N91-19307

Task Title	**Analytical Solutions with Generalized Impedance Boundary Conditions**
Investigators	**H.H. Syed and J.L. Volakis**
Period Covered	**Sept. 1990 - Feb. 1991**

ABSTRACT

Rigorous UTD (uniform Geometrical Theory of Diffraction) diffraction coefficients are presented for a coated convex cylinder simulated with generalized impedance boundary conditions. In particular, ray solutions are obtained which remain valid in the transition region and reduce uniformly to those in the deep lit and shadow regions. These involve new transition functions in place of the usual Fock-type integrals, characteristic to the impedance cylinder. A uniform asymptotic solution is also presented for observations in the close vicinity of the cylinder. As usual, the diffraction coefficients for the convex cylinder are obtained via a generalization of the corresponding ones for the circular cylinder.

OBJECTIVE

This task involves the use of higher order boundary conditions to generate new solutions in diffraction theory. In particular, diffraction coefficients will be developed for dielectric/magnetic layers and metal-dielectric junctions which are often encountered on airborne vehicles as terminations of coatings and conformal antennas. Solutions for both polarizations will be developed for fairly thick junctions and versatile computer codes will be written and tested. Creeping wave diffraction coefficients will be also developed for multilayered coated cylinders.

PROGRESS

1. Introduction

The problem of scattering by a smooth convex impedance cylinder has received much attention. Wang [1, 2] presented ray-optical solutions for the impedance and coated cylinders. His results are valid only in the deep lit and shadow regions and do not apply to the case where the observation point is in the transition region. Wait and Conda [3, 4] developed a solution which is valid in the transition region and for observation points on and off the surface. However, as pointed out by Pathak [5] it did not uniformly reduce to the ray solution [6, 7] exterior to the transition regions. Also, it is not valid on the portion of the surface in the transition region and these limitations were the primary motivation in Pathak's work [5] for the perfectly conducting convex cylinder. Recently, Kim and Wang [8] presented a solution applicable to a coated cylinder that remained valid in the transition region. They employed a heuristic approach to obtain the numerical values of the resulting transition integral applicable to a coated cylinder. Their solution is uniform but is not applicable to the close vicinity of the cylinder.

Here we develop a rigorous UTD solution of the diffraction by a coated cylinder simulated with generalized impedance boundary conditions. In addition, a uniform asymptotic solution is obtained which remains valid when the observation point is in close vicinity of the cylinder. An important aspect of the paper is also the use of second order generalized impedance boundary conditions (GIBC) for the simulation of the coating. Their derivation has already been given in [9] and [10] and are characterized by the inclusion of higher order field derivatives in their definition. Because of this they are less local which leads to an improved simulation (with respect to the standard impedance boundary condition - SIBC) of the coating in a manner analogous to the order of the highest derivative kept in the condition. Recently, they were successfully applied to a number of diffraction problems [11], [12] and have also been used in numerical simulations of multilayer coatings (see fig. 1) [13]. These applications provided a measure of the accuracy of the proposed GIBC and in particular accuracy criteria were derived in [13] for the second order conditions as a function of coating thickness and composition.

The UTD solution to be presented here parallels that given by Pathak [5] for the circular perfectly conducting cylinder. However, in the case of the coated cylinder the resulting UTD expressions are in terms of Fock-type integrals whose efficient evaluation is of primary interest. In the following we first present the eigenfunction solution based on the second order GIBC simulation of a circular coated cylinder. By employing Watson's transformation this is written in integral form which is then cast in a ray representation. They ray solution is subsequently generalized to the case of a general convex cylinder. Finally, the evaluation of the Fock-type integrals is discussed and some results are presented which validate the accuracy of the GIBC eigenfunction and ray solutions. In the process, we demonstrate the improved accuracy of the GIBC solution over the corresponding SIBC solution, and it is also shown how the presented UTD solution can be extended to treat multilayered coated cylinders.

The details of the analysis are described in the report 025921-13-T which was recently submitted to the sponsor. Below we only attach a few results which demonstrate the accuracy and utility of the derived formulae.

2. Numerical Results

The UTD expressions derived in the UM report 025921-17-T provide a complete set of equations for the computation of the total field in all regions of interest. Below, we present some calculated data which validate the accuracy of the derived expressions by comparison with data based on the moment method and eigenfunction solutions.

In figure 2 the eigenfunction solutions based on the GIBC and SIBC simulations are compared with the exact for a coated cylinder and this clearly demonstrates the improved simulation (with respect to the standard impedance boundary conditions - SIBC) achieved with the second order GIBC. To show the validity of the UTD solution in the case of the convex cylinder, a special case of an elliptical cylinder is considered in figure 3. Data based on the moment method are compared with those obtained from the UTD solution in conjunction with the second order low and high contrast boundary conditions.

Figure 4 verifies the asymptotic solution developed for the field point in the close vicinity of a convex cylinder. We remark, however, that the approximations used for the Hankel functions in the derivation of (42) and (43) become less accurate for some values of ε_r and μ_r associated with lossless coatings, and this can be avoided by using more accurate approximations for the Hankel functions. Finally, figure 5 demonstrates the use of GIBC in simulating multilayer coatings by simply redefining the material constants a_m and a_m' as discussed in [10, 13].

A difficulty in implementing the expressions derived in this paper was the evaluation of the Fock-type integrals $G(x,q)$, $g_1(D)$ and $g_2(D)$ as well as determination of the zeros corresponding to (21). The Fock-type integrals were evaluated by employing the method described in [16] and the zeros of (21) were determined using the routine given in [20].

Summary

Rigorous ray solutions of the scattered fields were presented for a coated convex cylinder. These were developed in the context of the uniform geometrical theory of diffraction and specific expressions were given for the scattered fields in the lit, shadow and transition regions as well as for observations in the near vicinity of the cylinder. That is, UTD expressions were derived for all regions exterior to the coated cylinder. These are suited for engineering computations and are given in terms of the generalized Pekeris or Fock-type functions whose evaluation was efficiently performed via the Fourier Trapezoidal rule suggested by Pearson [16].

