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NASA Technical Memorandum 103723

# Turbulent Fluid Motion

## Part I—The Phenomenon of Fluid Turbulence

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February 1991

**NASA**



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PART I - The Phenomenon of Fluid Turbulence

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SUMMARY

Some introductory material on fluid turbulence is presented. This includes discussions and illustrations of what turbulence is, and how, why, and where turbulence occurs.

TURBULENT FLUID MOTION

*The literature on turbulence is now far too voluminous for anything like a full presentation to be given in a moderately-sized series of reports. Rather it will be attempted here to give a coherent account of one line of development. Part of this has been given in abbreviated form in Chapter 7 of Handbook of Turbulence, Volume 1 (Plenum, 1977). The purpose of the present series is to expand on this material so as to make it reasonably self-contained, and to supplement it with more recent research. In particular, the scope of the work, which was somewhat limited by our inability to solve the fundamental nonlinear equations analytically, has been considerably increased by numerical solutions. Moreover, applications of dynamical systems theory in conjunction with numerical solutions have resulted in, among other things, a sharper characterization of turbulence and a delineation of routes to turbulence.*

*Throughout the series the emphasis will be on understanding the physical processes in turbulent flow. This will be done to a large extent by obtaining and interpreting analytical or numerical solutions of the equations of fluid motion. Although the analytical solutions tend to be for somewhat idealized cases, the numerical solutions are less so. No attempt will be made to either emphasize or avoid the use of mathematical analysis. Since most of the material is given in some detail, the student or research worker with a modest knowledge of fluid mechanics should not find the text particularly hard to follow. Some familiarity with Cartesian-tensor notation and Fourier analysis may be helpful, although background material in those subjects will be given.*

*The basis of the present account of turbulence is the Navier-Stokes and other continuum equations for fluids. It is hoped that the series will show how those equations can act as a unifying thread for such an account.*

THE PHENOMENON OF FLUID TURBULENCE

Consider the steady unidirectional flow of a fluid through a smooth pipe of length  $L$ . The pipe is long enough that the flow can be considered fully developed (velocity changes along the pipe, negligible). The flow is incompressible and constant-viscosity. We balance the pressure and shear forces on a cylinder of fluid of radius  $r$  and length  $L$ , and assume that the fluid is Newtonian ( $\sigma = -\mu dU/dr$ , where  $\sigma$  is the shear stress,  $\mu$  is the viscosity, and  $U$  is the velocity), and that  $U = 0$  at the wall. It is then easy to show that the pressure drop  $\Delta p$  along a pipe of diameter  $D$  and length  $L$  is

$$\Delta p = \frac{128 \mu Q L}{\pi D^4}, \quad (1-1)$$

where  $Q$  is the volume of fluid passing any cross-section of the pipe in unit time (ref. 1).

Equation (1-1) accurately predicts the pressure drop along a pipe provided the Reynolds number  $U_a D / \nu$  is less than about 2000 ( $U_a$  is the area-averaged or bulk velocity and  $\nu$  is the kinematic viscosity). For higher Reynolds numbers equation (1-1) underestimates the pressure drop, unless care is taken to eliminate disturbances. For instance, for Reynolds numbers of  $10^4$ ,  $10^5$ , and  $10^6$  equation (1-1) underestimates  $\Delta p$  by factors of about 5, 30, and 200, respectively. The deviations from equation (1-1), particularly at high Reynolds numbers, are far from being small. Similar deviations are found if a steady-state analysis is used to predict the heat transfer when a temperature difference occurs between the pipe wall and the fluid.

The deviations just noted were first explained by the classical experiment of Osborne Reynolds (ref. 2). He injected dye at the smooth entrance of a glass tube through which water flowed. Results similar to those shown in figure 1-1 were obtained. At the low Reynolds number the stream of dye remains intact, indicating that the flow is laminar and unidirectional. At the higher Reynolds number the dye is dispersed, showing that the flow is no longer unidirectional. The fluid evidently moves vigorously in the transverse directions, as well as longitudinally, in order to promote the high degree of mixing observed. Thus the high pressure drops noted in the last paragraph appear to be caused by lateral momentum transfer which occurs as the fluid moves transversely. This transverse mixing can produce an effective viscosity which is many times the laminar value, and according to equation (1-1) (if  $\mu$  designates an effective viscosity), the pressure drop must increase.

The use of equation (1-1) to explain the increase in pressure drop at high Reynolds numbers is, however, only roughly justified. Equation (1-1) is derived by assuming that the viscosity (or effective viscosity) is independent of radius. In actuality, the transverse fluid motions must be zero at the solid wall of the pipe, so that the effective viscosity equals the molecular or laminar value at the wall and increases with distance from the wall. Rather than the parabolic constant-viscosity profile shown in figure 1-2(a) the local-mean velocity exhibits a profile which is flattened in the center and steep at the wall, as in figure 1-2(b). Since the effective viscosity at the wall is  $\mu$ , as in laminar flow, the shear stress at the wall is still given by  $-\mu du/dr$  but is greater than that for laminar flow because the velocity gradient at the wall is greater. The pressure drop required to balance the increased shear stress at the wall is then greater than that for laminar flow.

### 1.1 WHAT IS TURBULENCE?

As suggested by the appearance of the dye in figure 1-1(b), fluid motion at higher Reynolds numbers tends to be haphazard and random. This randomness is seen more clearly in figure 1-3, where an instantaneous velocity component at a point in a high-Reynolds-number boundary layer is plotted against time. In addition to the random appearance of the plot, as indicated by a visual inspection, it is found that a very small perturbation of the initial velocity



fluctuations produces a completely different instantaneous velocity pattern a short time later, although the mean velocity at a point, and other mean quantities associated with the flow, are not appreciably changed. This is known as sensitive dependence on initial conditions. Moreover, a number of scales of motion appear to be present simultaneously in figure 1-3, where small-scale fluctuations are superimposed on the large-scale ones.

Finally, the skewness factor of the velocity derivative of the flow  $(\overline{\partial u_1 / \partial x_1})^3 / (\overline{\partial u_1 / \partial x_1})^2^{3/2} = S$  is generally negative for nonlinear flows (evidently  $0 \geq S > -1$  for turbulent flow (ref. 5, fig. 1)), where  $u_1$  is the fluctuating velocity component in the  $x_1$ -direction and the overbars indicate averaged values. A negative  $S$  indicates that in general, energy is passed from large scales of motion to small ones. The passing of energy among scales of motion could account for the range of length scales in figure 1-3. High-Reynolds-number flows with characteristics such as those described here and depicted in figures 1-1 to 1-3 are generally called turbulent. The apparently random fluctuations which occur in such flows comprise what is known as turbulence.

Although turbulence is generally characterized as random, it contains a deterministic element. This is because the equations of fluid motion which describe turbulence are deterministic (no random coefficients). Much of the recent work related to turbulence is on something called deterministic chaos, which arises in the solutions of deterministic differential equations. The solutions are chaotic because they are extremely sensitive to small changes in initial conditions. In fact, sensitive dependence on initial conditions, which we gave as a characteristic of turbulence, is, strictly speaking, a characteristic of deterministic chaos. However, it has been shown that, at least for typical cases, fluid turbulence is chaotic (refs. 6 and 7). It appears reasonable to consider chaoticity as a characteristic of turbulence in general. In order to distinguish between chaos and turbulence, chaos is sometimes designated as having complexity in time, and turbulence as having complexity in time and space. More will be said about deterministic chaos in a later report.

