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Recent Advances in Approximation Concepts for Optimum Structural Design

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This paper reviews the basic approximation concepts used in structural optimization. It also discusses some of the most recent developments in that area since the introduction of the concept in the mid-seventies. The paper distinguishes between local, medium-range and global approximations; it covers functions approximations and problem approximations. It shows that, although the lack of comparative data established on reference test cases prevents an accurate assessment, there have been significant improvements. The largest number of developments have been in the areas of local function approximations and use of intermediate variable and response quantities. It appears also that some new methodologies emerge which could greatly benefit from the introduction of new computer architectures.

INTRODUCTION

In the mid-seventies, Schmit and his coworkers showed that applications of nonlinear programming methods to large structural design problems could prove cost effective, provided that suitable approximation concepts were introduced (Schmit and Farshi (1974), Schmit and Miura (1976)). They combined the now familiar techniques of intermediate variable definition, explicit approximation, reduced basis and design variable linking as well as constraint deletion and regionalization. This paper reviews the basic ideas underlying approximation concepts as well as some recent results. The emphasis is on methods that are generic in that they are applicable to any engineering discipline and are largely independent of the details of the analysis methodology used. As a consequence, the paper will not cover the closely connected field of solution of perturbed analysis equations also known as approximate reanalysis techniques. There are many excellent reviews of this field including Arora (1976), Kirsch (1984) on the static equilibrium equation, and Murthy and Haftka (1988) on the eigenvalue problem.

We identify three general categories of approximations. In a *global approximation*, the approximation concept is valid for the whole design space or, at least, large regions of it. In *local approximation* the approximation is only valid in the vicinity of a point in the design space. Finally, some approaches attempt to give global qualities to local approximations,

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and we will refer to those as to *mid-range approximations*. We will also distinguish between *function approximation*, in which an alternate and explicit expression is sought for the objective function and/or constraints of the problem, and *problem approximation* where the focus is on replacing the original statement of the problem by one which is approximately equivalent but which is easier to solve. It is pointed out that, generally, approximation concepts can be combined to make for a very efficient problem formulation.

LOCAL APPROXIMATIONS

Local approximations are valid in the vicinity of the point at which they are generated. Typically, they are used to generate an approximate problem formulation which is solved for an optimum solution point. A new approximate problem is then generated at that point, and the process continues until convergence. Local function approximations are variations on the Taylor series expansion; local problem approximations try to reduce the size of the active constraint set.

Local Function Approximations

Local function approximations are probably the most popular approximations used in optimization. One of the first robust optimization algorithms is the Simplex algorithm developed by Dantzig in 1947 for the solution of linear optimization problems (Linear Programs). It was natural that people attempted to use this highly successful algorithm for nonlinear programs by linearizing the constraints and objective function about a trial design. This led to the Sequential Linear Programming (SLP) method and the wide use of the linear Taylor series approximations. Applications can be found in Zienkiewicz and Campbell (1973) who optimize the shape of dams and in Pedersen (1981) who finds the optimum design of space trusses, for example. Another reason for the popularity of local approximations is that, as discussed later, some global approximations become very expensive computationally when the dimensionality of the design space exceeds about 10.

Approximations Based on Zeroth- and First-Order Function Information The most commonly used local approximations to objective functions and constraints are based on the use of the function and its first derivatives at a single design point, say $\mathbf{X}_0$. The simplest is the *linear approximation* based on the Taylor series. Given a function $g(\mathbf{X})$, the linear approximation $g_L(\mathbf{X})$ is

$$g_L(\mathbf{X}) = g(\mathbf{X}_0) + \sum_{i=1}^{n} (x_i - x_{0i}) \left(\frac{\partial g}{\partial x_i} \right)_{\mathbf{X}_0} \tag{1}$$

For some applications the linear approximation is inaccurate even for design points $\mathbf{X}$ that are close to $\mathbf{X}_0$. Accuracy can be increased by retaining additional terms in the Taylor series expansion. This, however, requires the costly calculation of higher-order derivatives. Instead, many researchers tried to obtain other approximations that use only first derivatives but which can be more accurate than the linear approximation.

