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Solution of Three-Dimensional Afterbody Flow Using Reduced Navier-Stokes Equations

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ABSTRACT

The flow over afterbody geometries has been investigated using the reduced Navier-Stokes (RNS) approximation. Both pressure velocity flux-split and composite velocity primitive variable formulations have been considered. Pressure or pseudo-potential relaxation procedures are combined with sparse matrix or coupled strongly implicit algorithms to form a 3-D solver for general non-orthogonal coordinates. Three-Dimensional subsonic and transonic viscous/inviscid interacting flows have been evaluated. Solutions with and without regions of recirculation have been obtained.

INTRODUCTION

For a significant class of problems, exhibiting strong viscous-inviscid interaction, the reduced form of the Navier-Stokes (RNS) equations can provide an effective alternative to the full Navier-Stokes (NS) system for large Reynolds (Re) number numerical calculations, in both, two- and, three- dimensions. In a series of papers [1-4], Rubin and Khosla have developed and applied composite velocity primitive variable and pressure velocity formulations for a variety of sub-, trans- and supersonic flow problems. The RNS formulation, which represents a composite of Euler, higher-order boundary-layer triple deck equations, is consistent with all asymptotic large Re strong interaction theories and results in a uniformly valid single set of equations. The RNS system includes all normal and secondary flow diffusion terms in the surface momentum equations, but does not include normal or axial flow diffusion effects in the surface normal momentum equation. This approximation is self consistent for the large Re limit. Crossflow or secondary flow diffusion is retained in all equations as these effects are required to accurately model many three dimensional external and internal flows; see e.g. references [5-11], where it has been shown that explicitly added artificial viscosity can significantly destort secondary flow, heat transfer and surface stress predictions.

In the present paper, both the pressure and composite velocity primitive variable RNS formulations are considered for the computation of three-dimensional afterbody flows. In the pressure velocity formulation, for the velocity ($\bar{q}$) and the pressure (p), the dominant elliptic behavior and associated upstream influence, as manifested through an eigenvalue analysis [12], is associated with acoustic propagation and therefore the discretization requires an appropriate form of pressure velocity flux-split differencing. In the composite velocity formulation, the upstream influence is modelled by considering the velocity as a multiplicative composite of viscous or rotational velocities (U,W), and velocities derived from a pseudo potential (Φ). This representation of velocities is defined in the spirit of matched asymptotic expansions and is such that the resulting system reduces to the potential equation for inviscid irrotational regions. For rotational inviscid flows, a composite form of the Euler equations is recovered and a vorticity (U,W) term

appears on the right hand side of the pseudo-potential continuity equation. The composite formulation compliments the two-dimensional vorticity stream function formulation, but is more suitable for three-dimension problems, as the total number of unknowns is does not increase, as would be the case for the three-dimensional vorticity-stream function procedure.

Flux vector splitting [12] is the basis of the discretization for the $(\bar{q},p)$ system and flux biasing with appropriate upwinding [1] is specified for the (U,W,Φ) system. For supersonic flows the both RNS systems reduce to initial value PNS formulations. The $(\bar{q},p)$ system can be solved with a spatial "boundary-layer" type marching relaxation procedure. Since flux biasing is employed for the composite (U,W,Φ) system, supersonic flows are computed by a relaxation process with the Enquist-Osher compressibilty correction applied for the pseudo-potential Φ. It should be emphasized that both formulations are valid throughout the Mach number Reynolds number range, including $M_\infty \ll 1$ and $M_\infty \gg 1$, and do not require the addition of any explicitly added artificial viscosity. The inherent numerical viscosity associated with the discretization is sufficient to capture strong shocks over 3 to 4 mesh points. This numerical error is minimized with fine grids and a multi-grid strategy [1]. In earlier investigations of Rubin and Khosla, both pressure and composite velocity RNS codes have been applied for the computation of incompressible, transonic and supersonic flows with strong viscous-inviscid interaction, flow reversal and shock capturing e.g. see references [1-11]. Furthermore, the omitted diffusion terms have been incorporated directly, or via a deferred corrector approach, to obtain solutions of the complete NS equations. It has been shown that these effects are minimal for the problems considered. These procedures have been tested for the steady flow over an axisymmetric boattail and for the unsteady flow over a Joukowski airfoil. In the steady RNS formulations, outflow boundary conditions are required only for the pressure or pseudo potential and, as such, these are the only variables for which global storage of velocities is required outside of reverse flow regions. This is a significant simplification for steady three dimensional flow computations, and this can also be useful in the computation of complex three-dimensional flows on very fine meshes.