## 1.2 THE UBIQUITY OF TURBULENCE

Turbulence phenomena are by no means confined to flows in pipes and boundary layers. In fact, laminar flows in nature, as well as man-made laminar flows, appear to be the exception rather than the rule. For instance the boundary between a column of rising smoke and the surrounding atmosphere is generally irregular and contains a range of scales of motion, thus indicating the presence of turbulence (fig. 1-4). Similar, perhaps even more striking examples of multiscale turbulence, are shown by a volcano erupting into the atmosphere (fig. 1-5), and by the cloud accompanying a space shuttle launch (fig. 1-6). Note the multiscale granular structure of the turbulence in both of those figures. The atmosphere itself is often turbulent, as shown by the irregular appearance of many of the clouds present in it (figs. 1-7 and 1-8). Jets, wakes, and boundary layers are often turbulent (figs. 1-9 to 1-12), as are the regions downstream of a grid in a wind tunnel (fig. 1-13) or downstream of a waterfall. Finally, in order to indicate the all-pervasive character of turbulence, we note that most plasmas and flames, as well as most biological and astrophysical flows (fig. 1-14) are turbulent.



### 1.3 WHY DOES TURBULENCE OCCUR?

Before leaving this introductory discussion, it is instructive to attempt to give at least a rough explanation for the occurrence of turbulence. Turbulence occurs as a result of instabilities in a flow. An instability occurs when a perturbation of the flow grows with time. In the examples considered so far, turbulence appears to arise from instabilities produced by layers of fluid sliding over one another or by buoyancy forces.

The former of these is illustrated in figure 1-15. The fluid in the upper part of the sketch moves to the right, that in the lower part to the left. This motion might occur either because of a velocity discontinuity at the central plane or a continuous velocity gradient. In the latter case the fluid at the central plane or streamline is stationary. If, for some reason a portion of fluid moves upward, so that the central streamline becomes convex upward as shown, the velocity of the fluid in the upper part of the figure, as it flows over the curved streamline, tends to be increased (streamlines are closer together), and that in the lower part decreased (streamlines are farther apart). If we neglect the effect of viscosity, Bernoulli's equation ( $U^2/2 + p/\rho = \text{const}$ , where  $U$  is the velocity and  $\rho$  is the density) shows that the pressure above the streamline is reduced and that below it increased, so that the fluid tends to continue moving upward. Similarly a portion of fluid displaced downward tends to continue downward. That is, the flow is unstable, and conditions favorable for the development of turbulence exist.

Figure 1-16 shows how a density or temperature gradient (density increasing upward) can interact with the downward force of gravity. If a portion of the fluid moves upward ( $U_3$ , positive) it will be lighter than the surrounding fluid, and so buoyancy forces tend to cause it to continue moving upward. Similarly a portion of fluid which moves downward ( $U_3$ , negative) tends to continue moving downward, and once again destabilizing forces tend to promote turbulence.

Of course we have not considered the effect of viscosity, and viscous forces may in some cases be stronger than the destabilizing ones. In our earlier example, for instance, we indicated that turbulent flow is sustained in a pipe only for Reynolds numbers greater than about 2000. Thus our simplified analyses do not show that in a particular case turbulence will develop. They only point out that there may be destabilizing forces which tend to produce turbulence. The prediction of whether or not turbulence will actually be produced or maintained in a given situation is a much more complex problem and is an active field of research (see e.g., ref. 15, for *transition to turbulence*). More will be said about the *maintenance* of turbulence in a later report.

### 1.4 CLOSING REMARKS

Needless to say, the occurrence of turbulence greatly complicates the work of the fluid dynamicist. It appears to be responsible in large measure for maintaining fluid dynamics as an active branch of theoretical physics. In spite of important work in this field carried out over the last century by many eminent scientists (Reynolds, Taylor, Prandtl, Heisenberg, Von Kármán, Kampé DeFériet, and Kolmogorov, to name a few (refs. 2 and 16 to 21)) much work remains to be done.

The difficulty in the problem of turbulence lies not so much in determining what equations are appropriate, as in using those equations to study turbulence. The required equations of motion, the Navier-Stokes and other fluid equations, are already known (see Turbulent Fluid Motion Part III, to appear). Because of their nonlinearity exact analytical solutions have, however, been obtained only in a few cases (usually linear cases). Nevertheless, as will be seen, those solutions and their interpretation can form the basis for studying many processes occurring in turbulence. For more complicated cases approximate analyses which are only partially based on the Navier-Stokes equations, have been used. Recently, however, the nonlinear problem of turbulence, at least for lower Reynolds numbers, has been studied by obtaining numerical solutions of the full time-dependent equations of motion. It appears that numerical methods can greatly increase the applicability of the Navier-Stokes and other fluid equations to turbulence and it may be that future progress will come about mainly through the use of those methods.

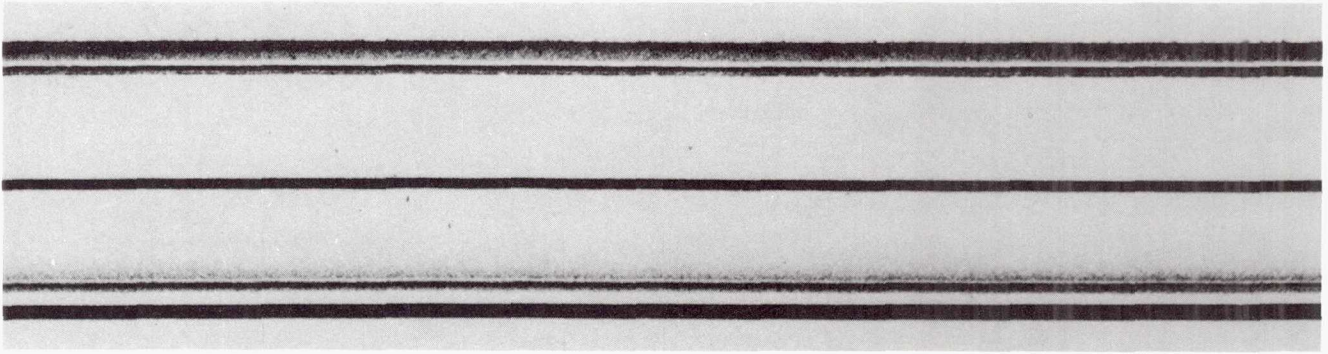
The present study of turbulence is couched in Cartesian-tensor notation, since the use of that notation greatly simplifies the analysis. Thus tensors and tensor notation will be considered in the next report of this series.

