92

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ANALYTICAL STUDY OF THE EFFECTS OF CLOUDS ON THE LIGHT PRODUCED BY LIGHTNING

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I. INTRODUCTION

We consider the scattering of light (visible or infrared) due to lightning by cubic, cylindrical, and spherical clouds. A typical cloud is represented by a statistically homogeneous ensemble of configurations of N identical and aligned spherical water droplets whose centers are uniformly distributed in its volume. The incident light is from point sources inside the penetrable cloud.

The optical effects of clouds on the light produced by lightning have received great interest for many years. Different techniques have been used in trying to explain the complicated nature of these effects. In particular, we mention the Monte Carlo method [9] which is a computer simulated technique. In a Monte Carlo program, we follow the path of the photons emitted into the cloud by lightning. A photon is said to be scattered if it escapes from the cloud after colliding with the spherical droplets. Otherwise, it is considered as being absorbed by the cloud [1 to 8]. The Monte Carlo method is time consuming, expensive and it is very difficult to obtain from it reliable statistics [9].

Here, we extend to cloud physics the work done by Twersky [10 to 12] for single and multiple scattering of electromagnetic waves. We solve the interior problem separately to obtain the bulk parameters for the scatterer equivalent to the ensemble of spherical droplets. With the interior solution or the equivalent medium approach, the multiple scattering problem is reduced to that of a single scatterer in isolation. Hence, the computing methods of Wiscombe [13] or Bohren [15] specialized to Mie scattering with possibility for absorption have been used to generate numerical results in short computer time.

II. MATHEMATICAL ANALYSIS

We model the incident point source as $\vec{\phi} = \hat{\mathbf{a}}\frac{e^{i\kappa r}}{r}, \kappa_1 = k\eta'$, and η' being the complex relative index of refraction for the host medium inside the cloud, the total outside solution

$$\vec{\psi} = \vec{\phi} + \mathbf{u}_o \tag{1}$$

satisfied the following differential equation obtained from Maxwell's equations after suppressing the harmonic time dependence

$$\left[\vec{\nabla} \times \vec{\nabla} \times +\kappa_1^2\right]\vec{\psi} = 0, \vec{\nabla} \cdot \vec{\psi} = 0. \tag{2}$$

Here,

$$\kappa_2 = \kappa_1 \eta'' = k\eta'\eta'', \tag{3}$$

with η'' being the complex relative index of refraction for the medium inside the spherical water droplet.

Similar to Twersky [11], we have from (1)

$$\vec{\psi} = \hat{\mathbf{a}}\frac{e^{i\kappa r}}{r} + \left\{\tilde{h}\left(\kappa_1|\mathbf{r}-\mathbf{r}'|\right), \mathbf{u}_0\left(\mathbf{r}'\right)\right\}. \tag{4}$$

Asymptotically, for $\kappa_1 r >> 1$, we can write

$$\mathbf{u}_0(\mathbf{r}) = h(\kappa_1 r)\mathbf{g}(\hat{\mathbf{r}}, \hat{\kappa}_1 : \hat{\mathbf{a}}), \hat{\mathbf{r}} \cdot \mathbf{g} = 0, \tag{5}$$

and the scattering amplitude

$$\mathbf{g}(\hat{\mathbf{r}}, \hat{\kappa}_1 : \hat{\mathbf{a}}) = \left\{\tilde{\mathbf{I}}_t e^{-i\vec{\kappa}_1 \cdot \mathbf{r}'}, \mathbf{u}_0\left(\mathbf{r}'\right)\right\} \tag{6}$$

are evaluated from Mie scattering theory.