In comparison to the solution given by Kim and Wang [8], the ray representations given here are based on a second order generalized impedance boundary condition which permits the simulation of thin multilayered coating as demonstrated in the included examples. Also, in our implementation of the transition fields we employed a rigorous rather than a heuristic evaluation of the Fock-type integrals. Further, we have presented accurate field representations for observations on or near the vicinity of the coated cylinder and these can also be used for computing the radiated fields by a source or an aperture on the surface of the convex cylinder.

REFERENCES

[1] N. Wang, "Regge poles, natural frequencies, and surface wave resonance of a circular cylinder with a constant surface impedance," *IEEE Trans. Antenna & Propagat.,* Vol. AP-30, Nov. 1982.

[2] ----, "Electromagnetic scattering from a dielectric coated circular cylinder," *IEEE Trans. Antenna & Propagat.,* Vol. AP-33, Sept. 1985.

[3] J.R. Wait and A.M. Conda, "Diffraction of electromagnetic waves by smooth obstacles for grazing angles," *J. Res. Bur. Stand.,* Vol. 63D, pp. 181-197, 1959.

[4] ----, "Pattern of an antenna on a curved lossy surface," *IRE Trans. Antennas & Propagat.,* Vol. AP-6, pp. 348-359, Oct. 1958.

[5] P.H. Pathak, "An asymptotic analysis of the scattering of plane waves by a smooth convex cylinder," *Radio Sci.,* Vol. 14, pp. 419-435, 1979.

[6] J.B. Keller, "Diffraction by a convex cylinder," *IEEE Trans. Antenna & Propagat.,* Vol. AP-24, pp. 312-321, 1956.

[7] ----, "Geometrical theory of diffraction," *J. Opt. Soc. Amer.,* Vol. 52, pp. 116-130, 1962.

[8] H. Kim and N. Wang, "UTD solution for electromagnetic scattering by a circular cylinder with thin lossy coatings," *IEEE Trans. Antenna & Propagat.,* Vol. AP-37, pp. 1463-1472, Nov. 1989.

[9] T.B.A. Senior and J.L. Volakis, "Derivation and application of a class of generalized boundary conditions," *IEEE Trans. Antenna & Propagat.,* Vol. AP-37, pp. 1566-1572, Dec. 1989.

[10] M.A. Ricoy and J.L. Volakis, "Derivation of generalized transition/boundary conditions for planar multiple layer structures," *Radio Sci.,* Vol. 25, pp. 391-405, July - August 1990.

[11] ----, "Diffraction by a symmetric material junction, Part I: general solution," submitted to *IEEE Trans. Antenna & Propagat.*

[12] ----, "Diffraction by a symmetric material junction, Part II: boundary conditions," submitted to *IEEE Trans. Antenna & Propagat.*

[13] J.L. Volakis and H.H. Syed, "Application of higher order boundary conditions to scattering by multilayered coated cylinders," *J. of Electromagnetic Waves and Applications.* (in press)

[14] J.J. Bowman, T.B.A. senior and P.L.E. Uslenghi, "Electromagnetic and Acoustic Scattering by Simple Shapes," Amsterdam: North-Holland, 1969.

[15] N.A. Logan and K.S. Yee, "Electromagnetic Waves," edited by R.E.Langer, University of Wisconsin Press, Madison.

[16] L.W. Pearson, "A scheme for automatic computation of Fock-type integrals," *IEEE Trans. Antenna & Propagat.,* Vol. AP-35, pp. 1111-1118, Oct. 1987.

[17] O.A. Tuck, "A simple 'Finlon-trapezoid' rule," *Math. Comp.,* Vol. 21, pp. 239-241, 1967.

[18] L. Felson and N. Marcuvitz, "Radiation and Scattering of Waves," Prentice Hall, 1973.

[19] R.G. Kouyoumjian and P.H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. IEEE,* Vol. 62, pp. 1448-1461, 1974.

[20] B.K. Singaraju, D.V. Giri and C.E. Baum, "Further development in the application of contour integration to the evaluation of the zero of analytical functions and relevent computer programs," Math. Note 24, March 1976.

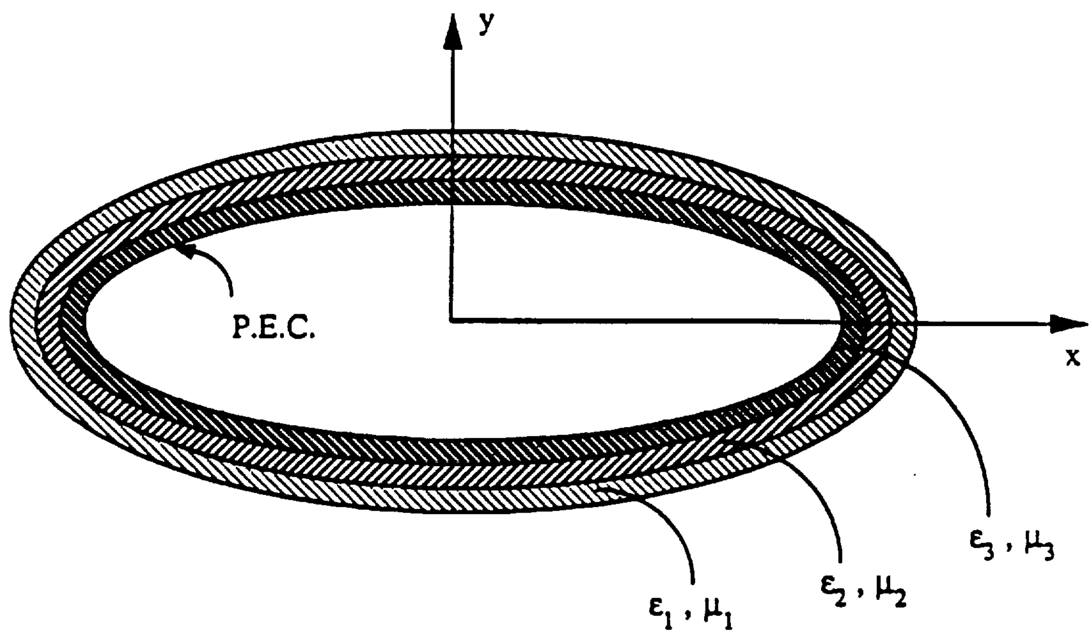


Fig. 1. Illustration of a three-layer coated cylinder.