#### REFERENCES

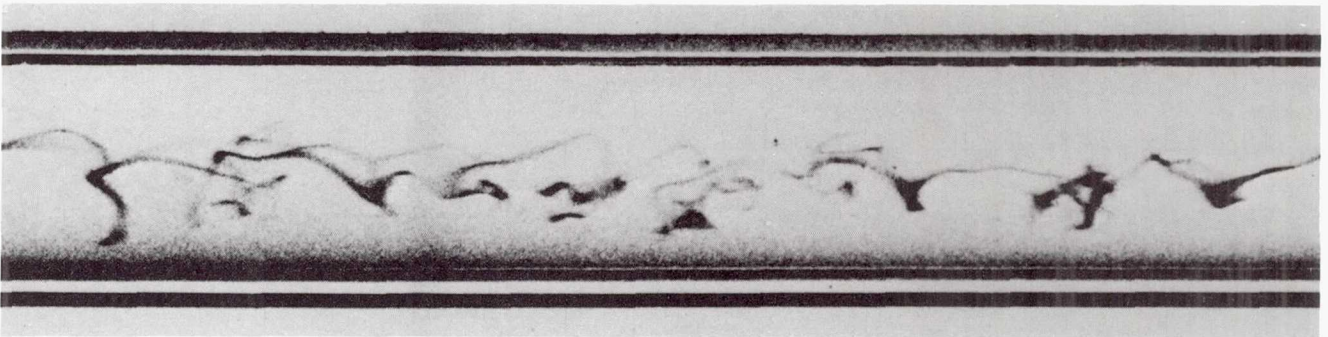
1. Goldstein, S.: Modern Developments in Fluid Dynamics. Vol. 1. Clarendon Press, Oxford, England, 1938, pp. 19-21.
2. Reynolds, O.: An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall be Direct or Sinuous, and the Law of Resistance in Parallel Channels. Philos. Trans. R. Soc. London, vol. 174, 1883, pp. 935-982.
3. Van Dyke, M.: An Album of Fluid Motion. Parabolic Press, Stanford, CA., 1982.
4. Frenkiel, F.N.; and Klebanoff, P.S.: Probability Distributions and Correlations in a Turbulent Boundary Layer. Phys. Fluids, vol. 16, no. 6, 1973, pp. 725-737.
5. Tavoularis, S.; Bennett, J.C.; and Corrsin, S.: Velocity-Derivative Skewness in Small Reynolds Number, Nearly Isotropic Turbulence. J. Fluid Mech., vol. 88, pt. 1, Sept. 13, 1978, pp. 63-69.
6. Deissler, R.G.: Is Navier-Stokes Turbulence Chaotic? Phys. Fluids, vol. 29, no. 5, 1986, pp. 1453-1457.
7. Brandstätter, A.; Swift, J.; Swinney, H.L.; Wolf, A.; Farmer, J.D.; Jen, E.; and Crutchfield, J.P.: Low-Dimensional Chaos in a Hydrodynamic System. Phys. Rev. Lett., vol. 51, no. 16, 1983, pp. 1442-1445.
8. Perry, A.E.; Lim, T.T.; Chong, M.S.; and Teh, E.W.: The Fabric of Turbulence. AIAA Paper 80-1358, July 1980.
9. Broad, W.: Threat to U.S. Airpower: The Dust Factor. Science, vol. 213, Sept. 25, 1981, p. 1476.



10. McGowan, J., ed.: NASA Activities. National Aeronautics and Space Administration, vol. 13, no. 5, 1982, Cover photograph.
11. Iribarne, J.V.; and Cho, H.R.: Atmospheric Physics. D. Reidel Publishing Company, 1980, p. 101.
12. Fiszdon, W.; Kucharczyk, P.; and Prosnak, W.J., eds.: Proceedings of the Fifteenth International Symposium on Fluid Dynamics. Polish Scientific Publishers, Warsaw, Poland, 1981.
13. Falco, R.E.: Coherent Motions in the Outer Region of Turbulent Boundary Layers. Phys. Fluids, vol. 20, no. 10, pt. II, Oct. 1977, pp. S124-S132.
14. Minkowski, R.: The Observational Background of Cosmical Gasdynamics, in Gasdynamics of Cosmic Clouds. Interscience Publishers Inc., New York; North Holland Publishing Company, Amsterdam; Elsevier Science Publishing Company, New York, 1955, p. 10.
15. Orszag, S.A.; and Kells, L.C.: Transition to Turbulence in Plane Poiseuille and Plane Couette Flow. J. Fluid Mech., vol. 96, pt. 1, Jan. 16, 1980, pp. 159-205.
16. Taylor, G.I.: Statistical Theory of Turbulence, pts. I-IV. Proc. R. Soc., A. vol. 151, 1935, pp. 421-478.
17. Prandtl, L.: Bericht über Untersuchungen Zur Ausgebildeten Turbulenz. Z. Angew. Math. Mech., vol. 5, no. 2, 1925, pp. 136-139.
18. Heisenberg, W.: Zur statistischen Theorie der Turbulenz. Z. Phys. A., vol. 124, 1948, pp. 628-657.
19. Von Kármán, T.: The Fundamentals of the Statistical Theory of Turbulence. J. Aeronaut. Sci., vol. 4, no. 4, Feb. 1937, pp. 131-138.
20. Kampé DeFériet, J.: Some Recent Researches on Turbulence. Proceedings of the Fifth International Congress for Applied Mechanics, J.P. Den Hartog and M. Peters, eds., Wiley, 1939, p. 352.
21. Kolmogorov, A.N.: Local Structure of Turbulence in an Incompressible Fluid at Very High Reynolds Numbers. Dokl. Akad. Nauk SSSR, vol. 30, no. 4, 1941, pp. 299-303.

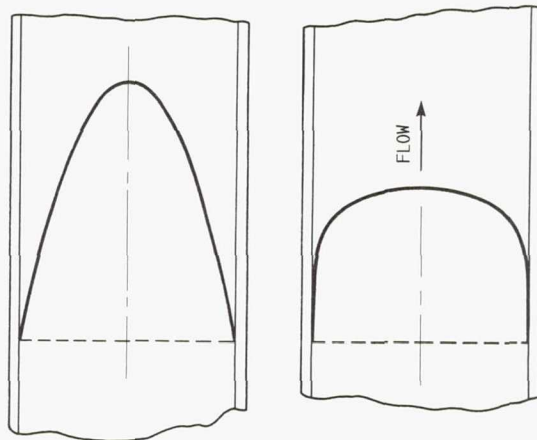


(a) LOW REYNOLDS NUMBER ( $\ll 2000$ ).



(b) HIGHER REYNOLDS NUMBER.

FIGURE 1-1. - EFFECT OF REYNOLDS NUMBER ON RIBBON OF DYE IN WATER FLOWING THROUGH A GLASS TUBE. PHOTOGRAPHS BY N.H. JOHANNESEN AND C. LOWE. REPRINTED BY PERMISSION OF PARABOLIC PRESS. (REF. 3).

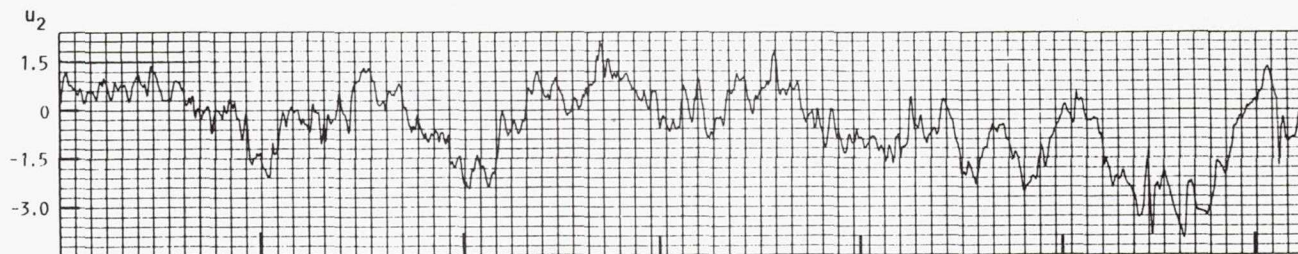


(a) LOW REYNOLDS NUMBER ( $\ll 2000$ ), PARABOLIC PROFILE.

