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## THE SEMANTICS OF FUZZY LOGIC

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### INTRODUCTION

In this brief paper,<sup>1</sup> we summarize the results of recent research on the conceptual foundations of fuzzy logic [9]. This research resulted in the formulation of several semantic models that interpret the major concepts and structures of fuzzy logic in terms of the more primitive notions of resemblance and similarity between "possible worlds," i.e., the possible states, situations or behaviors of a real-world system. The metric structures representing this notion of proximity are generalizations of the accessibility relation of modal logics [1].

Possibilistic reasoning methods may be characterized, by means of our interpretation, as approaches to the description of the relations of proximity that hold between possible system states that are logically consistent with existing evidence, and other situations, which are used as reference landmarks. By contrast, probabilistic methods seek to quantify, by means of measures of set extension, the proportion of the set of possible worlds where a proposition is true.

Our discussion will focus primarily on the principal characteristics of a model, discussed in detail in a recent technical note [3], that quantifies resemblance between possible worlds by means of a similarity function that assigns a number between 0 and 1 to every pair of possible worlds. Introduction of such a function permits to interpret the major constructs and methods of fuzzy logic: conditional and unconditional possibility and necessity distributions and the generalized modus ponens of Zadeh on the basis of related metric relationships between subsets of possible worlds.

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<sup>1</sup>The present paper is a slightly revised and expanded version of a communication appearing in the Proceedings of the 1990 Iizuka Conference on Fuzzy Logic and Neural Networks.

## THE APPROXIMATE REASONING PROBLEM

Our semantic model of fuzzy logic is based on two major conceptual structures: the notion of possible world, which is the basis for our unified view of the approximate reasoning problem [4], and a metric structure that quantifies similarity between pairs of possible worlds.

If a reasoning problem is thought of as being concerned with the determination of the truth-value of a set of propositions that describe different aspects of the behavior of a system, then a *possible world* is simply a function (called a valuation) that assigns a unique truth value to every proposition in that set and that, in addition, is consistent with the rules of propositional logic. The set of all such possible worlds is called the *universe of discourse*.

In any reasoning problem, knowledge about the characteristics of the class of systems being studied combined with observations about the particular system under consideration restricts the extent of possible worlds that must be considered to a subset of the universe of discourse, called the *evidential set*, which will be denoted  $\mathcal{E}$ .

The purpose of the inferential procedures utilized in any reasoning problem may be characterized as that of establishing if, for a given proposition  $\mathcal{H}$  (the *hypothesis*), either  $\mathcal{E} \Rightarrow \mathcal{H}$  or  $\mathcal{E} \Rightarrow \neg\mathcal{H}$ , i.e., whether existing evidence implies the hypothesis or it implies its negation. In approximate reasoning problems, as illustrated in Figure 1, such determination is, by definition, impossible: there are some possible worlds in the the evidential set where the hypothesis is true and some where it is false.

## SIMILARITY FUNCTIONS AND IMPLICATION

In the view of fuzzy logic proposed by our model the purpose of possibilistic methods is the description of the evidential set by characterization of the resemblance relations that hold between its elements and elements of other sets used as reference landmarks. By contrast, probabilistic methods (i.e., probabilities usually interpreted as frequencies or as degree of personal belief) seek to measure the relative extensions of the sets  $\mathcal{E} \cap \mathcal{H}$  and  $\mathcal{E} \cap \neg\mathcal{H}$ .

To represent similarity or resemblance between possible worlds we introduce a binary function  $S$  that assigns a value between 0 and 1 to every pair of possible worlds  $w$  and  $w'$ . A value of  $S$  equal to 1 means that  $w$  and  $w'$  are identical, while a value of  $S$  equal to 0

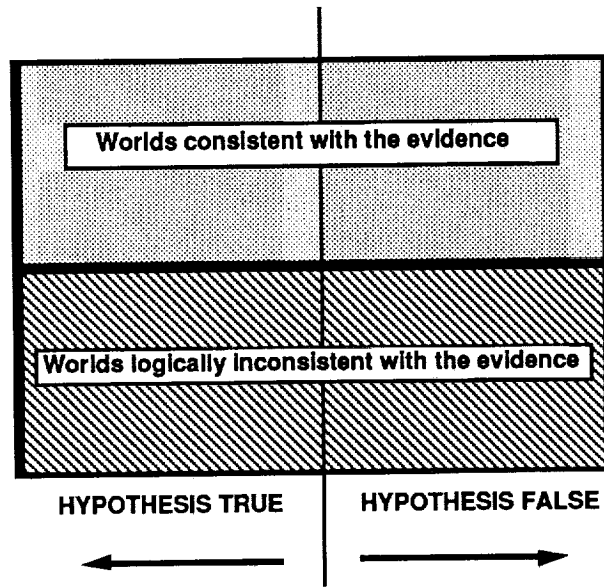


Figure 1: The approximate reasoning problem

indicates that knowledge of propositions that are true in one possible world does not provide any indication about the nature of the propositions that are true in the other.

In addition to the above requirement of reflexivity, i.e.

$$S(w, w) = 1,$$

we will need to impose additional axioms to assure that  $S$  captures the semantics of a similarity relation. In addition to assuming that  $S$  is symmetric, i.e.,

$$S(w, w') = S(w', w),$$

we will also require that  $S$  satisfies a form of transitivity that is motivated by noting that if  $w, w'$  and  $w''$  are possible worlds and if  $w$  is highly similar to  $w'$  and  $w'$  is highly similar to  $w''$ , then it would be surprising if  $w$  and  $w''$  were highly dissimilar. This consideration indicates that knowledge of  $S(w, w')$  and  $S(w', w'')$  should provide a lower bound for values of  $S(w, w'')$ , as expressed by the inequality

$$S(w, w'') \geq S(w, w') \otimes S(w', w''),$$

where  $\otimes$  is a binary operator used to represent a real function that produces the required bound. If reasonable requirements are imposed upon the function  $\otimes$ , it is easy to show

that it has the properties of triangular norms: a class of functions that play a major role in multivalued logics [6].

The generalized transitivity property expressed by the above inequality may be easier to understand as a classical triangular inequality if it is noted that the function  $\delta = 1 - S$  has the properties of a metric. When  $\otimes$  is the Lukasiewicz norm

$$a \otimes b = \max(a + b - 1, 0)$$

, then the transitivity property of  $S$  is equivalent to the well-known triangular property

$$\delta(x, z) \leq \delta(x, y) + \delta(y, z),$$

of distance functions. If  $\otimes$  corresponds to the Zadeh triangular norm  $a \otimes b = \min(a, b)$ , then  $\delta$  may be shown to satisfy the more stringent ultrametric inequality

$$\delta(x, z) \leq \max(\delta(x, y), \delta(y, z)).$$

The correspondence between propositions and subsets of possible worlds simplifies the interpretation of the classical rule of modus ponens as a rule of derivation based on the transitive property of set inclusion. If three propositions  $p$ ,  $q$  and  $r$  are such that the set of possible worlds where  $p$  is true is a subset of the set of possible worlds where  $q$  is true, and if such set is itself a subset of the set of worlds where  $r$  is true, then the modus ponens simply states that the set of  $p$ -worlds is a subset of the set of  $r$ -worlds.

The conventional relation of set inclusion, based on the binary truth-value structure of classical logic, allows only to state that a set of possible worlds is a subset of another or that it is not. Introduction of a metric structure in the universe of discourse, however, permits the quantification of the degree by which a set is included into another. Every set of possible worlds, as illustrated in Figure 2, is a subset of some neighborhood of any other set. The minimal amount of "stretching" that is required to include a set of possible worlds  $q$  in a neighborhood of a set of possible worlds  $p$ , given by the expression

$$\mathbf{I}(p | q) = \inf_{w' \vdash q} \sup_{w \vdash p} S(w, w'),$$

is called the *degree of implication*.

