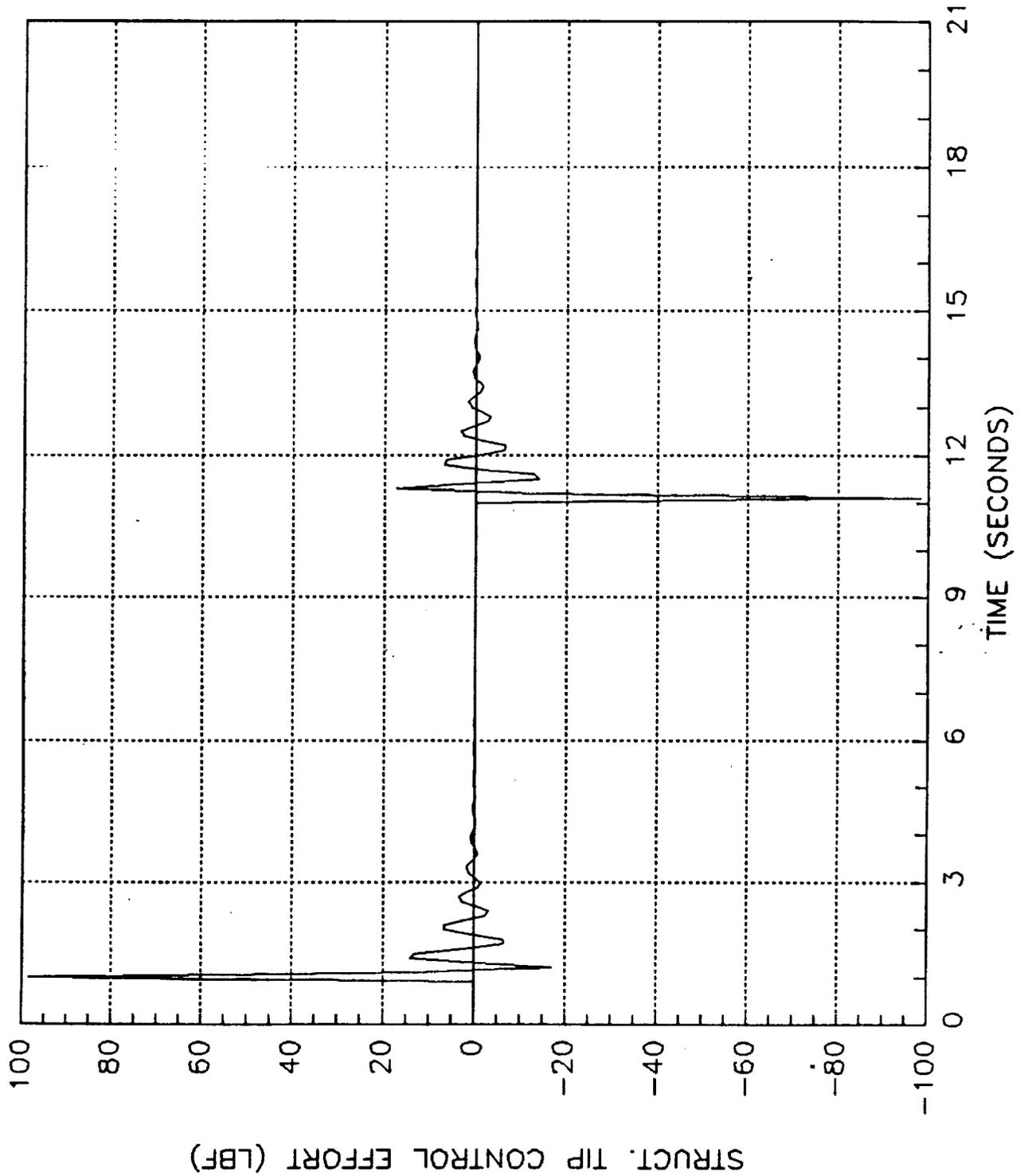


Figure 6.4: Control Effort Response of the AFWAL Structure for a 50 lb_f pulse (S-T Controller).

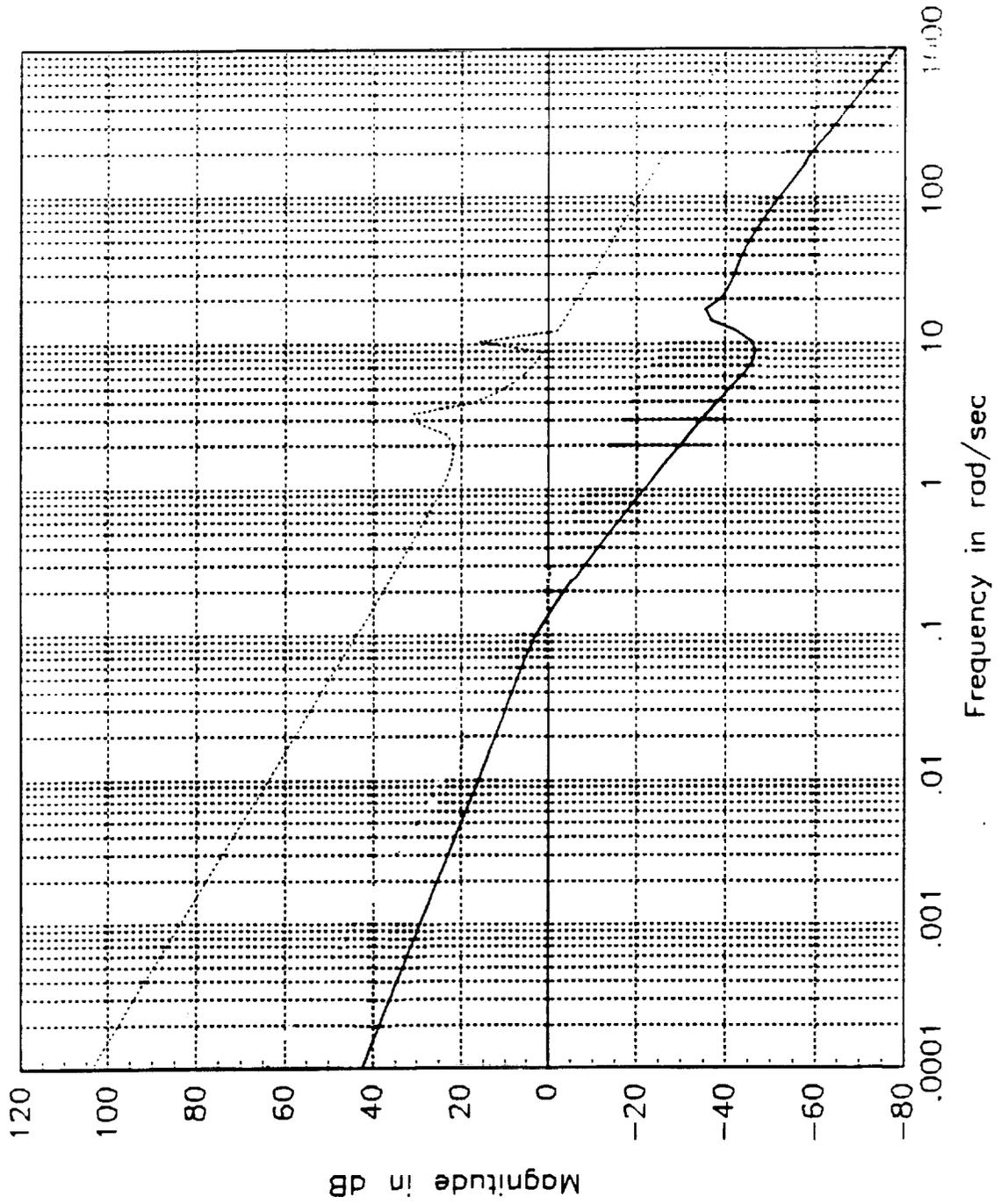


Figure 6.5: Frequency Response of the Forward Loop Transfer Function Matrix.

Chapter 7

Summary and Conclusions

This study presented a solution to the problem of optimal persistent disturbance rejection in flexible structures. In doing so a deterministic uncertainty modeling approach is presented and the controller synthesis is formulated as a nonlinear optimization problem. The main objective of the control problem is to keep the displacement at the tip of a flexible structure within a certain range, for satisfaction of pointing requirements, while rejecting the external persistent disturbances. In addition, this is accomplished by limiting the control effort to within the saturation limits of the force actuators and without requiring excessively fast controller response times. In other words, through the appropriate choice of the control and control rate constraints, the compensator bandwidth is limited to within a desired range. The controller structure used in this development lends itself to the earlier development of the Linear Quadratic Gaussian/Loop Transfer Recovery (LQG/LTR) method and it utilizes the so-called MBC structure. The flexible structure used is a two-bay truss model, representative of a

wide class of space structures. The structure is assumed to be mounted on a relatively rigid platform and it is being subjected to a bounded deterministic disturbance of unknown shape, which is affecting the structure at its tip (point 2 in Figure 3.1).

The ST design method is applied to a reduced order model of the structure. The results indicate that active control methods can be used, in combination with passive damping, to ensure accurate pointing of subsystems, such as payloads, which are mounted on larger structures. The proposed design technique uses only the available control to maximize the disturbance magnitude that can be tolerated by the structure, while keeping the tip and mid points of the structure within the desired ranges from their equilibrium positions. The ST analysis and the transient response simulations indicate that some of the obtained bound estimates are conservative. However, these can be improved, through a combination of iterative design and transient response simulations.

