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Oscillational Instabilities in Single Mode Acoustics Levitators

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ABSTRACT

An extension of standard results for the acoustic force on an object in a single-mode resonant chamber yields predictions for the onset of oscillational instabilities when objects are levitated or positioned in these chambers. Our results are consistent with experimental investigators. The present approach accounts for the effect of time delays on the response of a cavity to the motion of an object inside it. Quantitative features of the instabilities are investigated. We discuss the experimental conditions required for sample stability, saturation of sample oscillations, hysteretic effects, and the loss of ability to levitate.

I would like to discuss some progress that has been made in the study of a phenomenon that is both a technological challenge to the designers of acoustic positioners and an exciting research topic with relevance to fundamental issues in the mechanics of solid bodies and continuous media. This phenomenon is illustrated in the behavior of a sample levitated in an ACES module in Space Shuttle Flight STS31B, January 1984. The sample began to oscillate, and those oscillations ultimately resulted in the loss of positioning of the object. While the oscillations were initially orderly and periodic, or at least approximately so, they later became random, or in nonlinear dynamics terminology, chaotic. Oscillations have also been observed in ground-based levitators, specifically a triple-axis levitator. Here one sees two kinds of behavior. One can observe oscillations that decay away. They are clearly a transient feature of the motion of the levitated object, which eventually settles into a stable position. This position represents a "dynamically stable fixed point." As a second alternative, there are oscillations that do not decay. Rather they persist at a fixed amplitude for as long as the acoustic field in the chamber is excited. This kind of oscillation is called a dynamically stable limit cycle. A striking feature of this behavior is that it develops *spontaneously*. That is, the levitated object can start out essentially stationary, and then it will begin to oscillate with a steadily growing amplitude until the amplitude of the oscillations saturates at its limiting value.

It is possible to respond to this interesting phenomenon in a variety of ways (Figure 1). One can regard it as a problem to be overcome by appropriate modifications in the design of the positioning apparatus — for example, by utilizing some form of feedback to respond to changes in the position of the object by changing some aspect of the acoustic drive. Alternatively, one might attempt to understand the physical mechanism underlying the oscillations and utilize this knowledge to design more stable cavities, or operate them in such a way as to avoid the parameter range in which such oscillations occur.

I strongly feel that the superior approach is the one that is based on a clear, physical understanding of the mechanism for oscillational instabilities. A positioner designed in such a way as to avoid oscillations will ultimately prove more practical than one that relies on an external mechanism to detect and damp out oscillations via electronic feedback. I think that there is an even more fruitful approach to take to this phenomenon. Regardless of its possible negative, or even potentially positive, impact on the application of acoustic positioning technology, the dynamical instability that occurs in acoustic positioner represents a fertile research topic. One stands to gain a significantly improved understanding of a variety of important dynamical phenomena.

In the remainder of this article, I will touch on some of those topics, discussing what has already been accomplished and what remains to be done.

Figure 2 - Adiabatic invariance. The principle of adiabatic invariance as applied to the single mode levitator allows one to establish a connection between the object's effect on the resonant frequency of an isolated mode in a high Q cavity and the acoustic forces and torques that this mode exerts on the object. The principle of adiabatic invariance, as discovered by Boltzman and Ehrenfest, asserts the following proportionality:

$$\vec{F}_{acoustic} \quad \propto \quad \vec{\nabla}_{\vec{r}_0}\,\omega(\vec{r}_0)$$

Here ω is the resonant frequency of the cavity and r_0 is the position of the object of interest. The constant of proportionality can be calculated in a variety of interesting cases (Figure 3). It depends on quantities like the intensity of the acoustic field. This relationship has been derived and it has been tested experimentally. It provides a useful way of calibrating single mode levitators.

Figures 4 and 5 - The limits of adiabatic invariance. The above relationship applies when all changes in the position and orientation of the object in the chamber are *quasistatic*. Furthermore, it assumes that the acoustic mode is both undriven and undamped. For example, assume that we are in the real world in which acoustic modes are damped and must be driven. What if changes that occur in a system occur at a finite rather than infinitesimal rate? In the case of an acoustic positioner, we have the beginning of an answer to those questions. Garrett and Barmatz have shown that in an analogue system (the damped, driven harmonic oscillator), parametric changes occurring at a finite rate can lead to instabilities that are strikingly similar to the oscillational instabilities that are observed in acoustic positioners. Dr. Barmatz and I have verified the results of Garrett and Barmatz by performing a highly nonlinear calculation of the forces on a small spherical object that is moving inside a single mode chamber. We have begun to explore in a greater generality the effects of time delays and mechanical feedback on the motion of an object of arbitrary shape and size in a single mode chamber. Here we are investigating virgin territory in dynamics, in that we are attempting to understand in a systematic way the limits of adiabatic invariance and the novel consequences of the corrections to that important dynamical principle that must be introduced to describe the behavior of real, physical objects.