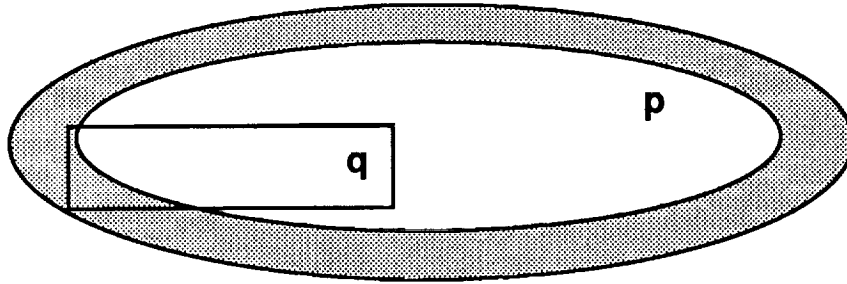


Figure 2: Degree of implication

The degree of implication function has the important transitive property expressed by

$$I(p | q) \geq I(p | r) \otimes I(r | q),$$

which is the basis of the generalized modus ponens of Zadeh. As illustrated in Figure 3, this important rule of derivation tells us how much the set of  $p$ -worlds should be stretched to encompass  $q$  on the basis of knowledge of the sizes of the neighborhoods of  $p$  that includes  $r$  and of  $r$  that includes  $q$ .

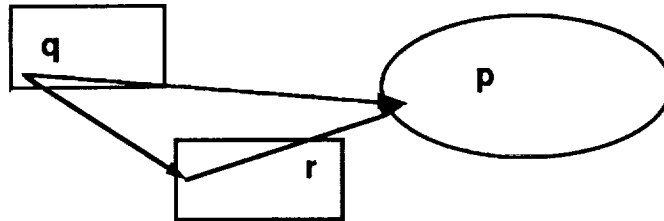


Figure 3: The generalized modus ponens

A notion dual to the degree of implication is that of *degree of consistence*, which quantifies the amount by which a set must be stretched to intersect another, and that is given by the expression

$$C(p | q) = \sup_{w' \vdash q} \sup_{w \vdash p} S(w, w').$$

## POSSIBILISTIC DISTRIBUTIONS

Although the transitive property of the degree of implication essentially provides the bases for the conceptual validity of the generalized modus ponens, this rule of derivation is typically expressed by means of necessity and possibility distributions.

An *unconditioned necessity distribution* given the evidence  $\mathcal{E}$  is any function defined over propositions that bounds by below the degree of implication function, i.e., any function satisfying the inequality

$$\mathbf{Nec}(p) \leq \mathbf{I}(p | \mathcal{E}).$$

Correspondingly, an *unconditioned possibility distribution* is any upper bound for the degree of consistence function, i.e.,

$$\mathbf{Poss}(p) \geq \mathbf{C}(p | \mathcal{E}).$$

The definition of conditional possibility and necessity distributions makes use of a form of inverse of the triangular norm denoted  $\odot$  and defined by the expression

$$a \odot b = \sup \{ c : b \otimes c \leq a \}.$$

Using this function, it is possible to define conditional possibilistic distributions as follows:

**Definition:** A function  $\mathbf{Nec}(\cdot | \cdot)$  is called a *conditional necessity distribution* for  $\mathcal{E}$  if

$$\mathbf{Nec}(q|p) \leq \inf_{w \vdash \mathcal{E}} [\mathbf{I}(q | w) \odot \mathbf{I}(p | w)].,$$

**Definition:** A function  $\mathbf{Poss}(\cdot | \cdot)$  is called a *conditional possibility distribution* for  $\mathcal{E}$  if

$$\mathbf{Poss}(q|p) \geq \sup_{w \vdash \mathcal{E}} [\mathbf{I}(q | w) \odot \mathbf{I}(p | w)].$$

## GENERALIZED MODUS PONENS

The *compositional rule of inference* or generalized modus ponens of Zadeh is a generalization of the corresponding classical rule of inference that may be used even when known facts do not match the antecedent of a conditional rule. The interpretation provided by our model explains the generalized modus ponens as an extrapolation procedure that uses

knowledge of the similarity between the evidence and a set of possible worlds  $p$  (the antecedent proposition), and of the proximity of  $p$ -worlds to  $q$ -worlds, to bound the similarity the latter to the evidential set. The actual statement of the generalized modus ponens for necessity and possibility distributions in terms of similarity structures makes use of a family  $\mathcal{P}$  of satisfiable propositions that partitions the universe of discourse:

**Theorem (Generalized Modus Ponens for Possibility Distributions):** Let  $\mathcal{P}$  be a partition and let  $q$  be a proposition. If  $\mathbf{Poss}(p)$  and  $\mathbf{Poss}(q|p)$  are real values, defined for every proposition  $p$  in  $\mathcal{P}$ , such that

$$\mathbf{Poss}(p) \geq C(p|\mathcal{E}), \quad \mathbf{Poss}(q|p) \geq \sup_{w \in \mathcal{E}} [\mathbf{I}(q|w) \odot \mathbf{I}(p|w)],$$

then the following inequality is valid:

$$\sup_{\mathcal{P}} [\mathbf{Poss}(q|p) \otimes \mathbf{Poss}(p)] \geq C(q|\mathcal{E}).$$

**Theorem (Generalized Modus Ponens for Necessity Distributions):** Let  $\mathcal{P}$  be a partition and let  $q$  be a proposition. If  $\mathbf{Nec}(p)$  and  $\mathbf{Nec}(q|p)$  are real values, defined for every proposition  $p$  in  $\mathcal{P}$ , such that

$$\mathbf{Nec}(p) \leq \mathbf{I}(p|\mathcal{E}), \quad \mathbf{Nec}(q|p) \geq \inf_{w \in \mathcal{E}} [\mathbf{I}(q|w) \odot \mathbf{I}(p|w)],$$

then the following inequality is valid:

$$\sup_{\mathcal{P}} [\mathbf{Nec}(q|p) \otimes \mathbf{Nec}(p)] \leq \mathbf{I}(q|\mathcal{E}).$$

## VARIABLES AND FUZZY RULES

If our attention is restricted to propositions of the form “ $X = x$ ,” describing the value of a variable  $X$ , and to logical combinations of these propositions, then a possibility distribution  $\Pi_{Y|X}$  may be regarded, as is well known, as an elastic constraint that restricts the values of a variable  $Y$  on the basis of general background information (the evidence  $\mathcal{E}$ ) and knowledge about possible values of another variable  $X$ .

In our similarity-based interpretation, this notion of elastic constraint is easier to understand (Figure 4) by means of the concept of compatibility relation that associates specific

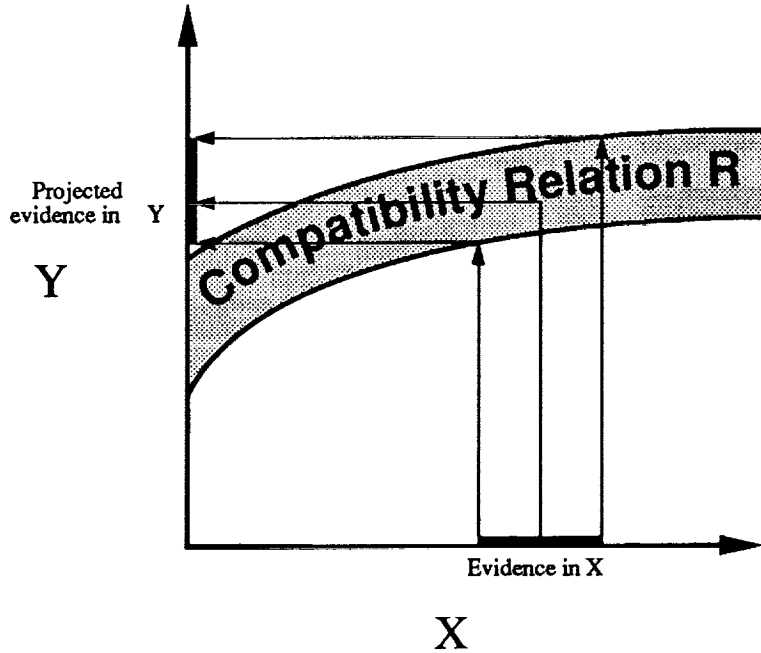


Figure 4: Compatibility Relation.

values of one variable ( $X$ ) with possible values of another ( $Y$ ). Using this basic notion, we may now describe two major interpretations of fuzzy rules as its similarity-based approximations by means of fuzzy-set theoretic structures.