It is believed that the proposed approach offers an alternative to stochastic design in addressing the problem of persistent disturbance rejection, especially when the statistics of the disturbances are hard to obtain or there is experimental evidence to suggest that the disturbances can not be well-represented by a stochastic model. In addition, the ubb approach makes explicit treatment of actuator saturation limits and system output constraints, guaranteeing that the final design satisfies all of the imposed compensator design specifications.

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Part IV

Active Rejection of Persistent Disturbances in Flexible Structures via Sensitivity Minimization

Chapter 8

Introduction

The disturbance rejection capability of a controller is usually of utmost concern when dealing with physical systems which are subjected to uncertain external perturbations. This is especially true for flexible objects, such as space structures, which must be controlled to perform a prespecified mission. In addition to good structural design and appropriate choice of structural materials, resulting in acceptable passive damping [5], insuring satisfactory structural performance usually requires the use of an active control law, i.e., a feedback controller. In fact, to obtain the best persistent disturbance rejection capability, it is important to select an active control law that could in some sense minimize the maximum amplification of the disturbance-to-output transfer function matrix. In other words, it is desired to find an active control law which will minimize the sensitivity of the system to persistent disturbances.

The H_∞ -norm has recently become one of the most popular performance measures in optimal control theory. The H_∞ -norm of a transfer

function matrix $\mathbf{G}(s)$ denotes the peak of the maximum singular value of $\mathbf{G}(j\omega)$ over all frequencies ω . In familiar terms, for instance in single-input, single-output (SISO) systems, the H_∞ -norm of a transfer function $g(s)$ is equal to the distance from the origin to the farthest point on its Nyquist plot. To appreciate the concept of the H_∞ -norm in control theory, it can be interpreted as the worst amplification that a bounded-energy input signal could undergo as a result of passing through a system with transfer function matrix $\mathbf{G}(s)$. Therefore, an H_∞ -optimal control law is considered one of the best available choices in dealing with the active rejection of persistent disturbances.

The objective of the H_∞ -optimal control problem is to find a control law that minimizes the H_∞ -norm of the transfer function matrix from the disturbances to the controlled output, while stabilizing the closed-loop system. One of the main reasons for developing the H_∞ -optimal control theory is to accommodate variations in the power spectra of the disturbances. Unlike the H_2 -optimal (or Linear Quadratic Gaussian) control problem, where fixed power spectra are usually assumed, the disturbances in the H_∞ -optimal problem are assumed to belong to the class of finite energy signals. This represents a more realistic treatment of real-world disturbances, because in most cases external excitations are neither Gaussian nor white, as it is assumed in the derivation of the Linear Quadratic Gaussian/Loop Transfer Recovery (LQG/LTR) controllers.

The state space method for continuous time systems, proposed by Doyle [2],[3], is for a standard H_∞ -optimal problem described below. The controller gain, a constant matrix, is obtained from a special form of the Ric-

cati equation. For multivariable systems the state space formulation of the H_∞ -optimal control problem is more convenient than alternate approaches, especially because of the ease in numerical computations. However, the controlled system model orthogonality assumptions, (elimination of the feedforward terms in the state-space model), used in the formulations presented in references [2] and [3], are hard to satisfy in physical systems. This is particularly true for system models obtained from model order reduction techniques which always include a feedforward term. More recently, the H_∞ -optimal control theory has been applied to a SISO system with structured parameter uncertainties [1]. The main objective of the latter paper, [1], was to design a controller that maximizes the closed-loop robustness with respect to parameter variations. However, the system model used in this study did not contain feedforward terms and it was not subjected to persistent external disturbances which must be rejected by the H_∞ -optimal controller.

The primary objective of this study is to demonstrate the feasibility of persistent disturbance rejection in flexible space structures via the H_∞ -optimal control theory. In doing so, a new formulation of the H_∞ -optimal control problem entirely in the time domain, which includes the state-space model feedforward terms, is presented, facilitating the treatment of MIMO systems. A low order approximation of a high order structural model is used to design a H_∞ -optimal compensator for the rejection of persistent disturbances, such as shuttle docking, in a flexible structure. Robustness to higher order unmodeled dynamics, an important issue in active structural control, is acknowledged, but investigated only via numerical simula-

tions. Further systematic treatment of controller robustness to parameter variations and unmodeled dynamics, is currently under investigation.

The “Standard Problem” considered for the H_∞ -optimal formulation has the following state-space representation [4]:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_2\mathbf{u}(t) + \mathbf{B}_1\mathbf{w}(t) \\ \mathbf{y}(t) &= \mathbf{C}_2\mathbf{x}(t) + \mathbf{D}_{21}\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{12}\mathbf{u}(t)\end{aligned}\tag{8.1}$$

where $\mathbf{x}(t) \in R^n$, $\mathbf{y}(t) \in R^p$, $\mathbf{u}(t) \in R^m$, $\mathbf{w}(t) \in R^r$, and $\mathbf{z}(t) \in R^l$ denote the state, the measured output, the control, the disturbance, and the controlled output vectors, respectively. A full state feedback control law is assumed, $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$, where \mathbf{K} is a constant control gain matrix. The disturbance $\mathbf{w}(t)$ typically consists of reference inputs, external plant disturbances, and sensor noise. The components of the controlled output $\mathbf{z}(t)$ are the tracking (regulation) errors, the control efforts, etc. The control objective is to find the H_∞ -optimal control law $\mathbf{u}(t)$, from the set of bounded energy signals, such that for the finite energy disturbance $\mathbf{w}(t)$, the L_2 -norm of the controlled output $\mathbf{z}(t)$ is minimized. Notice that since the controlled output $\mathbf{z}(t)$ consists of the tracking (regulation) errors and the control efforts, minimization of the controlled output sensitivity to worst case disturbances implies sensitivity minimization of its components for the same class of signals. Thus, given the fixed controller structure, the objective of the H_∞ -optimal control problem is consistent with the physical control system design problem of reducing the sensitivity of the closed-loop system to disturbances, while keeping the closed-loop system stable.

Part IV of this report is organized as follows: Chapter 9 presents a new formulation to the H_∞ -optimal control problem, namely using calculus of variations, thus simplifying the treatment of MIMO systems. This section concludes with a proposed algorithm for solving the H_∞ -control problem and with the formulation of the relevant H_∞ -optimal regulator and tracking problems. Chapter 10 presents the results obtained from the application of the developed H_∞ -optimal control algorithm to the flexible structure presented in chapter 3. Finally, chapter 11 summarizes the paper and presents the main conclusions of this study.

Chapter 9

A Variational Formulation of the H_∞ -Optimal Control Problem

9.1 The H_∞ -Optimal Control Problem

In the following discussion, the standard H_∞ -optimal control problem formulated using the system of equations (8.1) is considered, where \mathbf{A} , \mathbf{B}_1 , \mathbf{B}_2 , \mathbf{C}_1 , \mathbf{C}_2 , \mathbf{D}_{12} , \mathbf{D}_{21} are constant matrices of appropriate dimensions. The goal of this method is to find a control law $\mathbf{u}(t)$, optimal in the H_∞ -norm sense, which minimizes the worst amplification of the plant disturbance-to-controlled output transfer function matrix. The approach is to convert the H_∞ -optimal control problem into minimizing an objective functional, subjected to the system dynamic constraints, under the worst possible disturbance conditions. The main results are summarized in two theorems

and a corollary.

For a plant given in the form of equations (8.1), it is usually possible to rescale the controller $\mathbf{u}(t)$ and the measured output $\mathbf{y}(t)$ so that $\mathbf{D}_{12}^T \mathbf{D}_{12} = \mathbf{I}$, which constitutes one of the two assumptions imposed upon the “Standard Problem” of the system of equations (8.1). However, in many systems, especially in system models involving model-order reduction, it is difficult to form the standard problem satisfying the orthogonality condition, $\mathbf{D}_{12}^T \mathbf{C}_1 = \mathbf{0}$. This orthogonality assumption has been widely used in the literature when applying the H_∞ -optimal control theory. Therefore, to facilitate application of the H_∞ -optimal approach to physical problems, this orthogonality assumption must be removed. For this reason, the following theorem leading to a more general solution of the H_∞ -optimal control problem is developed.

