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Coerte V. Voorhies

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**On the Joint Inversion
of Geophysical Data for Models
of the Coupled Core-Mantle System**

Coerte V. Voorhies
NASA-Goddard Space Flight Center
Greenbelt, Maryland



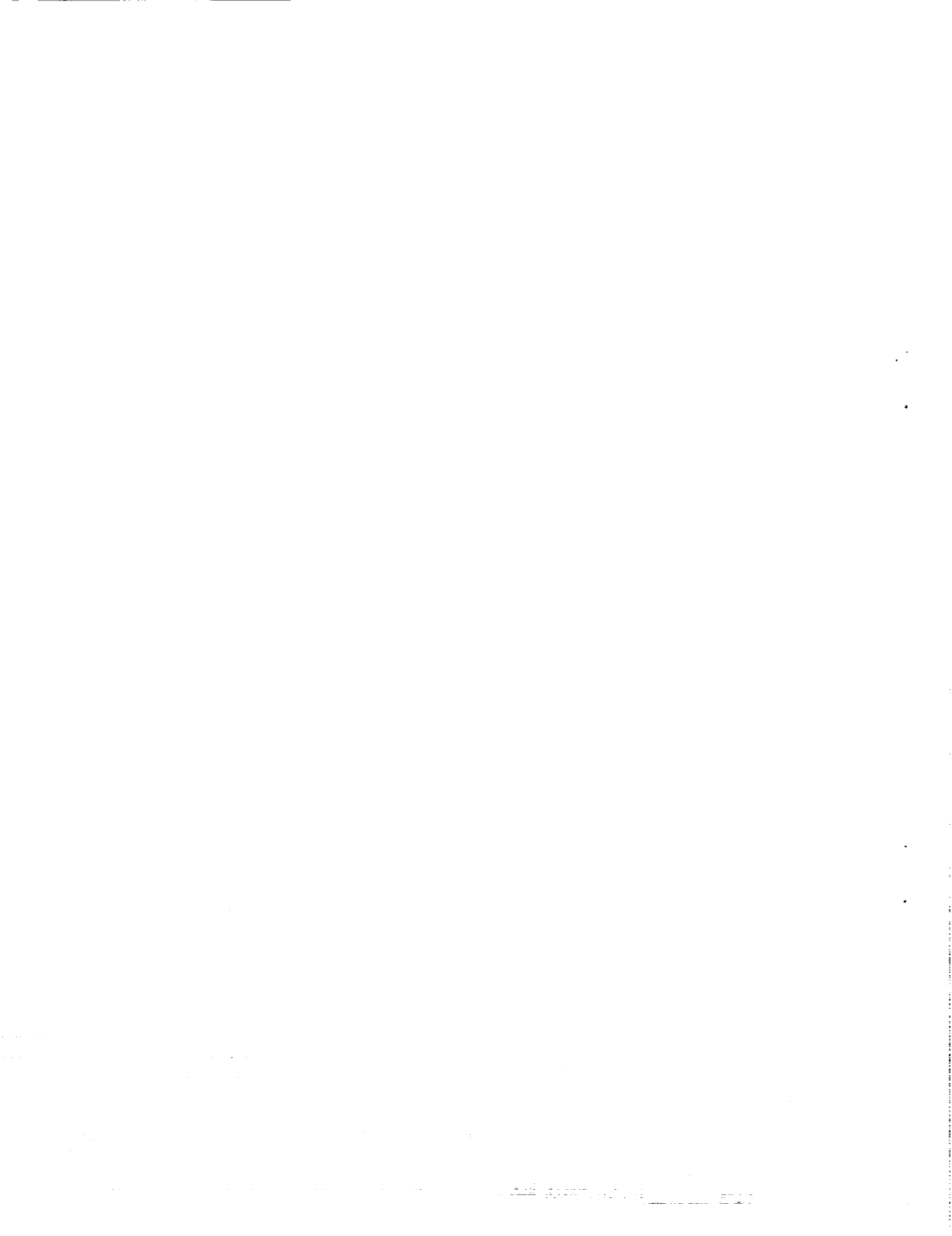
National Aeronautics and
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ABSTRACT

Joint inversion of magnetic, Earth rotation, geoid, and seismic data for a unified model of the coupled core-mantle system is proposed and shown to be possible. A sample objective function is offered and simplified by targeting results from independent inversions and summary travel time residuals instead of original observations. These "data" are parameterized in terms of a very simple, closed model of the topographically coupled core-mantle system. Minimization of the simplified objective function leads to a non-linear inverse problem; an iterative method for solution is presented. Parameterization and method are emphasized; numerical results are not presented.



1. INTRODUCTION

Geophysicists working with different types of data are probing Earth's deep interior (see, e.g., Lay, 1989). For example, geomagnetic data have been used to estimate fluid motions near the top of the core (Ball, Kahle & Vestine, 1969; Voorhies, 1984, 1986a,b, 1988; LeMouél, Gire & Madden, 1985; Whaler & Clarke, 1988); seismic data have been used to estimate laterally heterogeneous mantle structure and core-mantle boundary – hereafter denoted CMB – topography (Morelli & Dziewonski, 1987); gravity and geodetic data have been combined with seismic estimates of Earth structure to estimate CMB topography (Hager et al., 1985); and estimates of surficially geostrophic core motions have been combined with estimates of CMB topography to calculate the topographic torque exerted by the core on the mantle and the implied changes in "solid" Earth rotation (Speith et al., 1986). The latter uses results from independent or "disjoint" inversions of different geophysical data types to forwardly model decade fluctuations in solid Earth rotation.

I propose joint inversion of diverse geophysical data types for a unified model of the coupled core-mantle system. The plan merges magnetic, Earth rotation, geoid, and seismic data into one objective function which, when suitably weighted, constrained, and parameterized, can be minimized with respect to the parameters of a unified deep Earth model. The goal is to develop, parameterize, and test hypotheses about Earth's deep interior against all relevant types of data.

Curiously, the philosophical foundation for this type of inversion has been questioned. Clearly, much can be learned from experiments designed to isolate those data which are thought to be most sensitive to some particular property of the Earth. This approach can yield decisive

tests of particular hypotheses; yet one need not always lose sight of the forest for the trees. Some properties of Earth's deep interior (e.g., CMB topography) can contribute signals to many kinds of data yet are apparently not uniquely determined by any single kind of data. In such cases, more plausible estimates of the properties might be obtained by using more than one kind of data.