One approximation of this type is the *reciprocal approximation* which is a linear approximation in y_i, the reciprocal of x_i,

$$y_i = \frac{1}{x_i}. \tag{2}$$

Its frequent use reflects the fact that many of the early structural optimization studies were performed on structures consisting of truss or plane stress elements. The design variables in these studies were usually the cross-sectional areas of the truss elements and the thicknesses of the plane-stress elements. For statically determinate structures, stress and displacement constraints are linear functions of the reciprocals of these design variables. Even for statically indeterminate structures, using the reciprocals of the design variables still proved to be a useful device in making the constraints more linear (see, for example, Storaasli and Sobieszczanski, 1974, and Noor and Lowder, 1975a). The reciprocal approximation can be written in terms of the original design variables as

$$g_R(\mathbf{X}) = g(\mathbf{X}_0) + \sum_{i=1}^{n}(x_i - x_{0i})\frac{x_{0i}}{x_i}\left(\frac{\partial g}{\partial x_i}\right)_{\mathbf{X}_0}. \tag{3}$$

One of the attractive features of the reciprocal approximation, even for statically indeterminate structures, is that it preserves the property of scaling. That is, when the stiffness matrix is a homogeneous function of order h in the components of $\mathbf{X}$ the displacements are a homogenous function of order $-h$ in the components of $\mathbf{X}$. For truss and membrane elements $h = 1$, so that the displacements are a homogeneous function of the reciprocal of the design variables. If all the design variables are scaled by a factor, the displacement vector is scaled by the reciprocal of that factor. The reciprocal approximation preserves this scaling property, and therefore it is exact for scaling the design. Fuchs (1980), and Hajali and Fuchs (1989) have investigated the importance of the homogeneity property, and proposed a family of constraints that generalize the reciprocal approximation to any order of homogeneity.

One problem with the reciprocal approximation is that it becomes unbounded when one of the variables approaches zero. This is acceptable when the design variables are bounded away from zero, as is the case in many structural problems. However, it can result in large errors when one of the design variables becomes very small. To correct this deficiency Haftka and Shore (1979) proposed a modified reciprocal approximation given as

$$g_{mR}(\mathbf{X}) = g(\mathbf{X}_0) + \sum_{i=1}^{n}(x_i - x_{0i})(x_{0i} + x_{mi})\frac{x_{0i}}{(x_i + x_{mi})}\left(\frac{\partial g}{\partial x_i}\right)_{\mathbf{X}_0}, \tag{4}$$

where the values of x_{mi}'s are typically small compared to representative values of the corresponding x_i's. It is possible, however to take large values for some x_{mi}'s, and this results in an approximation which is closer to the linear approximation than the reciprocal approximation in these variables.

Another approximation, called the *conservative approximation* (Starnes and Haftka, 1979), is a hybrid form of the linear and reciprocal approximations which is more conservative than either. It is particularly suitable for interior and extended interior penalty

function methods which do not tolerate constraint violations well. To obtain the conservative approximation, we start by subtracting the reciprocal approximation from the linear approximation

$$g_L(\mathbf{X}) - g_R(\mathbf{X}) = \sum_{i=1}^{n} \frac{(x_i - x_{0i})^2}{x_i} \left(\frac{\partial g}{\partial x_i}\right)_{\mathbf{X}_0}. \tag{5}$$

The sign of each term in the sum is determined by the sign of the ratio $(\partial g/\partial x_i)/x_{0i}$ which is also the sign of the product $x_{0i}(\partial g/\partial x_i)$. Design variables for which this product is negative contribute to make the reciprocal approximation more positive than the linear approximation, and vice versa. Since the constraint is usually expressed as $g(\mathbf{X}) \leq 0$, a more positive approximation is more conservative. The conservative approximation, g_C, is created by selecting for each design variable the more positive contribution

$$g_C(\mathbf{X}) = g(\mathbf{X}_0) + \sum_{i=1}^{n} G_i(x_i - x_{0i}) \left(\frac{\partial g}{\partial x_i}\right)_{\mathbf{X}_0}, \tag{6}$$

where

$$G_i = \begin{cases} 1 & \text{if } x_{0i}(\partial g/\partial x_i) \geq 0, \\ x_{0i}/x_i & \text{otherwise.} \end{cases} \tag{7}$$

Note that $G_i = 1$ corresponds to a linear approximation, and $G_i = x_{0i}/x_i$ corresponds to a reciprocal approximation in x_i.