From the general reciprocity relation

$$\left\{\mathbf{\Psi}, \vec{\psi}_{\mathbf{a}}\right\}_t = 0 \tag{7}$$

for any arbitrary direction of incidence, we derive as in [11] the self-consistent integral equation for the multiple configurational scattering amplitude

$$\mathbf{G}_t(\hat{\mathbf{r}}) = \tilde{\mathbf{g}}_t(\hat{\mathbf{r}}, \hat{\kappa}_1) \cdot \hat{\mathbf{a}} + {\sum_m}' \int_c \tilde{\mathbf{g}}_t(\hat{\mathbf{r}}, \hat{\mathbf{r}}_c) \cdot \mathbf{G}_m(\hat{\mathbf{r}}_c) e^{i\vec{\kappa}_{1c} \cdot \mathbf{R}_{tm}}. \tag{8}$$

We take the average of (8) over a statistically homogeneous ensemble of configurations to obtain [11] the dispersion relation determining the coherent parameters

$$\begin{aligned} \mathcal{G}\left(\vec{\kappa}_1|\vec{\mathrm{K}}\right) &= -\frac{\rho}{c_o(\mathrm{K}^2 - \kappa_1^2)}\left\{\left[e^{-i\vec{\mathrm{K}}\cdot\mathbf{R}}, \mathsf{U}\right]\right\} \\ &+\rho \int_{V_\infty - v} [f(\mathbf{R}) - 1] e^{-i\vec{\mathrm{K}}\cdot\mathbf{R}} \mathsf{U} d(\mathbf{R}). \end{aligned} \tag{9}$$

Equation (9) solves formally the interior problem for the cloud. To obtain numerical results, one can apply stationary phase method [12] on (8) and reduce (9) to

$$K - \kappa_1 \sim -\frac{ig\sigma_o}{2\eta}\mathfrak{L}^{-1}$$

and

$$\mathfrak{L} = \left\{ 1 - \rho\frac{g\sigma_o}{2\eta}\int_0^\infty [f(\mathbf{R}) - 1]e^{i(\kappa_1 - K\ R)}d(\mathrm{R}) \right\}. \qquad (10)$$

From equation (10), the leading term approximation gives

$$\begin{aligned} \eta^2 &= \epsilon \\ \epsilon &= 1 + \frac{3\mathfrak{F}}{1-\mathfrak{F}}, \\ \mathfrak{F} &= \omega_o\left(\frac{\epsilon' - 1}{\epsilon' - 2}\right), \\ \omega_o &= \frac{\omega}{1+\omega}. \end{aligned} \qquad (11)$$

For

$$\epsilon' = (\eta_r' + \eta_i')^2, \qquad (12)$$

and the bulk index of refraction is

$$\begin{aligned} \eta^2 &= \epsilon, \\ \begin{matrix}\eta_r \\ \eta_i\end{matrix} &= \left\{ \frac{\epsilon_r}{2}\left[\left(1 + \frac{\epsilon_i^2}{\epsilon_r^2}\right)^{\frac{1}{2}}\right] \pm 1 \right\}^{\frac{1}{2}}. \end{aligned} \qquad (13)$$

The bulk parameters reduce the multiple scattering to a problem of a single equivalent scatterer. (See tables for numerical results).

III. CONCLUSION

Due to the complexity of the problem, only results for the leading term approximation are given here. The multiple scattering problem has been reduced to that of a single scatterer in isolation. Depending on the size parameter of the cloud particles as compared to the wave length of the incident light, either Rayleigh or Mie scattering technique can be used to determine Qext, Qscat, and Qbacs.

With the bulk parameters, we can use Wiscombe's computer code to obtain in short computer time, acceptable numerical results for a medium with a complex relative index of refraction which is an improvement of Bohren [15]. The equivalent medium approach gives naturally the polarizations and the angular distributions of photons which escape the cloud surface.

IV. REFERENCES

1. Aida, M., 1977: Scattering of solar radiation as a function of cloud dimensions and orientation. J. Quant. Spectrosc. Radiat. Transfer, 17, 303–310.

2. Bucher, E. A., 1973: Computer simulation of light pulse propagation for communication through thick clouds. Appl. Opt., 12, 2391–2400.

3. Danielson, R. E., D. R. Moore and H. C. van de Hulst, 1969: The transfer of visible radiation through clouds. J. Atmos. Sci., 26, 10778–1087.

4. Davies, R., 1978: The effect of finite geometry on the three dimensional transfer of solar irradiance in clouds. J. Atmos. Sci., 35, 1712–1725.

5. Davis, J. M., S. K. Cox and T. B. McKee, 1979: Vertical and horizontal distributions of solar absorption in finite clouds. J. Atmos. Sci., 36, 1976–1984.

