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LARGE GAP MAGNETIC SUSPENSION SYSTEM

Moustafa K. Abdelsalam, Y.M. Eyssa

Madison Magnetics Incorporated

216 Walnut Street

Madison

WI 53705

I. SYSTEM CONFIGURATION

I.1. Model Core.

The model core size envelope is 30.48 cm (12") long and 10.16 cm (4") OD. The model core can be either a permanent magnet of 1.2 tesla average remnant magnetism or a superconducting coil with or without a holmium core, in a liquid helium dewar.

I.1.1. Superconducting Option.

Epoxy-impregnated coils with current densities in excess of 20 kA/cm² at fields of 6-9 tesla may be used. Such coils do not contain much copper or cooled surfaces, and their ability to tolerate disturbances is limited to the adiabatic heat capacity of the conductor material. However, the absence of large amounts of copper and helium in the windings allows such coils to operate at current densities up to ten times as large as those for cryostable coils, which is needed for model cores.

Higher values of magnetic moments may be achieved through using holmium core if space permits. Holmium has superior magnetic properties at 4.2 K with a saturation magnetic moment of 3.9 tesla. Table I-1 lists the magnetization of holmium at 4.2K [1,2].

Table I-1.

Holmium Magnetization vs. Applied Field at 4.2 K.

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Magnetization force (T)	0	0.1	0.52	1.0	1.5	2.5	3.5	4.5	6.5
Magnetization (T)	0	1.6	2.48	2.9	2.98	3.12	3.25	3.35	3.7
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The total magnetic pole strength per unit mass vs. design maximum field, B, and operating current density, J, are listed in Tables I-2 and I-3 with and without holmium core. As shown, the presence of holmium does not add to the values of Q/M significantly, since there is limited space in the core. Table

I-4 lists the specifications of the model coil design. A gross current density of 30 kA/cm² at 6 T field with no holmium core is selected.

Table I-2.

Model Core Magnetic Pole Strength per Unit Mass Q/M vs. Design Maximum Field, B, and Operating Current Density, J. All cases have OD = 0.09 m, ID \geq 0.05 m, and mandrel thickness = 1.27 mm. MH is the holmium magnetization, RI is the winding inner radius, QM is the winding pole strength, QH is the holmium pole strength, and Q is the sum of QM + QH. The mass M is the mass of the winding and holmium in addition to 10 kg for the model, dewar, and helium mass.

B(T)	J(A/m**2)	MH(T)	RI(m)	QH	QM	Q(Am)	Q/M(Am/kg)
4.00	0.20E+09	3.23	0.29E-01	0.12E+04	0.14E+05	0.151E+05	0.925E+03
4.00	0.30E+09	3.23	0.34E-01	0.38E+04	0.16E+05	0.197E+05	0.118E+04
4.00	0.40E+09	3.23	0.37E-01	0.53E+04	0.17E+05	0.222E+05	0.132E+04
4.00	0.50E+09	3.23	0.39E-01	0.62E+04	0.18E+05	0.237E+05	0.140E+04
4.00	0.60E+09	3.23	0.40E-01	0.69E+04	0.18E+05	0.248E+05	0.146E+04
5.00	0.20E+09	3.46	0.25E-01	0.00E+00	0.16E+05	0.158E+05	0.974E+03
5.00	0.30E+09	3.46	0.32E-01	0.26E+04	0.19E+05	0.212E+05	0.129E+04
5.00	0.40E+09	3.46	0.35E-01	0.45E+04	0.20E+05	0.246E+05	0.147E+04
5.00	0.50E+09	3.46	0.37E-01	0.57E+04	0.21E+05	0.268E+05	0.159E+04
5.00	0.60E+09	3.46	0.38E-01	0.65E+04	0.22E+05	0.283E+05	0.167E+04
6.00	0.20E+09	3.63	0.21E-01	0.00E+00	0.17E+05	0.171E+05	0.107E+04
6.00	0.30E+09	3.63	0.29E-01	0.13E+04	0.21E+05	0.222E+05	0.136E+04
6.00	0.40E+09	3.63	0.33E-01	0.35E+04	0.23E+05	0.265E+05	0.160E+04
6.00	0.50E+09	3.63	0.35E-01	0.49E+04	0.24E+05	0.293E+05	0.175E+04
6.00	0.60E+09	3.63	0.37E-01	0.59E+04	0.25E+05	0.313E+05	0.186E+04
7.00	0.20E+09	3.75	0.17E-01	0.00E+00	0.18E+05	0.180E+05	0.113E+04
7.00	0.30E+09	3.75	0.26E-01	0.76E+02	0.23E+05	0.229E+05	0.141E+04
7.00	0.40E+09	3.75	0.31E-01	0.25E+04	0.26E+05	0.281E+05	0.171E+04
7.00	0.50E+09	3.75	0.34E-01	0.41E+04	0.27E+05	0.315E+05	0.190E+04
7.00	0.60E+09	3.75	0.36E-01	0.53E+04	0.29E+05	0.339E+05	0.203E+04
8.00	0.20E+09	3.82	0.13E-01	0.00E+00	0.19E+05	0.186E+05	0.117E+04
8.00	0.30E+09	3.82	0.24E-01	0.00E+00	0.24E+05	0.244E+05	0.151E+04
8.00	0.40E+09	3.82	0.29E-01	0.14E+04	0.28E+05	0.293E+05	0.179E+04
8.00	0.50E+09	3.82	0.32E-01	0.32E+04	0.30E+05	0.333E+05	0.202E+04
8.00	0.60E+09	3.82	0.34E-01	0.45E+04	0.32E+05	0.362E+05	0.218E+04
9.00	0.20E+09	3.83	0.92E-02	0.00E+00	0.19E+05	0.189E+05	0.120E+04
9.00	0.30E+09	3.83	0.21E-01	0.00E+00	0.26E+05	0.257E+05	0.160E+04
9.00	0.40E+09	3.83	0.27E-01	0.40E+03	0.30E+05	0.302E+05	0.186E+04
9.00	0.50E+09	3.83	0.31E-01	0.23E+04	0.33E+05	0.349E+05	0.212E+04
9.00	0.60E+09	3.83	0.33E-01	0.37E+04	0.35E+05	0.382E+05	0.231E+04

Table I-3.

Model Core Magnetic Pole Strength, per Unit Mass Q/M vs. Design Maximum Field, B, and Operating Current Density J. All cases have OD = 0.09 m. There is no holmium mandrel in the core. The mass M is the mass of the winding in addition to 10 kg for the model, dewar and helium mass.