The conservative approximation is not the only hybrid linear-reciprocal approximation possible. Sometimes physical considerations may dictate the use of linear approximation for some variables and the reciprocal for others (see, for example, Haftka and Shore, 1979, and Prasad 1984b). However, as can be easily checked, the conservative approximation has the advantage of being convex. If the objective function and all the constraints are approximated by the conservative approximation, the approximate optimization problem is convex. Convex problems are guaranteed to have only a single optimum, and they are amenable to treatment by dual methods. Fleury and Braibant (Braibant and Fleury, 1985, Fleury and Braibant, 1986) suggested taking advantage of this property and solving the approximate problem by dual methods. They also introduced the term convex linearization for the process of approximating the optimization by the conservative-convex approximation for the objective function and constraints. This approach has been used by many others (e.g., Ding and Esping, 1986, Ding, 1987).

There has been a systematic investigation of approximations based on using powers of the design variables (Prasad, 1983, 1984a,b, Woo, 1987). Many of these are conservative and/or convex approximations, but it is important to note that the one presented here and the others are not guaranteed to be conservative in an absolute sense (that is, we do not know that the approximation is more conservative than the exact constraint, $g_C(\mathbf{X}) \geq g(\mathbf{X})$). The conservative approximation presented here is only more conservative than either the linear or reciprocal approximations. Finally, it has been shown (e.g., Haftka, 1989) that the conservative-convex approximation tends to be less accurate than either the linear or the reciprocal approximation. Therefore, it should not be used unless its convexity or conservativeness are needed.

The reciprocal and conservative approximation destroy the linearity of the approximation, and the possibility of using sequential linear programming (SLP). Chan and Turlea (1978) used a nonlinear approximation that still permits the use of SLP. This is the *posynomial approximation* (Duffin, Peterson and Zener, 1967).

$$g_P(\mathbf{X}) = g(\mathbf{X}_0) \prod_{i=1}^{n} \left(\frac{x_i}{x_{0i}} \right)^{a_i}, \tag{8}$$

where a_i is the logarithmic derivative of g,

$$a_i = \frac{x_{0i}}{g(\mathbf{X}_0)} \left(\frac{\partial g}{\partial x_i} \right)_{\mathbf{X}_0}. \tag{9}$$

The logarithm of g_P is a linear function of the design variables, so if the objective function and constraints are approximated by this posynomial approximation, the optimization problem can be transformed into a linear problem by taking the logarithm of the constraint equations and objective function.

The posynomial approximation can be used without any transformation when the optimization method is geometric programming which actually requires such form. Applications of the posynomial approximations to structural optimization using geometric programming include Morris (1972, 1974), Templeman and Winterbotton (1974) and Hajela (1986).

Differential Equation Based Approximation Pritchard and Adelman (1990) recently introduced a new method that begins with the equation for the sensitivity of the quantity being approximated. It takes it as a constant coefficient differential equation, integrates it and derives a high-quality nonlinear equation. For example, in a dynamical system, the sensitivity of an eigenvalue ω^2 to change in a single design variable x is given by

$$\frac{d\omega^2}{dx} = \Phi^T \left[\frac{d\mathbf{K}}{dx} - \omega^2 \frac{d\mathbf{M}}{dx} \right] \Phi \tag{10}$$

assuming that

$$a = \Phi^T \frac{d\mathbf{K}}{dx} \Phi \text{ and } b = \Phi^T \frac{d\mathbf{M}}{dx} \Phi$$

are constant, the following differential equation results

$$\frac{d\omega^2}{dx} = a - b\omega^2$$

which, upon integration and specification of the boundary condition that for the original value x_0 of the variable, the eigenvalue is ω_0^2, yields the following approximation

$$\omega^2 = \left(\omega_0^2 - \frac{a}{b} \right) e^{-b(x - x_0)} + \frac{a}{b} \tag{11}$$

They extended this approach to several design variables as well as to approximation of eigenvectors, and displacements. They generally showed good approximation quality, by comparison with conventional linear Taylor series expansions.