In both RNS formulations, the axial convective terms are first or second-order upwind differenced. The solution procedure takes advantage of the flux split/upwind differencing and results in a boundary-layer type streamwise marching method that is imbedded in a global relaxation process. In certain strong interaction cases a sparse matrix direct solver is applied for the pressure variable cross plan solution; in the composite velocity velocity the coupled strongly implicit ILU inversion is applied.

Subsonic flow past afterbody configurations with elliptic and hyperelliptic cross sections are discussed in this paper. Comparison are given between the pressure and composite velocity solutions. Grid resolutions studies are used to assess the accuracy of the two solution procedures.

GOVERNING RNS SYSTEM AND SOLUTION PROCEDURE

As described in earlier investigations [1-11], the RNS approximation leads to a single composite system that includes the Euler, second-order boundary layer and triple deck equations. The governing RNS system is considered for three-dimensional generalized coordinates and is designed to allow for shock capturing and flow reversal. In nondimensionalized form, the RNS system for low-speed flow is given as

Continuity Equation

$$\frac{\partial}{\partial \xi} (\rho\sqrt{g}\ u) + \frac{\partial}{\partial \eta} (\rho\sqrt{g}\ v) + \frac{\partial}{\partial \zeta} (\rho\sqrt{g}\ w) = 0 \tag{1-a}$$

ξ - Momentum Equation

$$\frac{\partial}{\partial \xi} (\rho\sqrt{g}\ u^2) + \frac{\partial}{\partial \eta} (\rho\sqrt{g}\ vu) + \frac{\partial}{\partial \zeta}(\rho\sqrt{g}\ wu) + \text{curvature terms} =$$

$$- g^{11}\ p_\xi - g^{12}\ p_\eta - g^{13}\ p_\zeta + \text{viscous terms} \tag{1-b}$$

ζ - Momentum Equation

$$\frac{\partial}{\partial \xi} (\rho\sqrt{g}\ wu) + \frac{\partial}{\partial \eta} (\rho\sqrt{g}\ vw) + \frac{\partial}{\partial \zeta}(\rho\sqrt{g}\ w^2) + \text{curvature terms} =$$

$$- g^{31}\ p_\xi - g^{32}\ p_\eta - g^{33}\ p_\zeta + \text{viscous terms} \tag{1-c}$$

η - Momentum Equation

$$\frac{\partial}{\partial \xi} (\rho\sqrt{g}\ uv) + \frac{\partial}{\partial \eta} (\rho\sqrt{g}\ v^2) + \frac{\partial}{\partial \zeta}(\rho\sqrt{g}\ vw) + \text{curvature terms} =$$

$$- g^{21}\ p_\xi - g^{22}\ p_\eta - g^{23}\ p_\zeta \tag{1-d}$$

Energy Equation

$$\sqrt{g}\ u^i\ [\frac{\partial T}{\partial \xi_i} - (\gamma - 1)T\ \frac{\partial \rho}{\partial \xi_i}] = \frac{\gamma}{Re\ Pr}\ \frac{\partial}{\partial \xi_i}\ [\mu\ \sqrt{g}\ g^{ij}\ \frac{\partial T}{\partial \xi_j}] \tag{1-e}$$

Equation of State

$$p = \frac{\rho T}{\gamma M_\infty^2} \tag{1-f}$$

where g^{ij} are the contravariant form of the metrics, $\sqrt{g}$ is the Jacobian, ρ, u, v, w, p and T are the density, velocities, pressure and temperature, repectively and γ is the ratio of specific heats. In these equations all variables are non-dimensionalized with respect to their free stream conditions except for the pressure which is normalized with the free stream dynamic pressure.

a) Pressure/Velocity Formulation (P.V.)