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B(T)	J(A/m**2)	MH(T)	RI(m)	QH	QM	Q(Am)	Q/M(Am/kg)
4.00	0.20E+09	3.23	0.29E-01	0.00E+00	0.14E+05	0.139E+05	0.899E+03
4.00	0.30E+09	3.23	0.34E-01	0.00E+00	0.16E+05	0.159E+05	0.114E+04
4.00	0.40E+09	3.23	0.37E-01	0.00E+00	0.17E+05	0.169E+05	0.129E+04
4.00	0.50E+09	3.23	0.39E-01	0.00E+00	0.18E+05	0.175E+05	0.140E+04
4.00	0.60E+09	3.23	0.40E-01	0.00E+00	0.18E+05	0.180E+05	0.148E+04
5.00	0.20E+09	3.46	0.25E-01	0.00E+00	0.16E+05	0.158E+05	0.955E+03
5.00	0.30E+09	3.46	0.32E-01	0.00E+00	0.19E+05	0.186E+05	0.126E+04
5.00	0.40E+09	3.46	0.35E-01	0.00E+00	0.20E+05	0.201E+05	0.147E+04
5.00	0.50E+09	3.46	0.37E-01	0.00E+00	0.21E+05	0.211E+05	0.162E+04
5.00	0.60E+09	3.46	0.38E-01	0.00E+00	0.22E+05	0.218E+05	0.173E+04
6.00	0.20E+09	3.63	0.21E-01	0.00E+00	0.17E+05	0.171E+05	0.985E+03
6.00	0.30E+09	3.63	0.29E-01	0.00E+00	0.21E+05	0.209E+05	0.135E+04
6.00	0.40E+09	3.63	0.33E-01	0.00E+00	0.23E+05	0.230E+05	0.160E+04
6.00	0.50E+09	3.63	0.35E-01	0.00E+00	0.24E+05	0.244E+05	0.179E+04
6.00	0.60E+09	3.63	0.37E-01	0.00E+00	0.25E+05	0.253E+05	0.194E+04
7.00	0.20E+09	3.75	0.17E-01	0.00E+00	0.18E+05	0.180E+05	0.997E+03
7.00	0.30E+09	3.75	0.26E-01	0.00E+00	0.23E+05	0.228E+05	0.141E+04
7.00	0.40E+09	3.75	0.31E-01	0.00E+00	0.26E+05	0.256E+05	0.171E+04
7.00	0.50E+09	3.75	0.34E-01	0.00E+00	0.27E+05	0.274E+05	0.194E+04
7.00	0.60E+09	3.75	0.36E-01	0.00E+00	0.29E+05	0.286E+05	0.212E+04
8.00	0.20E+09	3.82	0.13E-01	0.00E+00	0.19E+05	0.186E+05	0.998E+03
8.00	0.30E+09	3.82	0.24E-01	0.00E+00	0.24E+05	0.244E+05	0.145E+04
8.00	0.40E+09	3.82	0.29E-01	0.00E+00	0.28E+05	0.279E+05	0.180E+04
8.00	0.50E+09	3.82	0.32E-01	0.00E+00	0.30E+05	0.301E+05	0.206E+04
8.00	0.60E+09	3.82	0.34E-01	0.00E+00	0.32E+05	0.317E+05	0.228E+04
9.00	0.20E+09	3.83	0.92E-02	0.00E+00	0.19E+05	0.189E+05	0.993E+03
9.00	0.30E+09	3.83	0.21E-01	0.00E+00	0.26E+05	0.257E+05	0.148E+04
9.00	0.40E+09	3.83	0.27E-01	0.00E+00	0.30E+05	0.298E+05	0.186E+04
9.00	0.50E+09	3.83	0.31E-01	0.00E+00	0.33E+05	0.326E+05	0.216E+04
9.00	0.60E+09	3.83	0.33E-01	0.00E+00	0.35E+05	0.345E+05	0.241E+04

Table I-4.

Model Core Coil Specifications (Superconducting Solenoid).

Winding outer radius (cm)	4.5
Winding inner radius (cm)	2.9
Mandrell thickness (cm)	0.127
Winding length (cm)	22.86
Winding current density (kA/cm ²)	30.00
Winding maximum field (T)	6.0

I.1.2. Permanent Magnet Material Option.

A new superior permanent magnet material Nd₁₅Fe₇₇B₈ is planned for the model core [3,4]. The magnetic properties are listed in Table I-5.

Table I-5.

Magnetic Properties of ND₁₅Fe₇₇B₈ Magnetic Material.

	Br (T)	Hc (kA/m)	(BH) max (kJ/m ³)	Tc (K)
Nd ₁₅ Fe ₇₇ B ₈	1.23	960	290	585
Nd ₁₅ (Fe _{0.9} Co _{0.1}) ₇₇ B ₈	1.23	800	290	670
Nd ₁₅ (Fe _{0.8} Co _{0.2}) ₇₇ B ₈	1.21	820	260	740

As shown in Fig. I-1, the new permanent magnet material has large values of M_r (residual magnetism) and H_c (demagnetization critical field). M_r stays well above 1.2 tesla for most of the demagnetizing field and well over 1.15 tesla up to $H_c = 960$ kA/m (1.21 tesla). With $M_r = 1.2$ tesla, the magnetic properties of the model core are listed in Table I-6.

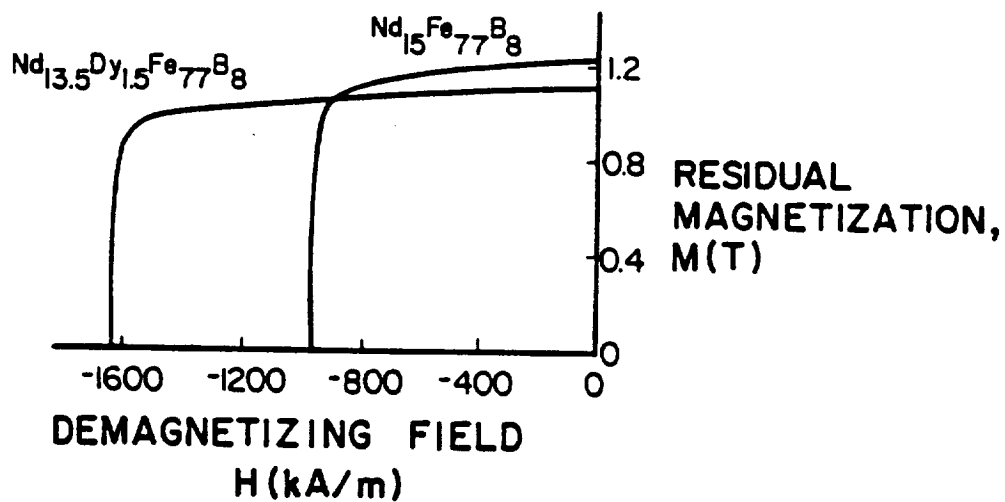


Figure I-1. Demagnetized curve of $Nd_{13.5}Dy_{1.5}Fe_{77}B_8$ sintered magnet

Table I-6.

Model Core Coil Specifications (Permanent Magnet).

Remnant magnetization (T)	1.2
Core length (cm)	30.48
Core diameter (cm)	10.16
Core mass (kg)	16.41
Model mass (kg)	6.7
Total mass, M (kg)	23.11
Pole strength, Q (Am)	7.742×10^3
Q/M (Am/kg)	335.

I.2. Levitation Magnet System.

The system under study is to levitate, position and control a 15 lb. model. The model is to be suspended 36" above the cryostat top plate. The array of magnets will control the position of the model in 5 degrees of freedom, namely the x, y, z displacements and the yaw and pitch rotations. Model rolling is controlled with eccentric weights.

I.2.1. Levitation Magnet System Configuration.

The system consists of "n" vertical solenoids arranged around the system center. The tops of the magnets are located as close as possible to the table surface. Because the model is allowed to assume any position between 0 and 360 degrees in the yaw direction, it is reasonable to assume that the magnet system should be arranged symmetrically around the vertical z-axis. The magnets may be arranged in one or more rings. Furthermore, each ring may perform a separate function. For example, a magnet array in one circle may be responsible for levitation and positioning while another array may be responsible for control and stability.

The first object of this study is to find the currents in the magnet array that satisfy a required F_z (lift), and control F_x , F_y , T_y and T_z at any position for the model. Since the number of forces and torques is 5, there is a need for at least 5 solenoids in the magnet array. A larger number of coils allows other constraints which depend on the nature of the system. For a superconducting magnet system, the minimum ampere-meters is

usually desired; while for resistive coils the criteria may be minimum ohmic heat losses in the coils. In the next section the procedure to optimize these two systems is analyzed.

I.2.2. Magnet System Optimization.

For both superconductive and resistive systems the optimization problem may be divided into two parts: first to find the optimum magnet dimensions and secondly to find the optimum current distribution in the magnet array that satisfies the force constraints. First, an approximate formula is used to derive closed form expressions for optimum magnet dimensions. Second, using these expressions for the dimensions, an "exact" approach is used to calculate the optimum current distribution in the magnet array.