Approximations Based on Higher-Order Function Information Higher order approximations are also used occasionally. For example, the quadratic approximation, g_Q is obtained by including the quadratic terms in the Taylor series expansion

$$g_Q(\mathbf{X}) = g(\mathbf{X}_0) + \sum_{i=1}^{n}(x_i - x_{0i})\left(\frac{\partial g}{\partial x_i}\right)_{\mathbf{X}_0} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - x_{0i})(x_j - x_{0j})\left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right)_{\mathbf{X}_0} . \quad (12)$$

The reciprocal quadratic approximation g_{QR} is obtained by using the quadratic approximation in terms of the reciprocal design variables to obtain (Haftka, Gürdal, and Kamat, 1990)

$$\begin{aligned} g_{QR}(\mathbf{X}_0) = g(\mathbf{X}_0) &+ \sum_{i=1}^{n}\left(\frac{x_{0i}}{x_i}\right)\left(2 - \frac{x_{0i}}{x_i}\right)(x_i - x_{0i})\left(\frac{\partial g}{\partial x_i}\right)_{\mathbf{X}_0} \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{x_{0i}}{x_i}\right)\left(\frac{x_{0j}}{x_j}\right)(x_i - x_{0i})(x_j - x_{0j})\left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right)_{\mathbf{X}_0} \end{aligned} \quad (13)$$

Quadratic approximations have been used primarily for eigenvalue problems where the linear approximation tends to be particularly limited in applicability. For example, Rommel (1983) used a quadratic approximation for flutter speeds and damping factors, while Miura and Schmit (1978) used a quadratic approximation for vibration frequencies. However, it should be noted that for eigenvalue problems the quadratic approximation is not efficient, because for nearly the same cost it is possible to obtain a cubic approximation by using a linear approximation to the eigenvectors in the Rayleigh quotient (Murthy and Haftka, 1988). Often the high cost of obtaining these derivatives dictates a compromise based on using only the diagonal second derivatives (e.g., Fleury, 1988, 1989a,b, Renwei and Peng, 1987). For stress constraints, the use of only diagonal second derivatives can be justified by invoking St. Venant principle and assuming that changes in the property of an element would affect only the stress in that element (Renwei and Peng, 1987). The use of diagonal second derivatives is also sometimes motivated by the desire for a separable approximation (e.g., Smaoui, Fleury and Schmit, 1988, Fleury and Smaoui, 1988, Fleury, 1989a,b). Haftka (1988) compared the performance of first- and second-order approximations in structural optimization. He found that the second-order approximations reduce the number of required optimization cycles by about 10-50 percent. This is marginal improvement when the cost of second derivatives is high. Jawed and Morris (1984, 1985) suggest the calculation of approximate second derivatives, which can make the use of quadratic approximations more attractive.

When second derivatives are available and it is still desirable to use SLP, it is possible to use the diagonal second derivatives to construct a better linear approximation to the constraint near the critical surface $g = 0$ (Mistree *et al.* 1981).

Local Problem Approximations

Technically, algorithms based on sequential approximations (linear programming, quadratic programming or convex linearization) can be viewed as problem approximation concepts as they replace the given nonlinear problem by a sequence of subproblems that are easier to solve. However, in this section, we will focus on techniques which have to do with reduction in the number of constraints or the number of design variables.

Constraint Number Reduction In order to improve the optimization algorithm efficiency, one may reduce the number of constraints retained at each iteration to those which are active or nearly active. This limits the necessary constraint gradient calculations and greatly reduces the cost of optimization per iteration. In addition to *constraint deletion*, Schmit and Miura (1976) also have advocated the use of *regionalization*. This is a technique where for each region of the structure, and for each load case, only the most active stress and displacement constraints are retained. Assuming that the most active constraint does not change during one iteration, then only that gradient must be found for that region. A region can be identified as the area described by a single variable in a design variable linking scheme (see subsection on global problem approximation).