The first interpretation, originally proposed by Zadeh [8] and further developed by Trillas and Valverde [6], is the formal translation of the statement

If  $\mu_A$  is a possibility for  $X$ , then  $\mu_B$  is a possibility distribution for  $Y$ .

Using our structures we may define this particular formulation by saying that

$$\mathbf{Poss}(y|x) = \mu_B(y) \circ \mu_A(x) \geq \mathbf{I}(y|w) \circ \mathbf{I}(x|w),$$

for every world  $w \vdash \mathcal{E}$ , i.e., that  $\mathbf{Poss}(\cdot|\cdot)$  is a conditional possibility distribution. This distribution expresses a basic relationship between the similarity between possible evidential worlds and the core of  $\mu_B$  as a “fraction” of their similarity with the core of  $\mu_A$ .

Under this interpretation, the fuzzy-rule based approximation to a compatibility relation may be depicted as done in Figure 5, where it has been assumed that the underlying metric



(i.e., dissimilarity) is proportional to the euclidean distance in the plane. As illustrated in that figure, the core of the corresponding conditional possibility distribution is an (upper) approximant of a classical compatibility relation (which fans outward from the Cartesian product of the cores of  $A$  and  $B$ ). Whenever several such rules are available, then each one of these rules will lead to a separate possibility distribution, which may be illustrated, as done in 7, as an approximating fuzzy relation. Combination of these estimates by intersection results in a sharper “integrated” estimate of the effect of a rule set.

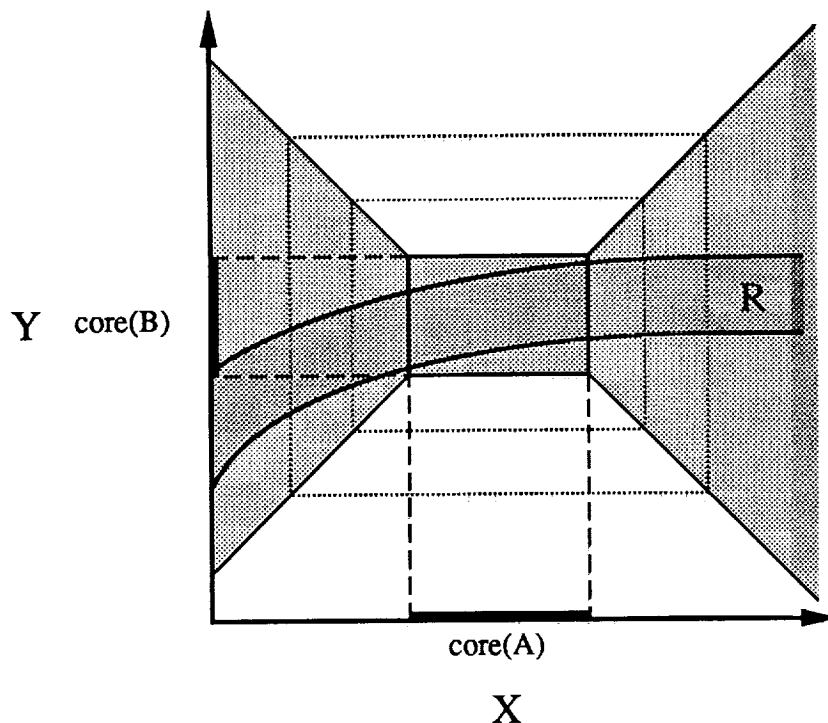


Figure 5: Rules as Possibilistic Approximants of a Compatibility Relation.

The second interpretation, originally propoded by Zadeh [7], was first applied by Mamdani and Assilian [2] to design fuzzy controllers, being also the basis for a wide variety of recent industrial products [5]. In this formulation, a number of statements of the form

$$\text{If } X \text{ is } A_k, \text{ then } Y \text{ is } B_k, \quad k = 1, 2, \dots, n,$$

are interpreted as a combined “disjunctive” description of the compatibility relation, rather

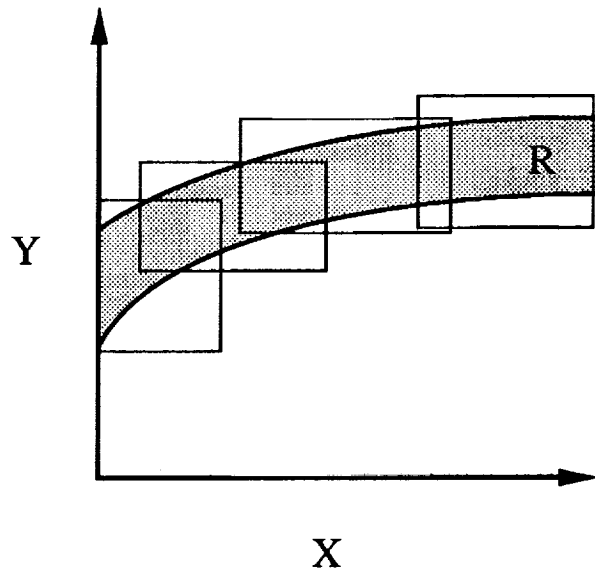


Figure 6: Rule-Sets as Possibilistic Approximants of a Compatibility Relation

than as a set of independently valid rules, as shown in Figure 6. In this case, each disjunctive approximant, corresponding to a fuzzy relation such as that illustrated in Figure 8 (with the relation “sloping” away from the cartesian products of the core of the fuzzy sets) is combined disjunctively by fuzzy set union with the other approximants.

## CONCLUSION

Models based on the logical notion of possible-world provide interpretations of the major concepts and structures of fuzzy logic in terms of primitive notions of similarity and resemblance. These interpretations clearly show the basic nature of the difference between possibilistic, which are based on metrics, and probabilistic methods, which are based on set measures.

## ACKNOWLEDGMENT

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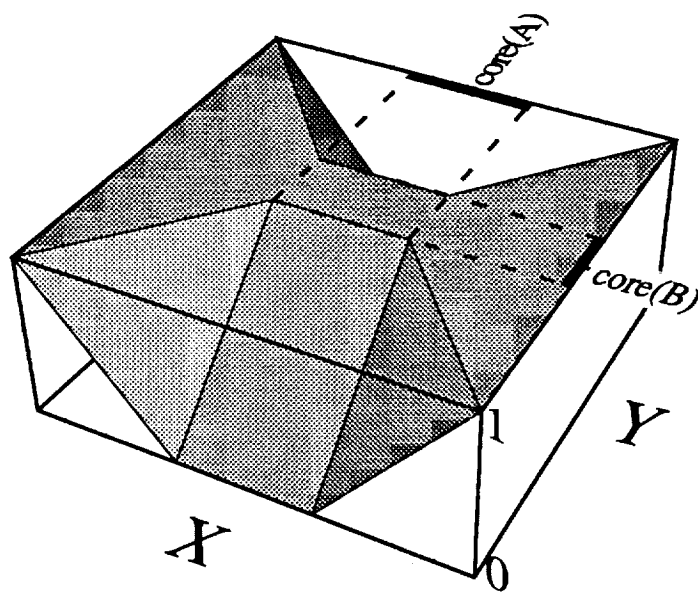