I.2.2.1. Superconducting Magnets Optimum Size.

The main function of the magnet array is to produce a lift force on the model. The lift force on the model due to one magnet in the array may be approximated by

$$F_z = KQ_1 (\pi/3) Jb^3 (1-\alpha)^3 [H^{-3} - (H+L)^{-3}] \quad (1)$$

where

K = constant for given locations of the magnet and the model

Q_1 = magnetic pole strength of the model

J = current density in the magnet

b = outer radius of the magnet

α = inner to outer diameter ratio of the coil

H = vertical distance between top of coil and center line of model, and

L = length of coil .

From Equation 1, we may define F_o to be:

$$F_o \equiv (F_z H^3 / K Q_1) = (\pi/3) J b^3 (1-\alpha^3) \chi, \quad \text{where}$$

$$\chi = 1 - (1+x)^{-3}$$

$$x = L/H.$$

The ampere-meters of the coil are

$$A = \pi J b^2 L (1-\alpha^2).$$

At any location the minimum of the ampere-meters A subject to the constraint $F_o = \text{constant}$ is achieved when the following conditions are met:

$$b \text{ is as large as possible for this location, and} \quad (2)$$

$$\alpha = \sqrt{m+(1/2)} - 1/2, \text{ and} \quad (3)$$

$$m = 2/(x^3+4x^2+6x+1). \quad (4)$$

I.2.2.2. Superconducting Magnets Optimum Current Distribution.

The forces and torques acting on the model due to the magnet array system may be presented as

$$\{F_i\} = [S_{ij}] \{I_j\} \quad (5)$$

where $\{F_i\}$ is the force vector, $i=1,5$

$[S_{ij}]$ is a pseudo-stiffness matrix whose elements

S_{ij} represent the force on the model in the i^{th} direction due to a unit current in the j^{th} coil.

The ampere-meters of the magnet array is

$$\Sigma A_j = \Sigma |I_j \ell_j|, \quad j=1,n$$

where I_j and ℓ_j are the current and conductor length of the j^{th} coil.

$$\text{For identical coils } \Sigma A_j = \ell \Sigma |I_j| \quad (6)$$

Equation 6 shows that for fixed coil dimensions, minimum ΣA_j occurs at minimum ΣI_j^2 .

Thus it is required to minimize ΣI_j^2 subject to the constraints of Equation 5.

Using Lagrange's approach, the problem reduces to minimizing an objective function G defined as

$$G = \Sigma I_j^2 + \Sigma \lambda_i \Sigma S_{ij} I_j - \Sigma \lambda_i F_i \quad (7)$$

where $i = 1$ to 5

$j = 1$ to n

This function has an optimum value at the set of currents I_j satisfied by the following (n+5) simultaneous equations:

$$(\partial G / \partial I_j) = 2 I_j + \Sigma \lambda_i \Sigma S_{ij} = 0 \quad j=1, n$$

$$(\partial G / \partial \lambda_i) = \Sigma S_{ij} I_j - F_i = 0 \quad i=1, 5$$

Arranging these equations in a matrix form we get

$$[B] \{x\} = \{c\} \quad (8)$$

where

$$[B] = \begin{bmatrix} 2[I]_{n \times n} & [S]^T_{n \times 5} \\ \hline [S]_{5 \times n} & [0]_{5 \times 5} \end{bmatrix}$$

where $[I]$ = identity matrix

$$\{x\} = \begin{bmatrix} I_1 \\ : \\ I_n \\ \lambda_1 \\ : \\ \lambda_5 \end{bmatrix}$$

$$\{c\} = \begin{Bmatrix} 0 \\ : \\ 0 \\ F_1 \\ : \\ F_5 \end{Bmatrix}$$

Manipulating Equation 8 and solving for the current distribution, we get

$$\{I_j\}_{nx1} = [S]_{nx5}^T [SS^T]_{5x5}^{-1} \{F_i\}_{5x1} \quad (9)$$

The elegance of equation (9) is that regardless of the number of magnets "n", the matrix to be inverted is always 5x5.

Solving Eq. (9) gives the current distribution in the magnet array that satisfies the force constraints and results in a minimum total ampere-meters in the coils.

I.2.2.3. Resistive Magnets Optimum Size

The power loss P_1 in one magnet in the array is

$$P_1 = I^2 R = \rho \pi J^2 b^2 L (1-\alpha^2), \text{ where} \quad (10)$$

J = the gross current density, and

ρ = the effective resistivity.

Minimizing the power dissipation subject to the force condition of Eq. (1) results in the following conditions:

b is as large as possible,

$$\alpha = 0.366 \quad \text{and} \quad (11)$$

$$L = 0.7373 H . \quad (12)$$

I.2.2.4. Resistive Magnets Optimum Current Distribution

The total power dissipation in the coil array is

$$P = \sum I_j^2 R_j \quad j = 1, n$$

For identical magnets the resistance R_j is the same,

$$P = R \sum I_j^2 \quad (13)$$

where R is the resistance of one coil.

Equation (13) shows that for an array of magnets with the same given dimensions, minimum power dissipation coincides with the minimum of $\sum I_j^2$. This is the same condition for minimum ampere-meters. Consequently, the current distribution given by equation (9) results in a minimum power dissipation in the magnet array.

II . SYSTEM OPTIONS

The six combinations of superconducting or permanent model magnets with superconducting or copper levitation magnets are listed in Table II-1.

Table II-1.

Model Core/Levitation Magnet Combinations.

Model	Magnets	Comment
S/C	S/C	smallest magnets
P/M	S/C	simple model
S/C	Cu	(LN ₂ cooled)
PM	Cu	magnets)
S/C	Cu	(H ₂ O cooled)
PM	Cu	magnets)

S/C ≡ superconducting
 PM ≡ permanent magnets
 Cu ≡ copper magnets

II.1. Superconducting Levitation Coils.

The superconducting magnet array is optimized for minimum total ampere-meters. Five and six arrays are studied at varying location radii for current densities ranging from 50 MA/m² to 300 MA/m². Table II-2 lists magnet parameters for an S/C model core.

The superconducting model in this table has a magnetic pole strength of 1350 Am/kg of total mass. The other parameters in this table are:

NC = Number of coils in the array

R = Location radius

L, RI, RO = Optimum coil dimensions (I = inner, O = outer)

Table II-2.

Minimum Ampere-Meters S/C Magnet Arrays
for Levitation of an S/C Model.

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MODEL Q/M	= 1.350E+03 Am/KG	FX/M	FY/M	FZ/M	TY/M	TN/M
MODEL LENGTH	= 3.000E-01 m	.000E+00	.000E+00	1.000E+00	.000E+00	.000E+00
MIN DISTANCE	= 5.000E-02 m					