Design Variable Reduction Hajela and Sobieski offered an interesting local design variable linking scheme (for more on design variable linking see the section on Global Problem Approximations) which they termed *controlled growth method* (Hajela and Sobieszczanski-Sobieski (1981), Hajela (1982)). This approach is applied at each optimization iteration. All the design variables are ranked according to their *combined measure of effectiveness (CME)*. Variables with low effectiveness are held constant during the current iteration, The remaining variable variations are linked to that of the variable with the highest CME, in effect replacing the original multi-variable optimization problem by a sequence of single variable subproblems. Hajela and Sobieszczanski-Sobieski (1981) showed reduction in analysis and gradient counts on conventional problems with up to 13 variables.

GLOBAL APPROXIMATIONS

Global approximations are valid for the whole design space or large areas of it. As such, they are used to modify the formulation of the problem from the outset and generate an alternate formulation that is more tractable. Global function approximations techniques include the generation of response surfaces; global problem approximations include the introduction of intermediate variable or response quantities as well methods to reduce the number of design variables in the problem.

Global Function Approximations

Response Surface Approach A natural approach to solving an optimization problem is to first build approximate analytical response surfaces giving the dependent variables as functions of the independent variables. Then an optimization algorithm can be used to optimize the approximate problem. Typically, these response surfaces are global in that they cover the whole design space, although, this is not necessary. Depending on the quality of the response surfaces, the resulting design can be used as a final solution or high-quality starting point for an ultimate optimization with direct coupling to a structural analysis. The main challenge in generating response surfaces is to do so without an excessive number of exact analyses.

Schoofs (1987) gave the following rationale for combining nonlinear optimization with response surface generation: i) both techniques can use the same variables, ii) both techniques aim at minimizing the number of expensive analyses, iii) once a global model is derived for a particular design problem, multiple optimization studies can be performed without additional analyses (as required for example when a multiobjective problem is solved or when a design problem formulation is being fine-tuned). In addition, gradients are not generally required for response surface generation although, if they are available, they can be used to enhance the process.

Construction of response surface (or model-building) relies heavily on the theory of experiments (see Box and Draper (1987)). It is an iterative process. A typical application begins with postulating a model for the relationship between dependent and independent variables. Although linear or quadratic polynomial approximations are by far the most common forms employed, other forms (e.g., polynomials in powers of trigonometric functions) have been used as well. The approximation contains a number of unknown parameters (such as polynomial coefficients) that must be adjusted for it to match the behavior of the system. To do so, analyses are performed at a number of carefully selected design points and a least-square solution is typically used to extract the parameter values from the analysis results. Then the approximate model is used to predict the response of the system at a number of selected test points and statistical measures are used to assess the goodness-of-fit, or the accuracy of the response surfaces. If the fit is not satisfactory, the process is restarted and further experiments are made or the postulated model is improved by removing and/or adding terms.

There is a limited number of examples of applications of response surface techniques in the structural optimization literature. Brown and Nachlas (1985) selected the orientation of the layers of composite fibers in the three sections of a missile exit cone. They used polynomials in trigonometric functions of the orientations to approximate the safety factors under selected load cases. With 4 design variables per section and up to 7 possible values for these variables, an exhaustive search of the design space would require 2401 analyses per section. Instead they selected 28 design points per section and generated a final design with a 37 lb weight reduction with respect to a metal baseline. White *et al.* (1985 and 1986) used the response surface approach in their study of passenger car crashworthiness. Most of the dependent variables optimized in their study were derived from deceleration data generated by simulation programs. They first approximated the deceleration time histories by polygonal profiles; they then constructed response surfaces to relate the parameters describing the polygonal profiles to their structural design variables. Using polynomials of up to third power, they required over 200 sets of simulation results for fitting 11 crash signature parameters with 7 design variables. After optimizing the problem they retained the model to conduct inexpensive univariate sensitivity studies. Schoofs (1987) described small mechanical engineering problems (design of pin joints, bearing joints, beam cross-sectional shape and heart leaflet valves). He also described at length the challenging design of the shape of a carillon bell to prescribed natural frequency ratios. The shape design variables were 7 radii, describing the bell cross-section in a vertical plane. Polynomial approximations of up