Figure 7: A Possibilistic Conjunctive Conditional Rule

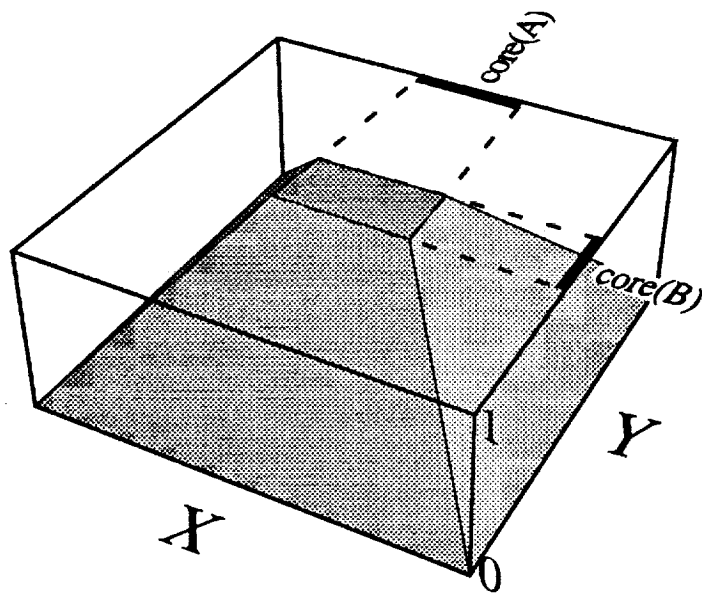


Figure 8: A Component of a Disjunctive Rule Set

No. DAAL03-89-K-0156. The views, opinions and/or conclusions contained in this note are those of the author and should not be interpreted as representative of the official positions, decisions, or policies, either express or implied, of the Air Force Office of Scientific Research, the Department of the Army, or the United States Government.

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# **THE SEMANTICS OF FUZZY LOGIC**

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**Fuzzy Logic and Neural Networks Workshop  
NASA JSC, Houston, April 1990**

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## MOTIVATION

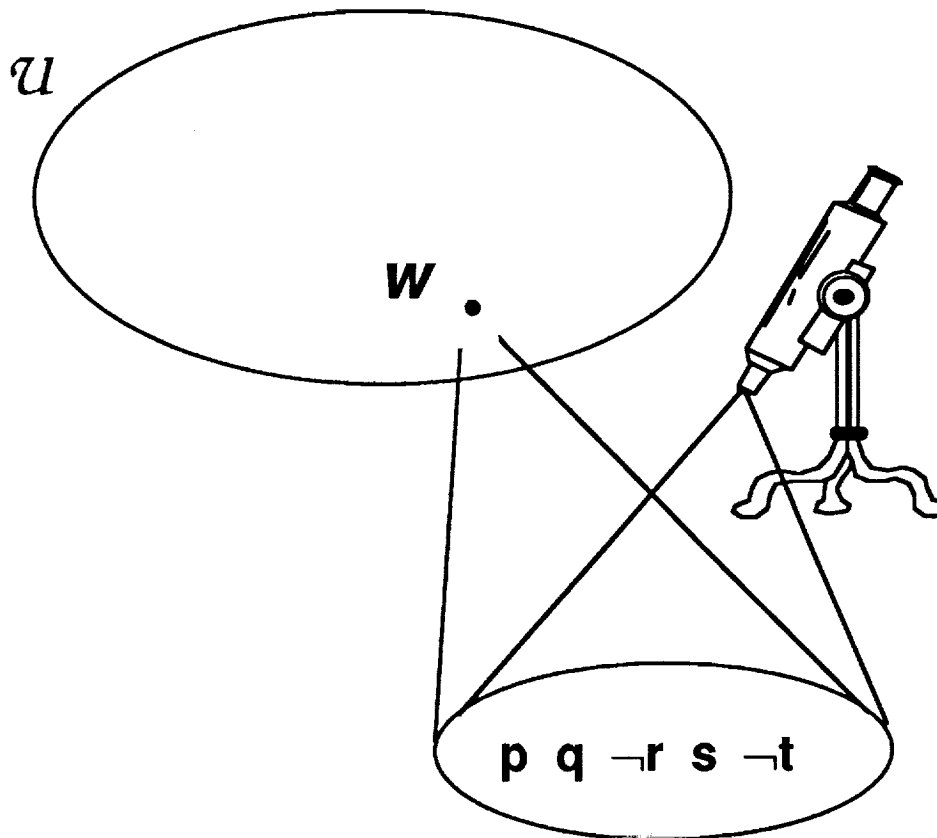
- **Provide basic characterization of Possibilistic concepts**
  - **Possibility and Necessity Distributions**
  - **Possibistic Calculus**
  - **Inferential Rules (GMP)**
- **Determine analogies and differences with Probabilistic Reasoning Methods**
  - **Unified Approach to Interpretation**
  - **Needs for specific formalisms/theoretical structures**

**FUZZY LOGIC MAY BE FORMALLY EXPLAINED  
BY METRIC CONCEPTS AND STRUCTURES:**

**Similarity, Proximity, Closeness, Resemblance**

## POSSIBLE WORLDS

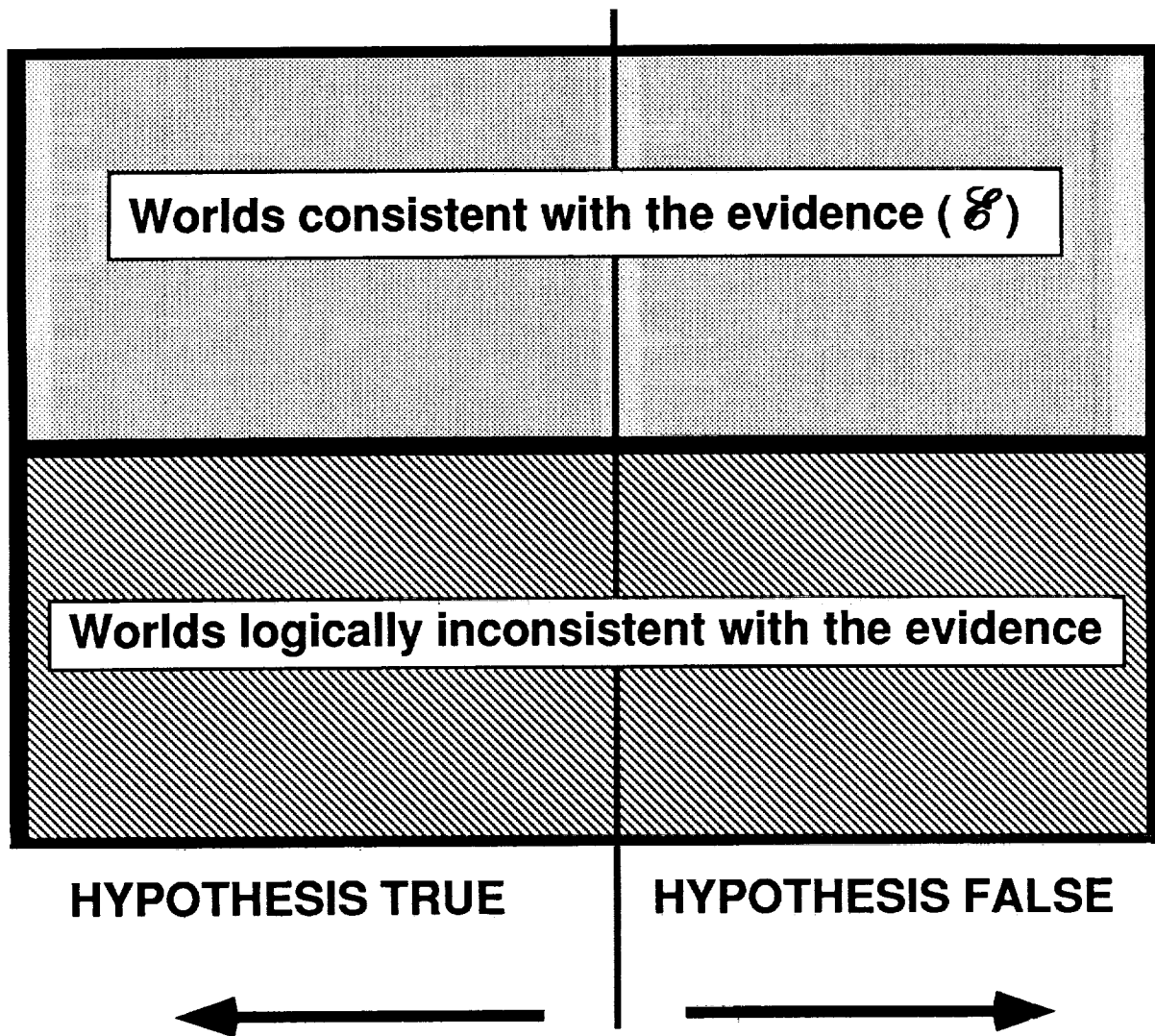
- Possible States, Behaviors, Trajectories of a Conceptual System that is being reasoned about  
*Examples: Weather System, Vehicle Control System, Portfolio Status*
- Formally equivalent to a Valuation:
- Assignment of truth-values (i.e., T, F) to all relevant propositions about the state of system
- Consistent with rules of logic
- Universe  $\equiv$  Set of all Possible Worlds