To do so, a merged data set may be compiled and used to estimate parameters of models of the Earth properties. One can hypothesize that signals from properties which are not modeled, and from parameters which are not estimated, do not vastly exceed the residuals indicated by a weighted least-squares fit of the modeled parameters to the data. This hypothesis can, in turn, be investigated by fitting more data and more types of data with more complete models of more Earth properties.

To this end, I offer a sample objective function and parameterize it in terms of a simplified, mechanically coupled, core-mantle system. The sample "data" considered are slowly varying geomagnetic potential coefficients, decade fluctuations in the angular velocity of the solid Earth, static gravitational potential coefficients, and summary seismic travel time residuals relative to a laterally homogeneous Earth model. The parameters describe a piecewise steady core surface velocity field, a perturbation density field in the mantle, and a CMB topography function. The system is closed by supposing superficially, indeed tangentially (Backus & LeMouél, 1986), geostrophic core motions and relations between perturbation seismic wave speeds and perturbation density in the mantle. Even for this simple Earth model, minimization of the sample objective function leads to a non-linear inverse problem; an iterated, linearized method of solution is presented.

This sample is intended to provide a foundation for more realistic deep Earth models which might include: a superior mean state; mantle dynamics and rheology; richer core dynamics; magnetic, viscous, and gravitational core-mantle coupling; and thermal and compositional core-mantle interactions. More work will be needed on the problems of how to parameterize such models, include more kinds of data (e.g., free oscillations and plate motions), and apply more constraints (e.g., from mineral physics and low-frequency gravity and deformation studies); and on problems of uniqueness, accuracy, and method.

2. AN OBJECTIVE FUNCTION SIMPLIFIED

Let \mathbf{r} be the position vector in geobarycentric spherical polar coordinates radius r , colatitude θ , and east longitude ϕ ; let t be time; and let observational data and Earth model predictions be denoted respectively by d and p subscripts on the following variables:

\mathbf{B} is the geomagnetic flux density vector;

$\mathbf{\Omega}$ is the apparent angular velocity vector at the surface of the solid Earth, technically including plate motions;

\mathbf{g} is the gravitational acceleration vector;

T is the travel time of seismic phase ζ from the source at (\mathbf{r}', t') to the receiver at (\mathbf{r}, t) ;

w^x is the weight function which is taken to be the inverse squared uncertainty in datum of type x at (\mathbf{r}, t) , but which can be generalized to a weight matrix for discrete data on the expectation of correlated errors; and

λ_j are multipliers giving weight and proper units to the

K_j which represent geophysical constraints (e.g., small density perturbations, finite surficially geostrophic core fluid velocity (Mach number < 1), smooth CMB topography, etc.).

One suitable objective function to be minimized in a joint inversion of geophysical data is

$$\Delta^2 = \{ \text{the weighted residual variance relative to the} \\ \text{magnetic + Earth rotation + geoid + seismic data} \} \\ + \{ \text{other geophysical constraints} \} \\ = \Delta_m^2 + \Delta_{er}^2 + \Delta_{ge}^2 + \Delta_s^2 + \lambda_i \Delta_{K_i}^2$$

or, in the foregoing notation,

$$\Delta^2 = \int_{r_i}^{r_o} \int_0^{2\pi} \int_0^\pi \int_{t_o}^{t_f} \{ [B_d(r,t) - B_p(r,t)]^2 W^m(r,t) \\ + [\Omega_d(r,t) - \Omega_p(r,t)]^2 W^{er}(r,t) \\ + [g_d(r,t) - g_p(r,t)]^2 W^{ge}(r,t) \\ + \left[\int_{r'_i}^{r'_o} \int_0^{2\pi} \int_0^\pi \int_{t'_o}^{t'_f} [T_d(\zeta; r', t'; r, t) - T_p(\zeta; r', t'; r, t)]^2 \right. \\ \left. W^s(\zeta; r', t', r, t) r'^2 \sin \theta' dt' d\theta' d\phi' dr' \right] \\ + [\lambda_i K_i(r, t)^2] \} r^2 \sin \theta dt d\theta d\phi dr .$$

It is convenient to view the results of measurements over small portions of the 4-volume of integration as discrete data. Then

$$\Delta^2 = [d_j - p_j] W_{jk}^{\text{merges}} [d_k - p_k] + \langle \lambda_i K_i \rangle$$

where d_j is an element of the merged data vector, p_j is an element of the merged prediction vector, and repeated subscripts denote summation.

This objective function could be generalized to include other types of data; yet it already seems too ambitious and the data have already been reduced and analyzed "disjointly". The suggestion is to build upon this rich tradition by replacing the diverse data types with either more tractable models thereof or data residuals relative to such models. One such approach begins with the following, de-subscripted, "data":

- (1) slowly varying, broad-scale spherical harmonic models B of the observed geomagnetic field (e.g., IAGA 1988);

- (2) length-of-day and polar motion data corrected as possible for nutation, precession, and tidal effects, either low-pass filtered or corrected for atmospheric and hydro-cryospheric effects (e.g., Stephenson & Morrison, 1984), and then fitted with, say, a piecewise linear function $\mathbf{Q}(t)$;
- (3) broad-scale spherical harmonic models of the steady part of the gravitational field \mathbf{g} (e.g., Marsh et al., 1988) – preferably corrected for surface topographic and crustal sources; and
- (4) summary travel time residuals T relative to a laterally homogeneous seismic Earth model (e.g., Dziewonski & Anderson, 1981) which specifies the axisymmetric mean state $(V_{P0}, V_{S0}, \rho_0, \rho_0, K_0, \mu_0)$ on reference ellipsoids (or spheres).

Effects of external fields on (1) and (3) are small. Effects of plate motions on (2) are omitted for now; the piecewise linear $\mathbf{Q}(t)$ fitted to corrected, low-pass filtered \mathbf{Q}_d should capture the decade fluctuations of interest here. Summary travel time residuals in (4) are averaged over closely spaced ray paths (Creager & Jordan, 1986) to reduce effects of small-scale structure, oversampling, and colinearity. Alternately, one might use (4b) a model of laterally heterogeneous phase speeds V_p and V_s , or (4c) spherical harmonic models of the travel time residuals at all (summary) receivers for each phase from each (summary) source.