Theorem 1: Consider a system in the standard form given by the system of equations (8.1), and suppose that the pair $(\mathbf{A}, \mathbf{B}_2)$ is controllable, the pair $(\mathbf{C}_1, \mathbf{A})$ is observable, and a scaling is performed such that $\mathbf{D}_{12}^T \mathbf{D}_{12} = \mathbf{I}$. Then, the full-state H_∞ -optimal control law $\mathbf{u}(t)$, minimizing $\|\mathbf{z}(t)\|_2$ for the worst finite energy disturbance $\mathbf{w}(t)$, is in the form:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t), \quad \mathbf{K} = -(\mathbf{B}_2^T \mathbf{k}_1 + \mathbf{D}_{12}^T \mathbf{C}_1) \quad (9.1)$$

where \mathbf{k}_1 is the positive definite solution of the following Algebraic Riccati Equation (ARE):

$$\begin{aligned} \mathbf{0} = & (\mathbf{A} - \mathbf{B}_2 \mathbf{D}_{12}^T \mathbf{C}_1)^T \mathbf{k}_1 + \mathbf{k}_1 (\mathbf{A} - \mathbf{B}_2 \mathbf{D}_{12}^T \mathbf{C}_1) + \mathbf{k}_1 (\mathbf{B}_1 \mathbf{B}_1^T - \mathbf{B}_2 \mathbf{B}_2^T) \mathbf{k}_1 + \\ & \mathbf{C}_1^T (\mathbf{I} - \mathbf{D}_{12} \mathbf{D}_{12}^T) (\mathbf{I} - \mathbf{D}_{12} \mathbf{D}_{12}^T) \mathbf{C}_1 \end{aligned} \quad (9.2)$$

As stated before, application of the H_∞ -optimal control synthesis required transformation of state-space models to the form given by the system of equations (8.1). Even though relatively straightforward, it is desirable to avoid this transformation in design studies. The following corollary presents the solutions of the H_2 -optimal and H_∞ -optimal full-state feedback control problems for the common state-space representation of linear-time-invariant systems. Whereas in subsection C of this section, the state-space model is transformed to the "Standard" form of equations (8.1), for the regulator and tracking problems.

Corollary: Consider the following regulator problem:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\mathbf{w}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}\tag{9.3}$$

where $\mathbf{x}(t) \in R^n$, $\mathbf{y}(t) \in R^p$, $\mathbf{u}(t) \in R^m$, $\mathbf{w}(t) \in R^r$, are the plant state, the plant output, the plant control, and the plant disturbance vectors, respectively. Suppose that the controlled output $\mathbf{z}(t)$, is given by:

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{y}(t) \\ \rho\mathbf{u}(t) \end{bmatrix}\tag{9.4}$$

where ρ is the weighting on the control effort $\mathbf{u}(t)$, (\mathbf{A}, \mathbf{B}) is a controllable pair and (\mathbf{C}, \mathbf{A}) is an observable pair. Then the following statements are true:

- (i) For the case that $\mathbf{w}(t)$ is a white Gaussian process, the H_2 -optimal (LQG) control law is given by

$$\mathbf{u}(t) = -\left(\frac{1}{\rho^2}\right)\mathbf{B}^T\mathbf{k}_2\mathbf{x}(t)\tag{9.5}$$

where \mathbf{k}_2 is the positive definite solution of the following ARE:

$$\mathbf{A}^T \mathbf{k}_2 + \mathbf{k}_2 \mathbf{A} - \left(\frac{1}{\rho^2}\right) \mathbf{k}_2 \mathbf{B} \mathbf{B}^T \mathbf{k}_2 + \mathbf{C}^T \mathbf{C} = \mathbf{0} \quad (9.6)$$

- (ii) For the case that the disturbance $\mathbf{w}(t)$ is any finite energy signal, the H_∞ -optimal control law is given by

$$\mathbf{u}(t) = -\left(\frac{1}{\rho^2}\right) \mathbf{B}^T \mathbf{k}_1 \mathbf{x}(t) \quad (9.7)$$

where \mathbf{k}_1 is the positive definite solution of the following ARE:

$$\mathbf{A}^T \mathbf{k}_1 + \mathbf{k}_1 \mathbf{A} + \mathbf{k}_1 (\mathbf{L} \mathbf{L}^T - \left(\frac{1}{\rho^2}\right) \mathbf{B} \mathbf{B}^T) \mathbf{k}_1 + \mathbf{C}^T \mathbf{C} = \mathbf{0} \quad (9.8)$$

Proof: The proof of this Corollary is similar to the proof of the previous Theorem and it is therefore omitted here.

9.2 Output Feedback and the H_∞ -Optimal Observer Problem

In this section the H_∞ -optimal observer for the standard problem given by equations (8.1) is formulated, based on the properties of the optimal H_∞ -regulators developed in the previous section. The duality between the optimal regulator problem (ORP) and the optimal observer problem (OOP) can be summarized as follows:

- (i) The matrix \mathbf{A} of the ORP equals the matrix \mathbf{A}^T of the OOP,
- (ii) The matrix \mathbf{C}_1 of the ORP equals the matrix \mathbf{B}_1^T of the OOP,

(iii) The matrix \mathbf{B}_2 of the ORP equals the matrix \mathbf{C}_2^T of the OOP,

(iv) The matrix \mathbf{D}_{12} of the ORP equals the matrix \mathbf{D}_{21}^T of the OOP,

Reference [6] is recommended for details and a proof of the duality properties. These properties enable solution of the H_∞ -optimal observer problem by adaptation of the H_∞ -optimal controller equations. Therefore, the H_∞ -optimal output feedback problem can easily be formulated. The main results are summarized in the following lemma:

Theorem 2: Consider a system in the standard form of equations (8.1), and suppose that the pairs $(\mathbf{A}, \mathbf{B}_1)$ and $(\mathbf{A}, \mathbf{B}_2)$ are controllable, the pairs $(\mathbf{C}_1, \mathbf{A})$ and $(\mathbf{C}_2, \mathbf{A})$ are observable, and the scaling $\mathbf{D}_{12}^T \mathbf{D}_{12} = \mathbf{I}$ and $\mathbf{D}_{21} \mathbf{D}_{21}^T = \mathbf{I}$ has been performed. If the system states are estimated using the following observer:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}_2\mathbf{u}(t) + \mathbf{H}(\mathbf{C}\hat{\mathbf{x}}(t) - \mathbf{y}(t)) + \mathbf{B}_1\mathbf{w}_{worst}(t), \quad \mathbf{w}_{worst}(t) = \mathbf{B}_1^T \mathbf{k}_1 \hat{\mathbf{x}}(t) \quad (9.9)$$

Then, the optimal observer gain \mathbf{H} is given by:

$$\mathbf{H} = -(\mathbf{I} - \mathbf{h}_\infty \mathbf{k}_1)^{-1} (\mathbf{h}_\infty \mathbf{C}_2^T + \mathbf{B}_1 \mathbf{D}_{21}^T) \quad (9.10)$$

where \mathbf{h}_∞ is the positive definite solution of the following ARE:

$$\mathbf{A}_p \mathbf{h}_\infty + \mathbf{h}_\infty \mathbf{A}_p^T + \mathbf{h}_\infty (\mathbf{C}_1^T \mathbf{C}_1 - \mathbf{C}_2^T \mathbf{C}_2) \mathbf{h}_\infty + \mathbf{B}_{1p} \mathbf{B}_{1p}^T = \mathbf{0} \quad (9.11)$$

where, $\mathbf{A}_p = \mathbf{A} - \mathbf{B}_1 \mathbf{D}_{21}^T \mathbf{C}_2$, and $\mathbf{B}_{1p} = \mathbf{B}_1 (\mathbf{I} - \mathbf{D}_{21}^T \mathbf{D}_{21})$. Now, considering the H_∞ -optimal controller gain \mathbf{K} of equation (9.1), the H_∞ -optimal dynamic compensator can be expressed as:

$$\dot{\hat{\mathbf{x}}}(t) = (\mathbf{A} + \mathbf{B}_2 \mathbf{K} + \mathbf{H} \mathbf{C}_2 + \mathbf{B}_1 \mathbf{B}_1^T \mathbf{k}_1) \hat{\mathbf{x}}(t) - \mathbf{H} \mathbf{y}(t) \quad (9.12)$$

$$\mathbf{u}(t) = \mathbf{K}\hat{\mathbf{x}}(t) \quad (9.13)$$

In terms of transfer function matrices, the dynamic compensator is expressed as:

$$\mathbf{u}(s) = \mathbf{K}(s)\mathbf{y}(s) \quad (9.14)$$

with

$$\mathbf{K}(s) = \begin{bmatrix} \mathbf{A}_5 & -\mathbf{H} \\ \mathbf{K} & \mathbf{0} \end{bmatrix}; \quad \mathbf{A}_5 = \mathbf{A} + \mathbf{B}_2\mathbf{K} + \mathbf{H}\mathbf{C}_2 + \mathbf{B}_1\mathbf{B}_1^T\mathbf{k}_1 \quad (9.15)$$

where the controller gain \mathbf{K} and \mathbf{k}_1 are computed from equations (9.1) and (9.2), respectively, and the observer gain \mathbf{H} is computed from equation (9.10).

Proof : The proof is immediate from the duality properties.

As it is indicated in the following subsections, the input to the compensator $\mathbf{K}(s)$ becomes the error signal $\mathbf{e}(s)$ instead of the output $\mathbf{y}(s)$ when considering tracking problems.

9.3 The Regulator and Tracking Problems

In this section, the regulation and the tracking problems are formulated in the standard form of equations (8.1), to facilitate controller synthesis using the algorithm of the previous section. Consider a plant, denoted by $(\mathbf{A}, \mathbf{B}, \mathbf{G}, \mathbf{C}, \mathbf{D}, \mathbf{E})$:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{w}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{E}\mathbf{w}(t) \end{aligned} \quad (9.16)$$

where $\mathbf{x}(t) \in R^n$, $\mathbf{y}(t) \in R^p$, $\mathbf{u}(t) \in R^m$, $\mathbf{w}(t) \in R^r$, are the state, the plant output, the control, and the exogenous input vectors, respectively. Without loss of generality, the constant matrix \mathbf{E} is set to the identity matrix in the following formulation. Note that to fit the standard problem of equations (8.1), the plant transfer function matrix (from $\mathbf{u}(s)$ to $\mathbf{y}(s)$) must be strictly proper. For many physical problems, the matrix \mathbf{D} is usually a null matrix. However, if $\mathbf{D} \neq \mathbf{0}$, as it is the case with systems resulting from model order reduction, the plant must be augmented to make it strictly proper. In the sequel, a strictly proper plant is formed by augmenting it with integrators as its input to bring it to the standard form. Additionally, even though the tracking problem is not used in this study, if the disturbance spectrum is known, the tracking formulation can be used to asymptotically reject the disturbance.