The pressure flux vector splitting, as described in reference [12], leads to the following discrete representation of the axial pressure gradient:

$$P_\xi = \omega_{i-1/2}(P_i - P_{i-1}) / \Delta\xi + (1-\omega_{i+1/2})(P_{i+1} - P_i)/\Delta\xi \qquad (2)$$

where

$$\omega \le \omega_M = \min[\gamma \, M /(1+(\gamma-1)M_\xi^2) \, , \, 1 \,] \qquad \text{for constant stagnation enthalpy}$$

and

$$\omega \le \omega_M = \min[M^2 \, , \, 1] \quad \text{for the full flux split energy equation.}$$

This pressure gradient splitting and the associated convective upwinding satisfies the major eigenvalue continuity constraints on the fluxes and flux derivatives. For $\omega = \omega_M$, one eigenvalue is alwaya zero so that sharp shocks are obtained. For regions of reverse, flow the condition $\omega=0$ is required. This ensures that the fluxes, flux derivatives and eigenvalues remain continuous throughout the flow. This form of flux splitting is designed to maintain a bias in the direction of convective fluxes and leads to a relaxation procedure that is solely acoustically driven in the subsonic regions.

As described in earlier investigations, the continuity equation is discretized at $(i,j-1/2,k)$. The streamwise ($\xi-$) and crossflow ($\zeta-$) momentum equations are discretized at (i,j,k) and the normal ($\eta-$) momentum equation is discretized at $(i,j+1/2,k)$. This discretization is consistent with the flux eigenvalues and with the appropriate boundary conditions required for u and p . For outflow boundaries without recirculation, only a pressure or pressure gradient condition is required. Far field boundary conditions, with positive outflow, are only required for u, p, and T. Inflow boundary conditions are specified for all flow variables or their gradients. Zero injection is assumed for all solid surface. The resulting algebraic system for the delta form of pressure $\Delta p = p^{n+1} - p^n$ is solved in each cross-plane using a coupled version of a sparse matrix direct solver. Additional details on the solver can be found in references [11,14]

b) Composite Velocity (U,W,Φ) Formulation (C.V.)

In the spirit of matched asymptotic expansions, the contravariant velocities are rewritten as:

$$u = (U+1) (g^{11}\Phi_\xi + g^{12}\Phi_\eta + g^{13}\Phi_\zeta) = (U+1)u_e$$

$$v = (g^{21}\Phi_\xi + g^{22}\Phi_\eta + g^{23}\Phi_\zeta) \qquad (3)$$

$$w = (U+1)(g^{13}\Phi_\xi + g^{32}\Phi_\eta + g^{33}\Phi_\zeta) + W$$

The composite representations for u and w, the axial and cross-flow velocity components, contain two types of terms, e.g. a rotational "pseudo" potential function Φ and viscous velocities U, W. The viscous no-slip boundary conditions are

introduced through the velocities U, W. The kinematical boundary conditions are satisfied through the pseudo potential. This is consistent with the potential and boundary-layer approximations. Substitution of equation (3) into equation (1) leads to the RNS composite system. Additional details and the resulting equations are given in references [1-4]. All derivatives are approximated using three point central differences except for the convective U_ξ, W_ξ terms: these are second-order upwinded. Upstream influence in attached flow regions arises solely through Φ_{i+1} and thus an outflow boundary condition on Φ, in lieu of p for the pressure formulation, is required. This leads to boundary-layer like streamwise marching for U, W with global relaxation for Φ. In reversed flow regions, w_ξ is upwind differenced and therefore the relaxation procedure includes both Φ and w, as is the case for p and (u,w) in the pressure split velocity formulation. The resulting algebraic system in each crossplane, for the delta form of Φ, $\nabla\Phi=\Phi^{n+1} - \Phi^n$ is solved using a consistent version of the coupled strongly implicit algorithm. This algorithm has previously been described in reference 10.

BOUNDARY CONDITIONS

The boundary conditions used for this investigation are; (i) uniform flow at the inflow; (ii) weak viscous/inviscid interaction at the outflow, thus at $\xi = \xi_m$, $\Phi_{\xi\xi}$= 0 or the negative eigenvalue fluxes are set to zero for the pressure velocity solver; (iii) the no slip and zero injection conditions are specified at the body surface; i.e. u = U+1 = 0, v = v_e = 0 and w = W = 0; (iv) far from the body the flow is assumed to be undisturbed; i.e. u = 1, w = 0, $\Phi = \Phi_{fs}$, T = 1 and the entropy is assumed to be constant.