As an example of the progress achieved so far, Figure 6 shows curves that we have calculated for the threshold and saturated amplitudes of oscillations in a single mode acoustic levitator operating in a gravitation-free environment. The amplitudes normalized to the dimension, L_Z, of the chamber are plotted against the frequency of the drive minus the resonant frequency of the chamber, normalized to the half-width of the resonance.

There are other modifications to adiabatic invariance that merit study (Figure 7). The investigations described above have been, or will be, carried out on cavities in which the relevant acoustic mode is isolated. The question of what happens when the positioning is accomplished by the excitation of degenerate, or nearly degenerate, modes also deserves our attention. The triple axis levitator utilizes three exactly degenerate plane wave modes in order to levitate and position the object inside. This levitator can rotate the object inside at a controllable rate. A triple-axis levitator was used in the Drop Dynamics Module (DDM) on Shuttle Flight. The rate of rotation is controlled by the relative phase of two of the plane wave modes. There is, as yet, no entirely satisfactory explanation for this phenomenon. A proper description will involve a detailed discussion of the energy exchange that occurs between two degenerate modes, most probably in violation of the principle of adiabatic invariance. It is interesting that a recent issue of *Physical Review Letters* contains an article describing anti-adiabatic behavior in a quantum mechanical system at the point of level crossing (or degeneracy).

Figure 8. Another aspect of the oscillational instability that deserves study is the possibility of chaotic motion by the moving object. Recall that the object in the videotaped ACES module executed motion that was far from regular. It is altogether likely that this motion fits the now-accepted definition of chaos — in that it is locally deterministic but unpredictable in the long run. However, any attempt to understand that motion will be complicated by the fact that one must take into account the fact that the acoustic drive was constantly being electronically adjusted to home-in on the resonant frequency of the cavity. This "external" feedback gives rise to a more complicated set of equations. We are currently simulating the motion of a small sphere in a single mode cavity using equations that properly describe the motion of the atmosphere in a cavity subject to a fixed-acoustic drive. We hope to be able to construct a full dynamical phase diagram of the object's motion, and, in the process, to explore the interplay between chaotic motion of a solid object and the concomitant behavior of the medium whose acoustic field is levitating it.

Figure 9. Finally, it is worth pointing out why it is a good idea to perform experiments on the motion of levitated object in space rather than in an earth-bound laboratory. The microgravity environment possesses three distinct advantages. First, the relatively low acoustic intensities required to position an object in microgravity (as opposed to the 155–160 dB that is needed in 1 g) make it possible to eliminate unwanted effects, such as rotational instabilities, and also to control nonlinearities in the system. Second, it is possible to utilize a less dense levitating medium in microgravity, so the system becomes more nearly Hamiltonian, because of the reduced viscous drag. Finally, in microgravity it is possible to position a sample at a velocity antinode and exploit the full symmetry of the levitating system.

FIGURE 1 TWO ASPECTS OF OSCILLATIONAL INSTABILITIES

- TECHNOLOGICAL
  - HOW DOES ONE AVOID OR EXPLOIT INSTABILITIES?
    - REACTIVE APPROACH
      - DESIGN THE ACOUSTIC DRIVE WITH FEEDBACK THAT ALLOWS IT TO RESPOND TO EVENTS IN THE POSITIONER
    - BASIC SCIENCE
      - APPROACH: UNDERSTAND THE MECHANISMS LEADING TO INSTABILITY
      - DESIGN AND OPERATE THE POSITIONER IN ACCORD WITH THE INSIGHT GAINED

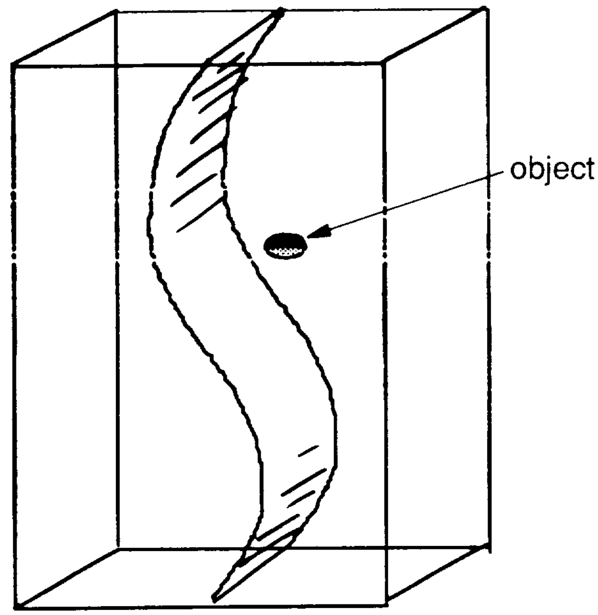
FIGURE 2 SCIENTIFIC IMPLICATIONS OF OSCILLATIONAL INSTABILITIES

- **ADIABATIC INVARIANCE**
  - BOLTZMANN - EHRENFEST PRINCIPLE

    APPLIES WHEN

    - THE OBJECT IS STILL OR MOVING ASYMPTOTICALLY SLOWLY ("QUASISTATIC")
    - THE POSITIONING MODE IS ISOLATED (NO ENERGY EXCHANGE WITH NEARBY ACOUSTIC MODES)
    - DELAY EFFECTS ARE UNIMPORTANT
    - THE POSITIONING MODE IS UNDAMPED AND UNDRIVEN