9.3.1 The Regulator Problem

For a regulator problem, the exogenous input $\mathbf{w}(t)$ represents the disturbances (and/or the noise) $\mathbf{d}(t)$ introduced to the system. To match the standard form of equations (8.1), first the plant is augmented at each input channel with an integrator and the augmented system is represented by $(\mathbf{A}_s, \mathbf{B}_s, \mathbf{G}_s, \mathbf{C}_s, \mathbf{0}, \mathbf{D}_s)$. The block diagram corresponding to the augmented plant of the regulator problem is shown in Figure 9.1.

The filter \mathbf{W}_s may be chosen as a strictly proper stable transfer function reflecting the frequency range over which the plant output is to be controlled; let \mathbf{W}_s be denoted by $(\mathbf{A}_3, \mathbf{B}_3, \mathbf{C}_3)$ in the state-space form. $\rho\mathbf{I}$ and $\rho_1\mathbf{I}$ are constant weighting matrices on the control rate and control ef-

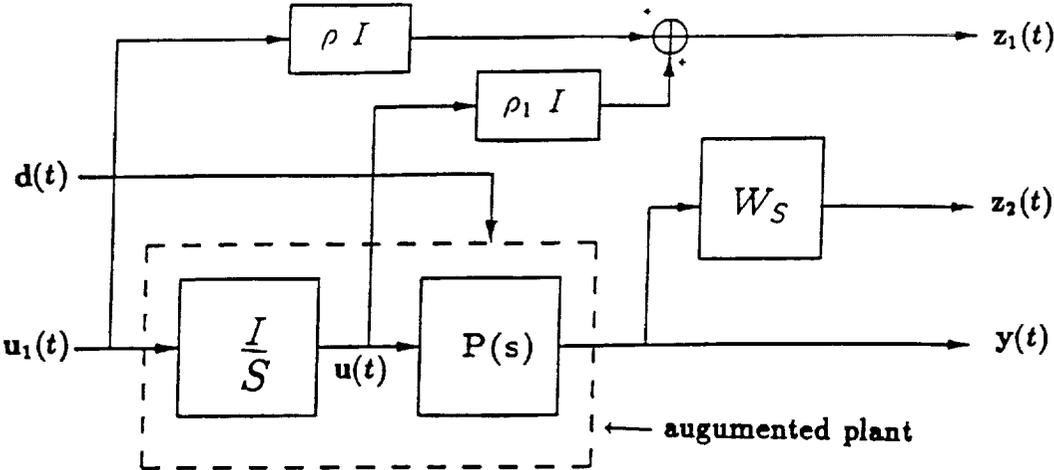


Figure 9.1: Regulator Problem Set-up.

forts, respectively. The controlled output $\mathbf{z}(t)$ and the disturbance (and/or noise) $\mathbf{d}(t)$ are defined as follows:

$$\mathbf{z}(t) = \begin{bmatrix} \rho \mathbf{u}_1(t) + \rho_1 \mathbf{u}(t) \\ \mathbf{W}_s \mathbf{y}(t) \end{bmatrix}, \quad \mathbf{w}(t) = \mathbf{d}(t) \quad (9.17)$$

where, $\mathbf{u}_1(t)$ and $\mathbf{u}(t)$ represent the integrator and the actual plant input, respectively. The former is also the plant input control rate. By rearrangement, the generalized plant can be written in the form of equations (8.1), as follows:

$$\begin{aligned} \dot{\bar{\mathbf{x}}}(t) &= \begin{bmatrix} \mathbf{A}_s & \mathbf{0} \\ \mathbf{B}_3 \mathbf{C}_s & \mathbf{A}_3 \end{bmatrix} \bar{\mathbf{x}}(t) + \begin{bmatrix} (1/\rho) \mathbf{B}_s \\ \mathbf{0} \end{bmatrix} \mathbf{u}_{1s}(t) + \begin{bmatrix} \mathbf{G}_s \\ \mathbf{B}_3 \mathbf{D}_s \end{bmatrix} \mathbf{w}(t) \\ \mathbf{y}(t) &= [\mathbf{C}_s \quad \mathbf{0}] \bar{\mathbf{x}}(t) + \mathbf{D}_s \mathbf{w}(t) \\ \mathbf{z}(t) &= \begin{bmatrix} [\rho_1 \mathbf{I} \quad \mathbf{0}] & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_3 \end{bmatrix} \bar{\mathbf{x}}(t) + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{u}_{1s}(t), \end{aligned} \quad (9.18)$$

where

$$\mathbf{A}_s \equiv \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}, \quad \mathbf{B}_s \equiv \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{G}_s \equiv \begin{bmatrix} \mathbf{0} \\ \mathbf{G} \end{bmatrix}, \quad \mathbf{C}_s \equiv [\mathbf{D} \quad \mathbf{C}], \quad \mathbf{D}_s = \mathbf{I}, \quad (9.19)$$

and where $\bar{\mathbf{x}}(t)$ is the new plant state vector augmented with the integrator and filter states.

Since \mathbf{D}_s is the identity matrix, the above state-space representation is now in the standard form of equations (8.1). Thus the formulae developed in previous sections can be applied. The rescaled control law $\mathbf{u}_{1s}(t)(= \rho \mathbf{u}_1(t))$ is obtained from the appropriate Riccati equations proposed in section III.A and III.B; so will the optimal control law $\mathbf{u}(t)$. Note that when the control input does not contribute to the plant output, i.e., $\mathbf{D} = \mathbf{0}$ in

equation (9.16), augmentation with integrators becomes redundant unless other specific performance requirements (e.g., steady state errors, bandwidth conditions, etc.) are imposed upon the designed compensator. In the H_∞ -optimal problem, if the states in equation (9.18) are not available for feedback, the weighting factor ρ_1 is usually set to zero. This is because in the output feedback case, the optimal observer gain is formulated in a similar manner.

9.3.2 The Tracking Problem

The tracking problem illustrated in Figure 9.2 is formulated in the state-space form for the same system, given by equations (9.16), except that the exogenous input $\mathbf{w}(t)$ now consists of disturbances (and/or noises) $\mathbf{d}(t)$ and the reference input $\mathbf{r}(t)$. The controlled output vector $\mathbf{z}(t)$ and the exogenous input $\mathbf{w}(t)$ are:

$$\mathbf{z}(t) = \begin{bmatrix} \rho \mathbf{u}_1(t) + \rho_1 \mathbf{u}(t) \\ \mathbf{W}_s \mathbf{e}(t) \end{bmatrix}, \quad (9.20)$$

$$\mathbf{w}(t) = \begin{bmatrix} \mathbf{r}(t) \\ \mathbf{d}(t) \end{bmatrix}. \quad (9.21)$$

The tracking error $\mathbf{e}(t)$, ($= \mathbf{r}(t) - \mathbf{y}(t)$), is weighted by a design filter \mathbf{W}_s , similar to that of the regulator problem formulated earlier. The augmented plant in the standard form of equations (8.1) then becomes:

$$\dot{\bar{\mathbf{x}}}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{A} & \mathbf{0} \\ -\mathbf{B}_3 \mathbf{D} & -\mathbf{B}_3 \mathbf{C} & \mathbf{A}_3 \end{bmatrix} \bar{\mathbf{x}}(t) + \begin{bmatrix} 1/\rho \mathbf{I} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{u}_{1s}(t) + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \\ \mathbf{B}_3 & -\mathbf{B}_3 \end{bmatrix} \mathbf{w}(t),$$

$$\bar{\mathbf{y}}(t) = [-\mathbf{D} \quad -\mathbf{C} \quad \mathbf{0}] \bar{\mathbf{x}}(t) + [\mathbf{I} \quad -\mathbf{I}] \mathbf{w}(t), \quad (9.22)$$

$$(9.23)$$

and

$$\mathbf{z}(t) = \begin{bmatrix} [\rho_1 \mathbf{I} \quad \mathbf{0}] & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_3 \end{bmatrix} \bar{\mathbf{x}}(t) + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{u}_{1s}(t); \mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_i(t) \\ \mathbf{x}(t) \\ \mathbf{x}_3(t) \end{bmatrix}$$