Let the reference surface of mean radius $|\mathbf{a}| = a \simeq 6.3712 \text{ Mm}$ enclose the internal sources of scaloidal \mathbf{B} and \mathbf{g} . On and above this surface we have

$$\mathbf{B}_d = \mathbf{B}(\mathbf{r}, t) + \delta\mathbf{b}(\mathbf{r}, t) \qquad \mathbf{B} = -\nabla V$$

with internal scalar magnetic potential

$$V = a \sum_{n=1}^{\infty} \left[\frac{a}{r} \right]^{n+1} \sum_{m=0}^n [g_n(t) \cos m\phi + h_n(t) \sin m\phi] P_n^m(\cos\theta)$$

and radial magnetic component

$$B_r(a, t) = - \frac{\partial V}{\partial r} = \sum_i g_i(a, t) S_i(\theta, \phi) \equiv g_i S_i \equiv \underline{g}^T \underline{S} . \quad (1)$$

Here the (g_n^m, h_n^m) are the Gauss coefficients, P_n^m is the Schmidt-normalized associated Legendre polynomial of degree n and order m , g_i is an element of the ordered column vector \underline{g} of radial magnetic field coefficients, S_i is an element of the ordered vector \underline{S} of spherical harmonics of degree $n(i)$ and order $m(i)$, and a T superscript indicates the transpose (Voorhies, 1986b). Moreover,

$$\Omega_d(t) = \Omega_0 + \Omega(t) + \delta\omega(t) \quad (2)$$

where Ω_0 is the mean angular velocity of the solid Earth and $\Omega(t)$ is the piecewise linear representation of the decade fluctuations. Furthermore, with $g_0(r) = -\nabla U_0$ being the mean gravity caused by the mean density $\rho_0(r)$, and with tidal and other time-varying effects represented by δg ,

$$g_d = g_0(r) + g(r) + \delta g(r, t) .$$

Steady $g(r) = -\nabla U$ has steady perturbation gravitational potential

$$U = \frac{GM_E}{a} \sum_{n=0}^{\infty} \left[\frac{a}{r} \right]^{n+1} \sum_{m=0}^n [c_n \cos m\phi + s_n \sin m\phi] P_n^m(\cos\theta)$$

and steady perturbation radial gravitational component

$$g_r(a) = - \frac{\partial U}{\partial r} = \sum_i c_i(a) S_i(\theta, \phi) \equiv c_i S_i \equiv \underline{c}^T \underline{S} . \quad (3)$$

Here G is the gravitational constant, M_E is Earth's mass, (c_n^m, s_n^m) are the steady perturbation gravitational potential coefficients, and c_i is an element of the ordered vector \underline{c} of steady perturbation radial gravitational field coefficients. Finally,

$$T(\zeta; \mathbf{r}', t'; \mathbf{r}, t) = T_d(\zeta; \mathbf{r}', t'; \mathbf{r}, t) - T_o(\zeta; \mathbf{r}', t'; \mathbf{r}, t) = T_i = T_{di} - T_{oi} \quad (4)$$

where, for each seismic phase ζ and each (summary) source-receiver pair denoted by $i = i(\zeta; \mathbf{r}', t'; \mathbf{r}, t)$, the (summary) travel time residual T_i is relative to the travel time predicted by the reference Earth model T_{oi} . Hopefully g_i , Ω_i , c_i , and T_i would be corrected for external and crustal effects; such corrections to low-degree geomagnetic main field models, piecewise linear fits to low-pass filtered Earth rotation data, a low-degree gravity model, and perhaps summary travel times should be small.

With the foregoing reduction of data types, and targeting the evolution of the radial magnetic component and the steady perturbation radial gravitational component on \mathbf{a} , the redefined objective function is

$$\begin{aligned} \Delta^2 \equiv & \int_{t_0}^{t_f} \int_0^{2\pi} \int_0^\pi [B_r(\mathbf{a}, t) - B_{rp}(\mathbf{a}, t)]^2 W^m(\mathbf{a}, t) \sin\theta d\theta d\phi dt \\ & + \int_{t_0}^{t_f} [\Omega(t) - \Omega_p(t)]^2 W^{er}(t) dt \\ & + \int_0^{2\pi} \int_0^\pi [g_r(\mathbf{a}) - g_{rp}(\mathbf{a})]^2 W^{ge}(\mathbf{a}) \sin\theta d\theta d\phi \\ & + [T_i - T_{pi}] W_{ij}^s [T_j - T_{pj}] + \langle \lambda_i K_i(\mathbf{r}, t)^2 \rangle . \quad (5) \end{aligned}$$

Scalar weight functions W^m and W^{ge} can be derived from the (increasingly realistic) error covariance matrices for the geopotential field models, while W^{er} should reflect uncertainty and error in the piecewise linear fit $\Omega(t)$. The diagonal elements of the matrix W^s should reflect the standard deviation of each summary travel time residual; hypocenter uncertainties may suggest non-trivial off-diagonal elements. With eqns. (1) and (3), $B_{rp}(\mathbf{a}, t) = \gamma_i S_i$, $g_{rp}(\mathbf{a}) = \xi_i S_i$, the three components of $\Omega_p(t)$ written $\Omega_{ip}(t) = \omega_i(t)$, and with $T_{pk} = \tau_k$, evaluation of the weighted surface integrals in eq. (5) yields

$$\begin{aligned}
\Delta^2 = & \int_{t_0}^{t_f} [(g_i - \gamma_i) W_{ij}^m (g_j - \gamma_j)] dt \\
& + \int_{t_0}^{t_f} [(\Omega_i - \omega_i) W_{ij}^{er} (\Omega_j - \omega_j)] dt \\
& + [(c_i - \xi_i) W_{ij}^{ge} (c_j - \xi_j)] \\
& + [(\tau_i - \tau_i) W_{ij}^s (\tau_j - \tau_j)] + \langle \lambda_i K_i(r,t)^2 \rangle . \quad (6)
\end{aligned}$$

Other types of geophysical data may be included by adding suitably weighted terms to the objective function given by eq. (6).

Select changes in the weight matrices of eq. (6) can be made so as to transform the objective function. For example: (i) geomagnetic and gravity coefficients can be fitted through degree and order 10 without biasing the higher degree γ_i and ξ_i by setting W_{ij}^m and W_{ij}^{ge} equal to zero for either i or j greater than 120; (ii) replacing W_{ij}^m with the inverse of the error covariance matrix for the radial magnetic coefficients targets the scalar geomagnetic potential instead of the radial component alone; or (iii) W_{ij}^s might be culled to restrict attention to particular phases like PKP or ScS.

3. PARAMETERIZATION

The parameterization of predictions γ_i , ω_j , ξ_k , and τ_l offered here is based on a very simple Earth model with the following attributes.