NC	R	L	RI	RO	JMAX	BMAX	ISTOT	N*IS
5	.400	.924	.069	.210	1.500E+07	2.54	5.576E+06	8.575E+06
5	.450	.387	.120	.240	1.500E+07	1.76	2.552E+06	3.918E+06
5	.500	.262	.160	.269	1.500E+07	1.34	1.882E+06	2.887E+06
5	.550	.204	.195	.298	1.500E+07	1.10	1.589E+06	2.435E+06
5	.600	.170	.229	.328	1.501E+07	.95	1.438E+06	2.202E+06
5	.650	.148	.260	.357	1.500E+07	.85	1.358E+06	2.079E+06
5	.700	.133	.291	.386	1.501E+07	.79	1.323E+06	2.024E+06
5	.750	.122	.321	.416	1.501E+07	.74	1.317E+06	2.015E+06
5	.800	.114	.350	.445	1.500E+07	.70	1.334E+06	2.040E+06
5	.400	.459	.097	.210	2.000E+07	2.42	3.257E+06	5.006E+06
5	.450	.273	.140	.240	2.000E+07	1.73	2.106E+06	3.233E+06
5	.500	.200	.177	.269	2.001E+07	1.36	1.677E+06	2.572E+06
5	.550	.162	.212	.298	2.001E+07	1.14	1.464E+06	2.243E+06
5	.600	.138	.244	.328	2.001E+07	1.00	1.350E+06	2.067E+06
5	.650	.122	.275	.357	2.001E+07	.91	1.291E+06	1.975E+06
5	.700	.110	.306	.386	1.999E+07	.84	1.266E+06	1.936E+06
5	.750	.102	.335	.416	2.001E+07	.80	1.267E+06	1.938E+06
5	.800	.096	.364	.445	1.998E+07	.76	1.289E+06	1.972E+06
6	.450	.758	.072	.200	1.500E+07	2.27	4.939E+06	7.459E+06
6	.500	.393	.112	.225	1.500E+07	1.70	2.793E+06	4.227E+06
6	.550	.280	.145	.250	1.500E+07	1.36	2.168E+06	3.288E+06
6	.600	.223	.174	.275	1.500E+07	1.16	1.881E+06	2.857E+06
6	.650	.189	.202	.300	1.500E+07	1.02	1.736E+06	2.639E+06
6	.700	.167	.228	.325	1.501E+07	.93	1.667E+06	2.537E+06
6	.750	.152	.253	.350	1.500E+07	.87	1.643E+06	2.503E+06
6	.800	.141	.278	.375	1.500E+07	.83	1.658E+06	2.528E+06
6	.400	1.349	.049	.175	2.000E+07	3.13	9.523E+06	1.436E+07
6	.450	.423	.096	.200	2.000E+07	2.20	3.245E+06	4.906E+06
6	.500	.277	.131	.225	2.000E+07	1.69	2.313E+06	3.503E+06
6	.550	.212	.161	.250	2.000E+07	1.38	1.919E+06	2.912E+06
6	.600	.175	.190	.275	2.001E+07	1.19	1.721E+06	2.615E+06
6	.650	.152	.217	.300	2.000E+07	1.07	1.616E+06	2.459E+06
6	.700	.136	.243	.325	2.000E+07	.99	1.572E+06	2.393E+06
6	.750	.125	.268	.350	2.000E+07	.92	1.563E+06	2.382E+06
6	.800	.117	.293	.375	2.001E+07	.89	1.587E+06	2.420E+06

JMAX = Maximum current density in the magnet array
BMAX = Maximum field in the magnets
ISTOT = Operational ampere-meters at the specified model
location
N*IS = Total ampere-meters capacity of the system.

It is seen that increasing the allowable current density in the magnets has a small effect on the total ampere-meters capacity.

The total ampere-meters in five- and six-coil S/C arrays are plotted versus array location radius for an S/C model (Fig. II-1). The optimum ampere-meters is at a location radius of 70 cm. The five-coil array uses less ampere-meters than the six-coil array. Table II-3 lists parameters for the S/C arrays for a permanent magnet model (PM). In Fig. II-2 the ampere-meters of the five-coil and six-coil S/C arrays are plotted versus magnet location radius. The optimum ampere-meters occurs around the 70 cm radius.

II.2. Resistive Levitation Coils.

The minimization of power consumption is the main goal for water-cooled or nitrogen-cooled coil designs. Water-cooled or cryocooled copper magnet arrays are shown to be feasible. Tables II-4 and II-5 list magnet array parameters for S/C and permanent magnet models. The significant parameters are:

Cu Ratio: ratio of copper in the windings

L, RI, RO: optimum magnet dimensions

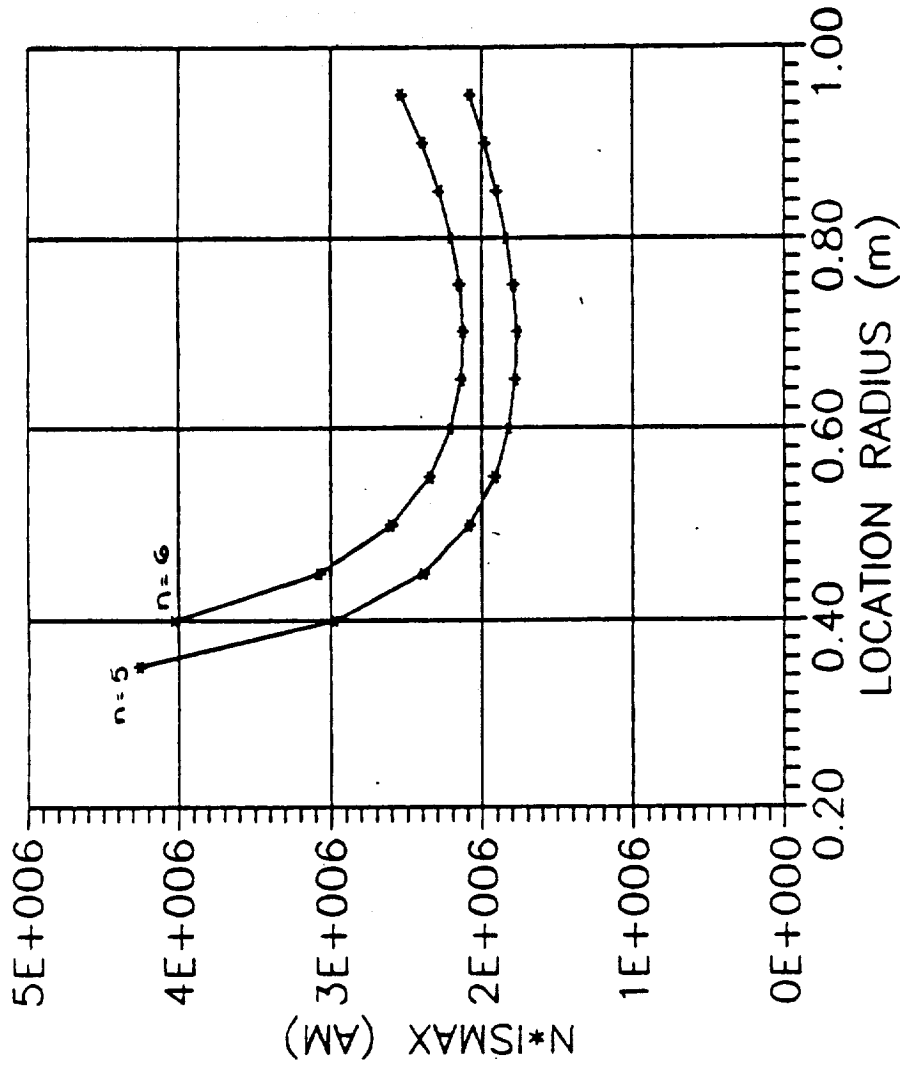


Figure II-1. Total ampere-meters of S/C magnet array at current density of 50 MA/m². Plots are for 5-coil and 6-coil arrays with 5 cm distance between adjacent coils. Optimum location radius is around 70 cm. Model is an S/C magnet.

Table II-3.

Minimum Ampere-Meter S/C Magnet Arrays
for Levitation of a PM Model.