where $\bar{\mathbf{x}}(t)$ is the augmented plant state vector, containing the integrator, plant and filter states, $\bar{\mathbf{y}}(t)$ denotes the tracking error $\mathbf{e}(t)$ and where $(\mathbf{A}_3, \mathbf{B}_3, \mathbf{C}_3)$ is the state space realization of \mathbf{W}_s . Now if we define $\mathbf{w}_1(t) = \mathbf{w}^{1/2}(t)$, the above state equations will be in the standard form of equations (8.1), with the following properties: $\mathbf{D}_{12}^T \mathbf{D}_{12} = \mathbf{I}$, $\mathbf{D}_{21} \mathbf{D}_{21}^T = \mathbf{I}$, $\mathbf{B}_1 \mathbf{D}_{21}^T = -\mathbf{G} \neq \mathbf{0}$ and $\mathbf{D}_{12}^T \mathbf{C}_1 = \rho_1 \mathbf{I}$. For this standard problem, the algorithm proposed in the following section can be used. When the command input consists of polynomials, exponentials and trigonometric functions, a suitable servo-compensator $\mathbf{S}_c(s)$ may be added to guarantee perfect asymptotic tracking. Let $\mathbf{S}_c(s)$ be denoted by $(\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c)$ in the state-space form. The plant given by equations (9.16) augmented with a servo-compensator $\mathbf{S}_c(s)$ takes the following form:

$$\begin{aligned} \dot{\mathbf{x}}_c(t) &= \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{B}_c \mathbf{C} & \mathbf{A}_c \end{bmatrix} \mathbf{x}_c(t) + \begin{bmatrix} \mathbf{B} \\ -\mathbf{B}_c \mathbf{D} \end{bmatrix} \mathbf{u}(t) + \begin{bmatrix} \mathbf{0} & \mathbf{G} \\ \mathbf{B}_c & -\mathbf{B}_c \mathbf{E} \end{bmatrix} \mathbf{w}(t) \\ &\equiv \mathbf{A}_a \mathbf{x}_c(t) + \mathbf{B}_a \mathbf{u}(t) + \mathbf{G}_a \mathbf{w}(t), \end{aligned} \quad (9.24)$$

$$\mathbf{y}(t) = [\mathbf{C} \quad \mathbf{0}] \mathbf{x}(t)_c + \mathbf{D} \mathbf{u}(t) + [\mathbf{0} \quad \mathbf{E}] \mathbf{w}(t)$$

$$\equiv \mathbf{C}_a \mathbf{x}(t)_c + \mathbf{D} \mathbf{u}(t) + \mathbf{E}_a \mathbf{w}(t), \quad (9.25)$$

where $\mathbf{w}(t)$ is the same as in equation (9.21). $(\mathbf{A}_a, \mathbf{B}_a, \mathbf{G}_a, \mathbf{C}_a, \mathbf{D}, \mathbf{E}_a)$ can now be viewed as a new plant, such as the one given by equations (9.16), accounting for the tracking requirements of the problem. The remaining task now is to stabilize the augmented system and to minimize $\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty}$. This can be accomplished by formulating the problem as in equations (9.20) through (9.22). For stabilization, it is necessary that the poles of $\mathbf{S}_c(s)$ do not cancel any transmission zeros of the plant.

9.4 A Computer Algorithm for the H_∞ -Optimal Control Problem

In view of the mathematical formulation presented in the previous sections, an algorithm for determining the compensator gains of the H_∞ -optimal control law can now be developed. The objective of the algorithm is to determine the control and observer gain matrices, such that the closed-loop system remains stable and $\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty} \leq \nu$, where, ν , the desired value of the H_∞ -norm is a design specification. It is possible to either minimize ν or set it at a sub-optimal value for satisfaction of additional design constraints, such as controller bandwidth. To comply with the assumptions made in the theoretical developments, it is assumed that the design objective is $\|\frac{1}{\nu} \mathbf{T}_{\mathbf{wz}}\|_{H_\infty} \leq 1$ or equivalently $\|\hat{\mathbf{T}}_{\mathbf{wz}}\|_{H_\infty} \leq 1$, guarantying a unique solution to the H_∞ -optimal control law, provided certain existence conditions are met [2], [3]. This transformation can be accomplished by scaling $\mathbf{w}(t)$ and/or $\mathbf{z}(t)$ such that the H_∞ -norm of $\mathbf{T}_{\mathbf{wz}}$ is always less than or equal to

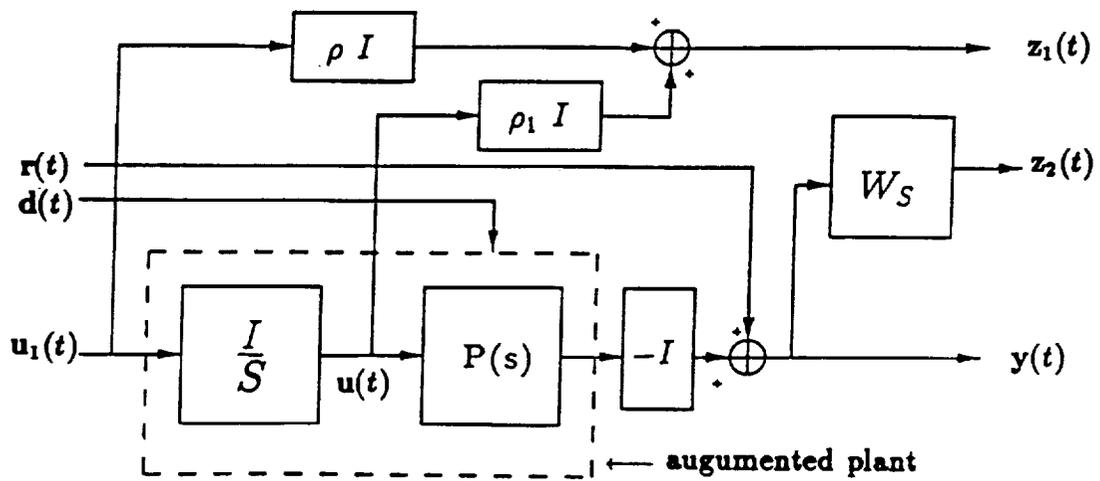


Figure 9.2: Tracking Problem Set-up.

1. This scaling of the disturbance and controlled output terms requires the following transformation of the matrices involved in the open-loop system dynamics of the standard form given by equations (1):

$$\mathbf{B}_1 \rightarrow \nu^{-1/2} \mathbf{B}_1, \mathbf{B}_2 \rightarrow \nu^{1/2} \mathbf{B}_2, \mathbf{C}_1 \rightarrow \nu^{-1/2} \mathbf{C}_1, \mathbf{C}_2 \rightarrow \nu^{1/2} \mathbf{C}_2 \quad (9.26)$$

Let us now make the following definition:

$$\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty} \equiv \delta \quad (9.27)$$

which at this point is an unknown variable to be minimized. Based on the two fundamental results of the previous subsections, and the aforementioned scaling, an H_∞ -optimal control law can be found by implementing the following iterative algorithm:

Step 1: Select the (control) weights ρ, ρ_1 and an initial value for ν . The initial value of ν is set at $\|\mathbf{T}_{\mathbf{wz}}\|_{H_2}$, though alternate choices are acceptable (see reference [4]).

Step 2: Apply the aforementioned scaling and solve the two Riccati equations given by equations (9.2) and (9.11) for the scaled system.

Step 3: Evaluate the gains \mathbf{k}_1 and \mathbf{h}_∞ corresponding to the unscaled system and calculate the $\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty}$, i.e. the value of δ .

Step 4: If $\mathbf{k}_1 \geq \mathbf{0}$ and $\mathbf{h}_\infty \geq \mathbf{0}$, then go to Step 5, else,

if $\delta \geq \nu$, then decrease ρ and/or ρ_1 and repeat Steps 2 through 4,

else,

set $\nu = \delta$ and repeat Steps 2 through 4.

Step 5: If $\lambda_{\max}(\mathbf{k}_1 \mathbf{h}_\infty) < 1$, then go to Step 6, else,

if $\delta \geq \nu$, then decrease ρ and/or ρ_1 and repeat Steps 2 through 4,
 else,
 set $\nu = \delta$ and repeat Steps 2 through 4.

Step 6: If $\delta < \nu$, then set $\nu = \delta$ and proceed to Step 7, else,

if $\rho \leq \epsilon$ and $\rho_1 \leq \epsilon$, ϵ a small number, then set $\nu = \delta$ and go to Step 8,
 else,
 decrease ρ and/or ρ_1 and repeat Steps 2 through 6.

Step 7: If ν is less than the minimum value of ν evaluated so far then repeat Steps 2 through 7 with a smaller value of ν , else, proceed to Step 8.

Step 8: Form the H_∞ -optimal compensator, $\mathbf{K}(s)$, which minimizes the H_∞ -norm of the resulting closed-loop transfer function from $\mathbf{w}(t)$ to $\mathbf{z}(t)$, while keeping the closed-loop system stable. Moreover, $\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty} \approx \nu$.

The above algorithm is organized such that the proposed approach is versatile enough to accommodate design specifications other than the minimization of the H_∞ -norm of $\mathbf{T}_{\mathbf{wz}}$. For instance, Step 7 suggests that an admissible controller may be obtained during the search. As in the LQG/LTR

design method, this controller may also be adjusted to meet certain additional performance specifications, such as bandwidth, while still keeping $\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty}$ lower than what is obtainable from the classical H_2 -optimal approach. It is clear that a controller obtained by stopping at Step 7 is not necessarily the H_∞ -optimal, minimizing $\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty}$. However, for physical problems, a trade off exists between the minimization of $\|\mathbf{T}_{\mathbf{wz}}\|_{H_\infty}$ and the achievable performance.