RESULTS

Afterbody configurations of elliptical (Figure 1a) and hyper-elliptical (Figure 1b) cross-sections have been examined. In view of the geometric symmetry, the calculations have been carried out in one quarter of the flowfield. A body fitted
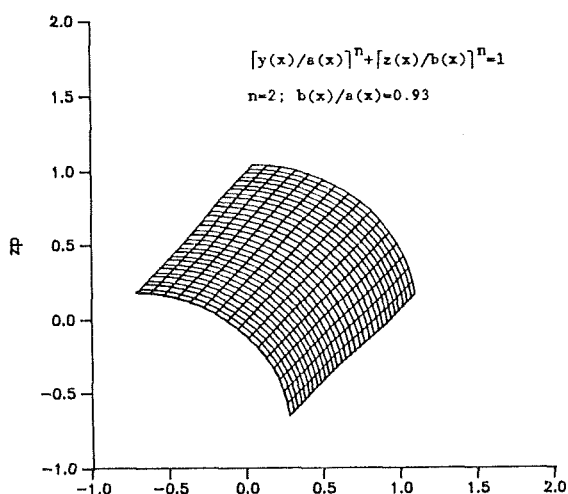

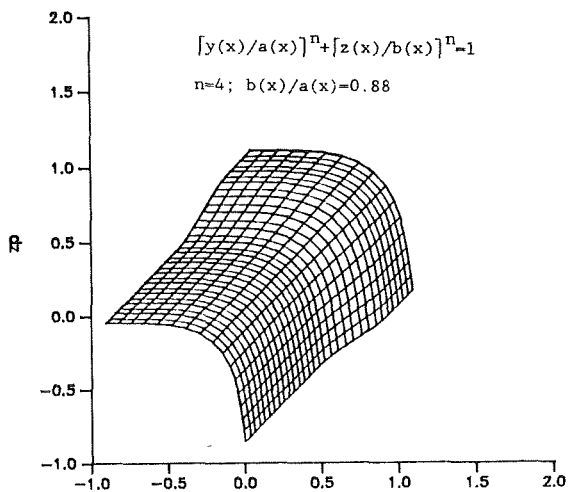Figure 1-a: Surface grid distribution
- ellipse

Figure 1-b: Surface grid distribution
- hyperellipse

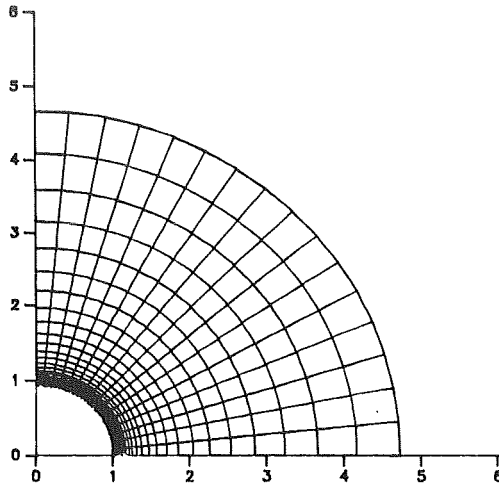computational grid e.g., see Figure 2 for an elliptic cross-section, was generated

Figure 2:

Ellipse - grid in the cross-plane 71x17

$\Delta\eta_{min}= 0.00175$
variable stretching

by using a shearing transformation. A stretched grid is employed in the normal direction while a uniform spacing is prescribed in the azimuthal direction. The axial boattail is generated by fitting a cubic polynomial between two appropriate cross-sections.