to third power in the variables were used. After several attempts, he concluded that 1220 analyses must be performed to obtain an adequate fit. In the process, he reported the design of bells with frequency ratio distributions that had eluded professional bell-founders for centuries. Lawson *et al.* (1989) discussed the design of a moving head disk drive actuator arm. They reviewed two approaches to selecting the sample analysis results and optimized a 4-variable model using quadratic response surfaces derived from as little as 15 full analyses and requiring as little as 27% of the time required for conventional optimization.

The experience with response surfaces in structural optimization is limited to problems of relatively small size. This is because the number of analyses required to construct response surfaces increases dramatically with the number of design variables. The usage of high-quality approximations has reduced the number of analyses required for the solution of most structural optimization problem to 15-20 regardless of the number of design variables. Assuming finite-difference calculations this translates to about $15n$ to $20n$ analyses. In the theory of experiments, the basic set of design points considered is called a *full factorial design.* Each variable (factor) is assumed to take on a number of possible discrete values (levels) and each possible combination of variable values is considered. A full factorial design with only two values per variables requires 2^n analyses. It permits to generate response surfaces linear in each design variables (and this would probably be inadequate for most structural responses). On this basis alone, response surface methodology is not competitive for problems with more than 8 design variables. However, direct comparison is not easy. First, in a typical design exercise, many conventional optimizations must be performed to develop a satisfactory problem formulation, to try and isolate a global optimum or to solve a multiobjective problem. With each additional optimization, a completely new set of analyses and sensitivity analyses must be performed. Second, in the theory of experiments, numerous methods exist to construct *fractional factorial designs* where the number of analyses is reduced drastically from the full factorial design while maintaining the required level of accuracy (see Box and Draper (1987), for example). Finally, a third point is that constructing response surfaces is an inherently parallel operation. While analysis (and, possibly, sensitivity analysis) results are needed sequentially in applications of conventional nonlinear programing methodology, they are needed all at one time when response surface methodology is used, thereby enabling better usage of multi-processor computers. It must be noted that the conventional analysis and sensitivity analysis process lends itself to parallel implementation if derivatives are found by finite difference. However, constructing a response surface offers even more parallelism since all the analyses are required at the outset. Further investigation of response surface methodology may show it to be a competitive alternative to using conventional optimization method for some problems.

Other Global Function Approximations Hajela and Berke (1990) have proposed using neural networks in optimization to provide fast approximate structural analysis. A neural network is a computer that attempts to mimic neurobiological processes. It is a massively parallel network of interconnected computing elements that processes input data and generates output. A neural network is trained by presenting it pairs of input

and output data and then iteratively adjusting weights in the connections between computing elements so that its output matches the known output data. Once trained, the network can be used to replace complex and time-consuming analysis procedures. In that sense, neural networks can be thought of as an alternate approach to global function approximation. Hajela and Berke described how neural networks are able to abstract key information and patterns present in their input sets. Also, they showed that networks are fault tolerant in that they are relatively insensitive to degeneration of a few computing elements or to corruption of a few data sets. Although very different conceptually, neural networks and response surfaces provide the same type of information and present a lot of the same advantages and disadvantages. On the one hand both methodologies i) require for input a number of analyses of the system considered, ii) can accommodate input from different sources (including analytical and experimental), iii) are adaptive in that they can be improved as more information becomes available, and, finally, iv) provide a rapid analysis capability that is global and can be reused many times at little or no additional cost. On the other hand, both require a significant amount of up-front computations. Hajela and Berke demonstrated the applicability of neural networks in optimization by considering simple truss and wing structures of 16 design variables or less. With as little as 100 training sets (complete analyses), and a significant amount of iterative training time, they showed optimization results that were comparable to those generated by conventional methods.