FIGURE 3



$U(\vec{r}_0, \vec{\theta}, s_1, s_2 \ldots)$ å $\Delta\omega_0(\vec{r}_0, \vec{\theta}, s_1, \ldots)$

- POSITIONING POTENTIAL — PERTURBATION OF THE MODE'S NATURAL FREQUENCY
- $\vec{r}_0$ = POSITION OF OBJECT — $\left(\vec{F} = -\vec{\nabla}_{r_0} U\right)$ POSITIONING FORCE
- $\vec{\theta}$ = ORIENTATION OF OBJECT — $\left(\vec{\tau} = -\vec{\nabla}_\theta U)\right)$
- s_1,...= SHAPE PARAMETERS (CORRESPONDING RELATIONSHIPS FOR SHAPING FORCES)
- GENERALIZES RESULTS OF KING & GOR'KOV
- ALLOWS FOR CALIBRATION OF LEVITATING MODULES

FIGURE 4 THE LIMITS OF ADIABATIC INVARIANCE

- SUPPOSE THE MODE IS DAMPED & DRIVEN DELAY EFFECTS ARE NON-NEGLIGIBLE AND THE POSITIONED OBJECT IS MOVING AT A FINITE RATE.

  - WHAT HAPPENS THEN?

    - S. GARRETT AND M. BARMATZ:
    - $m\frac{d^2}{dt^2} + \gamma \frac{dx}{dt} + k\,(t)x = Fe^{i\omega t} - Fe^{-i\omega t}$

  - RATE AT WHICH WORK MUST BE PERFORMED TO CHANGE THE SPRING CONSTANT, k (t), is

    - $= \frac{1}{2} \langle x^2 \rangle \frac{dk}{dt}$

FIGURE 5

SOLVING FOR x (t)

$$\frac{1}{2}\left\langle x(t)^2\right\rangle \frac{dk}{dt} = \frac{1}{2}\frac{F_0^2}{4\Omega^2 m^2}\frac{1}{\left(\frac{\gamma}{2m}\right)^2 + (\Omega - \omega)^2}\frac{dk}{dt}$$