Both laminar and turbulent flows have been investigated using the composite velocity (C.V.) formulation. Only laminar flow computations have been carried out using the pressure velocity (P.V.) formulation. Figure 3a and 3b depict the pressure
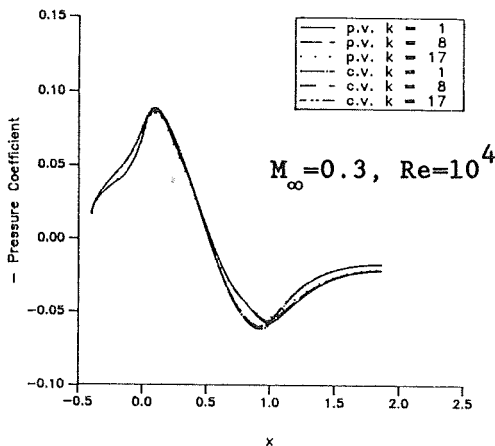
Figure 3-a: Pressure Coefficient
- comparison P.V. vs C.V.
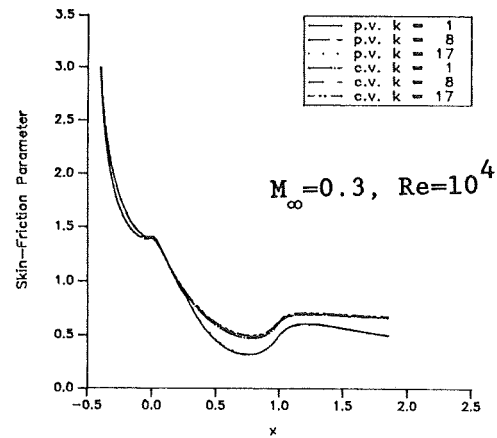
Figure 3-b: Skin-Friction Parameter
-comparison P.V. vs C.V.

coefficient and skin friction parameter for a laminar non-separated boattail of elliptical cross-section using both (P.V.) and (C.V.) formulations. The resonable agreement between the two solutions provides a self consistent evaluation of the two formulations. The metrics are calculated analytically for the P.V. solutions and numerically for the C.V. solutions. As the grid is refined the two solutions merge.

The pressure coefficient and skin friction parameter for turbulent flow computations for an afterbody of elliptical cross-section at $M_\infty=0.9$ using the (C.V.) formulation are shown in Figures 4-a and 4-b. As seen in the figure, a sharp weak transonic shock has been captured quite well.

The crossplane grid for an afterbody of hyper-ellipse cross-section is depicted in Figure 5. A comparison of the laminar P.V. flow solution for $Re=10^4$, on two grids
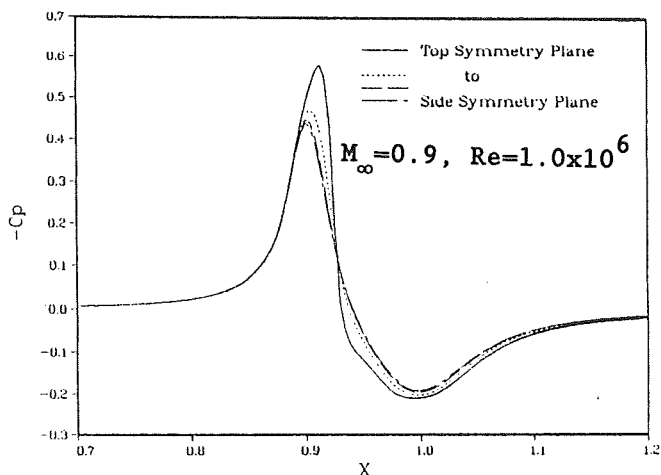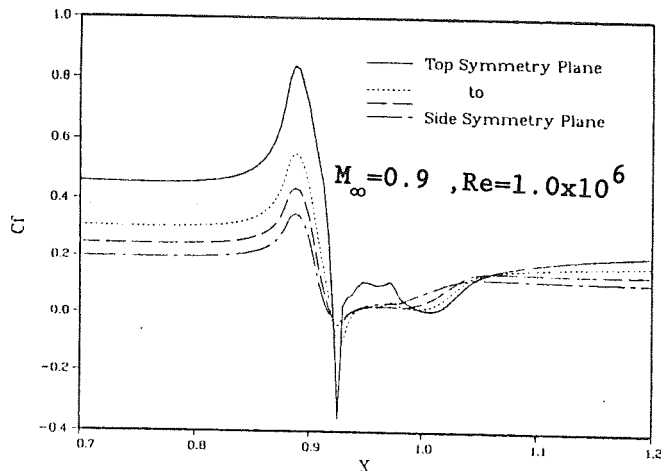