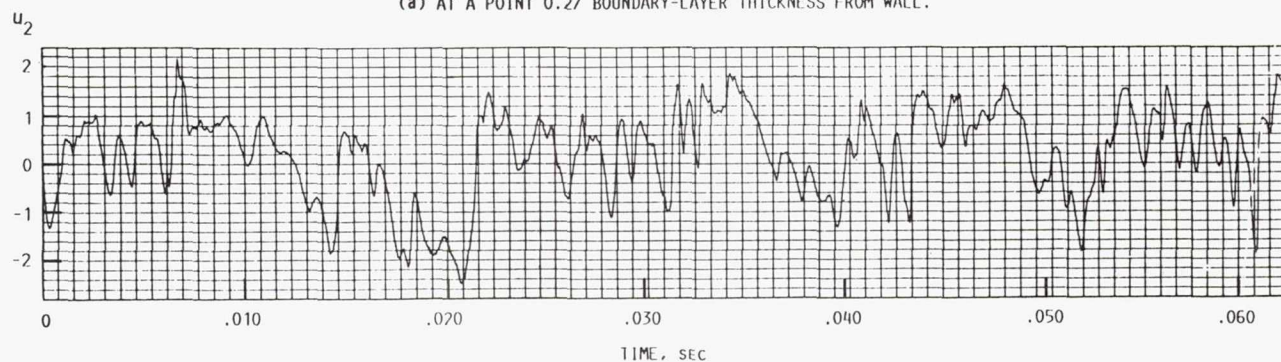
(b) HIGHER REYNOLDS NUMBER.

FIGURE 1-2. - LOCAL MEAN-VELOCITY PROFILES CORRESPONDING TO FLOWS IN FIGURE 1-1.





(a) AT A POINT 0.27 BOUNDARY-LAYER THICKNESSES FROM WALL.



(b) AT A POINT 0.0033 BOUNDARY-LAYER THICKNESSES FROM WALL.

FIGURE 1-3. - RECORDINGS OF INSTANTANEOUS TRANSVERSE VELOCITY COMPONENT IN A FLAT-PLATE TURBULENT BOUNDARY LAYER. NOTE THAT MOST OF THE SMALLER-SCALE FLUCTUATIONS ARE DAMPED OUT AT THE POINT CLOSER TO THE WALL. REPRINTED BY PERMISSION OF AMERICAN INSTITUTE OF PHYSICS. (REF. 4).

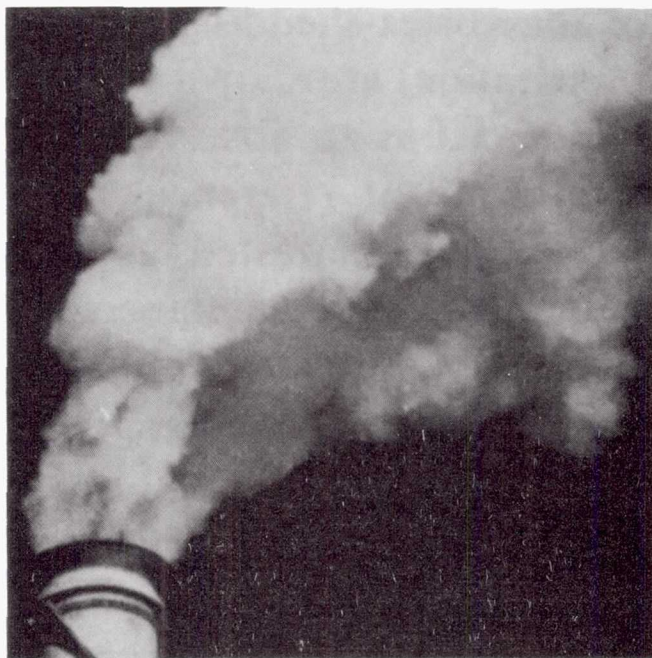


FIGURE 1-4. - COLUMN OF RISING TURBULENT SMOKE. REPRODUCED, WITH PERMISSION, FROM THE ANNUAL REVIEW OF FLUID MECHANICS, VOL. 13, 1981 BY ANNUAL REVIEWS INC. (REF. 8).

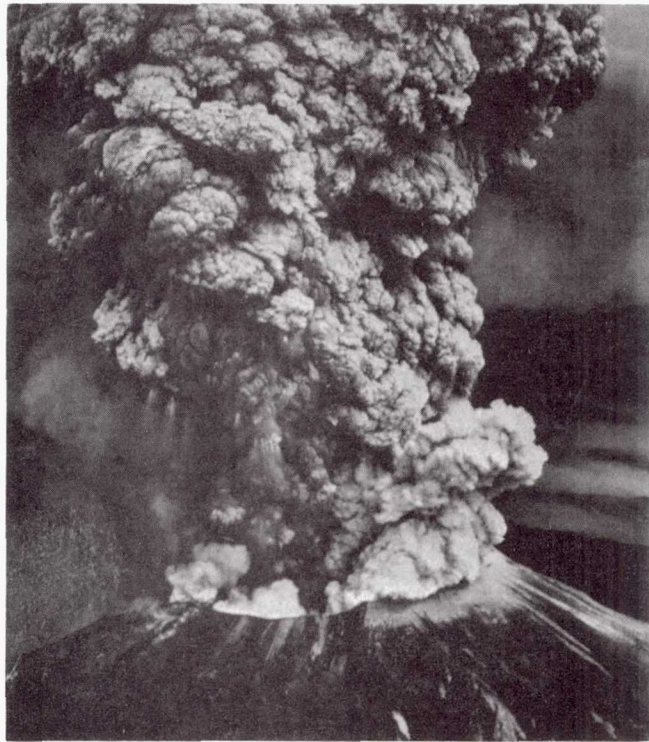


FIGURE 1-5. - TURBULENT ERUPTION OF MOUNT ST. HELENS ON MAY 18, 1980. NOTE THE MULTISCALE GRANULAR STRUCTURE OF THE TURBULENCE IN THIS AND THE FOLLOWING FIGURE. REPRINTED BY PERMISSION OF AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE. (REF. 9).





FIGURE 1-6. - MULTISCALE TURBULENT CLOUD AT A SPACE SHUTTLE LAUNCH (REF. 10).