=====

MODEL Q/M	=	3.350E+02 Am/KG	FX/W	FY/W	FZ/W	TY/W	TN/W
MODEL LENGTH	=	3.000E-01 m	.000E+00	.000E+00	1.000E+00	.000E+00	.000E+00
MIN DISTANCE	=	5.000E-02 m					

NC	R	L	RI	RO	JMAX	BMAX	ISTOT	N*IS
5	.600	1.397	.090	.328	1.500E+07	4.28	2.129E+07	3.268E+07
5	.650	.678	.136	.357	1.500E+07	3.45	1.135E+07	1.739E+07
5	.700	.493	.173	.386	1.500E+07	2.92	9.072E+06	1.390E+07
5	.750	.403	.205	.416	1.500E+07	2.57	8.117E+06	1.243E+07
5	.800	.348	.234	.445	1.500E+07	2.33	7.679E+06	1.175E+07
5	.550	1.168	.088	.298	2.000E+07	5.00	1.941E+07	2.980E+07
5	.600	.579	.135	.328	2.000E+07	3.91	1.057E+07	1.621E+07
5	.650	.418	.173	.357	2.000E+07	3.24	8.385E+06	1.285E+07
5	.700	.339	.206	.386	2.000E+07	2.81	7.446E+06	1.140E+07
5	.750	.291	.237	.416	2.000E+07	2.52	6.996E+06	1.071E+07
5	.800	.260	.266	.445	2.000E+07	2.33	6.812E+06	1.042E+07
6	.700	.980	.104	.325	1.500E+07	3.82	1.735E+07	2.629E+07
6	.750	.663	.135	.350	1.500E+07	3.35	1.289E+07	1.956E+07
6	.800	.530	.162	.375	1.500E+07	3.02	1.128E+07	1.714E+07
6	.600	2.124	.066	.275	2.000E+07	5.18	3.773E+07	5.707E+07
6	.650	.745	.109	.300	2.000E+07	4.23	1.446E+07	2.191E+07
6	.700	.525	.141	.325	2.000E+07	3.64	1.118E+07	1.697E+07
6	.750	.423	.168	.350	2.000E+07	3.23	9.874E+06	1.500E+07
6	.800	.364	.193	.375	2.000E+07	2.95	9.298E+06	1.414E+07

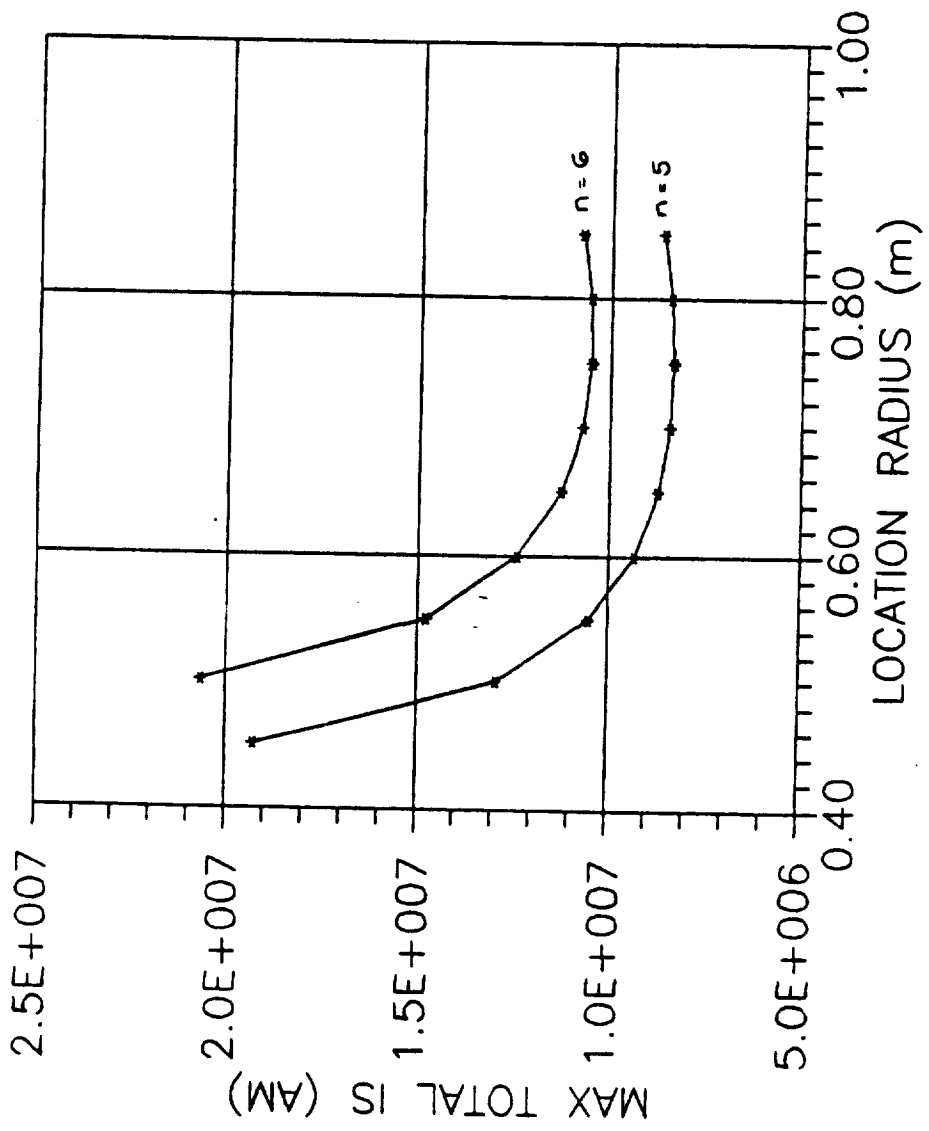


Figure II-2. Total ampere-meters of S/C magnet array at current density of 50 MA/m². Plots are for 5-coil and 6-coil arrays with 5 cm distance between adjacent coils. Optimum location radius is around 70 cm. Model is a PM.

Table II-4.

Minimum Power Copper Magnet Arrays
for Levitation of an S/C Model.