Figure 4-a: Pressure Coefficient
- ellipse

Figure 4-b: Skin-Friction Parameter
- ellipse
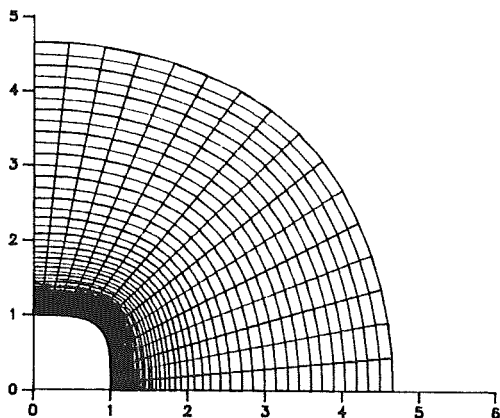
Figure 5: Hyperellipse

- grid in the cross-plane 79x17

$\Delta\eta_{min}=0.0005$

Stretching factor $\sigma=1.09$

61x79x17 and 121x79x17, is shown in Figures 6a and 6b. The pressure coefficient and skin friction parameter along a symmetry plane and the location of maximum azimuthal curvature are used for this comparison. A region of large recirculation, along the boattail, has been computed by this technique. The sensitivity to grid refinement is quite evident. Further grid refinement in both the axial and azimuthal directions are most likely required to obtain an acceptable level of accuracy. The axial velocity vectors in the recirculation region are depicted in Figures 7a and 7b. The effect of different subsonic Mach numbers is shown in Figures 8a and 8b. The recirculation is slightly increased. This is consistent with previous subsonic calculation on axisymmetric boattails [2].
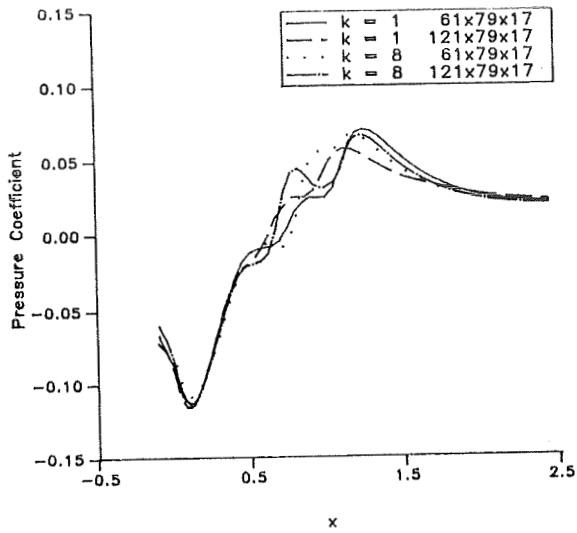
Figure 6-a: Pressure Coefficient for $M_\infty=0.3$, $Re=10^4$ (hyperellipse)
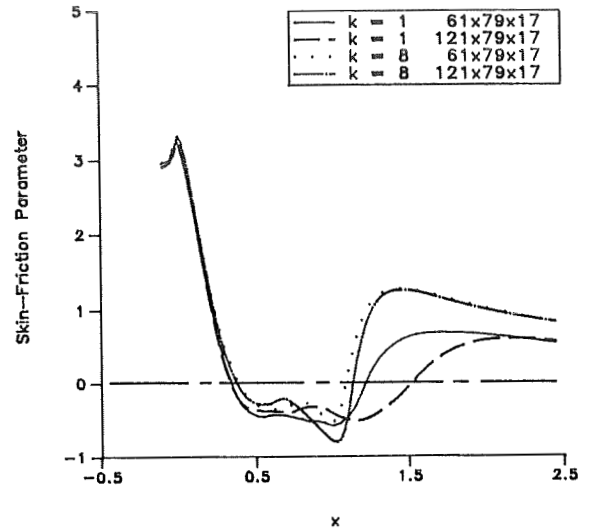- effect of grid

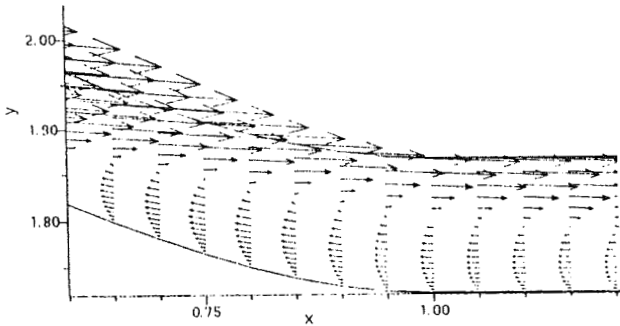Figure 6-b: Skin-Frition Parameter for $M_\infty=0.3$, $Re=10^4$ (hyperellipse)
- effect of grid