Chapter 10

Computer Simulation Results

Based on the H_∞ -optimal formulation given in the previous section, an H_∞ -optimal controller is designed for the reduced-order model given by the system of equations (3.9). The desired system bandwidth is approximately $10 \frac{rad}{sec}$, and thus the filter $\mathbf{W}_s(s)$ is chosen as $\mathbf{W}_s(s) = diag(1/(0.1s + 1))$.

Following the regulator formulation and the computer algorithm proposed in previous sections, and defining the controlled output as in equation (9.17), a compensator $\mathbf{K}(s)$ is designed for the flexible space structure under consideration. Following several design iterations, the parameters ρ, ρ_1, ν are set at 0.005, 0., and 0.05, respectively, and the compensator $\mathbf{K}(s)$ is given by:

$$\mathbf{K}(s) = \begin{bmatrix} \mathbf{A}_5 & -\mathbf{H} \\ \mathbf{K} & \mathbf{0} \end{bmatrix}, \quad \mathbf{A}_5 = \mathbf{A} + \mathbf{B}_2\mathbf{K} + \mathbf{H}\mathbf{C}_2 + \mathbf{B}_1\mathbf{B}_1^T\mathbf{k}_1 \quad (10.1)$$

where \mathbf{K} is the controller gain and \mathbf{H} is the observer gain. The numerical values of \mathbf{K} , \mathbf{H} , and \mathbf{A}_5 are given in Table 10.1. The disturbance, $\mathbf{d}(t)$, applied to the plant is a $50 lb_f$ pulse that contributes to the system

a period of 10 seconds. The plant output $\mathbf{y}(t)$, which is subjected to the described disturbance starting at $t = 1$ sec., is shown in Figure 10.1. Simulations, using the reduced-order model, show that after the disappearance of the disturbance, the controlled vertical displacement of the truss is within ± 0.01 inches in approximately 3 seconds, and thereafter it decays to zero. The histories of the control forces, shown in Figure 10.2, indicate that the maximum required force is less than $8 lb_f$, far less than the control power limit of $100 lb_f$.

Examination of the bode plots of the transfer function matrix from the disturbance $\mathbf{d}(t)$ to the controlled output $\mathbf{z}(t)$, \mathbf{T}_{dz} , reveals that the maximum amplification of the \mathbf{T}_{dz} has been suppressed to about -45 dB. Hence, the proposed control law appears to have good disturbance rejection properties. Of course, there is always a trade-off between the minimization of the maximum closed-loop sensitivity and the required control energy. Besides, there may be other performance specifications that must be fulfilled in designing such a controller. For instance, in designing the above controller, the loop-gain cross-over frequency is set at about $10 \frac{rad}{sec}$. The point to be emphasized here is that the approach proposed in this paper is not only used in seeking the controller that minimizes the $\|\mathbf{T}_{dz}\|_{H_\infty}$, but it is also desired to have enough degrees of freedom to satisfy other design specifications, as it is the case with other MIMO control system design methodologies [7].

The dynamic compensator $\mathbf{K}(s)$ is designed using the reduced-order system model given by equation (3.9). As higher order modes will always affect the controller performance, it is desired to investigate the closed-loop

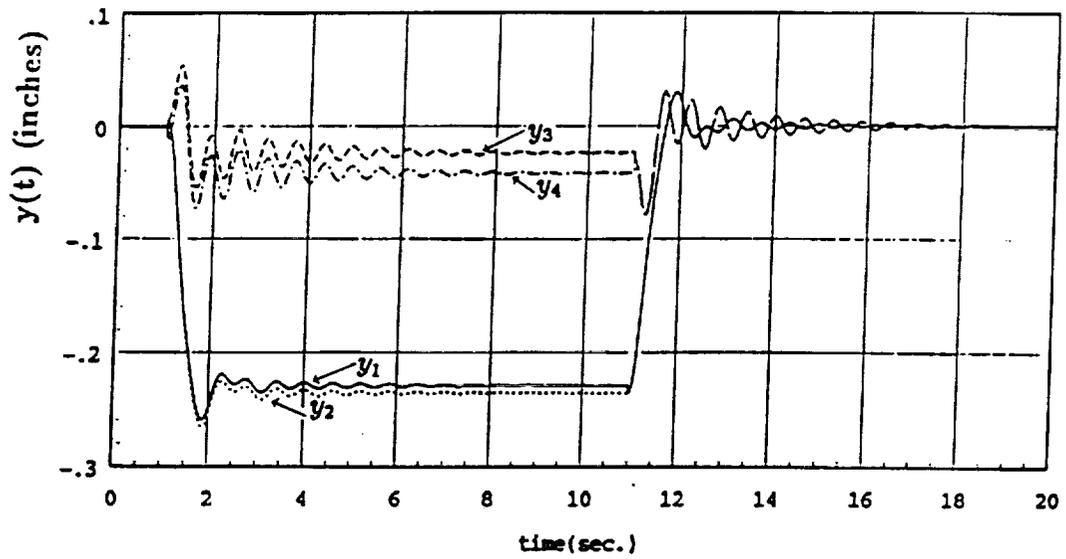


Figure 10.1: Transient Response of the Two-Mode Model to 50 lb_f Pulse Disturbance.

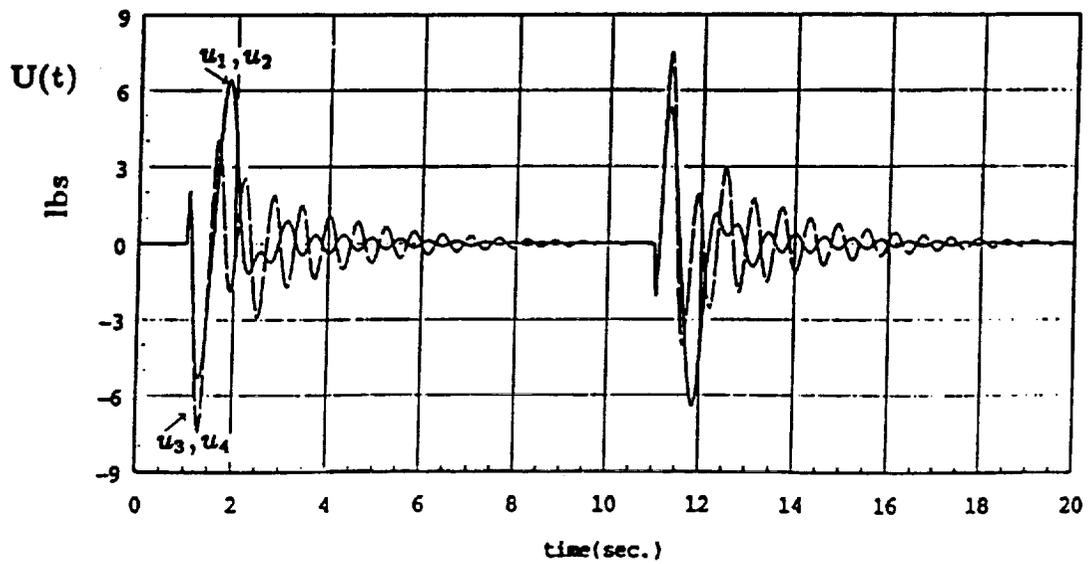


Figure 10.2: History of the Control Forces for the Two-Mode Model.

system response when the controller $\mathbf{K}(s)$ is used in combination with the original eight-mode model given by equation (3.2). Figure 10.3 shows that the truss displacement based on the eight-mode model decays following a similar trajectory as in Figure 10.1, when subjected to the same disturbance $\mathbf{d}(t)$; the control histories for this simulation are shown in Figure 10.4. Because of the presence of the higher order modes, it is not surprising that more chattering occurs in the control force and in the displacement histories shown in the above figures. Nevertheless, the peak value of the truss vertical displacement is about 0.27 inches, while the required maximum control force is approximately 13 lb_f . Examination of the Bode plots of $\mathbf{T}_{\mathbf{d}\mathbf{z}}$ indicates that the H_∞ -norm of the transfer function from $\mathbf{d}(t)$ to $\mathbf{z}(t)$ is still lower than -40 dBs and therefore the closed-loop system still exhibits good disturbance rejection properties.