# THE APPROXIMATE REASONING PROBLEM

- Conventional deductive methods fail to unambiguously determine the truth-value of a proposition of interest (hypothesis).



## APPROXIMATE REASONING METHODS

- Describe properties of the Evidential Set  $\mathcal{E}$

### Probabilistic Reasoning :

- Based on the use of additive set measures
- Concerned with (objective or subjective) proportions of occurrence of certain events, e.g.,

$$\mu(H | \mathcal{E}) + \mu(\neg H | \mathcal{E})$$

### Possibilistic Reasoning:

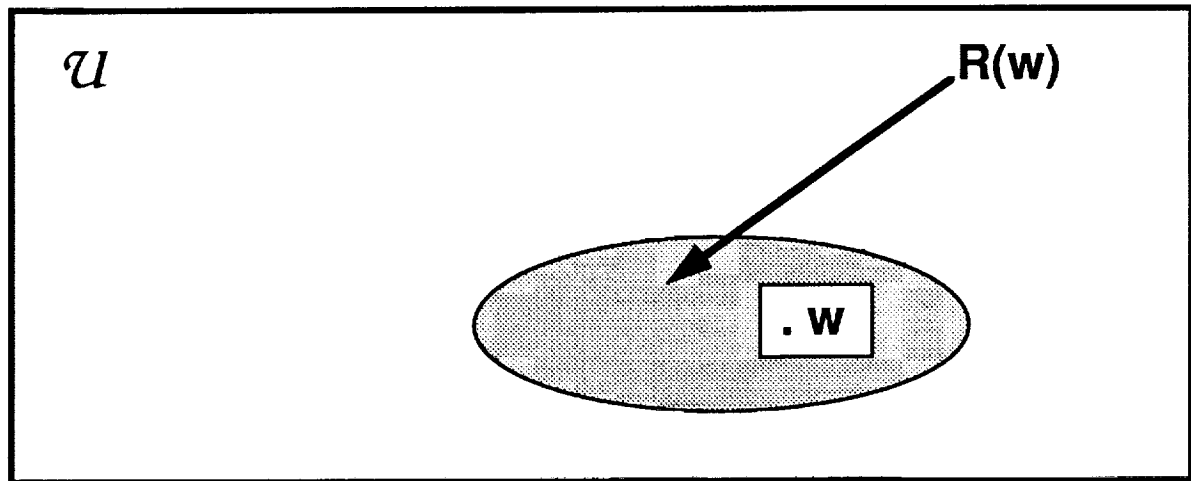
- Based on metric notions (distance, similarity, proximity)
- Uses measures of resemblance between subsets of possible worlds
- Oriented toward characterization of conceptual flexibility, typicality, proximity, degree of fitness

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## Semantic Formulation of Modal Logic

- Basic Elements:
  - $\mathcal{U}$  : A set of possible worlds (the universe)
  - $\mathcal{V}$  : a valuation (mapping pairs of possible worlds and propositions into truth values), e.g.,  
 $(w, \text{"it rains"}) \rightarrow T$
  - R: A binary relation (between pairs of possible worlds) called the conceivability, reachability, or accessibility relation
- POSSIBILITY AND NECESSITY :
  - p is possible in w (  $w \models \Diamond p$  )  
if and only if p is true in some world w'  
that is related to w
  - p is necessary in w (  $w \models \Box p$  )  
if and only if p is true in every world w'  
that is related to w
  - Different properties of R lead to different modal systems (T, S4, S5)

## INTERPRETING ACCESSIBILITIES



- $R = U \times U$  : Conventional notion of logical necessity
- $O = \{\text{a propositional subset}\}$  (the “observables”)  
 $R(w, w')$  if  $w$  and  $w'$  share the same “observations”  
 ( Necessity then models rational knowledge)
- Inevitability: Two worlds are related if they are identical up to some point in time
- Cognitive capability
- Moral Necessity
- Linguistic Modalities

**WE ARE INTERESTED IN MODELING THE ABILITY  
 OF POSSIBLE WORLDS TO EXEMPLIFY CERTAIN  
 CONDITIONS**

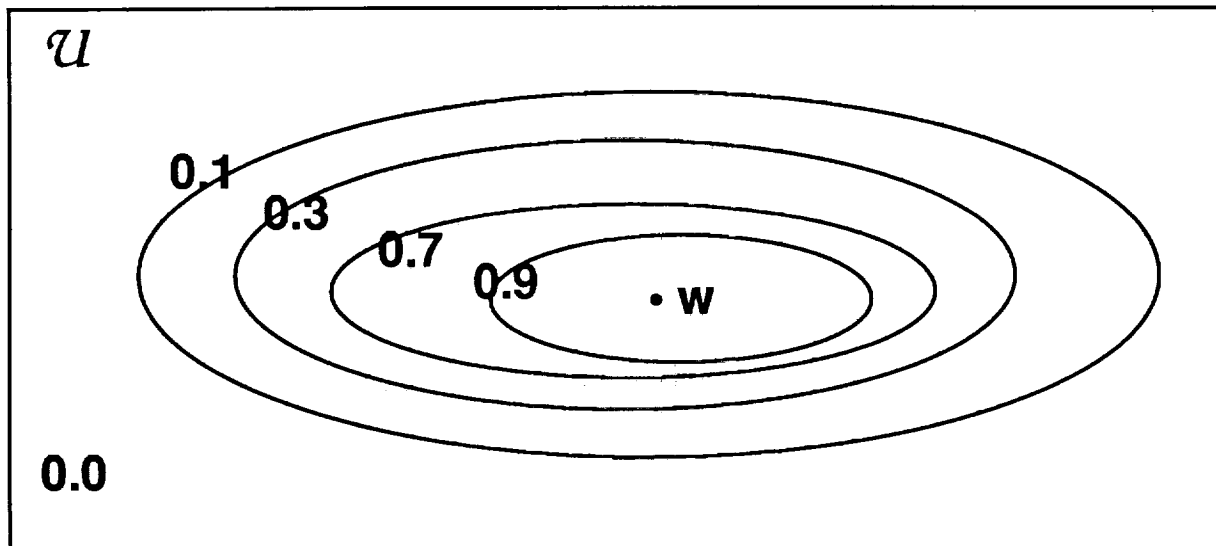
## MULTIPLE ACCESSIBILITY RELATIONS

- Generalize notion of accessibility relation by consideration of a family of relations indexed by a numerical parameter between 0 and 1
- Modeling Objective:

Define resemblance between situations so as to allow a form of analogical reasoning

Example: *Investment advice for S*  
(Wealth=\$1,000,000) is valid (to some extent) for  
*S'* (Wealth=\$999,999)

We want to be able to describe behavioral rules  
that are valid in neighborhoods of sets of  
possible worlds



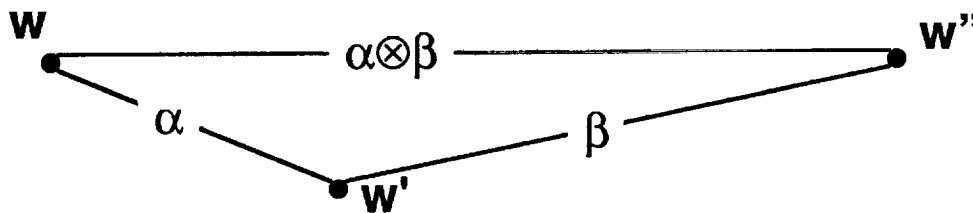
- Defined by means of a similarity function

# SIMILARITY FUNCTIONS

- Assigns a similarity value to pairs of possible worlds

$$S: W \times W \rightarrow [0,1]$$

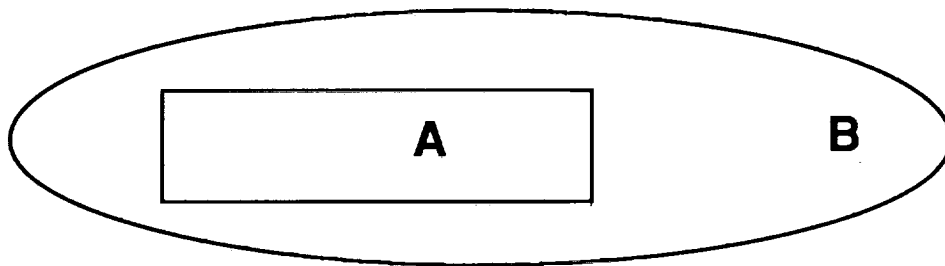
- $S(w,w')=1$  means that  $w$  is identical to  $w'$
- $S(w,w')=0$  means that  $w$  and  $w'$  are completely dissimilar
- Properties of Similarity Relations:
  - $S(w,w') = 1$  if and only if  $w=w'$  (Reflexivity)
  - $S(w,w') = S(w',w)$  (Symmetry)
  - $S(w,w'') \geq S(w,w') \otimes S(w',w'')$



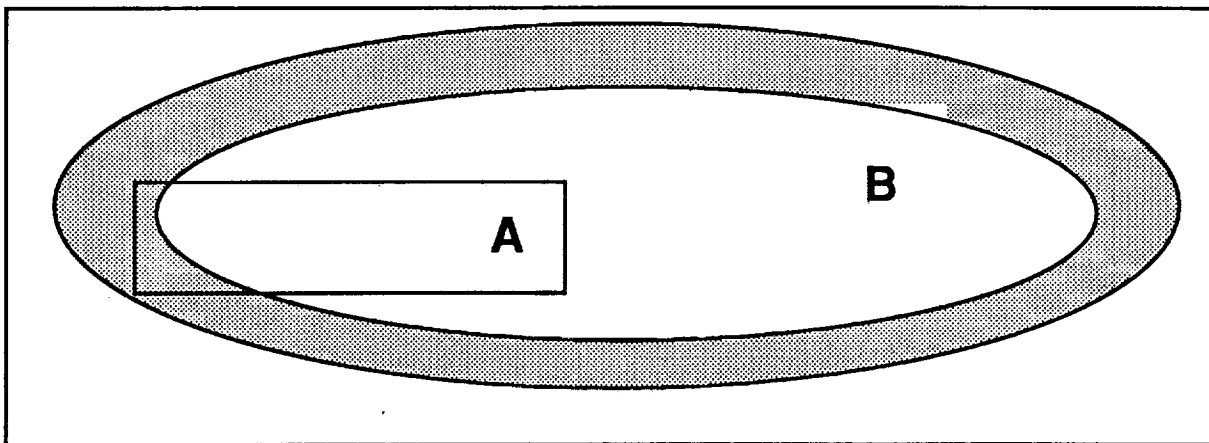
- Imposition of reasonable requirements indicates that  $\otimes$  should have the properties of a continuous triangular norm (T-norm).
- $\delta = 1 - S$ , is a distance function.

## LOGIC and METRICS

- Metric structures allow to characterize implications between propositions (i.e., subset inclusions) in terms of similarities between subset elements:
  - If  $B \supseteq A$ , then every point of A has one point of B (i.e., itself) that is similar to it to the degree 1



- In general, every point of a subset A is similar to some degree to a point of B (i.e., falls in some neighborhood of B)



## MODELS (Good, Bad, and otherwise)

- MODELS (MODAL LOGIC) :

- q is a model of p ( $q \models p$ ) if and only if every q-world is a p-world, i.e.,

$$q \Rightarrow p$$

- GENERALIZED MODELS :

- q is a necessary model of p to the degree  $\alpha$  if and only if every q-world is  $\alpha$ -similar to a p-world, i.e.

$$q \Rightarrow \Pi_{\alpha} p$$

(“De Re” Interpretation)

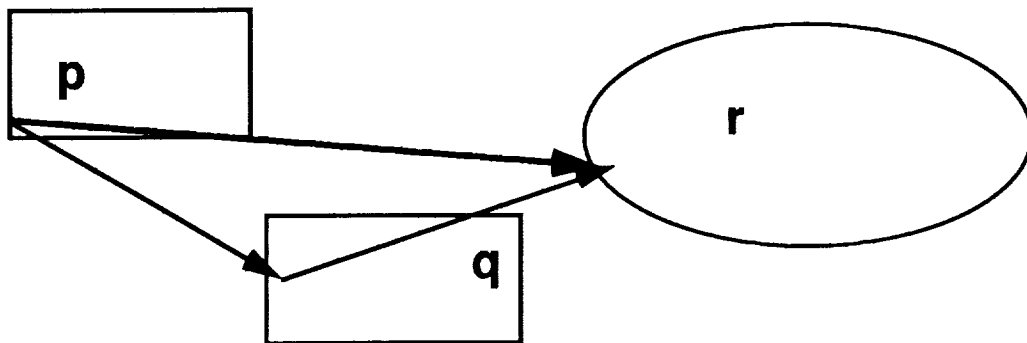
- To what degree q is a necessary model or a good example of p? ( “Degree of Implication” )

$$I(p | q) = \inf_{w|q} \sup_{w'|p} S(w, w')$$



## DEGREE OF IMPLICATION

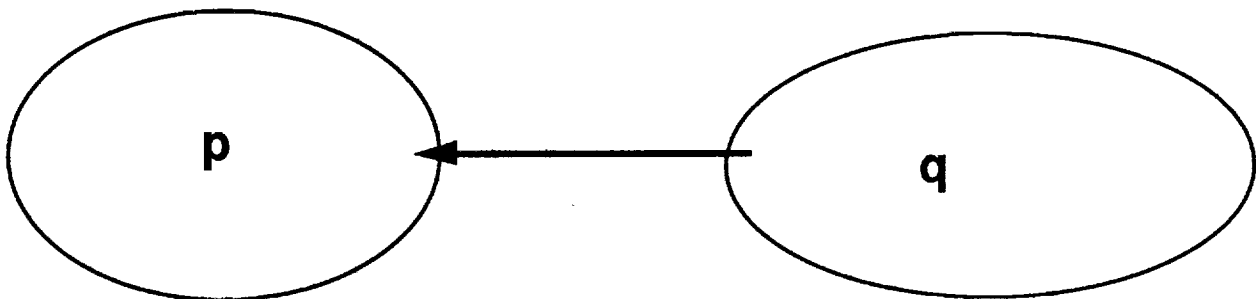
- $I(p|q)$  measures the extent by which any  $q$ -world resembles some  $p$ -world
- Degree by which what is true in one set must apply on another
- Properties of  $I(p|q)$ :
  - If  $p \Rightarrow r$ , then  $I(p|q) \leq I(r|q)$ ,
  - If  $q \Rightarrow r$ , then  $I(p|q) \geq I(p|r)$ ,
  - $I(p|q) \geq I(p|r) \otimes I(r|q)$ ,
  - $I(p|q) = \sup_r [I(p|r) \otimes I(r|q)]$
- Basis for generalized modus ponens