For problems in which some constraints are quite expensive to calculate, bounds can be developed for the constraints which are significantly easier to calculate and which will help provide bounds on the optimum solution. These bounds then replaced the original constraints in their optimization problem. One such example was given by Mills-Curran and Schmit (1983) in an application of optimization under dynamic behavior constraints. In that application, they developed time-dependent upper bounds for dynamic displacement and stress constraints which are valid for lightly damped systems away from resonant forcing conditions.

Global Problem Approximations

One of the most direct approaches to approximating a problem formulation is the use of simplified analysis models. On the one hand, the simplification can be to obtain the numerical solution with, for example, a coarser finite element mesh discretization. Haftka and Starnes (1976) looked at the effect of varying the number of degrees-of-freedom and design variables in wing models. They showed CPU times increasing linearly with the number of design variables for a given number of degrees of freedom and almost quadratically with the number of degrees of freedom for a given number of design variables. Salama *et al.* (1984) also examined the benefit of varying the number of degrees-of-freedom and design variables in problems of optimization of simple beams and built-up truss structures. On the other hand, the analysis can be performed with a simpler model, for example a plate model of a wing instead of a complex built-up finite element model. McCullers and Lynch (1972) used such a model to develop the program TSO which has remained a classic preliminary design tool for flexible wings. Sobieszczanski-

Sobieski and Loendorf (1972) developed a simple lumped property model of a fuselage for use in a two-level sizing procedure; Ricketts and Sobieszczanski-Sobieski (1977) developed a complete lumped model for aircraft sizing under flutter constraints. The focus of this section, however, remains on methods that are more generic in nature.

Intermediate Variables and Response Quantities In their quest for improving the quality of function approximations, researchers occasionally resort to using intermediate response quantities and intermediate variables. When a particular response $R(X)$ can be written in terms of an *intermediate response* (or response vector) R_I and of an *intermediate variable* (or variable vector) X_I, then, for example,

$$R(X) = R(R_I(X_I(X))) \tag{14}$$

If the relationships $R_I = R_I(X_I)$ and $X_I = X_I(X)$ are known analytically and if a very accurate approximation $\widetilde{R}_I(X_I)$ exists, then, a very accurate approximation $\widetilde{R}(X)$ is given by

$$R(X) \cong \widetilde{R}(X) = R\left(\widetilde{R}_I(X_I(X))\right) \tag{15}$$

In general, all three nested relationships may be approximated. In this case,

$$R(X) \cong \widetilde{R}(X) = \widetilde{R}\left(\widetilde{R}_I\left(\widetilde{X}_I(X)\right)\right) \tag{16}$$

While all reported applications are local approximations, the idea of resorting to intermediate design variables or intermediate response quantities to improve approximation quality is applicable to all forms of approximations.

Among the earliest introduction of intermediate design variables has been that of the reciprocal variable (see previous section on local approximations). For trusses, the intermediate variable of choice for displacement constraints is the reciprocal of the cross-sectional area (see Bennett (1981), for example). This reciprocal approximation is exact for statically determinate trusses and, in general, very good for all trusses. Similarly, the reciprocal of the thickness is an appropriate intermediate variable for approximations of the response of built up structures made of membrane elements.

For built-up structures made of beams of simple cross-section, Fleury and Sander (1983) recommended i) the reciprocal of the wall thickness for structures made of thin walled beam, tubes or sandwich beams with constant cross-section but variable thickness, ii) the reciprocal of the square of the dimensions for beams with uniformly varying cross-sections and iii) the reciprocal of the cube of the height for built-up structures made of beams of varying height. For plates in bending, they suggested i) the reciprocal of the face sheet thickness for sandwich plates of constant core depth and ii) the reciprocal of the cube of the plate thickness for solid plates. When both extensional and bending behaviors are present, they derived approximations based on the reciprocal of the thickness, its second and third power for displacement, stress, frequency and buckling constraints. In this latter case, however, the approximations were not simple linearizations any more but were derived on the basis of energy considerations.