- (1) Geomagnetic secular variation is attributed to piecewise steady (Voorhies & Backus, 1985) frozen-flux motional induction by a tangentially geostrophic (Backus & LeMouél, 1986) fluid velocity field v_s at the top of a roughly spherical core of mean radius $|c| = c \approx 3.48$ Mm (no subscript or underscore) which is surrounded by a comparatively rigid and magnetically source-free mantle.

- (2) Decade fluctuations in the angular velocity of the solid Earth are attributed to the mechanical torque L exerted by the perturbation pressure $p'(c, \theta, \phi; t)$ associated with v_s on the topography $h(\theta, \phi)$ of the CMB. The CMB is the locus of points r_c : $r_c = c + h(\theta, \phi)$.
- (3) The perturbation gravity field is attributed to the perturbation density in the solid Earth $\rho'(r \geq c + h, \theta, \phi)$ and the effect of CMB topography h . Density perturbations within the core are omitted.
- (4) Seismic travel time residuals are due to h , ρ' , and perturbations in the bulk and shear moduli K and μ .

3.1 Magnetic

In this very simple model, within each subinterval during which steady flow is presumed the time rate of change of the predicted radial field at the CMB is, to order zero in h ,

$$\dot{B}_{rp}(c, t) = \nabla_s \cdot [B_{rp}(c, t) v_s(c)] = \dot{\Gamma}_j S_j$$

where the over-dot indicates the partial time derivative and Γ_j is an element of the ordered vector of predicted radial magnetic field coefficients at c . With the diagonal upward continuation matrix Υ_{ij} having elements $(c/a)^{n(i)+2}$ for harmonic degree $n(i)$,

$$\dot{B}_{rp}(a, t) = \dot{\gamma}_i S_i = (\Upsilon_{ij} \dot{\Gamma}_j) S_i .$$

The surficial fluid velocity is expressed in terms of the streamfunction $-T^m$ and the velocity potential $-U^m$

$$v_s(c) = r \times \nabla_s T^m + \nabla_s U^m \quad T^m = \alpha_j S_j \quad U^m = \beta_j S_j .$$

These expressions imply

$$\begin{aligned} \dot{\gamma}_m &= \Upsilon_{mk} \{ [\Gamma_i X_{ijk}] \alpha_j + [\Gamma_i Y_{ijk}] \beta_j \} \\ &= \Upsilon_{mk} \{ X_{ijk} \Gamma_i \alpha_j + Y_{ijk} \Gamma_i \beta_j \} = A_m | v | \end{aligned} \quad (7)$$

where, in the last step, the sums over i and k have been performed and v_j is an element of the concatenated vector of streamfunction coefficients α_j and velocity potential coefficients β_j . Equation (7) is but $\dot{\underline{\gamma}} = \underline{A}\underline{v}$. The time-varying elements A_{mj} of matrix \underline{A} depend on Γ_j , hence B_{rp} , and thus upon the velocity field coefficients v_j . The inverse problem is therefore non-linear; for the iterative linearized approach suggested in section 4, the A_{mj} are first calculated from the g_j and are recalculated on each deep iteration (Voorhies, 1987a,b, 1988). Formulae for $X_{ijk} = -X_{jik}$, Y_{ijk} , and A_{mj} are given elsewhere for the linear case (Voorhies, 1986b); formulae for the non-linear case have been presented (Voorhies, 1987a), posted (Voorhies, 1988), and are in typescripts (available by request) detailing the steady core flow estimation methods applied routinely at GSFC.

Tangential geostrophy (Ball et al., 1969; Backus & LeMou el, 1986) seems awkward to enforce. In contrast, it is easy to damp departures from a geostrophic radial vorticity balance (Voorhies, 1986b,c). Then $\nabla_S \cdot [v_S \cos \theta] \simeq 0$ and downwelling implies poleward flow (Voorhies, 1987c). Such flows are but "surficially geostrophic" (Voorhies, 1990); subsequent supposition of tangential geostrophy allows calculation of the perturbation pressure field on the sphere c from the v_j . Steady perturbation pressure "maps" so derived at GSFC show fair agreement with those derived from the work of C. Gire and J.-L. LeMou el - the first, I believe, to produce such maps (D. Jault, 1989, personal communication).

3.2 Earth Rotation

The reference mantle has steady principle moments of inertia ($A, B \simeq A, C$) in geobarycentric Cartesian coordinates (x, y, z) with Ω_0 parallel to the z axis; the time rate of change of the predicted angular

velocity vector is, according to the Euler equations,

$$\begin{aligned}\dot{\omega}_x &= [L_x - (\Omega_0 + \omega_z)(C-B)\omega_y]/A \simeq [L_x - \Omega_0(C-A)\omega_y]/A \\ \dot{\omega}_y &= [L_y + (\Omega_0 + \omega_z)(C-A)\omega_x]/A \simeq [L_y + \Omega_0(C-A)\omega_x]/A \\ \dot{\omega}_z &= [L_z - \omega_x\omega_y(B-A)]/C \quad \simeq L_z/C\end{aligned}$$

with the approximations good to first order in $|\omega_i|/\Omega_0 \ll 1$. Voorhies (1990) shows that, to first-order accuracy in the asphericity $|h|/c \ll 1$ and in $|V_S h| \ll 1$, the topographic torque is (omitting Lorentz terms)

$$L \simeq 2\Omega_0\rho c^3 \int_0^{2\pi} \int_0^\pi [h(\theta, \phi) v_S(r_c \simeq c, \theta, \phi) \cos\theta] \sin\theta d\theta d\phi$$

With $h(\theta, \phi) = h_i S_i$ one has, to first-order accuracy,

$$L_k = [h_i Q^*_{ijk}] v_j = Q^*_{ijk} h_i v_j$$

$$\dot{\omega}_k(t) = Q_{ijk} h_i v_j + E_{kj} \omega_j(t) \equiv Q_{ijk} h_i v_j + q_k(t) \quad (8)$$

or simply $\dot{\underline{\omega}} = \underline{Q}^T \underline{h} \underline{v} + \underline{q}$. Because $\dot{\omega}_k(t)$ depends on piecewise steady $h v_S$, hence $h_i v_j$, and because $q_k(t)$ depends on $\omega_j(t)$, the inverse problem is non-linear. In the iterative linearized approach (see section 4), the q_k , and perhaps $A, B \neq A, C$ and off diagonals of the inertia tensor, should be recalculated on each deep iteration. Then it is convenient to introduce the matrices $Q^v_{kj} \equiv Q_{ijk} h_i$ and $Q^h_{ki} \equiv Q_{ijk} v_j$.