One of the interesting and important problems that has been investigated only, however, via numerical simulations, is the ability and extent of the compensator $\mathbf{K}(s)$ to tolerate parameter variations of the flexible truss structure. Simulation results show that the closed-loop system remains stable even if the mass of the truss is increased by up to 80%. However, only 1.8% or more reduction on the truss mass results in unacceptable system performance. The transient responses for $\mathbf{y}(t)$ corresponding to +80% and -1.8% mass matrix perturbations, using the same disturbance $\mathbf{d}(t)$ as before, are given in Figures 10.5 and 10.6, respectively. These results imply that effective use of the proposed control law requires a conservative design for the flexible structure, which corresponds to the lightest anticipated structural configuration. Keeping this in mind, one could design a controller

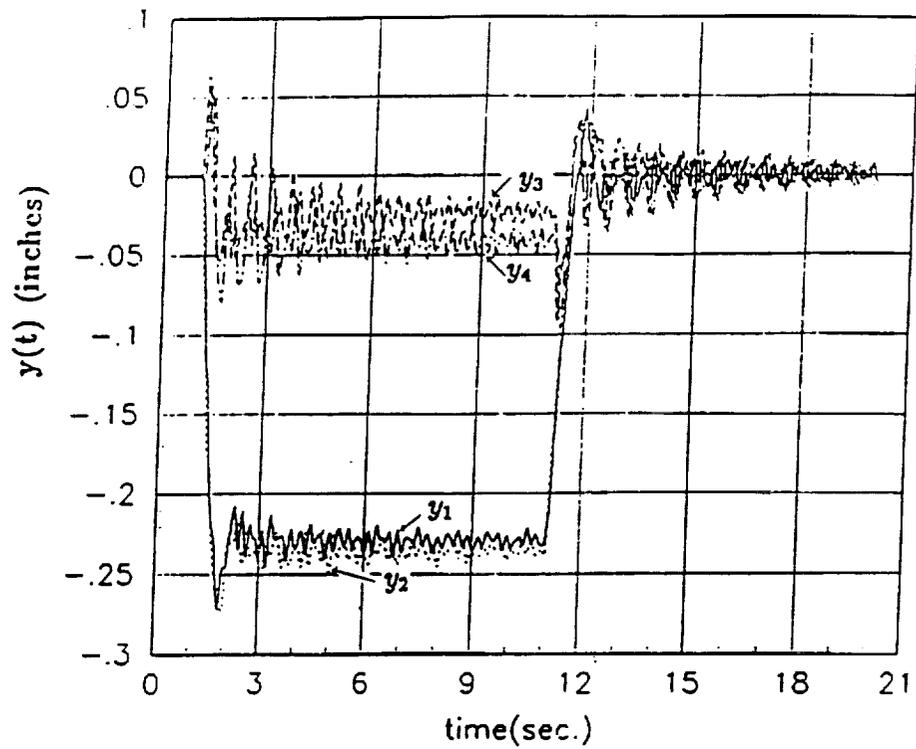


Figure 10.3: Transient Response of the Eight-Mode Model to 50 lb_f Pulse Disturbance.

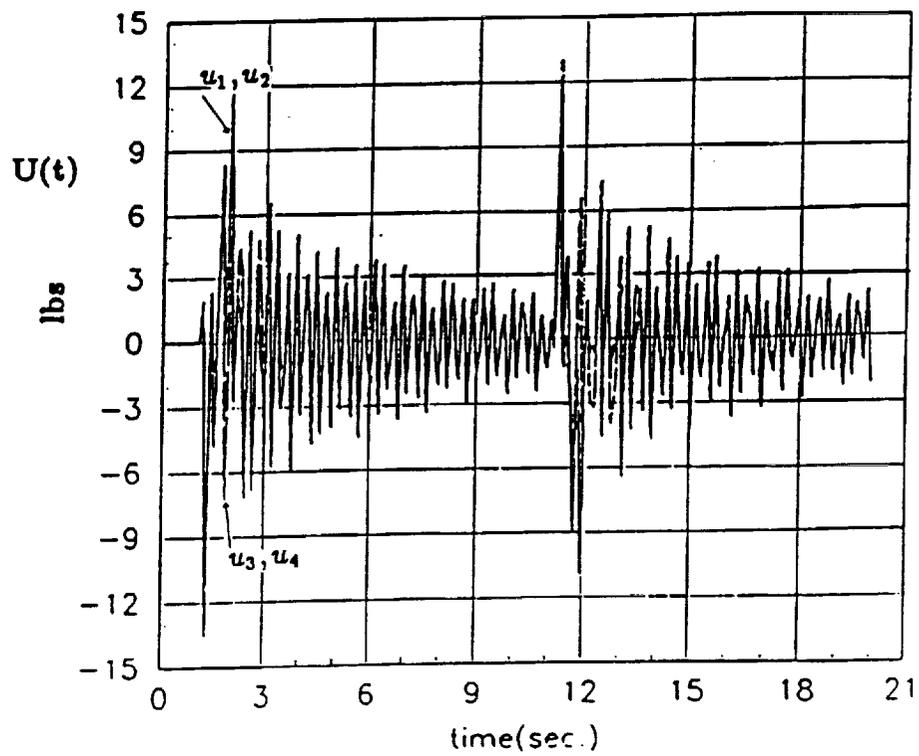


Figure 10.4: History of the Control Forces for the Eight-Mode Model.

using the proposed approach, which exhibits good stability robustness to structural parameter variations.

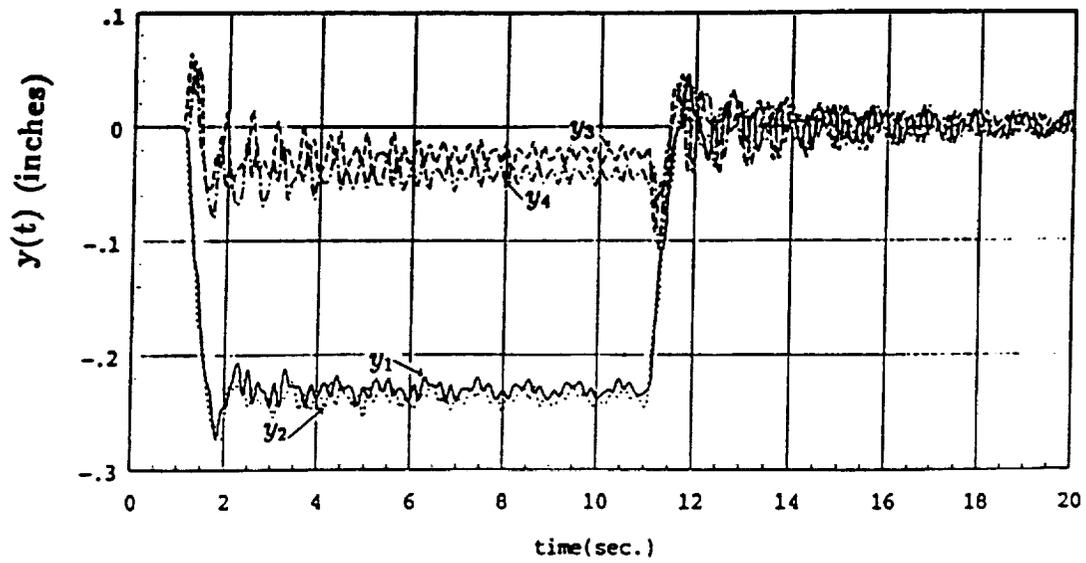


Figure 10.5: Transient Response of the Eight-Mode Model with 80% Mass Increase.



**TABLE 10.1 Numerical Values for the H_∞ -Optimal
Compensator**

A5 (12×12 matrix) =

Columns 1 thru 8

-4.3074	-4.1347	-1.0838	-1.0892	55.3546	-0.8037	74.9634	40.9583
-4.1347	-4.3074	-1.0892	-1.0838	55.3544	-0.8037	74.9632	40.9573
-1.0838	-1.0892	-1.5355	-1.3727	19.0405	3.2464	27.0301	-124.1648
-1.0892	-1.0838	-1.3727	-1.5356	19.0400	3.2464	27.0296	-124.1675
-0.3623	-0.3245	-0.1676	-0.0625	-0.0119	-0.0126	-40.8025	65.8052
0.1080	0.1039	-0.3278	-0.3392	-0.0408	-0.0774	3.2575	-115.0120
-0.0040	0.0035	-0.0103	0.0108	1.0000	0.0000	-6.2144	13.1980
0.0048	-0.0042	0.0122	-0.0128	0.0000	1.0000	7.3464	-15.6022
0.0002	-0.0002	-0.0007	0.0007	-0.0001	0.0000	0.5978	0.5421
-0.0007	0.0007	-0.0007	0.0007	0.0000	0.0000	0.5924	0.5540
0.0001	-0.0001	0.0008	-0.0008	-0.0001	0.0000	0.5955	0.4574
0.0002	-0.0002	0.0000	0.0000	0.0001	-0.0001	0.5807	0.4900

Columns 9 thru 12

20.4618	37.6372	0.3982	0.0118
37.5851	20.5137	-0.1301	0.5403
0.9840	0.4543	21.5394	37.6880
0.4557	0.9821	37.6929	21.5352
0.0223	0.0327	-0.0327	-0.0044

TABLE 10.1 (cont.)

-0.0468	-0.0685	0.0685	0.0091
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
-10.0001	-0.0001	0.0001	0.0000
0.0000	-10.0000	0.0000	0.0000
-0.0001	-0.0001	-9.9999	0.0000
0.0001	0.0002	-0.0002	-10.0000

H (12×4 matrix) =

0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
-47.8556	-0.3994	-39.4312	90.1956
5.1996	0.0434	4.2842	-9.7998
-9.5980	-0.0801	-7.9084	18.0898
11.3464	0.0947	9.3490	-21.3850
-0.6209	-0.0052	-0.5116	1.1702
-0.6296	-0.0053	-0.5187	1.1866
0.3946	0.0033	0.3251	-0.7437
0.3709	0.0031	0.3056	-0.6990

TABLE 10.1 (cont.)