MODEL Q/M = 1.350E+03 Am/kg		FX/W	FY/W	FZ/W	TY/W	TN/W											
		.000E+00	.000E+00	1.000E+00	.000E+00	.000E+00											
MODEL LENGTH = 3.000E-01 m	CU RATIO = 7.500E-01	X	Y	Z	YAW	PITCH											
MIN DISTANCE = 5.000E-02 m		.000E+00	.000E+00	1.000E+00	.000E+00	.000E+00	NC	R	L	RI	RO	JMAX	BMAX	CU-WT	N*IS	PTOT*	N*PMAX*
5	.300	.737	.055	.151	4.779E+07	5.54	1.544E+03	1.098E+07	7.029E+06	1.399E+07							
5	.350	.737	.066	.181	2.620E+07	3.58	2.201E+03	8.583E+06	3.016E+06	5.997E+06							
5	.400	.737	.077	.210	1.604E+07	2.50	2.976E+03	7.104E+06	1.530E+06	3.039E+06							
5	.450	.737	.088	.240	1.068E+07	1.87	3.866E+03	6.143E+06	8.817E+05	1.749E+06							
5	.500	.737	.098	.269	7.584E+06	1.46	4.873E+03	5.500E+06	5.612E+05	1.112E+06							
5	.550	.737	.109	.298	5.675E+06	1.19	5.997E+03	5.064E+06	3.870E+05	7.664E+05							
5	.600	.737	.120	.328	4.431E+06	1.00	7.237E+03	4.772E+06	2.849E+05	5.638E+05							
5	.650	.737	.131	.357	3.584E+06	.86	8.593E+03	4.583E+06	2.215E+05	4.380E+05							
5	.700	.737	.141	.386	2.986E+06	.76	1.007E+04	4.473E+06	1.802E+05	3.562E+05							
5	.750	.737	.152	.416	2.552E+06	.69	1.166E+04	4.426E+06	1.524E+05	3.011E+05							
5	.800	.737	.163	.445	2.228E+06	.63	1.336E+04	4.429E+06	1.332E+05	2.631E+05							
5	.850	.737	.174	.475	1.981E+06	.59	1.518E+04	4.477E+06	1.198E+05	2.365E+05							
6	.300	.737	.046	.125	7.036E+07	6.82	1.264E+03	1.323E+07	1.235E+07	2.483E+07							
6	.350	.737	.055	.150	3.797E+07	4.37	1.820E+03	1.028E+07	5.171E+06	1.041E+07							
6	.400	.737	.064	.175	2.297E+07	3.05	2.477E+03	8.469E+06	2.574E+06	5.188E+06							
6	.450	.737	.073	.200	1.515E+07	2.26	3.235E+03	7.295E+06	1.460E+06	2.948E+06							
6	.500	.737	.082	.225	1.069E+07	1.77	4.095E+03	6.511E+06	9.182E+05	1.856E+06							
6	.550	.737	.091	.250	7.952E+06	1.44	5.055E+03	5.982E+06	6.269E+05	1.268E+06							
6	.600	.737	.101	.275	6.182E+06	1.21	6.117E+03	5.627E+06	4.580E+05	9.275E+05							
6	.650	.737	.110	.300	4.984E+06	1.05	7.279E+03	5.399E+06	3.539E+05	7.175E+05							
6	.700	.737	.119	.325	4.143E+06	.93	8.543E+03	5.267E+06	2.868E+05	5.818E+05							
6	.750	.737	.128	.350	3.534E+06	.84	9.908E+03	5.211E+06	2.419E+05	4.911E+05							
6	.800	.737	.137	.375	3.083E+06	.77	1.137E+04	5.219E+06	2.112E+05	4.291E+05							
6	.850	.737	.146	.400	2.742E+06	.72	1.294E+04	5.281E+06	1.900E+05	3.862E+05							
7	.300	.737	.038	.105	1.007E+08	8.27	1.044E+03	1.564E+07	2.089E+07	4.199E+07							
7	.350	.737	.046	.127	5.339E+07	5.25	1.519E+03	1.207E+07	8.533E+06	1.718E+07							
7	.400	.737	.054	.149	3.190E+07	3.64	2.082E+03	9.885E+06	4.171E+06	8.409E+06							
7	.450	.737	.062	.170	2.083E+07	2.69	2.735E+03	8.479E+06	2.333E+06	4.710E+06							
7	.500	.737	.070	.192	1.458E+07	2.10	3.476E+03	7.542E+06	1.450E+06	2.932E+06							
7	.550	.737	.078	.214	1.078E+07	1.71	4.307E+03	6.910E+06	9.815E+05	1.987E+06							
7	.600	.737	.086	.235	8.341E+06	1.44	5.226E+03	6.486E+06	7.120E+05	1.443E+06							
7	.650	.737	.094	.257	6.698E+06	1.24	6.234E+03	6.214E+06	5.473E+05	1.110E+06							
7	.700	.737	.102	.279	5.552E+06	1.10	7.330E+03	6.056E+06	4.417E+05	8.965E+05							
7	.750	.737	.110	.300	4.726E+06	1.00	8.516E+03	5.989E+06	3.716E+05	7.548E+05							
7	.800	.737	.118	.322	4.117E+06	.92	9.790E+03	5.998E+06	3.239E+05	6.585E+05							
7	.850	.737	.126	.344	3.659E+06	.86	1.115E+04	6.072E+06	2.913E+05	5.924E+05							

*PTOT & N*PMAX are for water-cooled copper magnets at 60°C. For liquid nitrogen-cooled magnets, divide by a factor of 10.

JMAX: maximum overall current density in the array
BMAX: maximum field
PTOT: optimum total power consumption at this configuration
N*P_{MAX}: number of coils times the maximum power consumption of any of them (reflects the size of the power supplies and serves as an upper bound on the power requirements).

From Tables II-4 and II-5 it is seen that the six-coil array requires 55% more power than the five-coil array; and in either case, the power consumption is not prohibitive. Figure IV-3 shows the upper bound for the power requirements versus the location radius for the five- and six-coil arrays. It is clear from this sketch that the larger the location radius of the magnets, the less power consumption of the system.

Table II-6 lists comparisons between the six different options discussed earlier. Ampere-meters in the magnets relate to forces on the model which may be a 20,000 Am pole strength S/C model or a 7,740 Am pole strength PM model. The S/C magnets are optimized for minimum ampere-meters and the copper magnets are optimized for minimum power. The power supply is required to charge (for S/C coils) and to maintain I^2R losses (for Cu coils).

Water-cooled copper magnets seem to offer no benefits. LN_2 cooled copper appears interesting with S/C models. The S/C + S/C and S/C + PM models seem attractive, particularly for extrapolation to larger systems.

Table II-5.

Minimum Power Copper Magnet Arrays for Levitation of a PM Model.

MODEL Q/M = 3.350E+02 Am/kg											
		FX/W	FY/W	FZ/W	TY/W	TN/W					
		.000E+00	.000E+00	1.000E+00	.000E+00	.000E+00					
MODEL LENGTH = 3.000E-01 m	CU RATIO = 7.500E-01	X		Y	Z	YAW	PITCH				
MIN DISTANCE = 5.000E-02 m		.000E+00	.000E+00	.000E+00	1.000E+00	.000E+00	.000E+00				
NC	R	L	RI	RO	JMAX	BMAX	CU-WT	N*IS	PTOT*	N*PMAX*	
5	.300	.737	.055	.151	1.926E+08	22.34	1.544E+03	4.424E+07	1.141E+08	2.272E+08	
5	.350	.737	.066	.181	1.056E+08	14.41	2.201E+03	3.459E+07	4.898E+07	9.739E+07	
5	.400	.737	.077	.210	6.465E+07	10.09	2.976E+03	2.863E+07	2.485E+07	4.936E+07	
5	.450	.737	.088	.240	4.303E+07	7.52	3.866E+03	2.476E+07	1.432E+07	2.841E+07	
5	.500	.737	.098	.269	3.056E+07	5.89	4.873E+03	2.216E+07	9.114E+06	1.806E+07	
5	.550	.737	.109	.298	2.287E+07	4.79	5.997E+03	2.041E+07	6.285E+06	1.245E+07	
5	.600	.737	.120	.328	1.786E+07	4.03	7.237E+03	1.923E+07	4.627E+06	9.156E+06	
5	.650	.737	.131	.357	1.444E+07	3.48	8.593E+03	1.847E+07	3.597E+06	7.114E+06	
5	.700	.737	.141	.386	1.203E+07	3.08	1.007E+04	1.803E+07	2.926E+06	5.785E+06	
5	.750	.737	.152	.416	1.028E+07	2.77	1.166E+04	1.783E+07	2.475E+06	4.890E+06	
5	.800	.737	.163	.445	8.977E+06	2.54	1.336E+04	1.785E+07	2.163E+06	4.273E+06	
5	.850	.737	.174	.475	7.985E+06	2.36	1.518E+04	1.804E+07	1.945E+06	3.841E+06	
6	.300	.737	.046	.125	2.836E+08	27.49	1.264E+03	5.333E+07	2.006E+08	4.032E+08	
6	.350	.737	.055	.150	1.530E+08	17.60	1.820E+03	4.143E+07	8.397E+07	1.690E+08	
6	.400	.737	.064	.175	9.258E+07	12.27	2.477E+03	3.413E+07	4.180E+07	8.426E+07	
6	.450	.737	.073	.200	6.106E+07	9.13	3.235E+03	2.940E+07	2.372E+07	4.787E+07	
6	.500	.737	.082	.225	4.306E+07	7.14	4.095E+03	2.624E+07	1.491E+07	3.013E+07	
6	.550	.737	.091	.250	3.204E+07	5.81	5.055E+03	2.411E+07	1.018E+07	2.060E+07	
6	.600	.737	.101	.275	2.491E+07	4.89	6.117E+03	2.267E+07	7.437E+06	1.506E+07	
6	.650	.737	.110	.300	2.008E+07	4.23	7.279E+03	2.176E+07	5.748E+06	1.165E+07	
6	.700	.737	.119	.325	1.669E+07	3.75	8.543E+03	2.122E+07	4.657E+06	9.448E+06	
6	.750	.737	.128	.350	1.424E+07	3.38	9.908E+03	2.100E+07	3.928E+06	7.975E+06	
6	.800	.737	.137	.375	1.242E+07	3.11	1.137E+04	2.103E+07	3.430E+06	6.968E+06	
6	.850	.737	.146	.400	1.105E+07	2.90	1.294E+04	2.128E+07	3.085E+06	6.271E+06	
7	.300	.737	.038	.105	4.058E+08	33.34	1.044E+03	6.302E+07	3.392E+08	6.819E+08	
7	.350	.737	.046	.127	2.152E+08	21.15	1.519E+03	4.862E+07	1.386E+08	2.790E+08	
7	.400	.737	.054	.149	1.285E+08	14.66	2.082E+03	3.984E+07	6.773E+07	1.366E+08	
7	.450	.737	.062	.170	8.395E+07	10.85	2.735E+03	3.417E+07	3.789E+07	7.650E+07	
7	.500	.737	.070	.192	5.875E+07	8.46	3.476E+03	3.039E+07	2.356E+07	4.762E+07	
7	.550	.737	.078	.214	4.345E+07	6.88	4.307E+03	2.784E+07	1.594E+07	3.226E+07	
7	.600	.737	.086	.235	3.361E+07	5.79	5.226E+03	2.614E+07	1.156E+07	2.343E+07	
7	.650	.737	.094	.257	2.699E+07	5.01	6.234E+03	2.504E+07	8.887E+06	1.802E+07	
7	.700	.737	.102	.279	2.237E+07	4.44	7.330E+03	2.440E+07	7.173E+06	1.456E+07	
7	.750	.737	.110	.300	1.905E+07	4.02	8.516E+03	2.414E+07	6.034E+06	1.226E+07	
7	.800	.737	.118	.322	1.659E+07	3.70	9.790E+03	2.417E+07	5.261E+06	1.069E+07	
7	.850	.737	.126	.344	1.474E+07	3.45	1.115E+04	2.447E+07	4.730E+06	9.621E+06	