With initial conditions $g_i(\mathbf{a}, t_0) = \gamma_i(\mathbf{a}, t_0)$ and $\Omega_j(t_0) = \omega_j(t_0)$, and simply supposing steady flow from t_0 to t_f , the magnetic and Earth rotation portion of eq. (6) is

$$\begin{aligned}\int_{t_0}^{t_f} [(g_i - \gamma_i) W_{ij}^m (g_j - \gamma_j)] + [(\Omega_i - \omega_i) W_{ij}^{er} (\Omega_j - \omega_j)] dt \\ = \int_{t_0}^{t_f} \left\{ \left[\int_{t_0}^t (g_i - \gamma_i) dt' \right] W_{ij}^m \left[\int_{t_i}^t (g_j - \gamma_j) dt' \right] + \right.\end{aligned}$$

$$\left[\int_{t_0}^t (\dot{\Omega}_i - \dot{\omega}_i) dt' \right] W_{ij} \left[\int_{t_0}^t (\dot{\Omega}_j - \dot{\omega}_j) dt' \right] \} dt$$

or, by eqns. (7) and (8),

$$= \overline{\overline{[g - A\underline{v}]^T \underline{m} \cdot [g - A\underline{v}]} + [\dot{\Omega} - \underline{Q}^T \underline{h\underline{v}} - \underline{q}]^T \text{er} \cdot [\dot{\Omega} - \underline{Q}^T \underline{h\underline{v}} - \underline{q}]} \quad (9)$$

where the number of underscores denotes tensor rank, the first over-bar indicates dummy time integration from t_0 to t , and the second over-bar indicates time integration from t_0 to t_f . Besides the explicit non-linear dependence on $\underline{h\underline{v}}$, there are non-linearities implicit in $\underline{A}(\underline{\gamma}(\underline{v}))$ and $\underline{g}(\underline{\omega}(\underline{h}, \underline{v}))$.

3.3 Geoid

The suggested geoid parameterization is in terms of spherical harmonic coefficients for a perturbation density which is independent of radius within each of K layers

$$\rho'(r_k < r \leq r_{k+1}, \theta, \phi) = [\rho(r_k < r \leq r_{k+1})]_i S_i(\theta, \phi) = \rho_{ki} S_i$$

and the mean state density contrast across the CMB, $\Delta\rho = \rho_c - \rho_m$, times the CMB topography

$$\rho'(r_0=c, \theta, \phi) = \Delta\rho h_i S_i .$$

The perturbation geopotential is

$$U(r) = -G \iiint \frac{\rho'(r')}{|r - r'|} r'^2 \sin\theta' d\theta' d\phi' dr' .$$

Now $g_r = -U_{,r}$; with $|a| = |r| > |r'|$, expanding $|r - r'|^{-1}$ yields

$$g_r(a) = -G \int_c^a \int_0^{2\pi} \int_0^\pi \{ [\rho'(r')]_i S_i(\theta', \phi') \} \{ [n(j)+1] \left(\frac{r'}{a}\right)^{n(j)+2} S_j(\theta', \phi') S_j(\theta, \phi) \} \sin\theta' d\theta' d\phi' dr'$$

$$\begin{aligned}
&= -\int_c^a [\rho'(r')]_i \left\{ \frac{4\pi G[n(j)+1]}{[2n(j)+1]} \left(\frac{r'}{a}\right)^{n(j)+2} \delta_{ij} \right\} S_j(\theta, \phi) dr' \\
&= \sum_{k=1}^K \rho_{ki} \left\{ \frac{-4\pi a G [n(j)+1]}{[2n(j)+1][n(i)+3]} \left[\left(\frac{r_{k+1}}{a}\right)^{n(i)+3} - \left(\frac{r_k}{a}\right)^{n(i)+3} \right] \delta_{ij} \right\} S_j \\
&\quad + \Delta \rho h_i \left\{ \frac{-4\pi a G [n(j)+1]}{[2n(j)+1][n(i)+3]} \left(\frac{c}{a}\right)^{n(i)+3} \delta_{ij} \right\} S_j \\
&\equiv [\rho_{ki} G_{ikj}] S_j + [h_i H_{ij}] S_j = [\rho_{ki} G_{ikj} + H_{ij} h_i] S_j
\end{aligned}$$

where δ_{ij} is the Kroenecker delta. Because G_{ikj} (and H_{ij}) are zero for $i \neq j$, the connection between ρ' and $g_r(\mathbf{a})$ is harmonically pure (as is that between h and $g_r(\mathbf{a})$). It is convenient to reorder the ρ_{kj} into a single vector ρ_l with $l = (k-j)j_{\max} + j$ so that $\rho_{ki} G_{ikj} = F_{jl} \rho_l$ and

$$g_r(\mathbf{a}) = [F_{jl} \rho_l + H_{ij} h_i] S_j \equiv \xi_j S_j \equiv \underline{\xi}^T \underline{S} . \quad (10)$$

The geoid contribution to eq. (6) is thus

$$\begin{aligned}
&[(c_i - \xi_i) W_{ij}^{ge} (c_j - \xi_j)] \\
&= \{c_i - F_{il} \rho_l - H_{im} h_m\} W_{ij}^{ge} \{c_j - F_{jl} \rho_l - H_{jm} h_m\} \\
&= [c - F\rho - Hh]^T W^{ge} [c - F\rho - Hh] . \quad (11)
\end{aligned}$$