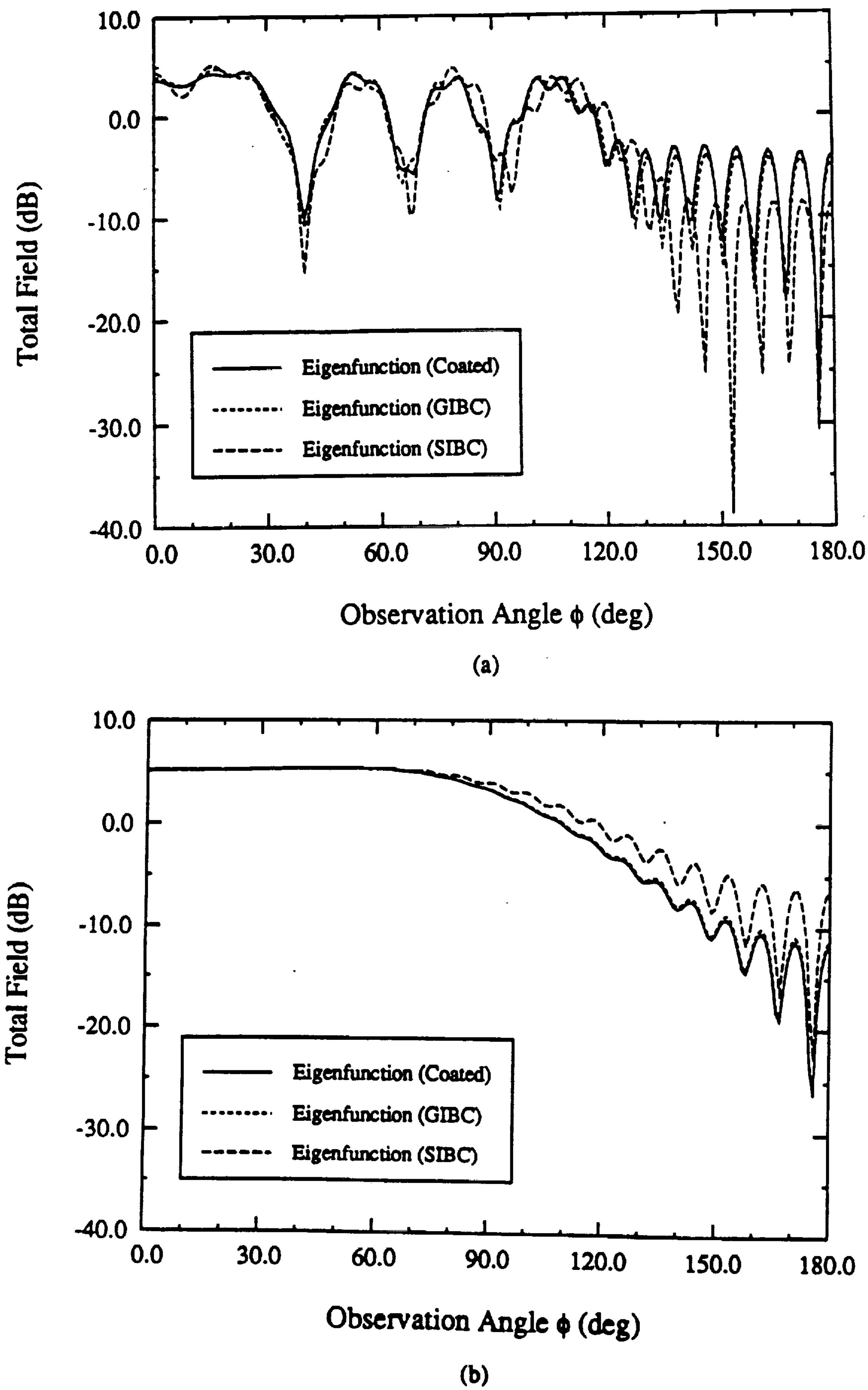
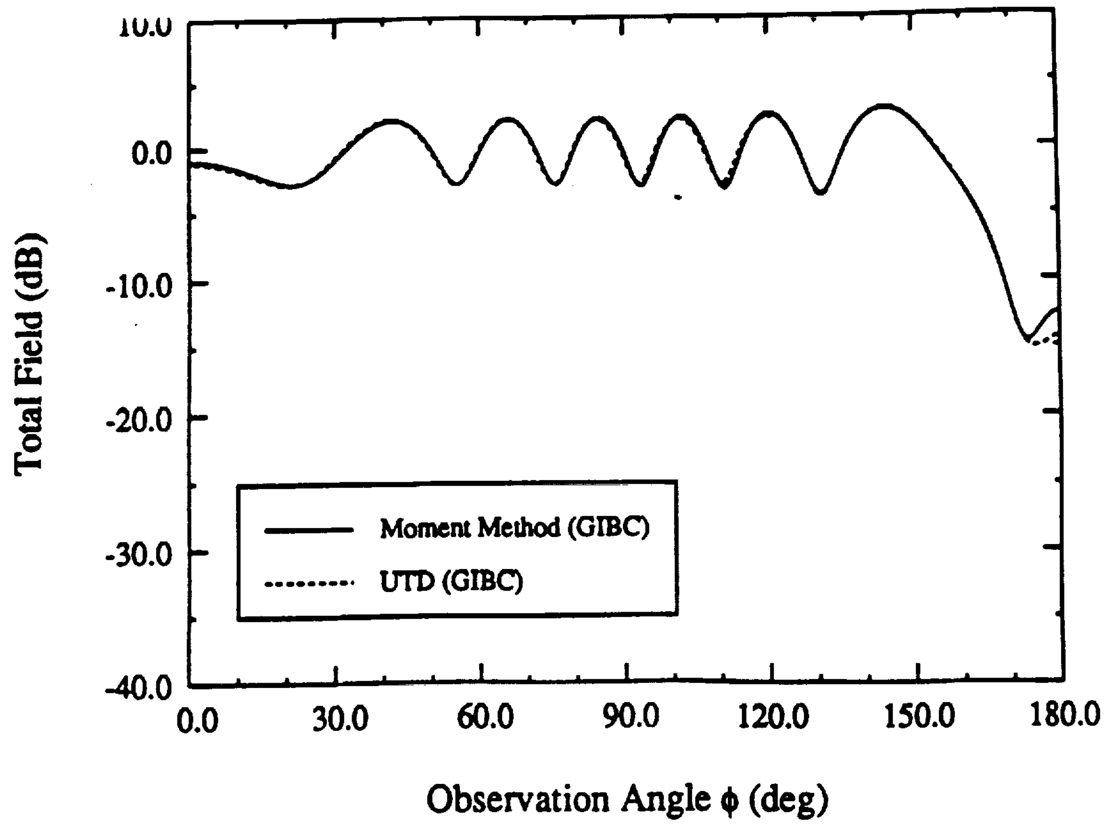
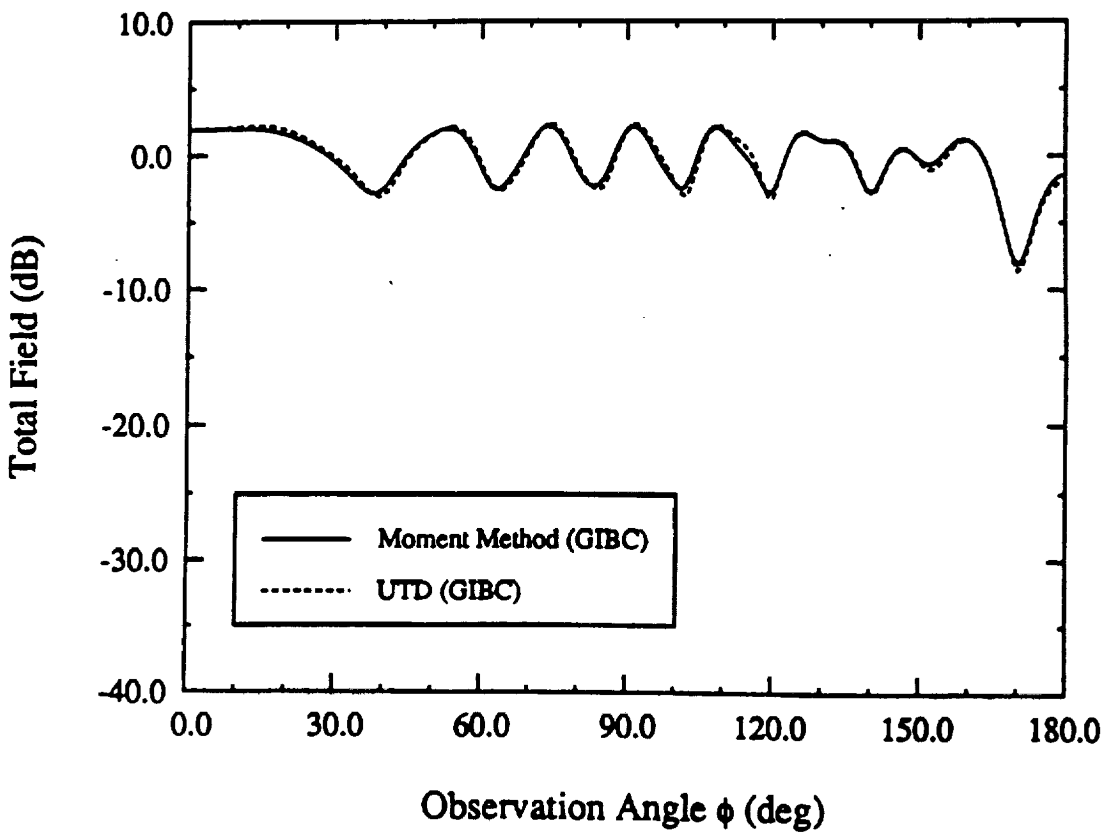