6. McKee, T. B., and S. K. Cox, 1974: Scattering of visible radiation by finite clouds. J. Atmos. Sci., 31, 1885–1892.

7. Plass, G. N., and G. W. Kattawar, 1968: Monte Carlo calculations of light scattering from clouds. Appl. Opt., 1, 126–130.

8. Van Blerkom, D., 1969: Diffuse reflection from clouds with horizontal inhomogeneities. Astrophys. J., 166, 235–242.

9. Thomason, L. W., and E. P. Krider, 1982: The effects of clouds on the light produced by lightning. J. Atmos. Sci., 39, 2051–2065.

10. Twersky, V., 1977: Coherent scalar field in pair-correlated random distributions of aligned scatterers. J. Math. Phys., 18, 2468–2486.

11. Twersky, V., 1978: Coherent electromagnetic waves in pair-correlated random distributions of aligned scatterers. J. Math. Phys., 19, 215–230.

12. Twersky, V., 1983: Propagation in correlated distributions of large-spaced scatterers. J. Opt. Soc. Am., 73, 313–320.

13. Wiscombe, W. J., 1979. Mie scattering calculations: Advances in technique and fast, vector-speed computer codes, NCAR/TN-140 +STR, National Center for Atmospheric Research, Boulder, Colo.

14. Twersky, V., 1983: Wavelength-dependent refractive and absorptive terms for propagation in small-spaced correlated distributions. J. Opt. Soc. Am., 73, 1562–1567.

15. Twomey, S., and C. F. Bohren, 1980: Simple approximations for calculations of absorption in clouds. J. Atmos. Sci., 37, 2086–2094

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SPHERE SCATTERING PROGRAM

      REFMED=  0.1000E+01   REFRE=  0.500000E+00   REFIM=  0.000000E+00
      SPHERE RADIUS = 15.000   WAVELENGTH = 0.4880
      SIZE PARAMETER=  0.1931E+03


QSCA=  0.205700E+01   QEXT=  0.205700E+01   QBACK=  0.233708E+00

 ANGLE         S11            POL            S33            S34

  0.00    0.100000E+01  -0.542434E-13   0.100000E+01   0.355165E-14

  9.00    0.101441E-03  -0.240238E+00   0.958169E+00   0.155556E+00

 18.00    0.809887E-05  -0.679672E+00  -0.337390E-01  -0.732740E+00

 27.00    0.151475E-04   0.524835E+00   0.821119E+00   0.224302E+00

 36.00    0.468224E-04  -0.431056E+00   0.807469E+00  -0.402721E+00

 45.00    0.459087E-04  -0.490314E+00   0.814878E+00  -0.309139E+00

 54.00    0.506346E-05   0.860585E-01   0.181473E+00  -0.979623E+00

 63.00    0.334143E-04  -0.523285E+00   0.311479E+00  -0.793192E+00

 72.00    0.172027E-04  -0.359514E+00   0.371604E+00  -0.855956E+00

 81.00    0.258653E-04   0.271532E+00   0.770260E+00  -0.577036E+00

 90.00    0.165587E-04   0.313426E-01   0.366425E-01  -0.998837E+00

 99.00    0.171157E-04   0.358241E+00  -0.168439E-01  -0.933477E+00

108.00    0.378746E-04   0.464737E-01   0.602422E+00  -0.796824E+00

117.00    0.112762E-04   0.220812E+00   0.704808E+00  -0.674157E+00

126.00    0.355094E-05   0.933251E+00   0.354882E+00  -0.556950E-01

135.00    0.366420E-05   0.734118E+00  -0.677763E+00  -0.413374E-01

144.00    0.132568E-05   0.468369E+00  -0.751838E+00  -0.464080E+00

153.00    0.318972E-05   0.106089E+00  -0.724534E+00  -0.681026E+00

162.00    0.127502E-05  -0.103156E+00  -0.968058E+00  -0.228521E+00

171.00    0.315999E-05   0.264331E+00  -0.962842E+00  -0.553618E-01

180.00    0.590261E-05  -0.236712E-10  -0.100000E+01   0.313251E-10
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        SPHERE RADIUS = 0.480   WAVELENGTH = 0.8600
        SIZE PARAMETER=  0.3507E+01