3.4 Seismic

The predicted (summary) travel time perturbation τ_i is the perturbation slowness $\psi^i(r)$ integrated along the ray path $L(i)$ for phase $\zeta(i)$ from source $r'(i)$ to receiver $r(i)$

$$\tau_i = \int_{r'(i)}^{r(i)} \psi^i(r) dL_i$$

No sum is performed over superscripts. For local (P or S) phase speed $V = V_0 + \Delta V = (s_0 + \psi)^{-1}$, $s_0 \simeq 1/V_0$ and $\psi \simeq -\Delta V/V_0^2$ to first order in $|\Delta V|/V_0 \ll 1$. The suggestion here is to represent $\psi(r)$ in a manner similar to $\rho'(r)$. Then

$$\psi^i(r_k < r \leq r_{k+1}, \theta, \phi) = [\psi^i(r_k < r \leq r_{k+1})]_j S_j(\theta, \phi) = \psi_{kj}^i S_j$$

where ψ_{kj}^i is the j th spherical harmonic coefficient of the slowness in the k th layer for phase $\zeta(i)$. The path $L(i)$ will depend upon $\psi^i(r)$, so the inverse problem is non-linear. I suggest both descending and ascending path segments of respective lengths L_{ki}^d and L_{ki}^a in layer k be determined initially from the mean state and subsequently from the results of the previous deep iteration. These ray path segment lengths are assigned to the mean location of the segment $(\theta_{ki}^i, \phi_{ki}^i)$. In this approximation, with the sum over k made explicit for descending and ascending segments,

$$\tau_i = \sum_{k=1}^K [\psi_{kj}^i S_j(\theta_k, \phi_k)] L_{ki}^d + \sum_{k=K}^1 [\psi_{kj}^i S_j(\theta_k, \phi_k)] L_{ki}^a - u_i h_j S_j(\theta_u, \phi_u) - v_i h_j S_j(\theta_v, \phi_v)$$

where, to account for the effect of CMB topography on the path length, (i) $u_i = v_i = 0$ for paths not touching the CMB (e.g., P, S, PP, PS, etc.); (ii) $u_i = s_0(r_0=c^+)/\sin\lambda_u$ and $v_i = s_0(r_0=c^+)/\sin\lambda_v$ for reflections with incident angle λ_u and reflected angle λ_v at $(\theta_u, \phi_u) = (\theta_v, \phi_v)$ (e.g., PcP, PcS, etc.); and (iii) $u_i = [s_0(c^-) - s_0(c^+)/\sin\lambda_u$ and $v_i = [s_0(c^-) - s_0(c^+)/\sin\lambda_v$ for phases leaving the mantle with incident angle λ_u at (θ_u, ϕ_u) and reentering the mantle with exit angle λ_v at (θ_v, ϕ_v) (e.g., PKP, PKS, etc.). Note λ_u and λ_v can be corrected using previous estimates of h .

The foregoing expression for τ_i can be rewritten as

$$\tau_i = \sum_{j,k}^d \psi_{kj}^i + \sum_{j,k}^a \psi_{jk}^i - M_{ij}^u h_j - M_{ij}^v h_j$$

with the understanding that for seismic phase i on path $L(i)$ between source $r^i(i)$ and receiver $r^i(i)$ Z_{ijk}^d (or Z_{ijk}^a) is the length of the

descending (or ascending) ray path segment in layer k times spherical harmonic j evaluated at the segment midpoint, while MU_{ij} (or MV_{ij}) is the travel time correction due to CMB topography at the point of core entry (or exit). This expression is more compactly represented by concatenating (i) the $\underline{\underline{Z}}^d$ and $\underline{\underline{Z}}^a$ tensors into $\underline{\underline{Z}}$, (ii) the $\underline{\underline{\psi}}^i$ matrices for P and S slownesses into $\underline{\underline{\psi}}$, and (iii) the $\underline{\underline{M}}^u$ and $\underline{\underline{M}}^v$ matrices into $\underline{\underline{M}}$; then reorder the elements of $\underline{\underline{\psi}}$ (and $\underline{\underline{Z}}$) into vector $\underline{\psi}$ (and matrix \underline{D}):

$$\tau_i = D_{ij}\psi_j + M_{ij}h_j . \quad (12a)$$

There remains the thorny problem of relating perturbation slowness to perturbation density. The differential slownesses for P-waves or S-waves are

$$\begin{aligned} d\psi^P &= d(v_p)^{-1} = -v_p^{-2}dv_p \\ d\psi^S &= d(v_s)^{-1} = -v_s^{-2}dv_s . \end{aligned}$$

With bulk and shear moduli K and μ , $v_p^2 = (K + 4\mu/3)/\rho$ and $v_s^2 = \mu/\rho$, so

$$\begin{aligned} d\rho &= v_s^{-2}d\mu - 2\rho v_s^{-1}dv_s \\ d\rho &= v_p^{-2}dK + (4/3)v_p^{-2}d\mu - 2\rho v_p^{-1}dv_p \end{aligned}$$

which, upon linearization about the mean state $(v_{p0}, v_{s0}, \rho_0, K_0, \mu_0)$, are viewed as two equations in the three unknowns $d\rho$, dK , and $d\mu$. We can solve for

$$d\mu = \rho K^{-1}[v_s^2 dK + 2\rho v_p v_s (v_s dv_s - v_p dv_p)]$$

where $(v_p^2 - 4v_s^2/3) = K/\rho = (\partial p/\partial \rho)_{ad}$ is the adiabatic sound speed.

Unfortunately, $d\rho$ cannot be determined without additional information on dK (or $d\mu$).

The suggestion is to treat perturbation slowness as if directly proportional to perturbation density; however, the constant of

proportionality may vary radially (with the mean state). Lateral variations of this 'constant' are omitted for simplicity – as are anisotropies in the fourth rank-tensor of elastic constants and the complexities of attenuation. Then the perturbation density coefficient for spherical harmonic j in layer k is

$$\rho_{kj} = [C_{kk'}^p]^{-1} \psi_{k'j}^p = [C_{kk'}^s]^{-1} \psi_{k'j}^s$$

or, in the vector notation,

$$\psi_j = C_{jj}^{-1} \rho_j \quad (12b)$$

The elements of the diagonal matrix C are the different constants in each layer – be it mainly olivene; olivene-spinel; ferro-magnesian silicate perovskite; stishovite, non-stoichiometric ferro-magnesian wüstites, and ferro-silicides; or iron (Knittle & Jeanloz, 1989).

If, within each layer, perturbations in temperature, pressure, and composition were directly proportional to density perturbations and if the partial derivatives of K with respect to temperature, pressure, and composition were laterally homogeneous, then (12b) would be fully justified. Hopefully, the information on dK needed for a more realistic equation of state can be obtained from mineral physics or mantle dynamics. Because $d\rho$, $d\mu$, and dK , hence dV_s and dV_p , are caused by temperature, pressure, and composition perturbations associated with departures from the mean state of hydrostatic equilibrium with an adiabatic temperature gradient, both a perturbation equation of state and the equations of motion for the mantle will likely be needed. Then mantle circulation will have to be parameterized, the parameters included, and estimates thereof constrained to fit plate motion and

deformation data. (A parameterization of mantle circulation might also be tied to a parameterization of elastic anisotropy).