Fig. 2: **Bistatic H-polarization scattering pattern for a circular cylinder having $b = 3\lambda$, (a) $\epsilon_r = 4$, $\mu_r = 1$, $\delta = 0.07\lambda$, $\rho = 5\lambda$ (b) $\epsilon_r = 8$, $\mu_r = 1$, $\delta = 0.2\lambda$, $\rho = 3.05\lambda$.**

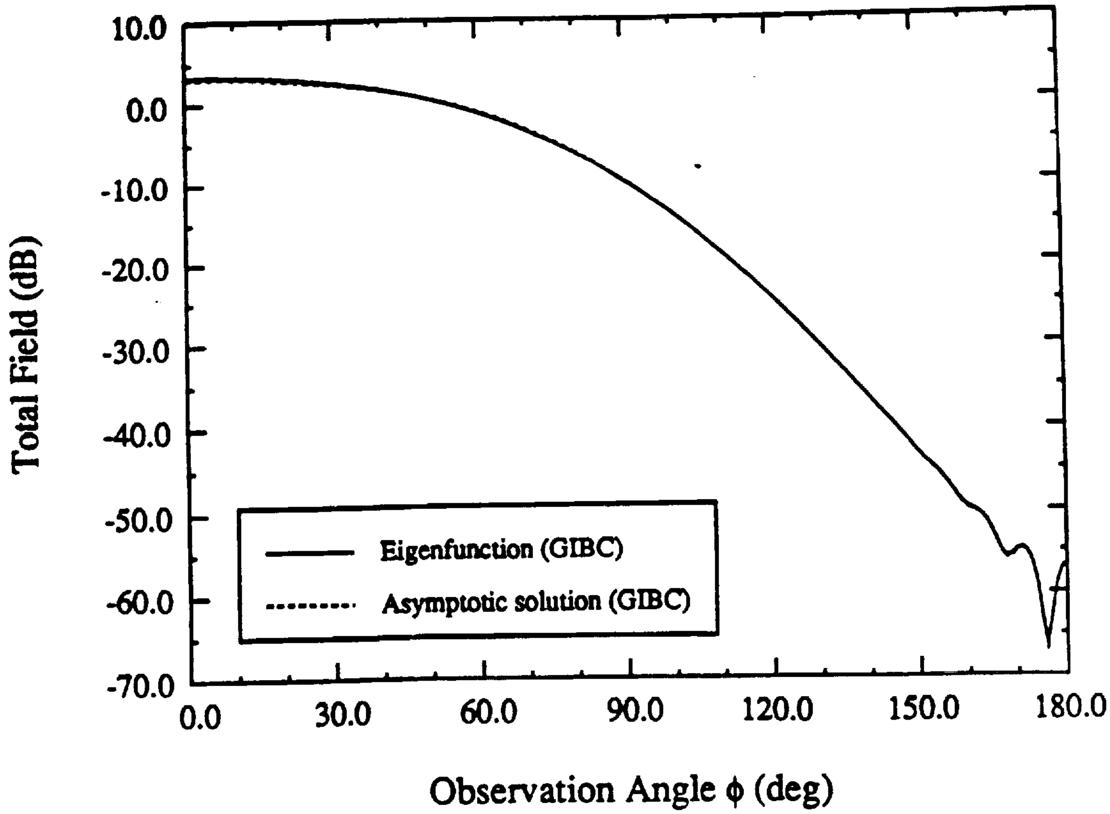


(a)