Figure 7-a: Velocity Field in the Separated Region for Hyperellipse
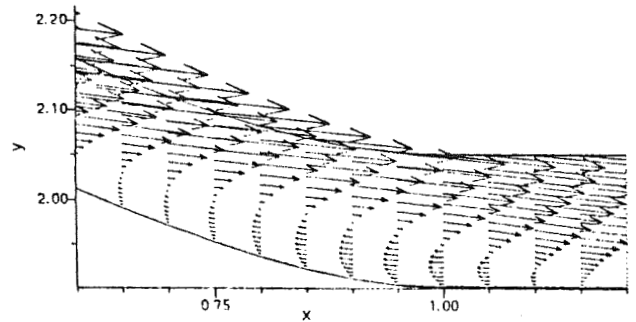$M_\infty=0.3$, $Re=10^4$, $k=1$

Figure 7-b: Velocity Field in the Separated Region for Hyperellipse
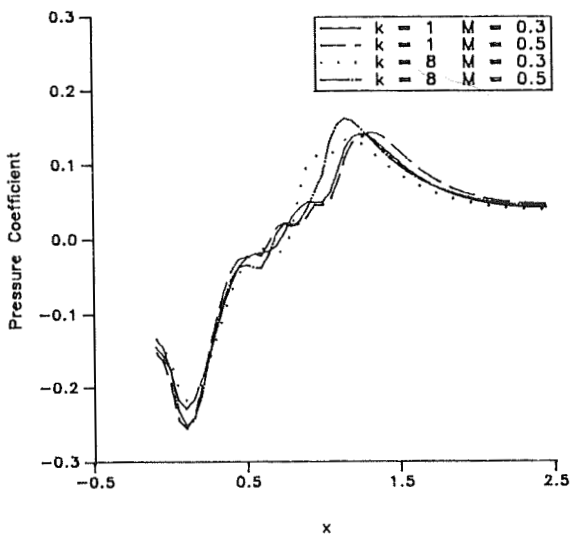$M_\infty=0.3$, $Re=10^4$, $k=8$

Figure 8-a: Pressure Coefficient for $Re=10^4$ (hyperellipse)
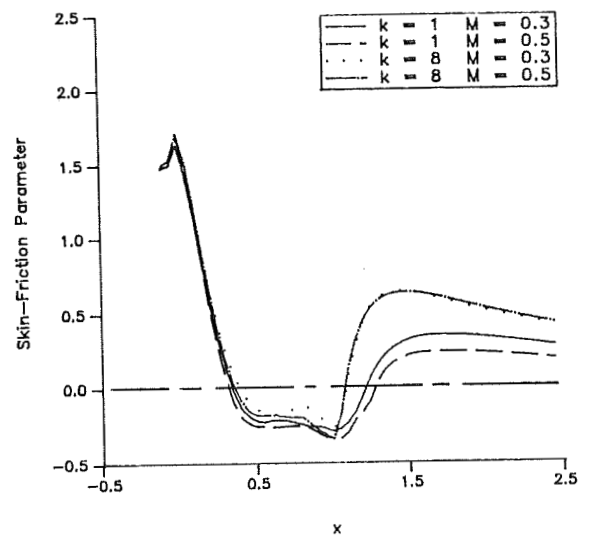- effect of Mach number

grid: 61x79x17

Figure 8-b: Skin-Friction Parameter for $Re=10^4$ (hyperellipse)
- effect of Mach number

grid: 61x79x17

From the above disscusion, it is clear that the present RNS techniques have the ability to compute fairly complex flow fields. The computational cost for the C.V. solution is modest, e.g., with 81923 grid points and 5 coupled unknowns this is of the order of 30 to 60 minutes on the Cray Y-MP. The computational cost of the P.V. solutions, on the other hand, is much greater. This is due of the fact that that sparse matrix direct solver, which requires 8 to 9 times the CPU of the CSIP, is applied for much of the cross plane inversion. Strategies to reduce the overall computational effort by minimizing the need for multiple LU (direct solver) inversions have been investigated. The computational times can be reduced considerably with bach subsitution. Recent studies have provided factors of less than 2 over C.V. computer time . A stabilized version of the consitent coupled strongly implicit algorithm for the P.V. formulation is also currently under investigation. This will result in further reductions in the P.V. CPU time.