In the interim, eqns. (12b) and (12a) allow the seismic portion of eq. (6) to be written

$$\begin{aligned}
 [(T_i - \tau_i) W_{ij}^S (T_j - \tau_j)] &= [T_i - D_{ij} C_{111}' \rho_1' - M_{im} h_m] W_{ij}^S \\
 &\quad [T_j - D_{jl} C_{111}' \rho_1' - M_{jm} h_m] \\
 &= [T - DC\rho - Mh] T W^S [T - DC\rho - Mh]
 \end{aligned} \tag{13}$$

4. INVERSION

With parameterization eqns. (9), (11), and (13), the constraint enforcing the geostrophic radial vorticity balance written $(\underline{Bv})^T \underline{\Lambda}_m (\underline{Bv})$, and optional biases towards prior estimates of the fluid flow \underline{v}^0 , topography \underline{h}^0 , and density $\underline{\rho}^0$ (possibly from 'disjoint' inversions), the objective function given by eq. (6) becomes

$$\begin{aligned}
 \Delta^2 = & \overbrace{[\underline{\dot{g}} - \underline{Av}]^T \underline{W}^m [\underline{\dot{g}} - \underline{Av}]} + \overbrace{[\underline{\dot{\Omega}} - \underline{QThv} - \underline{q}]^T \underline{W}^{er} [\underline{\dot{\Omega}} - \underline{QThv} - \underline{q}]} \\
 & + \overbrace{[\underline{c} - \underline{F\rho} - \underline{Hh}]^T \underline{W}^{ge} [\underline{c} - \underline{F\rho} - \underline{Hh}]} \\
 & + \overbrace{[\underline{T} - \underline{DC\rho} - \underline{Mh}]^T \underline{W}^S [\underline{T} - \underline{DC\rho} - \underline{Mh}]} \\
 & + \overbrace{(\underline{Bv})^T \underline{\Lambda}_m (\underline{Bv})} + \overbrace{(\underline{v} - \underline{v}^0)^T \underline{\Lambda}_v (\underline{v} - \underline{v}^0)} \\
 & + \overbrace{(\underline{h} - \underline{h}^0)^T \underline{\Lambda}_h (\underline{h} - \underline{h}^0)} + \overbrace{(\underline{\rho} - \underline{\rho}^0)^T \underline{\Lambda}_\rho (\underline{\rho} - \underline{\rho}^0)}.
 \end{aligned} \tag{14}$$

In addition to the non-linearity explicit in the Earth rotation term ($\underline{Qh}v = \underline{Q}^h v = \underline{Q}v h$), recall that \underline{A} depends upon $\underline{\gamma}$, hence \underline{v} ; \underline{q} depends upon \underline{w} , hence both \underline{h} and \underline{v} ; and \underline{D} and \underline{M} depend upon $\underline{\psi}$, hence upon $\underline{\rho}$ and \underline{h} . The minimization of Δ^2 is therefore a profoundly non-linear problem.

The attack on this problem offered here is based on iterative solution of the linearized problem.

Initial estimates of \underline{A} , \underline{q} , \underline{D} and \underline{M} can be calculated from either the 'data' or the mean state. Initial estimates of \underline{Q}^h and \underline{Q}^v can be calculated from models of \underline{h} and \underline{v} obtained by 'disjoint' inversion. Use the initial estimates \underline{v}^i , \underline{h}^i , and $\underline{\rho}^i$ with $i = 1$ (or 0 if necessary) to solve the forward problems of piecewise steady motional induction, changes in Earth rotation, geoid determination, and calculation of travel times. Such forward calculations give the predictions $\underline{\gamma}$, $\underline{\omega}$, $\underline{\xi}$, and $\underline{\tau}$. The differences between the 'data' \underline{g} , $\underline{\Omega}$, \underline{c} , and \underline{T} and these predictions are the residuals $\delta\underline{g}$, $\delta\underline{\Omega}$, $\delta\underline{c}$, and $\delta\underline{T}$. These residuals provide the 'data' for the first joint inversion ($\delta\underline{g}$, $\delta\underline{\Omega}$, $\delta\underline{c}$, and $\delta\underline{T}$). They also define the residual objective function, which is parameterized in terms of prior estimates (\underline{v}^0 , \underline{h}^0 , $\underline{\rho}^0$), initial values (\underline{v}^i , \underline{h}^i , $\underline{\rho}^i$), and parameter corrections ($\delta\underline{v}^i$, $\delta\underline{h}^i$, $\delta\underline{\rho}^i$). The residual objective function (not shown) is minimal only if it is extreme, so parameter corrections can be estimated by setting the partial derivatives of it with respect to the parameter corrections equal to zero.

In the linearized treatment, the elements of \underline{A} , \underline{q} , \underline{Q}^h , \underline{Q}^v , \underline{D} , and \underline{M} in the residual objective function are treated as if independent of the parameter corrections. Then the partial differentiation is easy and, to first order, the corrected parameters are the initial parameters (\underline{v}^i , \underline{h}^i , $\underline{\rho}^i$) plus

$$\delta\underline{v} = \left\{ \begin{array}{c} \underline{A}^T \underline{W} \underline{A} + \underline{Q}^h \underline{T} \underline{W} \underline{Q}^h + \underline{B}^T \underline{\Lambda} \underline{M} \underline{B} + \underline{\Lambda} \underline{V} \end{array} \right\}^{-1} \left\{ \begin{array}{c} \underline{A}^T \underline{W} \delta\underline{g} + \underline{Q}^h \underline{T} \underline{W} [\delta\underline{\Omega} - \underline{Q}^v \delta\underline{h} - \underline{q}] - \underline{B}^T \underline{\Lambda} \underline{M} \underline{B} \underline{v}^i - \underline{\Lambda} \underline{V} [\underline{v}^i - \underline{v}^0] \end{array} \right\} \quad (15a)$$

$$\delta \underline{h} = \left\{ \underline{Q} \underline{v} \underline{T} \underline{W}^{er} \underline{Q} \underline{v} + \underline{H} \underline{T} \underline{W}^{ge} \underline{H} + \underline{M} \underline{T} \underline{W}^s \underline{M} + \underline{\Lambda}_h \right\}^{-1} \left\{ \underline{Q} \underline{v} \underline{T} \underline{W}^{er} [\delta \dot{\underline{n}} - \underline{Q} \underline{h} \delta \underline{v} - \underline{q}] + \underline{H} \underline{T} \underline{W}^{ge} (\delta \underline{c} - \underline{F} \delta \underline{\rho}) + \underline{M} \underline{T} \underline{W}^s [\delta \underline{T} - \underline{D} \underline{C} \delta \underline{\rho}] - \underline{\Lambda}_h [\underline{h}^i - \underline{h}^0] \right\} \quad (15b)$$

$$\delta \underline{\rho} = \left\{ \underline{F} \underline{T} \underline{W}^{ge} \underline{F} + [\underline{D} \underline{C}] \underline{T} \underline{W}^s [\underline{D} \underline{C}] + \underline{\Lambda}_s \right\}^{-1} \left\{ \underline{F} \underline{T} \underline{W}^{ge} [\delta \underline{c} - \underline{H} \delta \underline{h}] + [\underline{D} \underline{C}] \underline{T} \underline{W}^s [\delta \underline{T} - \underline{M} \delta \underline{h}] - \underline{\Lambda}_\rho [\underline{\rho}^i - \underline{\rho}^0] \right\} \quad (15c)$$