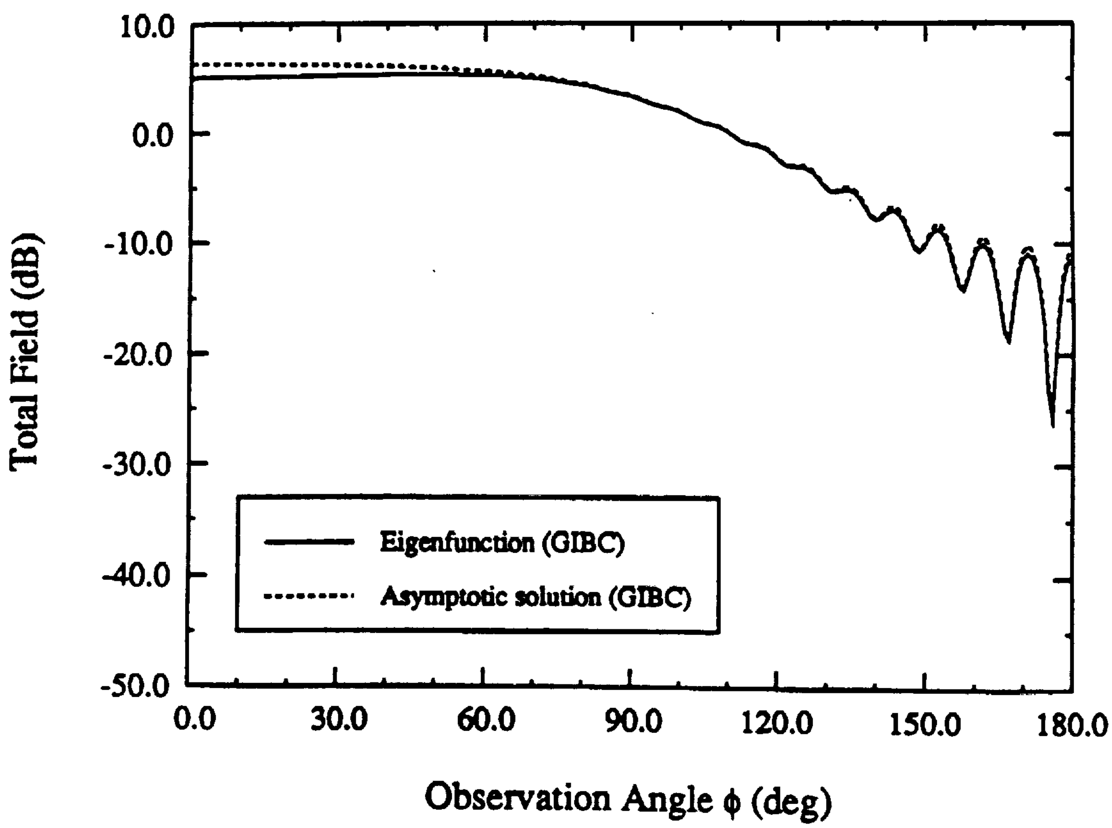


(b)

Fig. 3: **Bistatic scattering pattern for an elliptical cylinder having $a = 2\lambda$, $b = 1\lambda$, $\rho = 5\lambda$, $\phi_i = 0$ (a) E-polarization, $\epsilon_r = 4$, $\mu_r = 1$, $\delta = 0.07\lambda$ (b) H-polarization, $\epsilon_r = 8$, $\mu_r = 1$, $\delta = 0.2\lambda$.**

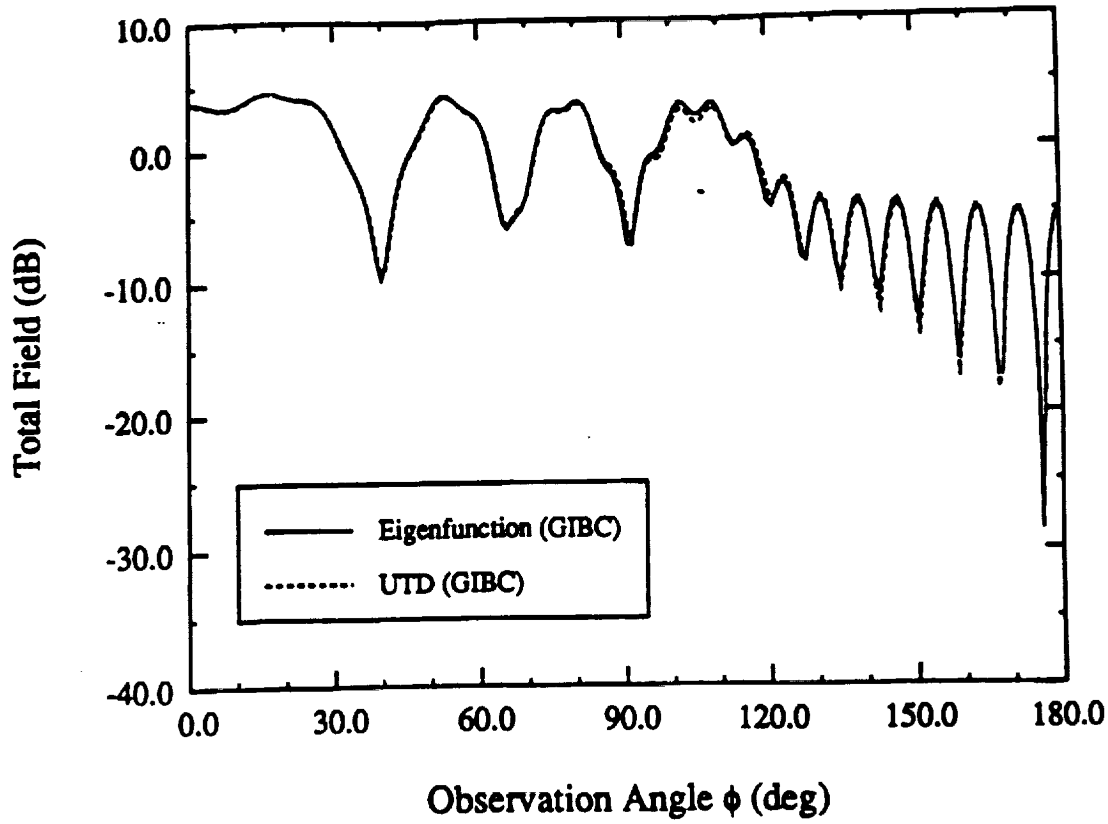


(a)