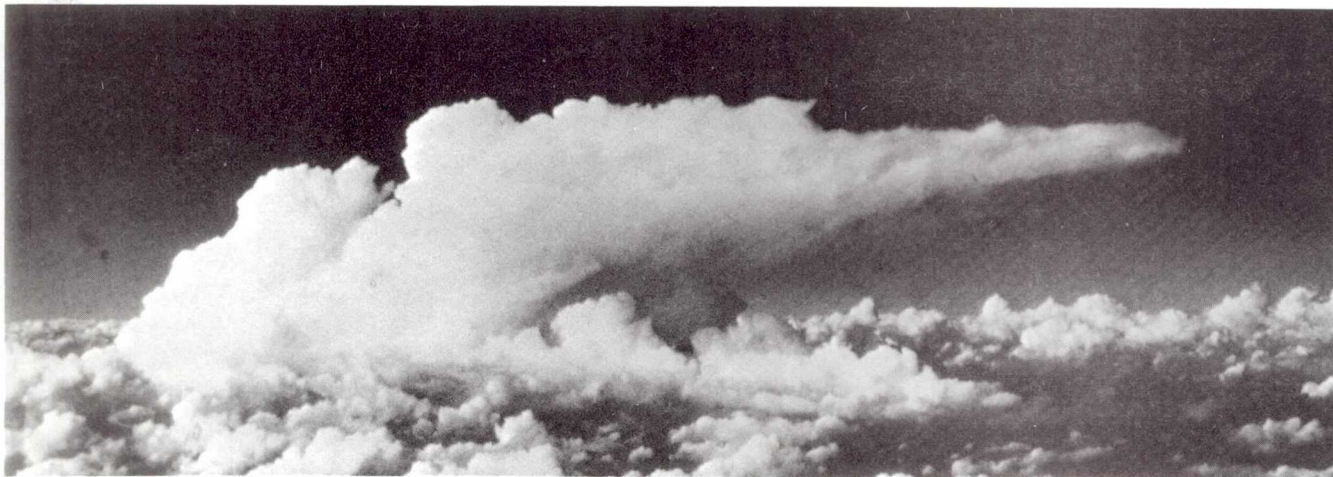


FIGURE 1-7. - TURBULENT ATMOSPHERIC CLOUDS. THE SHAPE OF THE LARGE CLOUD (ANVIL) IS DUE TO RISING AIR MASSES WITH A SUPERIMPOSED WIND SHEAR. REPRINTED BY PERMISSION OF KLUWER ACADEMIC PUBLISHERS. (REF. 11).

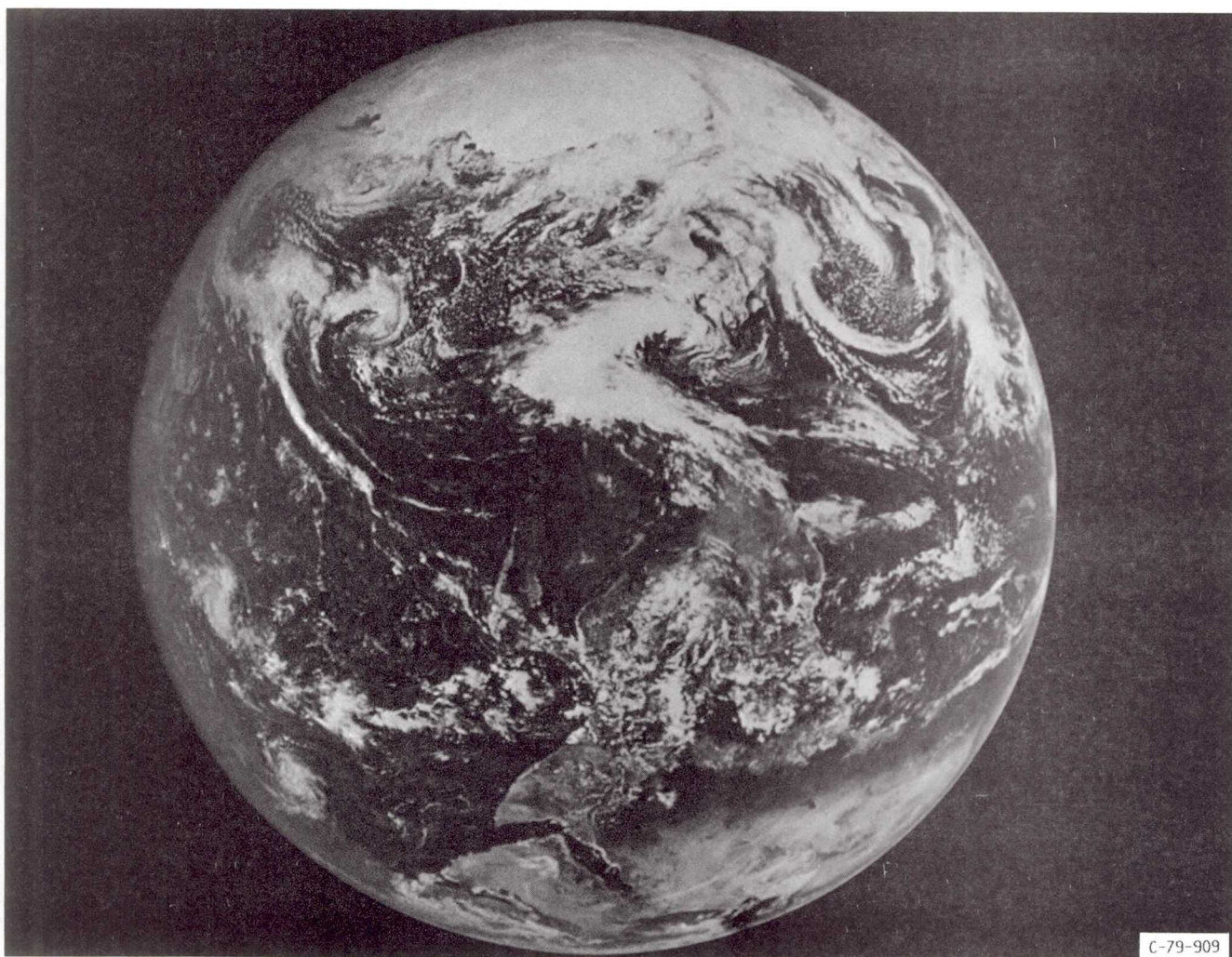


FIGURE 1-8. - TURBULENT EARTH (NASA PHOTOGRAPH).