## DEGREE OF CONSISTENCE

$$C(p | q) = \sup_{w|q} \sup_{w'|p} S(w, w')$$

- “Dual” of the degree of implication function
- Measures extent by which true propositions in one set may apply on another



## UNCONDITIONED POSSIBILITY DISTRIBUTIONS

- Upper bounds of  $C(p|\mathcal{E})$

$$C(p|\mathcal{E}) \leq \text{Poss}(p)$$

## UNCONDITIONED NECESSITY DISTRIBUTIONS

- Lower bounds of  $I(p|\mathcal{E})$

$$\text{Nec}(p) \leq I(p|\mathcal{E})$$

$$\text{Nec}(p) \leq I(p|\mathcal{E}) \leq C(p|\mathcal{E}) \leq \text{Poss}(p)$$

## Inverse of a T-Norm

$$a \oslash b = \sup\{c: b \otimes c \leq a\}$$

$a \otimes b$	$a \oslash b$
$\max(a + b - 1, 0)$	$\min(1 + a - b, 1)$
$ab$	$a / b, \text{ if } b > a,$ $1, \text{ otherwise}$
$\min(a, b)$	$a, \text{ if } b > a,$ $1, \text{ otherwise}$

## Conditional Distributions

- Conditional Necessity

$$\text{Nec}(q|p) \leq \inf_{w \in \mathcal{S}} [I(q|w) \ominus I(p|w)]$$

- Conditional Possibility

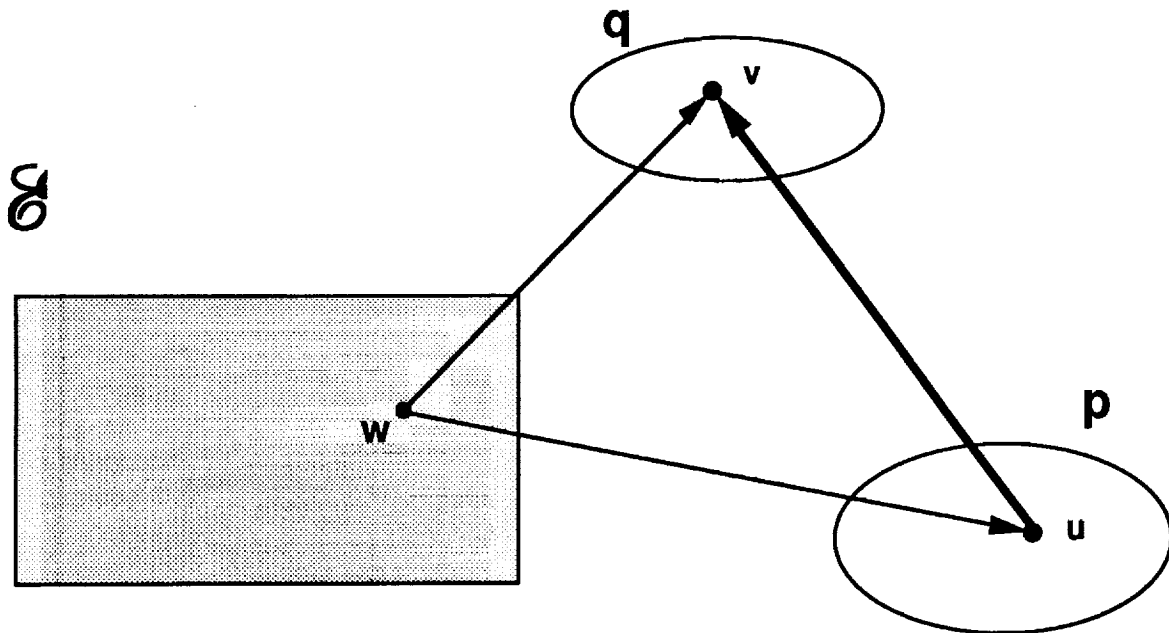
$$\text{Poss}(q|p) \geq \sup_{w \in \mathcal{S}} [I(q|w) \ominus I(p|w)]$$

The conditional distributions measure the extent by which similarity to the consequent must or may (respectively) exceed the similarity to the antecedent

## Generalized Modus Ponens

$$\sup_{\mathcal{F}} [\text{Nec}(q|p) \otimes \text{Nec}(p)] \leq I(q|\mathcal{E})$$

$$\sup_{\mathcal{F}} [\text{Poss}(q|p) \otimes \text{Poss}(p)] \geq C(q|\mathcal{E})$$



## POSSIBLE WORLDS and VARIABLES

- Possible Worlds will be characterized by means of a number of variables  $X, Y, \dots$
- Each variable  $X$  takes values in a well-defined domain  $\mathcal{D}(X)$ , e.g.,

$$\mathcal{D}(\text{Color}) = \{ \text{Green, Red, Blue, } \dots \}$$

- Possible worlds correspond to a complete specification of variable values
- Partial specification of variable values defines a subset of possible worlds
- The propositions of interest are those of the form:

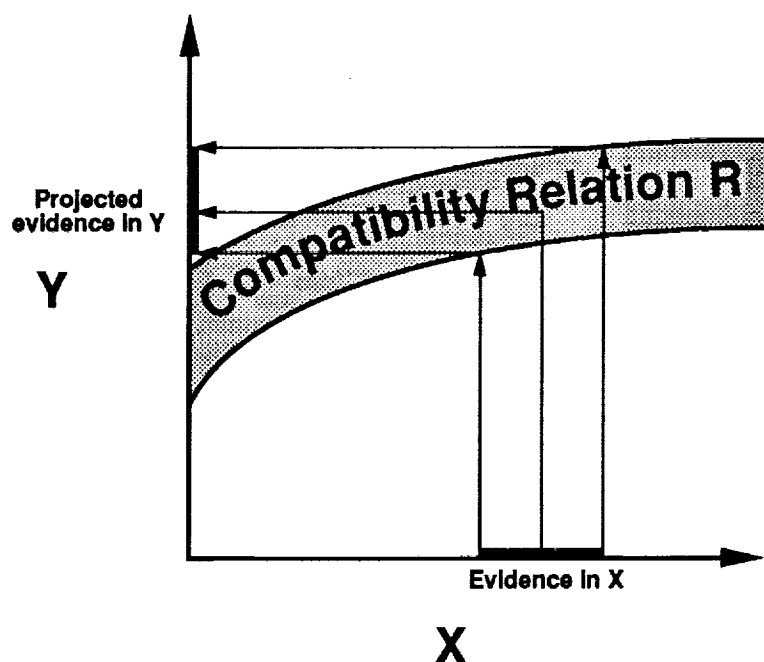
“ $X$  is  $x$ ,” “ $Y$  is  $y$ ,” .....

and their logical combinations (conjunctions, disjunctions, ... )

- $[X=x]$  denotes the subset of all worlds where  $X=x$

## Compatibility Relations

- Define relationship between values of two system variables (in the “actual” world)
- Permits the derivation of possible values of Y from knowledge of possible values of X
- Constrain the extent of the evidential set