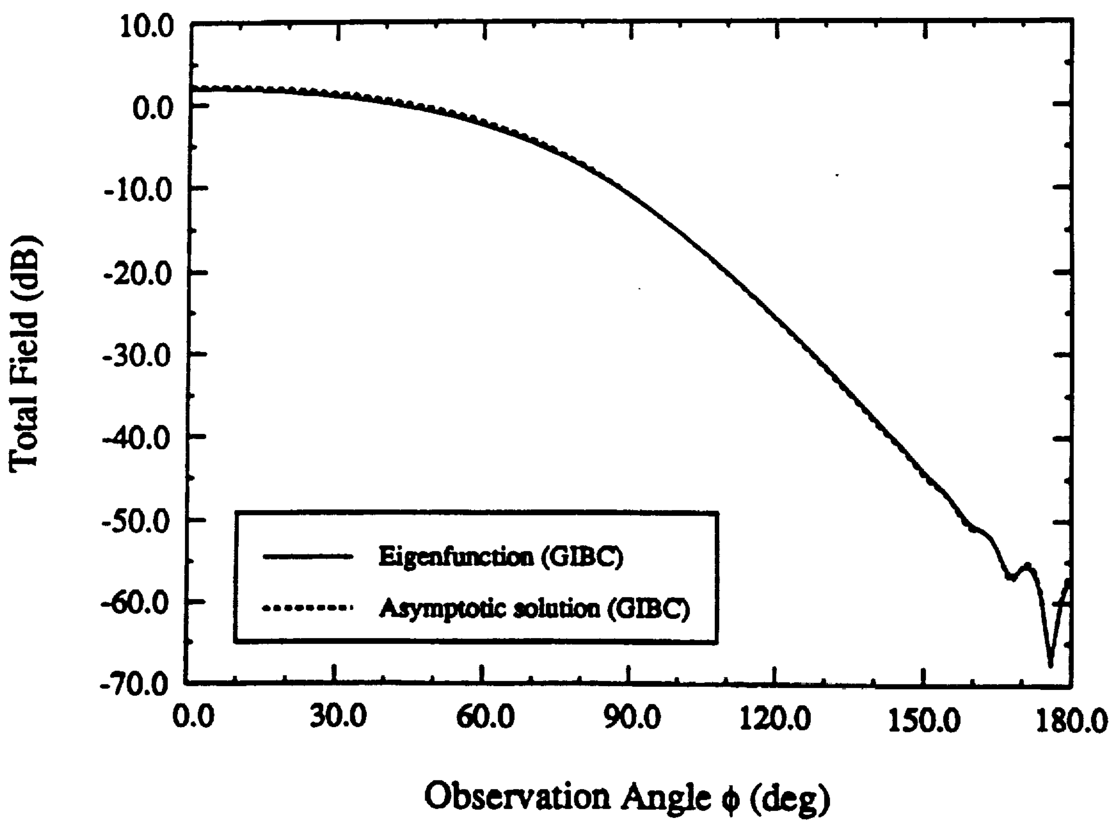


(b)

Fig. 4: **Bistatic scattering pattern of a circular cylinder having $b = 3\lambda$, $\rho = 3.05\lambda$, $\phi_i = 0$ (a) E-polarization, $\epsilon_r = 4$, $\mu_r = 1$, $\delta = 0.07\lambda$ (b) H-polarization, $\epsilon_r = 8$, $\mu_r = 1$, $\delta = 0.2\lambda$.**



(a)



(b)

Fig. 5: **Bistatic scattering pattern of a three-layer coated circular cylinder having $b = 3\lambda$, $\phi_i = 0$, $\epsilon_{r1} = 3 - j0.1$, $\epsilon_{r2} = 4 - j0.3$, $\epsilon_{r3} = 7 - j1.5$, $\mu_{r1} = \mu_{r2} = \mu_{r3} = 1$, $\delta_1 = 0.01\lambda$, $\delta_2 = 0.02\lambda$, $\delta_3 = 0.03\lambda$ (a) H-polarization, $\rho = 5\lambda$ (b) E- polarization, $\rho = 3.05\lambda$.**

PAPER WRITTEN (TO DATE) UNDER THIS GRANT

1. J.-M. Jin and J.L. Volakis, "A New Technique for Characterizing Diffraction by Inhomogeneously Filled Slot of Arbitrary Cross Section in a Thick Conducting Plane," *IEE Electronic Letters*, Vol. 25, No. 17, pp. 1121-1122, 17th Aug. 1989.

2. J-M. Jin, J.L. Volakis and V.V. Liepa, "A Moment Method Solution of Volume-Surface Integral Equation Using Isoparametric Elements and Point-Matching," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-37, pp. 1641-1645, Oct. 1989.

3. J.L. Volakis and M.A. Ricoy, "H-polarization Diffraction by a Thick Metal-Dielectric Join," *IEEE Trans on Antennas and Propagat.*, Vol. AP-37, pp. 1453-1462, Nov. 1989.

4. T.B.A. Senior and J.L. Volakis, "Derivation and Application of a Class of Generalized Boundary Conditions," *IEEE Trans. on Antennas and Propagat.*, Vol. AP-37, PP. 1566-1572, Dec. 1989.

5. K. Barkeshli and J.L. Volakis, "On the Implementation and Accuracy of the Conjugate Gradient Fourier Transform Method," *IEEE Transactions Antennas and Propagation Magazine*, Vol. 32, No.2, pp. 20-26, April 1990.