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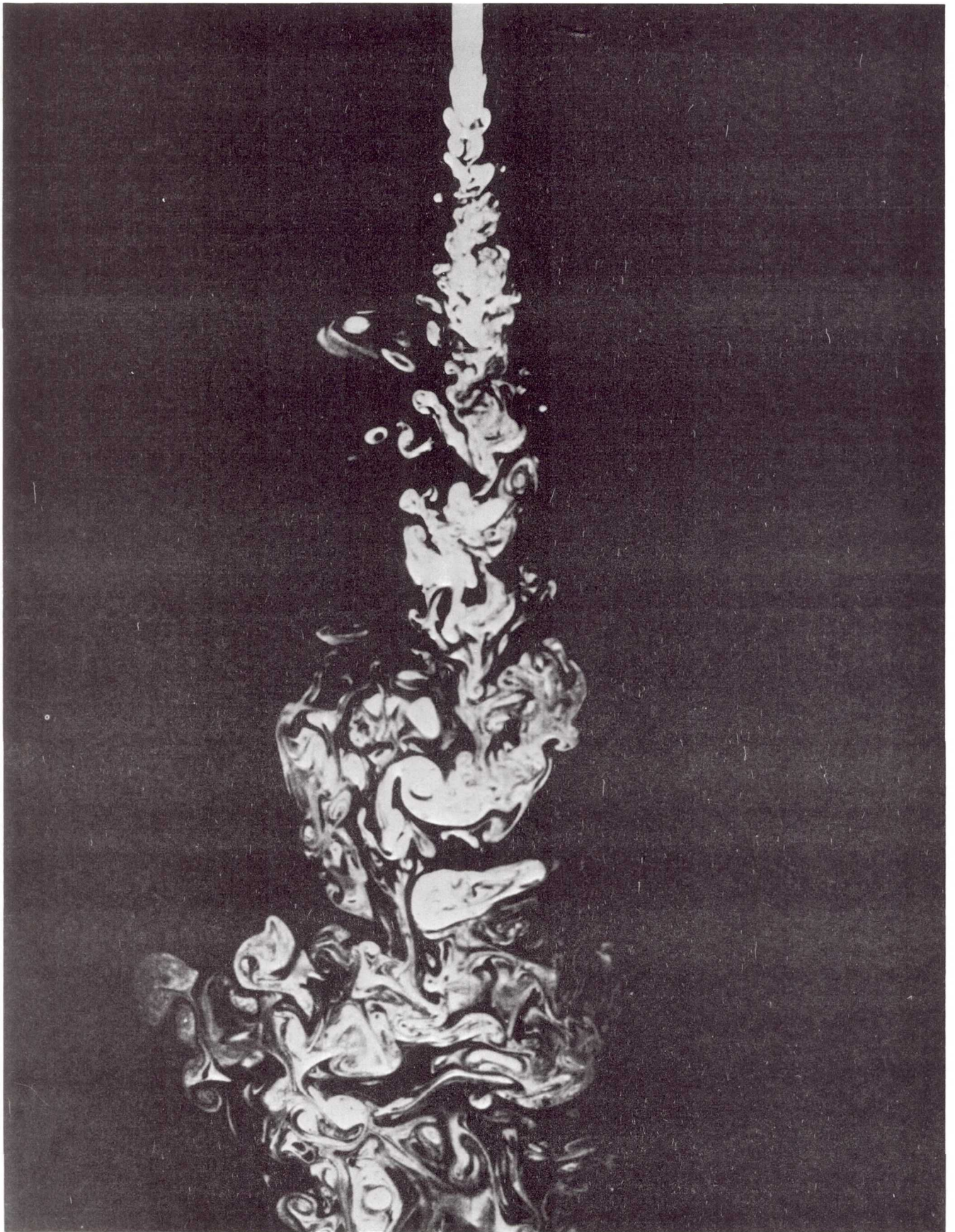
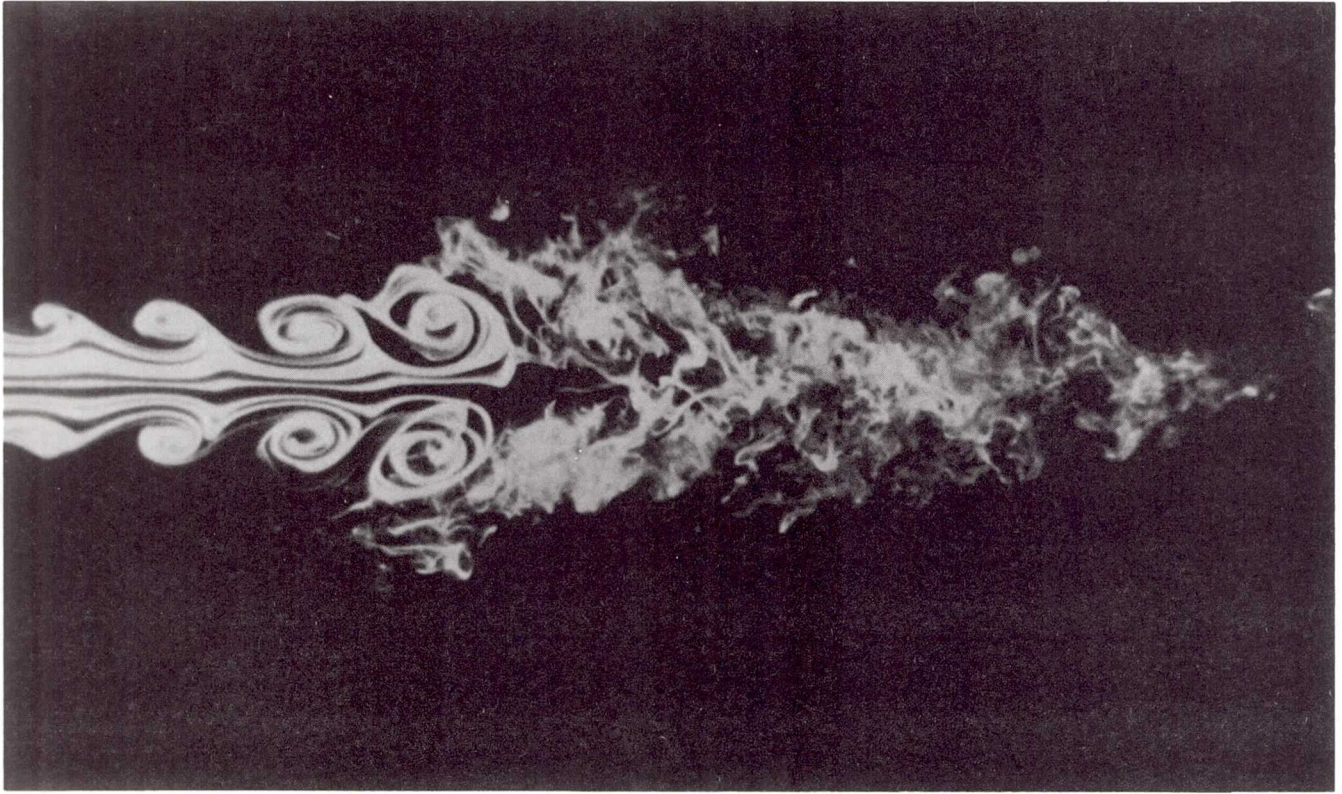


FIGURE 1-9. - AXISYMMETRIC WATER JET BECOMING TURBULENT AS IT IS DIRECTED DOWNWARD INTO WATER. REYNOLDS NUMBER  $\approx 2000$ . PHOTOGRAPH BY P.E. DIMOTAKIS, R.C. LYE AND D.Z. PAPANTONIOU. (REF. 12).





FLOW →

FIGURE 1-10. - AXISYMMETRIC JET OF AIR BECOMING TURBULENT. REYNOLDS NUMBER  $\sim 10\ 000$ . PHOTOGRAPH BY R. DRUBKA AND H. NAGIB. NOTE THAT THE TURBULENCE HERE HAS A FINER STRUCTURE THAN THAT AT THE LOWER REYNOLDS NUMBER IN THE PRECEDING FIGURE. REPRINTED BY PERMISSION OF PARABOLIC PRESS. (REF. 3).