## CONDITIONAL POSSIBILITIES from FUZZY RULES

• *If X is A, then Y is B*

• Interpretation:

• If  $\mathcal{E}$  is such that

$$\text{Poss}(x|\mathcal{E}) = \mu_A(x) \geq C(x|\mathcal{E}).$$

then

$$\mu_B(y) \geq C(y|\mathcal{E}).$$

• The function  $\Pi(y|x)$  defined by

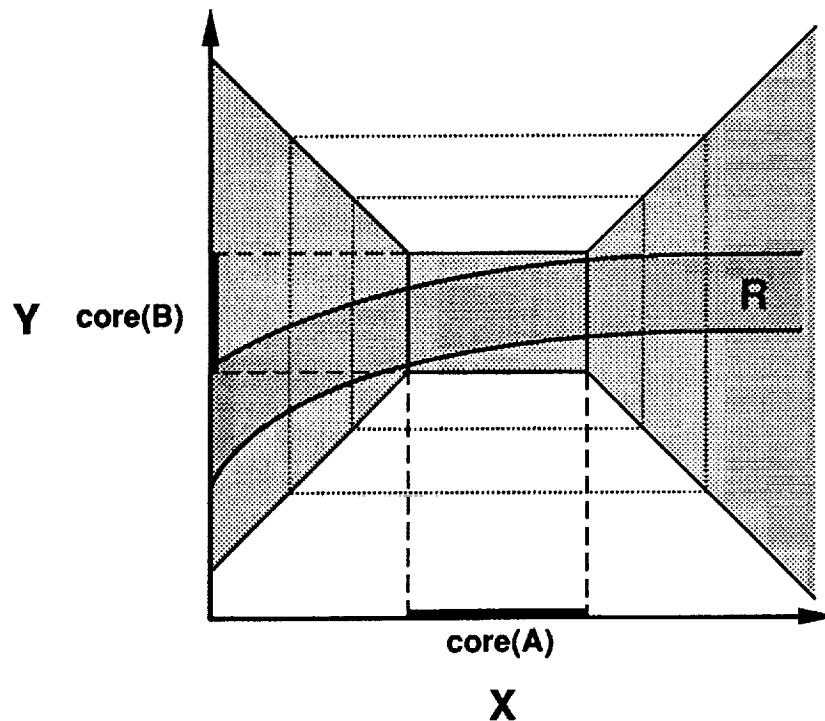
$$\Pi(y|x) = \mu_B(y) \oslash \mu_A(x),$$

is a conditional possibility for y given x

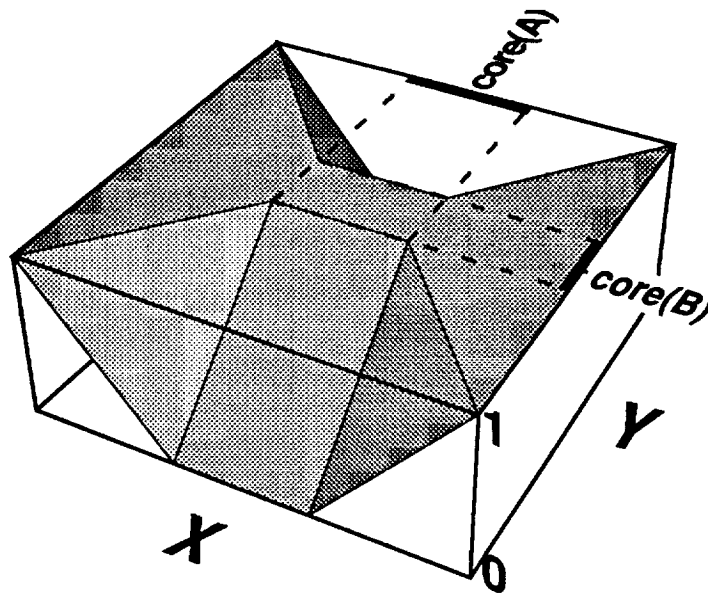
## Logical Interpretation of a Conditional Rule

(Zadeh-Trillas-Valverde)

- The conditional possibility is an “enclosing” approximation of the compatibility relation



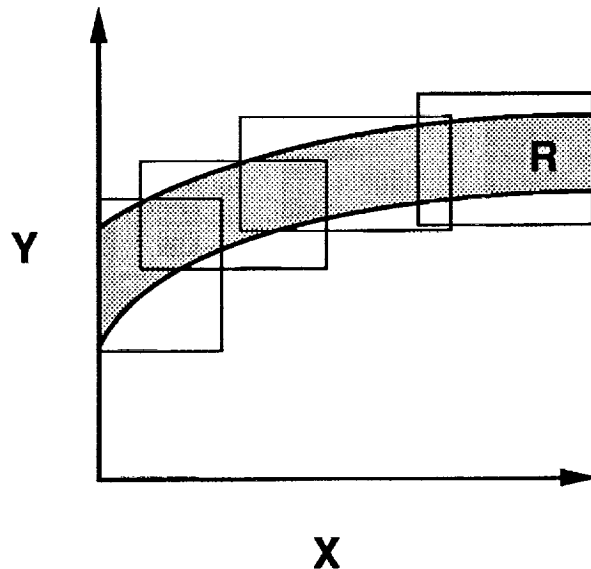
## ZTV Interpretations as Fuzzy Relations



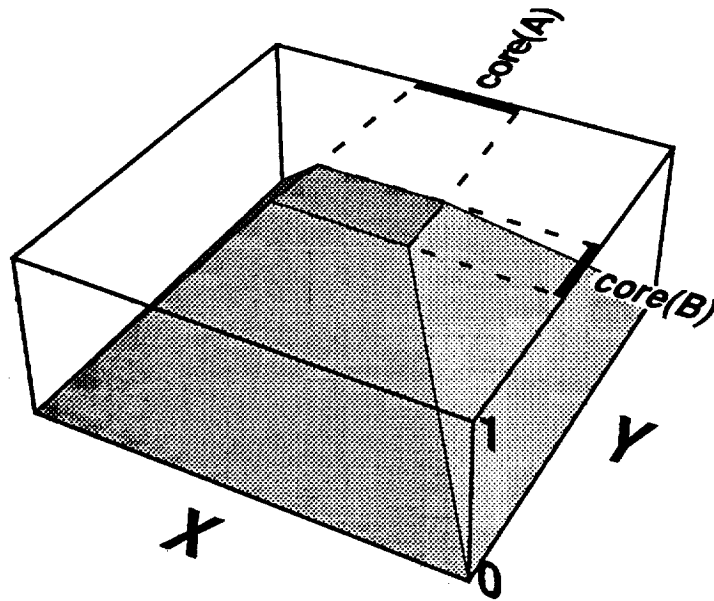
## Disjunctive Interpretations of Conditional Possibility Relations

(Zadeh-Mamdani-Assilian)

- “If X is A, then Y is B” is interpreted as one of a set of regions that must be combined (by disjunction) to approximate the compatibility relation
- Relation is characterized as a “set of points” rather than as the intersection of constraining regions



## ZMA Disjunctive Approximants as Fuzzy Relations





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## **Autonomous Robotics Research at the Artificial Intelligence Center**

- **FLAKEY**
  - **Successor of the pioneer autonomous robot SHAKEY**
  - **Technological Emphasis:**
    - **Autonomous Navigation**
    - **Autonomous Planning/Replanning**
    - **Multiple Intercommunicating Agents**
    - **Explicit Representation of Knowledge States**
    - **Integration of Sensing Activities into Plans**
    - **Learning**
  - **Fuzzy Logic/Neural Network Investigations:**
    - **Rule-based “blending” of Local Behaviors**
    - **Flexible Navigation**
    - **Flexible Planning/Replanning**
    - **NN-Based Learning**