6. J.-M. Jin and J.L. Volakis, "TM Scattering by an Inhomogeneously Filled Aperture in a Thick Ground Plane," *IEE Proceedings, Part H.* Vol. 137, No. 3, pp. 153-159, June 1990.

7. J.-M. Jin and J.L. Volakis, 'TE Scattering by an Inhomogeneously Filled Aperture in a Thick Conducting Plane," *IEEE Trans. Antennas and Propagat.*, Vol AP-38, pp. 1280-1286, August 1990.

8. K. Barkeshli and J.L. Volakis, "TE Scattering by a Two-Dimensional Groove in a Ground Plane Using Higher Order Boundary Conditions," *IEEE Trans. Antennas and Propagat.*, pp.1421-1428, October 1990.

9. M.A. Ricoy and J.L. Volakis, "Derivation of Generalized Transition/Boundary Conditions for Planar Multiple Layer Structures," *Radio Sci.*, Vol. 25, pp. 391-405, July-Aug. 1990.

10. J.D. Collins and J.L. Volakis and J.M. Jin, "A Finite Element-Boundary Integral Formulation for Solution via CGFFT of Two-Dimensional Scattering Problems," *IEEE Trans. Antennas and Propagat.*, Vol. AP-38, pp. 1852-1858, November 1990.

11. J.L. Volakis and H.H. Syed, "Application of Higher Order Boundary Conditions to Scattering by Multi-layer Coated Cylinders," *J. of Electromagnetic Waves and Applications.*, Vol. 4, No. 12, pp. 1157-1180, 1990

12. J. D. Collins, J.-M. Jin and J.L. Volakis, "A Combined Finite Element-Boundary Element Formulation for Solution of Two-Dimensional Problems via CGFFT," *Electromagnetics*, Vol 10, pp. 423-437, 1990.

13. J.M. Jin and J.L. Volakis, " A Finite Element-Boundary Integral Formulation for Scattering by Three-Dimensional Cavity-Backed Apertures," *IEEE Trans. Antennas and Propagation.*, Vol. AP-39, pp. January 1991.

14. M.A. Ricoy and J. L. Volakis, "Diffraction by a Multilayered Slab Recessed in a Ground Plane via a Generalized Impedance Boundary Condition," *Radio Science,* March-April, 1991.

15. J.M. Jin and J.L. Volakis, "Electromagnetic Scattering by and Transmission through a Three-Dimensional Slot in a Thick Conducting Plane," *IEEE Trans. Antennas and Propagat..*, Vol. AP-39, April 1991

16. H.H. Syed and J.L. Volakis, "An Approximate Diffraction Coefficient for an Impedance Wedge at Skew Incidence," submitted to *IEEE Trans. Antennas and Propagat.*

17. J.M. Jin, J.L. Volakis and J.D. Collins, "A Finite Element-Boundary Element Method for Scattering by Two and Three Dimensional Structures," submitted to *IEEE Antennas and Propagat. Magazine.*

18. T.B.A. Senior and J.L. Volakis "Generalized Impedance Boundary Conditions in Scattering," accepted in*Proceedings of the IEEE.*

19. J.-M. Jin and J.L. Volakis, "A Unified Approach to Integral Equation Formulations for Electromagnetic Scattering," tutorial paper submittted to *IEEE Trans. Antennas and Propagat.*

20. M.A. Ricoy and J.L. Volakis, "Diffraction by a Symmetric Material Junction, Part I: General Solution," submitted to *IEEE Trans. Antennas and Propagat.*

21. M.A. Ricoy and J.L. Volakis, "Diffraction by a Symmetric Material Junction, Part II: Resolution of Non-Uniqueness Associated with Higher Order Boundary Conditions," submitted to *IEEE Trans. Antennas and Propagat.*

22. H.H. Syed and J.L. Volakis, "High Frequency Scattering by a Smooth Coated Cylinder Simulated with Generalized Impedance Boundary Conditions," submitted to Radio Science

23. J.M. Jin and J.L. Volakis, "Scattering by a Finite Frequency Selective Surface," submitted to the *IEEE Trans. Antennas and Propagat*

CONFERENCE PAPERS PRESENTED/WRITTEN TO DATE

1. J.D. Collins and J.L. Volakis, "A Boundary Integral Conjugate Gradient Fast Fourier Transform Method for Solving Two Dimensional Scattering Problems," 1989 IEEE URSI/AP-S symposium, Session 50, San Jose, CA. URSI Digest p. 216.

2. H.H. Syed and J.L. Volakis, "An Approximate Diffraction Coefficient for an Impedance Wedge at Skew Incidence," 1989 IEEE AP-S/URSI symposium, Session 57, San Jose, CA. AP-S Digest pp. 1286-1289.

3. M.A. Ricoy and J.L. Volakis, "Application of Higher Order Boundary Conditions in Scattering by Material Discontinuities," 1989 IEEE AP-S/URSI symposium, Session 63, San Jose, CA. URSI Digest p. 279.

4. J.L. Volakis (invited), "Application of CG and FFT to Simulations with Higher Order Boundary Conditions," 1989 IEEE AP-S/URSI symposium, San Jose, CA.

5. J.-M. Jin, V.V. Liepa and J.L. Volakis, "Applications of Isoparametric elements in the Numerical Solution of Electromagnetic Field Problems," ISAE 1989, China.

6. J.L. Volakis, "Numerical Implementation of Generalized Impedance Boundary Conditions," 1989 URSI Electromagnetic Theory Symposium, Aug. 1989, Stockholm, Sweden. Symposium Digest pp. 434-437.

7. K. Barkeshli and J.L. Volakis, "TE Scattering by a Two-Dimensional Groove in a Ground Plane Using Higher Order Boundary Conditions," 1990 IEEE AP-S/URSI Symposium, Dallas, Tx., URSI Digest p. 43.

8. M.A. Ricoy and J.L. Volakis, "Application of Generalized Impedance Boundary Conditions to Diffraction by a Multilayered Metal-Dielectric Junction," 1990 IEEE AP-S/USRI Symposium, Dallas, Tx., URSI Digest, p.62.

9. H.H. Syed and J.L. Volakis, "Diffraction by a Smooth Coated Cylinder simulated with Generalized Impedance Boundary Conditions," 1990 IEEE AP-S/URSI Symposium, Dallas, Tx., URSI Digest p. 176.