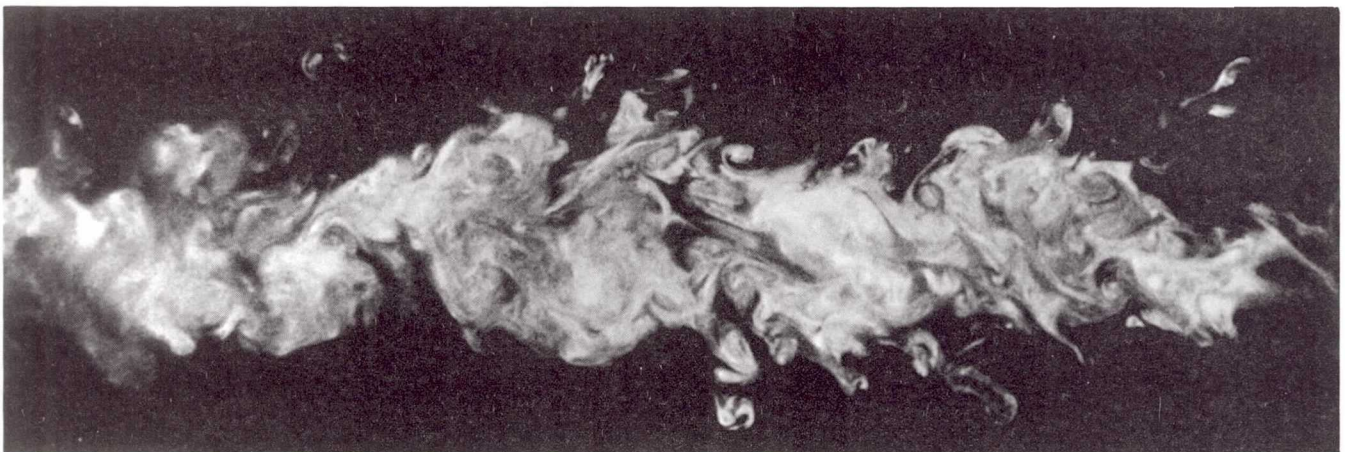


FIGURE 1-11. - TURBULENT WAKE OF A CYLINDER. PHOTOGRAPH BY R.E. FALCO. REPRINTED BY PERMISSION OF PARABOLIC PRESS. (REF. 3).



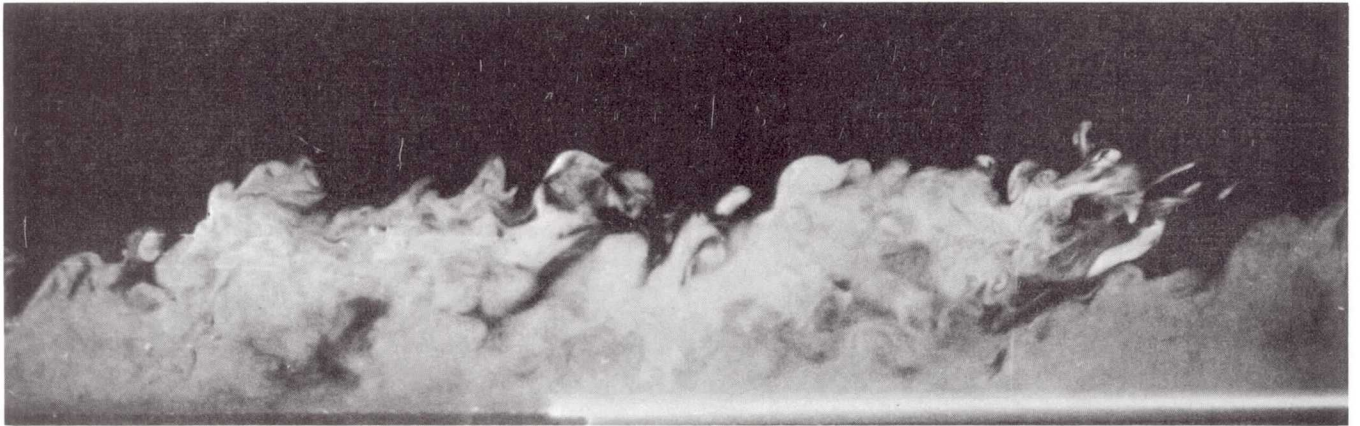


FIGURE 1-12. - TURBULENT BOUNDARY LAYER ON THE FLOOR OF A WIND TUNNEL. REPRINTED BY PERMISSION OF AMERICAN INSTITUTE OF PHYSICS. (REF. 13).

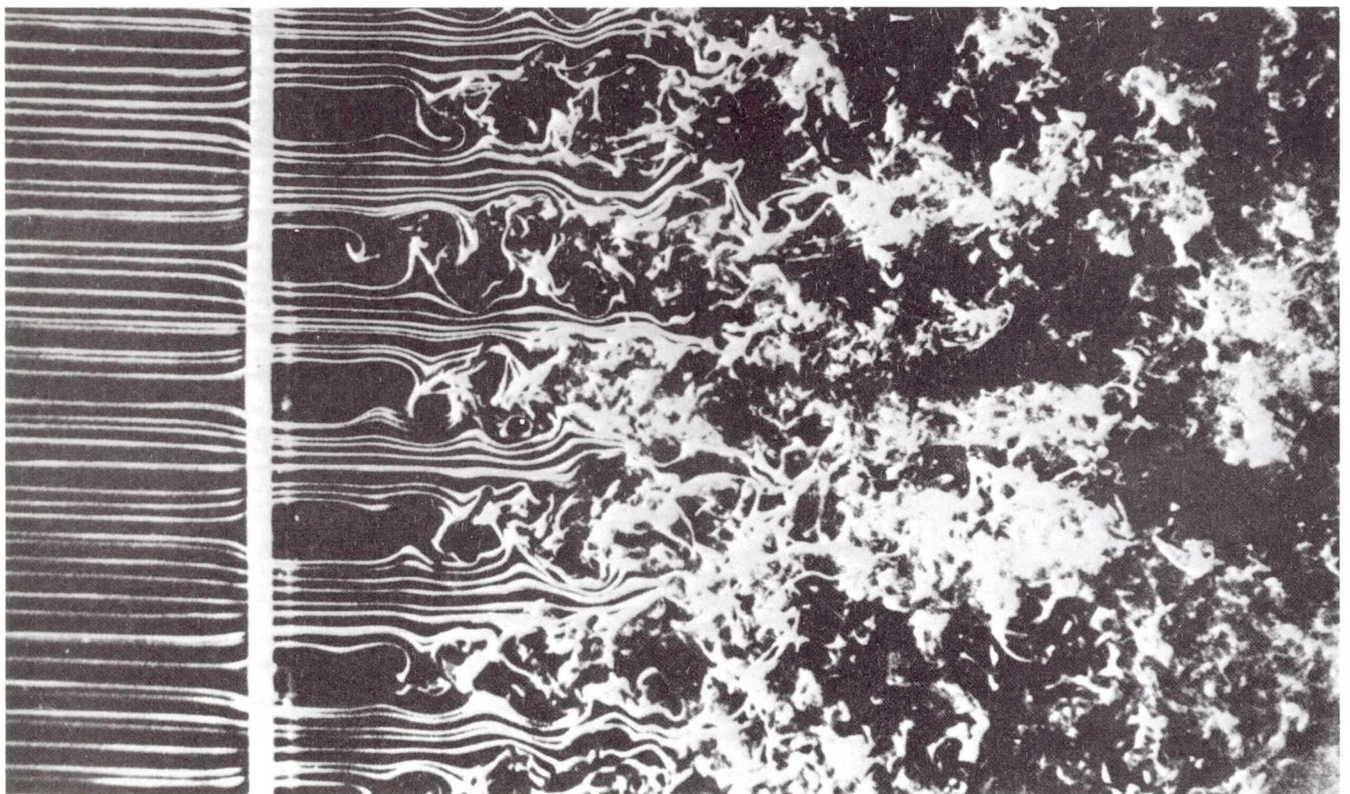
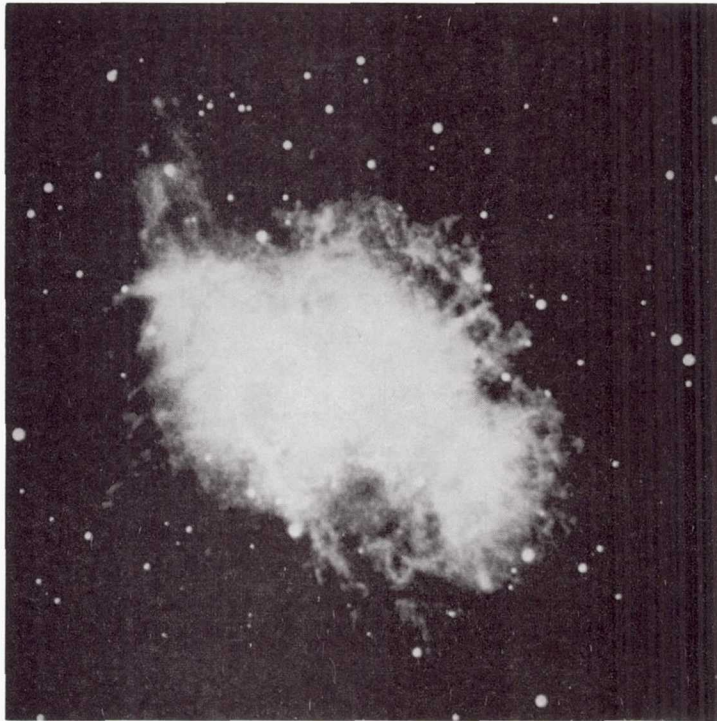
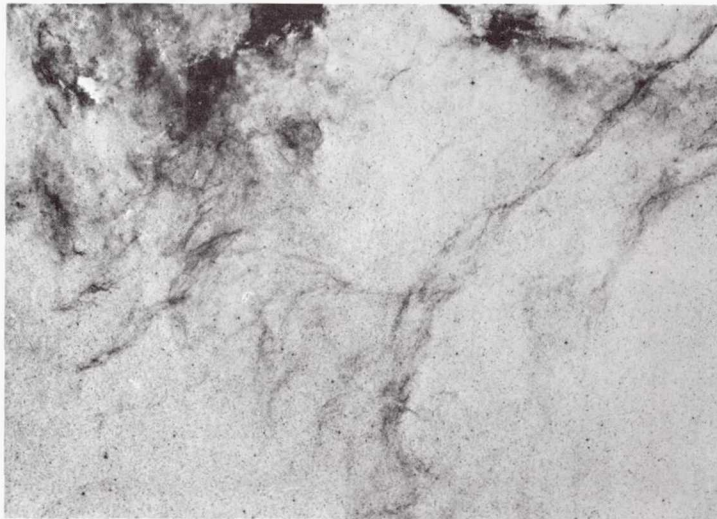