*PTOT & N*PMAX are for water-cooled copper magnets at 60°C. For liquid nitrogen-cooled magnets, divide by a factor of 10.

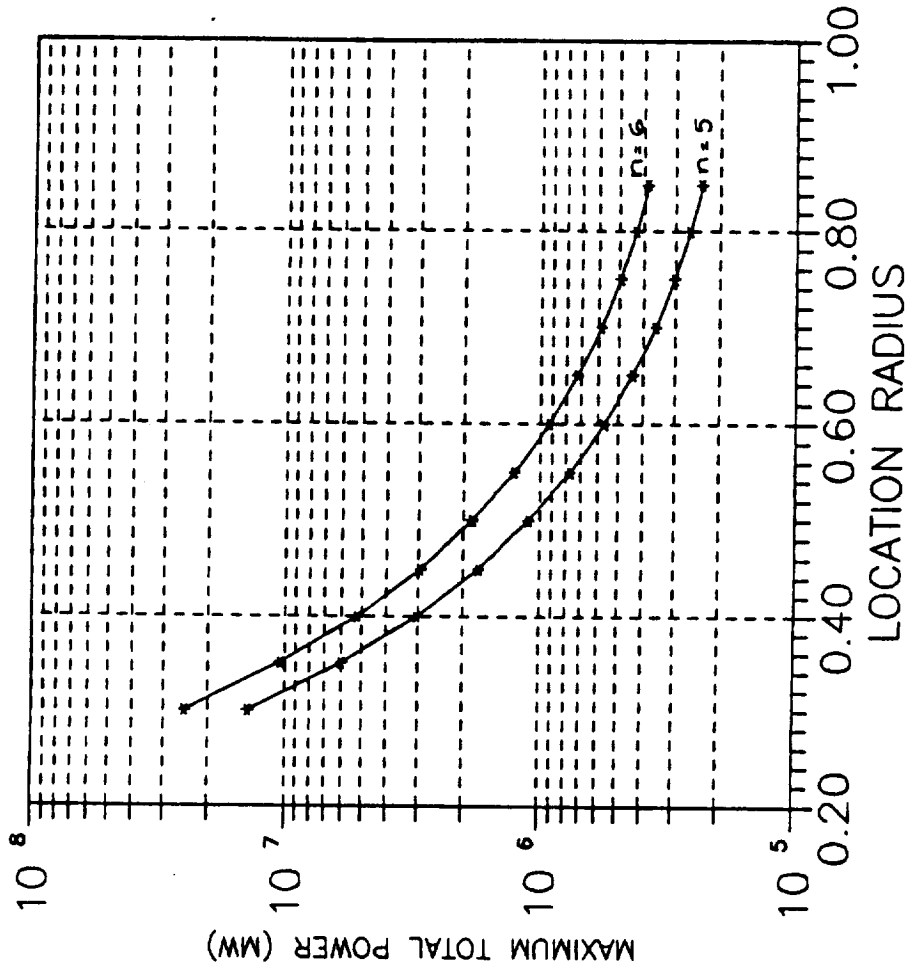


Figure II-3. Maximum power requirements for an array of water-cooled copper magnets. The plots are for 5-coil and 6-coil arrays with 5 cm distance between adjacent coils. The model is a superconducting magnet.

II-3. Levitation Coils Dimensions.

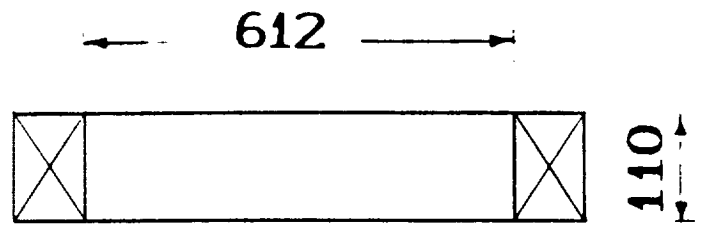
Three representative magnet cross-sections are sketched in Fig. II-4 for the 5 levitation coils. The top sketch is for S/C model and S/C magnets at 2 kA/cm². The middle sketch is for a PM model and S/C magnets at 2 kA/cm². The bottom sketch is for an S/C model with low current density copper magnets or for a PM model with higher current density copper magnets. Either copper magnet set could be cooled with water or liquid nitrogen.

Table II-6.

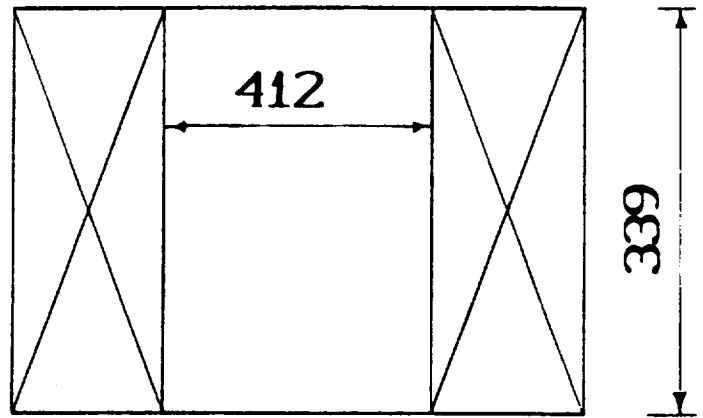
System Comparison.