CONCLUSION

Two formulations of the three-dimensional RNS equations have been investigated for the computation of laminar/turbulent subsonic, transonic flows with large recirculation regions. The technques are quite efficient in terms of storage and computational times. Future applicaltions to complex internal/external geometries and supersonic free streams are under investigation.

ACKNOWLEDGEMENT

REFERENCES

1. Gordnier, R.E. and Rubin,S.G., "3-D Composite Velocity Solutions for Subsonic/Transonic Flows over Afterbodies", AIAA paper no. 89-1837, Presented at the 20th Fluid Dynamics, Plasma Dynamics and Lasers Conference, Buffalo, New York, June 12-14, 1989.

2. Swanson, R.C., Rubin, S.G. and Khosla, P.K., "Calculation of Afterbody Flows with a Composite Velocity Formulation", AIAA paper no. 83-1736, Presented at the AIAA 16th Fluid and Plasma Dynamics Conference, Danvers, Massachusetts, July 12-14,1983.

3. Khosla, P.K. and Rubin, S.G., "A Composite Velocity Procedure for the Compressible Navier-Stokes Equations", AIAA Journal, Vol. 21, No. 11, November 1983, pp.1546-1551.

4. Rubin, S.G., Celestina, M. and Khosla, P.K., "Second Order Composite Velocity Solution for Large Reynolds Number Flows", AIAA paper no. 84-0172, Presented at the AIAA 22nd Aerospace Sciences Meeting, Reno, Nevada, January 9-12, 1984.

5. Reddy, D.R. and Rubin, S.G., "Consistent Boundary Conditions for Reduced Navier-Stokes (RNS) Scheme Applied to Three-Dimensional Viscous Flows", Journal of Fluids Engineering, Vol. 110, September 1988, pp. 306-314..

6. Rubin, S.G. and Reddy, D.R., "Analysis of Global Pressure Relaxation for Flows with Strong Interaction and Separation", Computers and Fluids, Vol. 11, 1983, pp. 281-306.

7. Khosla, P.K. and Lai, H.T., "Global PNS Solutions for Transonic Strong Interaction Flows", AIAA 22nd Aerospace Science Meeting, AIAA paper no. 84-0458, January, 1984.

8. Cohen, R. and Khosla, P.K., "Three-Dimensional Reduced Navier Stokes Equations solution for Subsoinc Separated and Non-Separated Flows Using a Global Pressure Relaxation Procedure", International Journal for Numerical Methods in Fluids, Vol. 9, 1989, pp. 1087-1098.

9. Rubin, S.G. and Khosla, P.K., "Navier-Stokes Calculations with a Coupled Strongly Implicit Procedure", Comput. Fluids 9, 1979, pp. 163-180.

10. Khosla, P.K. and Rubin, S.G., "Consistent Strongly Implicit Iterative Procedures for Two-Dimensional Unsteady and Three-Dimensional Space-Marching Flow Calculations", Computers and Fluids, Vol. 15, No. 4, 1987, pp. 361-377.

11. Bender, E.E. and Khosla, P.K., "Two-Dimensional Navier-Stokes Equations using Sparse Matrix Solvers", AIAA paper no. 87-0603, Presented at the 25th Aerospace Sciences Meeting, Reno, Nevada, January 12-15, 1987.

12. Rubin, S.G., "RNS/Euler Pressure Relaxation and Flux Vector Splitting", Technical Note, Computers & Fluids, Vol. 16, No. 4, 1988, pp. 485-490.

13. Pordal, H.S., Khosla, P.K. and Rubin, S.G., "A Flux-Split Solution Procedure for Unsteady Inlet Flows", AIAA paper no. 90-0585, Presented at the 28th Aerospace Sciences Meeting, Reno, Nevada, January 8-11. 1990.

14. Himansu, A., Khosla, P.K. and Rubin, S.G., "Relaxation/Sparse-Matrix Solvers for Three-Dimensional Recirculating Flows", AIAA paper no. 89-0552, Presented at the 27th Aerospace Sciences Meeting, Reno, Nevada, January 9-12, 1989.