   QSCA=  0.227020E+01   QEXT=  0.227020E+01   QBACK=  0.395052E+00

    ANGLE        S11            POL            S33            S34

     0.00   0.100000E+01   0.000000E+00   0.100000E+01   0.000000E+00

     9.00   0.922420E+00   0.563351E-02   0.999969E+00   0.550071E-02

    18.00   0.721860E+00   0.217153E-01   0.999490E+00   0.23414E-01

    27.00   0.474803E+00   0.448205E-01   0.997282E+00   0.584701E-01

    36.00   0.257936E+00   0.645601E-01   0.990661E+00   0.120096E+00

    45.00   0.112784E+00   0.485908E-01   0.973895E+00   0.221737E+00

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    63.00   0.135274E-01  -0.498173E+00   0.849299E+00   0.174686E+00

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    81.00   0.945430E-02   0.197477E+00   0.763149E+00  -0.615421E+00

    90.00   0.897089E-02   0.567128E+00   0.716241E+00  -0.406650E+00

    99.00   0.734485E-02   0.710871E+00   0.699117E+00  -0.767998E-01

   108.00   0.561621E-02   0.574907E+00   0.782048E+00   0.240588+00

   117.00   0.432635E-02   0.147739E+00   0.941330E+00   0.303432E+00

   126.00   0.360259E-02  -0.404131E+00   0.912331E+00  -0.658133E-01

   135.00   0.362801E-02  -0.715242E+00   0.365618E+00  -0.595611E+00

   144.00   0.468960E-02  -0.586591E+00  -0.393778E+00  -0.707707E+00

   153.00   0.682983E-02  -0.314769E+00  -0.829044E+00  -0.462175E+00

   162.00   0.953647E-02  -0.124406E+00  -0.970944E+00  -0.204428E+00

   171.00   0.118259E-01  -0.284236E-01  -0.998368E+00  -0.495403E-01

   180.00   0.127243E-01  -0.877489E-14  -0.100000E+01  -0.102374E-13
```

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  QSCA=  0.226582E+03   QEXT=  0.441131E+01   QBACK=  0.647478E+03

   ANGLE        S11            POL            S33             S34

    0.00    0.100000E+01  -0.203204E-11   0.100000E+01  -0.425569E-12

    9.00    0.310327E-01  -0.998188E+00  -0.531018E-01  -0.283038E-01

   18.00    0.716488E-02  -0.981093E+00   0.172648E+00  -0.874618E-01

   27.00    0.212244E-02  -0.979733E+00   0.173917E+00   0.993747E-01

   36.00    0.142130E-02  -0.956971E+00   0.189002E+00  -0.220192E+00

   45.00    0.205954E-03  -0.933447E+00  -0.251622E+00   0.255661E+00

   54.00    0.433484E-03  -0.775018E+00   0.601301E+00   0.194380E+00

   63.00    0.114486E-03   0.532359E+00   0.662261E+00  -0.527262E+00

   72.00    0.413065E-04   0.700565E-01  -0.642471E+00  -0.763101E+00

   81.00    0.142686E-03   0.858127E+00   0.958247E-02   0.513349E+00

   90.00    0.241022E-03  -0.241273E+00   0.391307E+00  -0.888069E+00

   99.00    0.119656E-03   0.655108E+00   0.702851E+00  -0.277189E+00

  108.00    0.130419E-02  -0.769865E+00  -0.404472E+00  -0.493669E+00

  117.00    0.463362E-02  -0.938115E+00  -0.178932E-02   0.346320E+00

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  135.00    0.147604E-02  -0.730593E+00   0.300318E+00   0.613224E+00

  144.00    0.102740E-02  -0.433089E+00  -0.693427E+00   0.575841E+00

  153.00    0.723232E-03  -0.170305E+00  -0.525661E+00  -0.833473E+00

  162.00    0.346430E-03   0.373583E+00  -0.920732E+00  -0.112645E+00

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