K (4×12 matrix) =

Columns 1 thru 8

-4.3074	-4.1347	-1.0838	-1.0892	55.3546	-0.8037	74.9634	40.9583
-4.1347	-4.3074	-1.0892	-1.0838	55.3544	-0.8037	74.9632	40.9573
-1.0838	-1.0892	-1.5355	-1.3727	19.0405	3.2464	27.0301	-124.1648
-1.0892	-1.0838	-1.3727	-1.5356	19.0400	3.2464	27.0296	-124.1675

Columns 9 thru 12

20.4618	37.6372	0.3982	0.0118
37.5851	20.5137	-0.1301	0.5403
0.9840	0.4543	21.5394	37.6880
0.4557	0.9821	37.6929	21.5352

Chapter 11

Summary and Conclusions

The problem of persistent disturbance rejection is an important aspect of structural control problems. The amplitude of the structural vibrations must be controlled to prevent structural damage and to insure the successful completion of a desired mission. In space structures, structural vibrations usually arise from commanded inputs and/or external disturbances. Moreover, disturbances in large space structures are in general unknown, though bounded. As an H_∞ -optimal control law is designed to reject the worst finite energy disturbances, implementation of this control law guarantees that the maximum amplification between the disturbances and the controlled output is bounded by the design parameter ν .

To facilitate the treatment of MIMO systems, such as flexible space structures, the H_∞ -optimal control problem is formulated without the orthogonality assumptions previously reported in the literature [4]. As demonstrated in previous sections, the H_∞ -optimal approach does not only seek the minimization of $\|\mathbf{T}_{dz}\|_{H_\infty}$, but it can also serve as a useful tool for

satisfaction of other design criteria, such as constraints on the controller cross-over frequency. However, such design specifications can only be met through iterative design. Additionally, the persistent disturbance rejection problem in the presence of unmodeled dynamics and parameter uncertainties can be treated as a multiple objective optimization problem. The objectives to be optimized are the controller robustness and the controller performance. In real world applications a trade-off exists between the two, leading to compromises in control system design. Further studies are necessary to investigate the feasibility of this approach.

It is also worth mentioning that since the structure of the optimal observer used is similar to that of the Model Based Compensator (MBC) used in the LQG/LTR method, matching of the singular values of the open-loop transfer function matrix $\mathbf{G}(s)\mathbf{K}(s)$ at low/high frequencies can be achieved through techniques similar to those used in the LQG/LTR method. This will allow more effective loop shaping when dealing with MIMO systems. In summary, in addition to the robust disturbance rejection properties to finite energy external inputs (e.g., disturbances), the H_∞ -optimal approach developed in this study may also serve as an useful tool in achieving certain demanded compensator performance, which mostly remains to be explored.

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Appendix A

Proof of Theorem 1

Define the objective functional:

$$J = \int_0^{\infty} \|\mathbf{z}(t)\|_2^2 dt . \quad (\text{A.1})$$

Using $\mathbf{D}_{12}^T \mathbf{D}_{12} = \mathbf{I}$ and equations (8.1), the above functional can be rearranged as:

$$J = \int_0^{\infty} \left[\mathbf{x}^T(t) \mathbf{C}_1^T \mathbf{C}_1 \mathbf{x}(t) + \mathbf{x}^T(t) \mathbf{C}_1^T \mathbf{D}_{12} \mathbf{u}(t) + \mathbf{u}^T(t) \mathbf{D}_{12}^T \mathbf{C}_1 \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{u}(t) \right] dt \quad (\text{A.2})$$

subjected to the dynamic constraint

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B}_2 \mathbf{u}(t) + \mathbf{B}_1 \mathbf{w}(t) . \quad (\text{A.3})$$

The constraint can be lifted from the problem by augmenting equations (A.2) and (A.3) via a Lagrange multiplier $\mathbf{G}(t)$ yielding the following form:

$$\begin{aligned} J &= \int_0^{\infty} F(\mathbf{x}(t), \mathbf{u}(t), \mathbf{G}(t)) dt \\ &= \int_0^{\infty} [1/2(\mathbf{x}^T(t) \mathbf{C}_1^T \mathbf{C}_1 \mathbf{x}(t) + \mathbf{x}^T(t) \mathbf{C}_1^T \mathbf{D}_{12} \mathbf{u}(t) + \mathbf{u}^T(t) \mathbf{D}_{12}^T \mathbf{C}_1 \mathbf{x}(t) + \\ &\quad \mathbf{u}^T(t) \mathbf{u}(t)) + (\mathbf{x}^T(t) \mathbf{A}^T + \mathbf{u}^T(t) \mathbf{B}_2^T + \mathbf{w}^T(t) \mathbf{B}_1^T - \dot{\mathbf{x}}(t)) \mathbf{G}(t)] dt \end{aligned} \quad (\text{A.4})$$

Employing the standard necessary condition for the functional J to be a minimum, the following conditions are obtained:

$$\dot{\mathbf{G}}(t) = -\mathbf{C}_1^T \mathbf{C}_1 \mathbf{x}(t) - \mathbf{C}_1^T \mathbf{D}_{12} \mathbf{u}(t) - \mathbf{A}^T \mathbf{G}(t) \quad (\text{A.5})$$

$$\mathbf{u}(t) = -\mathbf{B}_2^T \mathbf{G}(t) - \mathbf{D}_{12}^T \mathbf{C}_1 \mathbf{x}(t) \quad (\text{A.6})$$

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B}_2 \mathbf{u}(t) + \mathbf{B}_1 \mathbf{w}(t) . \quad (\text{A.7})$$

Equations (A.5) through (A.7) can be combined, yielding

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{B}_2 \mathbf{D}_{12}^T \mathbf{C}_1) \mathbf{x}(t) - \mathbf{B}_2 \mathbf{B}_2^T \mathbf{G}(t) + \mathbf{B}_1 \mathbf{w}(t) \quad (\text{A.8})$$

$$\begin{aligned} \dot{\mathbf{G}}(t) &= -\mathbf{C}_1^T (\mathbf{I} - \mathbf{D}_{12} \mathbf{D}_{12}^T) \mathbf{C}_1 \mathbf{x}(t) - (\mathbf{A}^T - \mathbf{C}_1^T \mathbf{D}_{12} \mathbf{B}_2^T) \mathbf{G}(t) \\ &= -\mathbf{C}_1^T (\mathbf{I} - 2\mathbf{D}_{12} \mathbf{D}_{12}^T + \mathbf{D}_{12} \mathbf{D}_{12}^T) \mathbf{C}_1 \mathbf{x}(t) - (\mathbf{A}^T - \mathbf{C}_1^T \mathbf{D}_{12} \mathbf{B}_2^T) \mathbf{G}(t) \\ &= -\mathbf{C}_1^T (\mathbf{I} - \mathbf{D}_{12} \mathbf{D}_{12}^T) (\mathbf{I} - \mathbf{D}_{12} \mathbf{D}_{12}^T) \mathbf{C}_1 \mathbf{x}(t) - \\ &\quad (\mathbf{A}^T - \mathbf{C}_1^T \mathbf{D}_{12} \mathbf{B}_2^T) \mathbf{G}(t) . \end{aligned} \quad (\text{A.9})$$

Remark: When the exogenous input $\mathbf{w}(t)$ (e.g., the disturbance) is a white Gaussian noise, then the optimal control problem becomes the standard H_2 -optimal (LQG) problem with the control law given by:

$$\mathbf{u}(t) = \mathbf{G}_{H_2} \mathbf{x}(t) \quad (\text{A.10})$$

where

$$\mathbf{G}_{H_2} = -(\mathbf{B}_2^T \mathbf{k} + \mathbf{D}_{12}^T \mathbf{C}_1) . \quad (\text{A.11})$$

In equation (A.11), \mathbf{k} is the solution of the following ARE:

$$\mathbf{A}_i^T \mathbf{k} + \mathbf{k} \mathbf{A}_i - \mathbf{k} \mathbf{B}_2 \mathbf{B}_2^T \mathbf{k} + \mathbf{C}_i^T \mathbf{C}_i = \mathbf{0} , \quad (\text{A.12})$$

where

$$\mathbf{A}_t = \mathbf{A} - \mathbf{B}_2 \mathbf{D}_{12}^T \mathbf{C}_1 \quad (\text{A.13})$$

$$\mathbf{C}_t = (\mathbf{I} - \mathbf{D}_{12} \mathbf{D}_{12}^T) \mathbf{C}_1 . \quad (\text{A.14})$$

If the exogenous input is unknown but is in a subset of the finite energy signal space, then the worst exogenous input $\mathbf{w}(t)$ can be shown to be [6]

$$\mathbf{w}(t) = \mathbf{B}_1^T \mathbf{G}(t) . \quad (\text{A.15})$$

Substituting equation (A.15) into equation (A.8), together with equation (A.9), and using arguments widely common in the optimal control literature, it can be concluded that the H_∞ -optimal state feedback control law is

$$\mathbf{u}(t) = \mathbf{K} \mathbf{x}(t) , \quad (\text{A.16})$$

where

$$\mathbf{K} = -(\mathbf{B}_2^T \mathbf{k}_1 + \mathbf{D}_{12}^T \mathbf{C}_1) \quad (\text{A.17})$$

In equation (A.17), \mathbf{k}_1 is the positive definite solution of the following ARE:

$$\mathbf{A}_t^T \mathbf{k}_1 + \mathbf{k}_1 \mathbf{A}_t + \mathbf{k}_1 (\mathbf{B}_1 \mathbf{B}_1^T - \mathbf{B}_2 \mathbf{B}_2^T) \mathbf{k}_1 + \mathbf{C}_t^T \mathbf{C}_t = \mathbf{0} , \quad (\text{A.18})$$

where \mathbf{A}_t and \mathbf{C}_t are given by equations (A.13) and (A.14). Further details of this proof can be found in reference [6].



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