Clearly, equations (15) are not fully reduced because of the terms $\underline{Q} \underline{v} \delta \underline{h}$ (15a), $\underline{Q} \underline{h} \delta \underline{v}$ and $\underline{F} \delta \underline{\rho}$ (15b), and $\underline{H} \delta \underline{h}$ and $\underline{M} \delta \underline{h}$ (15c) on the right. However, (15a) can be used to eliminate $\delta \underline{v}$ from (15b) and (15c) can be used to eliminate $\delta \underline{\rho}$ from (15b). The resulting expression can be solved for $\delta \underline{h}$. Then $\delta \underline{v}$ and $\delta \underline{\rho}$ can be determined by back substitution into (15a) and (15c).

To make this clear, symbolically rewrite eqns. (15a-c) as

$$\delta \underline{v} = \underline{V}^{-1} [\underline{G} - \underline{E} \delta \underline{h}] \quad (16a)$$

$$\delta \underline{h} = \underline{T}^{-1} [\underline{C} - \underline{U} \delta \underline{v} - \underline{S} \delta \underline{\rho}] \quad (16b)$$

$$\delta \underline{\rho} = \underline{R}^{-1} [\underline{P} - \underline{X} \delta \underline{h}] \quad (16c)$$

Then

$$\delta \underline{h} = [\underline{T} - \underline{U} \underline{V}^{-1} \underline{E} - \underline{S} \underline{R}^{-1} \underline{X}]^{-1} [\underline{C} - \underline{U} \underline{V}^{-1} \underline{G} - \underline{S} \underline{R}^{-1} \underline{P}] \quad (17)$$

Substitution of eq. (17) into (16a) yields $\delta \underline{v}$; substitution of (17) into (16c) yields $\delta \underline{\rho}$. Clearly CMB topography is assigned the pivotal role.

The corrected parameters can be used to solve the forward problem again and obtain new residuals. Then the corrections can be estimated again. After a satisfactory number of such "shallow" iterations, the

elements of \underline{A} , \underline{q} , \underline{Q}^v , \underline{Q}^h , \underline{D} , and \underline{M} can be recalculated using the predicted values $\underline{\gamma}(t)$, $\underline{w}(t)$, $\underline{\xi}$, and $V_p(r)$ and $V_s(r)$. Such recalculation is termed "deep" iteration; it might even include earthquake relocation. Then shallow iteration can be tried again, followed by another deep iteration, etc.. Shallow iteration is merely intended to provide a refined set of corrected parameters for use in the seemingly more burdensome recalculation of matrix elements required for deep iteration; it is considered optional for the small parameter corrections anticipated. Shallow iteration allows more use of residuals calculated by accurate numerical solution of the (time-dependent, non-linear) forward problem; such residuals ought not be replaced by coarse linear approximations. Clearly, the iteration process can be repeated until either an adequate fit is obtained or the Earth model is abandoned.

Convergence of the iteration scheme is, of course, assured if the biases favoring prior estimates \underline{v}^0 , \underline{h}^0 , and $\underline{\rho}^0$ (typically measured by the diagonal elements of positive definite $\underline{\Lambda}_v$, $\underline{\Lambda}_h$, and $\underline{\Lambda}_\rho$) are strong enough. Convergence is apparently neither prohibited nor guaranteed when either confidence in the prior estimates is eroded or when the prior estimates are replaced with the most recent estimate. In the latter case (which seems consistent with deep iteration), the $(\underline{\Lambda}_v, \underline{\Lambda}_h, \underline{\Lambda}_\rho)$ might serve as convergence factors which keep the corrections so small as to avoid severe violation of the linearization when seeking small weighted residual variance. More sophisticated methods of non-linear optimization are possible, but lie outside the scope of this paper. Even in the linearized case, care is needed to avoid baseless bias, over-parameterization, and confusion of anticipated parameter error estimates (from the covariance) with the significance of the residuals.

5. DISCUSSION

To obtain such a very simple deep Earth model complete through harmonic degree and order 10 with a 9-layer mantle, at least 1,440 parameters need to be determined by iterative linearized least squares: 120 coefficients representing the CMB topography, 240 coefficients representing the core velocity field per time interval, and 1,080 coefficients describing the perturbation density. Symmetric storage of a 1440 x 1440 symmetric matrix requires but 10^6 words of computer storage. If more than one interval is considered, then more core flow coefficients are needed. For a 10th degree, single-interval model with 29 layers, there would be 3,840 parameters; symmetric storage of a 3,840 x 3,840 matrix would require 7.4×10^6 words. Such matrices can be manipulated and, if well-conditioned, inverted with existing computers.

The condition of the matrix depends upon the "data" selected, the assigned weights, the number of parameters estimated, and the confidence assigned to prior estimates. Note that truncation of the model need not suppose that higher degree parameters are zero; only that such unmodeled parameters contribute to the residuals and may contribute to model error. If the weights are to reflect covariance of unmodeled signal as well as data error covariance, expectations regarding unmodeled signal may be developed by studying the residuals obtained during numerical experimentation and, of course, the unmodeled processes themselves.

In the very simple Earth model considered here, CMB topography provides the essential link between the diverse geophysical data types. Of course it is by no means clear that the connection between the magnetic side of the problem and either the geoid or the seismic sides of the problem is strong enough to warrant detailed calculations. This

connection is provided only through the three components of the decade fluctuations in earth rotation. Yet this connection can be strengthened by including the kinematic effect of CMB topography on core flow and thus the predicted secular geomagnetic variation. Furthermore, this connection might be strengthened upon parameterization of a more realistic Earth model, inclusion of more data types, and application of more physical constraints – as outlined in the introduction.

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