Resonant frequency of the oscillator

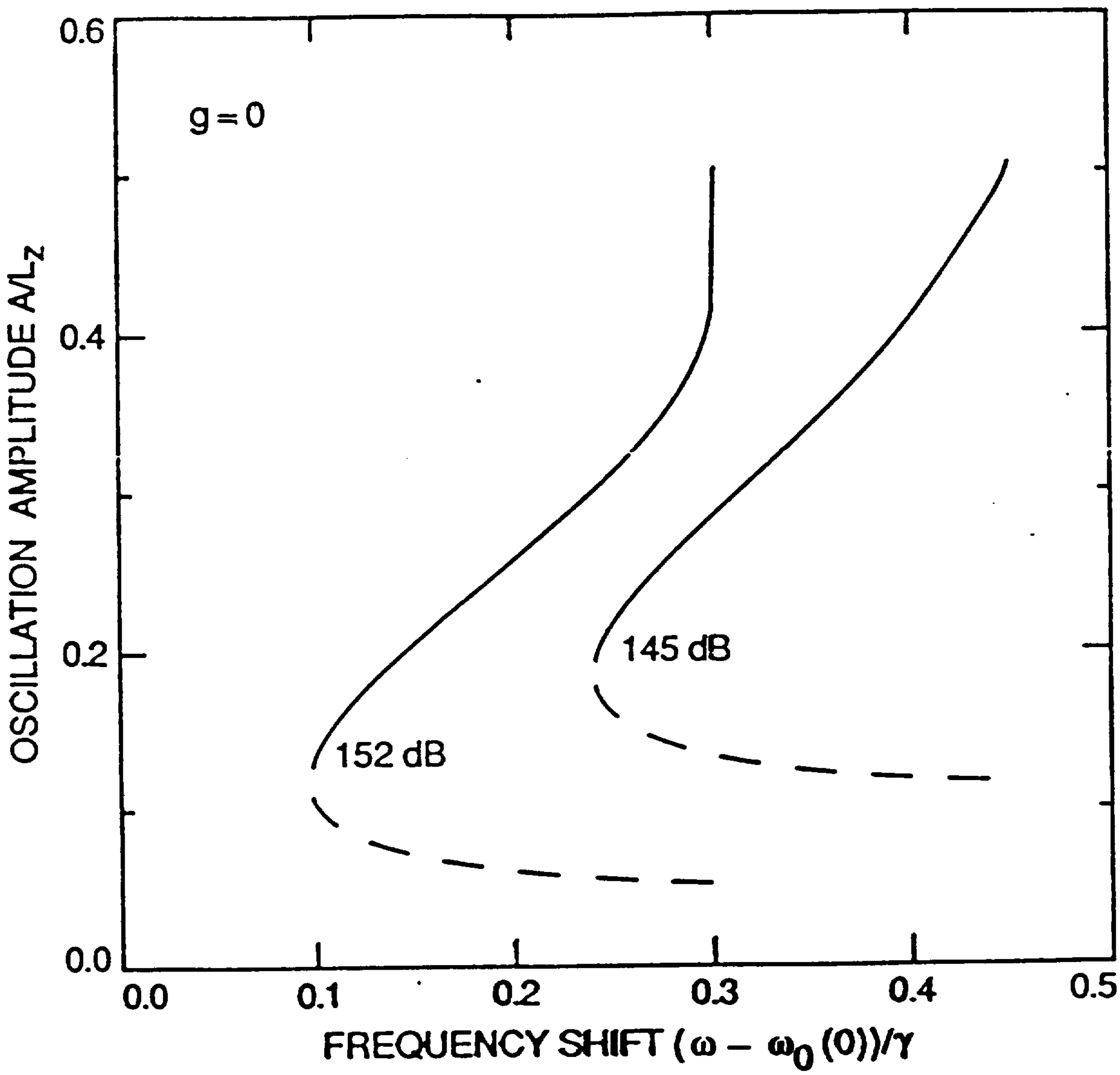
$$+\frac{4F_0^2}{[2\Omega m]^2}\frac{\frac{\gamma}{2m}(\Omega - \omega)}{\left[\left(\frac{\gamma}{2m}\right) + (\Omega - \omega)^2\right]^3}\left(\frac{dk}{dt}\right)^2$$

A velocity
dependent force

- ENERGY IS FED INTO THE SYSTEM IF $\omega < \Omega$
- ENERGY IS EXTRACTED FROM THE SYSTEM IF $\omega < \Omega$

- J. RUDNICK AND M. BARMATZ
  - VERIFIED BY EXPLICIT CALCULATION THAT THIS APPROACH HOLDS FOR SMALL SPHERES IN A LEVITATING OR POSITIONING CHAMBER

FIGURE 6



0.6
0.4
0.2
0.0
g = 0
152 dB
145 dB
OSCILLATION AMPLITUDE A/L_z
0.0
0.1
0.2
0.3
0.4
0.5
FREQUENCY SHIFT (ω − ω_0 (0))/γ

FIGURE 7

- UNANSWERED QUESTIONS:

  - DOES THE APPROACH OF GARRETT AND BARMATZ HOLD FOR AN ARBITRARILY SHAPED SAMPLE OF ARBITRARY SIZE?

  - CAN ONE PRODUCE A GENERALIZED PRINCIPLE LIKE THE ONE FOLLOWING FROM BOLTZMANN-EHRENFEST ADIABATIC INVARIANCE TO DESCRIBE OSCILLATIONAL, ROTATIONAL, AND DISTORTIONAL INSTABILITIES FOR LEVITATED OBJECTS?

  - WHAT HAPPENS WHEN THERE ARE TWO (OR MORE) DEGENERATE MODES

    - TRIPLE AXIS LEVITATOR
    - SOURCE OF ROTATIONAL CONTROL
    - ANTI ADIABATICITY

FIGURE 8

- MOTION OF A LEVITATED (OR POSITIONED) OBJECT
    - FIXED POINT (STABLE LEVITATION)
    - LIMIT CYCLE (STABILIZED OSCILLATION)
    - CHAOS (???)

      INTERPLAY BETWEEN MOTION OF A COMPACT OBJECT AND THE MODE(S) THAT SUPPORT IT

FIGURE 9 WHY WORK IN SPACE

- INTENSITY OF POSITIONING FIELD CAN BE REDUCED.
  155-160dB THE MINIMUM REQUIRED AT GROUND LEVEL. UNDESIRABLE EFFECTS ARE ELIMINATED (I.E., ROTATIONAL INSTABILITIES).

- ENVIRONMENT CAN BE MORE FULLY CONTROLLED (LESS DENSE MEDIUM - LESS DRAG)

- FULL SYMMETRY OF THE CAVITY CAN BE EXPLOITED (OBJECTS ARE POSITIONED AT CENTER)