10. J.M. Jin and J.L. Volakis, "A FEM/BEM Formulation for a CG-FFT Solution of 2-D Scattering by Grooves and Thick Slots," 1990 IEEE AP-S/URSI Symposium, Dallas, Tx., URSI Digest p. 260.

11. J.D. Collins and J.L. Volakis, "A Combined Boundary Integral Finite Element Formulation for Solution of Three Dimensional Scattering via Conjugate Gradient Fast Fourier Transform Method," 1990 IEEE AP-S/URSI Symposium, Dallas, Tx., URSI Digest p. 263.

12. J.L. Volakis and H.H. Syed, "Application of Higher Order Bounary Conditions to Scattering by Multilayered Coated Cylinders," 1990 IEEE AP-S/URSI Symposium, Dallas, Tx., AP-S Digest pp. 586-589.

13. J. M. Jin and J.L. Volakis, "A FEM/BEM Formulation for a CG-FFT Solution of 3-D Scattering by a Cavity ," 1990 IEEE AP-S/URSI Symposium, Dallas, Tx., AP-S Digest pp. 1726-1729.

14. J.M. Jin, J.L. Volakis and J.D. Collins, "A Finite Element Boundary Integral Formulation for Scattering by Two and Three DImensional Structures," to be presented at the 1990 URSI General Assembly, Session B4, Prague, Chechoslovakia; Conference Proceedings p. 384.

15. M.A. Ricoy and J.L. Volakis, "On the Analytical Implementation of Generalized Inpedance Boundary Conditions,"to be presented at the 1990 URSI General Assembly, Session B5, Prague, Chechoslovakia; Conference Proceedings p. 391.

16. J.M. Jin and J.L. Volakis, "A Finite Element-Boundary Integral Formulation for Scattering by A Three Dimensional Aperture in a Thick Conducting Plane," Submitted to the 4th Biennial IEEE Conference on Electromagnetic Field Computation, Oct. 1990, Toronto.

17. M.A. Ricoy and J.L. Volakis, "Diffraction by Material Junctions," Journess Internationales De Nice Sur Les Antennes (JINA), Nov. 1990, Nice France; Conference Proceedings pp. 34-44.

REPORTS WRITTEN TO DATE UNDER THIS GRANT

J.L. Volakis, T.B.A. Senior and J.-M. Jin, "Derivation and Application of a Class of Generalized Impedance Boundary Conditions - II," University of Michigan Radiation Laboratory, Technical Report 025921-1-T, February 1989, 47 pp.

K. Barkeshli and J.L. Volakis, "Scattering by a Two-Dimensional Groove in a Ground Plane," University of Michigan Radiation Laboratory, Technical Report 025921-2-T, February 1989, 31 pp.

H.H. Syed and J.L. Volakis, "An Approximate Skew Incidence Diffraction Coefficient for an Impedance Wedge," University of Michigan Radiation Laboratory, Technical Report 025921-4-T, September 1989, 31 pp.

M.A. Ricoy and J.L. Volakis, "Derivation of Generalized Transition/Boundary Conditions for Planar Multiple Layer Structures," University of Michigan Radiation Laboratory, Technical Report 025921-5-T, September 1989, 36 pp.

M.A. Ricoy and J.L. Volakis, "Diffraction by a Multilayered Slab Recessed in a Ground Plane via the Generalized Impedance Boundary Conditions," University of Michigan Radiation Laboratory, Technical Report 025921-8-T, Dec.1989, 43 pp.

H.H. Syed and J.L. Volakis, "An Asymptotic Analysis of the Plane Wave Scattering by a Smooth Convex Impedance Cylinder," University of Michigan Radiation Laboratory, Technical Report 025921-9-T, Jan. 1990. 31pp.

M.A. Ricoy and J.L. Volakis, "Electromagnetic Scattering from Two-Dimensional Thick Material Junctions," University of Michigan Radiation Laboratory 025921-14-T, August 1990. 166 pp.

H.H. Syed and J.L. Volakis, "High Frequency Scattering by a Smooth Coated Cylinder Simulated with Generalized Impedance Boundary Conditions," University of Michigan Radiation Laboratory Report 025921-17-T, January 1991. 32 pp.

J.L. Volakis, "Semi-Annual Progress Report for NASA Grant NAG-2-541," The University of Michigan Radiation Laboratory, Technical Report 025921-3-T, 45 pp.

J.D. Collins and J.L. Volakis, "A Combined Finite Element and Boundary Integral Formulation for Solution via CGFFT of Two-Dimensional Scattering Problems," University of Michigan Radiation Laboratory, Technical Report 025921-6-T, September 1989, 73 pp.

J.L. Volakis, "Semi-Annual Report for NASA Grant NAG-2-541, "University of Michigan Radiation Laboratory, Report 025921-7-T, September 1989, 46 pp.

J.M. Jin and J.L. Volakis, "A Finite Element-Boundary Integral Formulation for Scattering by Three-Dimensional Cavity Backed Apertures," University of Michigan Radiation Laboratory Technical Report 025921-10-T, February 1990. 28 pp.

J.D. Collins, J.M. Jin and J.L. Volakis, "A Combined Finite Element-Boundary Element Formulation for Solution of Two-Dimensional Problems via CGFFT," University of Michigan Radiation Laboratory Technical Report 025921-11-T, February 1990. 45 pp.

J.L. Volakis, "Semi-Annual Report for NASA Grant NAG-2-541, "University of Michigan Radiation Laboratory Report 025921-12-T, February 1990. 60 pp.

J.L. Volakis, "Semi-Annual Report for NASA Grant NAG-2-541, "University of Michigan Radiation Laboratory Report 025921-13-T, September 1990. 56 pp.

C. Hua, "Users Manual for AUTOMESH-2D A Program of Automatic Mesh Generation for Two-Dimensional Scattering Analysis by the Finite Element Method," University of Michigan Radiation Laboratory Report 025921-15-T, August 1990, 38 pp.

J. Zapp, "Examples of Finite Mesh Generation using SDRC IDEAS," University of Michigan Radiation Laboratory Report 025921-16-T, September 1990, 52 pp.

J.D. Collins and J.L. Volakis, "A Combined Finite Element-Boundary Element Formulation for Solution of Axially Symmetric Bodies," University of Michigan Radiation Laboratory Report 025921-18-T, January 1991. 64 pp.