Model Core Levitation Magnets	S/C S/C	PM S/C	S/C CU WATER COOLED	PM CU COOLED	S/C CU L. NITROGEN	PM CU
AMPERE-METERS (MAm)	2.1	14.7	4.5	18.0	4.5	18.0
CURRENT DENSITY (kA/cm ²)	1.5	1.5	0.3	1.2	0.3	1.2
DC POWER SUPPLY (kW)	0.0	0.0	360.0	5800.0	36.0	580.0

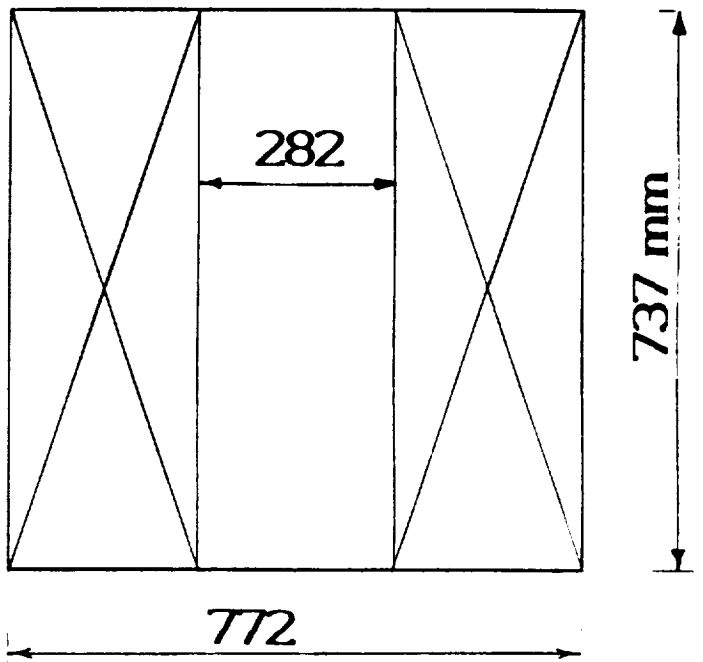
S/C MODEL
 S/C MAGNETS₂
 $J = 2.0 \text{ KA/cm}^2$



P M MODEL
 S/C MAGNETS₂
 $J = 2.0 \text{ KA/cm}^2$



S/C MODEL
 CU MAGNETS
 $J = 0.3 \text{ KA/cm}^2$



P M MODEL
 CU MAGNETS
 $J = 1.2 \text{ KA/cm}^2$

Figure II-4. Support Magnets for S/C and PM Models.

III. LEVITATION MAGNET SYSTEMS

Two options have been chosen for magnet system design. The first is superconducting coils and permanent magnet model. The second is nitrogen cooled coils and permanent magnet model.

III.1. Superconducting Levitation Magnets

Table III.1.

Specifications for the 5 Superconducting Levitation Magnets.

Number of Magnets	5.0
Location radius (m)	0.7
Magnet inner radius (m)	0.206
Outer radius (m)	0.386
Height (m)	0.339
Magnet top to model distance (m)	1.0
Maximum current density (A/m ²)	2.0 x 10 ⁷
Maximum winding field (T)	3.0
Magnet current (A)	500.0

Table III.1 lists the specifications of the 5 superconducting solenoids used to levitate the permanent magnet core. The coils are optimized to have the least ampere-meters. The 500 A conductor chosen for this design is a one triplex of an 11 kA ac conductor used in our previous suspension designs.^{1,2} The 500 A triplex has a twist pitch of 2.2 cm. Each part of the triplex is a seven-strand conductor. The seven-strand conductor is six OFHC copper wires twisted around a superconducting center conductor and all soldered with Staybrite. Since the requirements of low ac losses and cryostability conflict with each other, the basic principle chosen for this conductor is to achieve cryostability

within the basic cable. Each superconducting strand has a diameter of 0.051 cm and contains 2041 filaments of 6.7 μm dia with a twist pitch of 1.27 cm. The copper-to-superconductor ratio for each superconducting strand is 1.8. The reported losses on this conductor at a cycle that has $\dot{B} = 9 \text{ T/s}$ during charge and discharge are summarized in TableIII2.

TableIII2.

ac Losses of the 500 A Triplex Conductor.

Eddy current losses (J/cycle/m)	0.21
Hysteresis loss (J/cycle/m)	10^{-5}

Exact losses have not been calculated because finding the rms value of the correction currents is beyond the scope of this work. $|\Delta I/I|$ ranges between 10^{-4} to 10^{-2} depending on the yaw position and off-center signals. An estimate of ac losses for 500 A triplex conductor is about 200 W. The inductance matrix for the 5 levitation magnets is listed in Table V-3, as based on single turn coils. The maximum possible force between magnets is $18.5 \times 10^4 \text{ N}$.

A distance of 9 cm from the top of the cryostat to the tip of the magnets is assumed for calculation purposes. It now appears that 13 cm is a more practical choice for final design.

Table III3.

Inductance Matrix in H for the 5 Superconducting Levitation Magnets Based on Single Turn Coils.

	Coil #1	2	3	4	5
Coil #1	2.99×10^{-7}	--	--	--	--
2	1.09×10^{-8}	2.99×10^{-7}	--	--	--
3	2.49×10^{-9}	1.09×10^{-8}	2.99×10^{-7}	--	--
4	2.49×10^{-9}	2.49×10^{-9}	1.09×10^{-8}	2.99×10^{-7}	--
5	1.09×10^{-9}	2.49×10^{-9}	2.49×10^{-9}	1.09×10^{-8}	2.99×10^{-7}

III. 2. Nitrogen Cooled Magnet System

Table III4 lists the specifications for 5 nitrogen cooled copper solenoids used to levitate the permanent magnet model core. The coils are optimized for minimum ohmic heating. Each of the 5 levitation coils is a stack of pancakes of rectangular OFHC copper turns. The cooling surface is the surface between pancakes. For the optimized dimensions listed in Table V.1, the maximum heat flux q in W/cm^2 at the cooling surface for N pancakes is

$$q = 14.2/N .$$

For $N = 30$ the heat flux is less than $0.5 W/cm^2$ (which is small). Each pancake is 12 turns of 6kA square conductor 1.95 cm high. The separation between pancakes is 0.524 cm. There is 1 mm of insulation between turns. The maximum turn to turn ohmic voltage is 0.5 volt. The space between pancakes allows outward flow in the radial direction.

Table III 4.

Specification of LN₂-Cooled Levitation
Coils with PM Model Core

Number of magnets	5
Location radius (m)	0.7
Inner radius (m)	0.141
Outer radius (m)	0.386
Magnet height (m)	0.737
Magnet top to model distance (m)	1.0
Maximum gross current density (A/m ²)	1.203 x 10 ⁷
% Copper volume	75
Total copper weight (kg)	1.007 x 10 ⁴
Total power for 5 magnets (w)	2.926 x 10 ⁵
Maximum power per magnet (w)	1.157 x 10 ⁵
Magnet current (kA)	5

Each coil will be cooled as follows:

1. Variable flow along bore (single phase)
2. Constant radial outward flow (two phase)
3. Variable two phase flow along outer circumference.

Using low pressure 2 atm. boiling nitrogen cooling the flow rate for each coil is 124 gallon/m (994 ft³/hr) at 10% exit quality. The pressure drop along the central bore (1) is negligible compared to the pressure drop between pancakes (2) which is 0.05 psi. The pressure drop in the outer region (3) is 0.0077 psi/ft. This is so low compared to the 0.05 psi drop across pancakes that the flow will be close to uniform.

Preliminary design of the liquid nitrogen cryostat is aimed at determining minimum practical spacing from the top of the magnets to the upper surface of the vacuum jacket plate. Results of these calculations are presented in Table III 5.

Table III 5.

Magnet to Warm Surface Spacing for Liquid
Nitrogen-Cooled Solenoids.

Item	Space cm	Accumulated Space cm
Top of magnet	0	0
Flow space	0.3	0.3
Cold structure	2.0	2.3
Pressure deflection	0.3	2.6
Composite cold top plate	2.5	5.1
Insulation space	2.5	7.6
Top plate deflection	0.8	8.4
Composite top plate	2.0	10.4
Minimum total space from top of magnet		10.4