FIGURE 1-13. - GENERATION OF TURBULENCE DOWNSTREAM FROM A GRID. PHOTOGRAPH BY T. CORKE AND H. NAGIB. REPRINTED BY PERMISSION OF PARABOLIC PRESS. (REF. 3).



(a) CRAB NEBULA. NASA PHOTOGRAPH, NASA EP-167.



(b) REGION IN MILK WAY (REF. 14). REPRINTED BY PERMISSION OF ELSEVIER SCIENCE PUBLISHERS.

FIGURE 1-14. - TURBULENT ASTROPHYSICAL CLOUD.



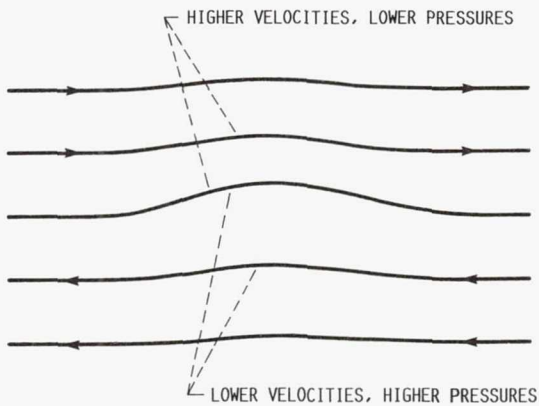


FIGURE 1-15. - SKETCH ILLUSTRATING UNSTABLE FLOW PRODUCED BY SHEAR. A SMALL PERTURBATION TENDS TO GROW.

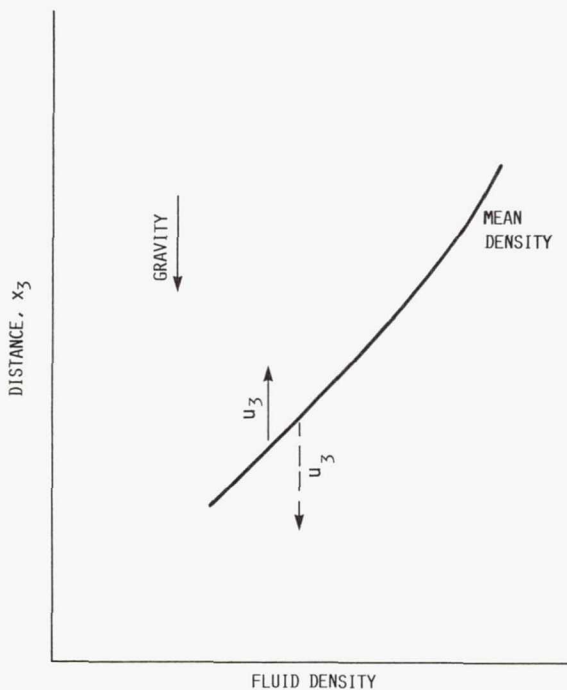


FIGURE 1-16. - SKETCH ILLUSTRATING BUOYANCY-PRODUCED INSTABILITY. A SMALL PERTURBATION EITHER UPWARD ( $u_3$ , POSITIVE) OR DOWNWARD ( $u_3$ , NEGATIVE) TENDS TO GROW.

1. Report No. NASA TM-103723		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle Turbulent Fluid Motion Part I—The Phenomenon of Fluid Turbulence			5. Report Date February 1991		
			6. Performing Organization Code		
7. Author(s) Robert G. Deissler			8. Performing Organization Report No. E-5956		
			10. Work Unit No. 505-90-01		
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191			11. Contract or Grant No.		
			13. Type of Report and Period Covered Technical Memorandum		
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001			14. Sponsoring Agency Code		
15. Supplementary Notes Responsible person, Robert G. Deissler, (216) 433-5823.					
16. Abstract Some introductory material on fluid turbulence is presented. This includes discussions and illustrations of what turbulence is, and how, why, and where turbulence occurs.					
17. Key Words (Suggested by Author(s)) Turbulence; Fluids; Flow; Navier-Stokes; Reynolds number			18. Distribution Statement Unclassified—Unlimited Subject Category 34		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of pages 18	22. Price* A03