In their work on frameworks made of beams of complex cross-sections, Schmit and his coworkers recognized that while a beam cross-section is fully described by all its *cross-section dimensions (CSD)*, (wall thickness, beam height and width, for example), its global behavior (nodal displacements, member forces) is described in terms of *reciprocal section properties (RSP)* (reciprocal of area, moment of inertia). Therefore, they recommended to include the latter as intermediate design variables. Mills-Curran *et al.* (1983) restated the conventional frame design problem in terms of the RSP taken as intermediate variables, eliminating the CSD during optimization and recovering them through linear approximations. Lust and Schmit (1986) compared frame optimization results where the problem was stated in terms of CSD only to results where intermediate variables including the RSP were included. Their results were slightly better when RSP were used as intermediate variables. Salajegheh and Vanderplaats (1986, Vanderplaats and Salajegheh 1988) recommended linearizing framework frequencies in terms of the *direct section properties (DSP)*.

Zhou and Xhia (1990) proposed a two-level approximation to solve framework problems. At the first level, the relevant behavior variables (e.g. stresses, displacements) are linearly approximated in terms of intermediate variables (called generalized variables) which include the RSPs as well as entries in the stress matrices. At the second level, the resulting approximate problem is replaced by a sequence of quadratic programming problems in terms of the CSDs which they solve using a dual approach. They showed good analysis count improvement over traditional implementations.

The first notable introduction of intermediate response quantity was due to Salajegheh and Vanderplaats (1986/1987, Vanderplaats and Salajegheh 1989) who recommended the use of linear approximation of forces in terms of areas for trusses and in terms of section properties for frameworks. Stress constraints were then derived exactly from the approximate forces. Kodiyalam and Vanderplaats (1989) extended this idea to the problem of shape optimization of a three-dimensional continuum and reported improved convergence of the optimization process. However, the repeated exact calculation of element stresses is quite expensive and offsets somewhat the improvement in approximation quality. Vanderplaats and Kodiyalam (1990) resolved the issue of stress recovery cost by using a two-level approximation. For each structural analysis, a linear force approximation is constructed. A sequence of linear problems is then solved. At the beginning of each problem a new linear stress approximation is derived from the linear force approximation and the sequence of inner linear problems is continued until convergence. At that point, a new structural analysis is conducted and a new linear force approximation is constructed and the process is continued until convergence. Hansen and Vanderplaats (1990) applied the force approximation in configuration optimization of trusses and demonstrated a markedly reduced analysis count. Moore and Vanderplaats (1990) showed that the linear force approximation yields stress approximates that include higher order terms than the conventional linear stress approximation. They then developed a simplified force approximation including only a subset of these higher-order terms and demonstrated promising results.

In his optimization with crashworthiness constraints, Lust (1990) needed to calculate

the nonlinear crash response of a car. He used a spring-mass model with nonlinear component force-displacement characteristics. He selected those force-displacement characteristics as intermediate response quantities and then proposed a linear scaling relation for them in terms of the design variables. He demonstrated that the resulting approximation permits very efficient calculations, while the same problem cannot be solved with conventional Taylor series expansions because of the low quality of the resulting approximations.

Canfield (1990) used a Rayleigh quotient approximation to improve approximations of frequencies in a cantilever beam and several truss examples. Each frequency is replaced by its Rayleigh quotient and the corresponding modal strain and kinetic energies serve as intermediate response quantities. They are in turn linearized with respect to the design variables, assuming that the normal modes remain constant. He demonstrated stable and smooth convergence in less iterations than other existing approaches. A more general discussion of approximating eigenvalues of modified matrices was taken up by Murthy and Haftka (1988) who covered approximations based on eigenvalue derivatives, generalized Rayleigh quotient and the trace theorem.

Thomas and coworkers combined intermediate variables and intermediate response quantities to derive high-quality approximations in control-augmented structural synthesis. Thomas and Schmit (1990) used DSP as intermediate design variables for trusses and frameworks. Their intermediate response quantities included the individual components of actuator forces and dynamic displacements, and the components of the complex pseudo-modal strain and kinetic energies for constraints on complex frequencies. They showed that using this combination of approximations for a mass minimization problem reduces the number of iterations from 24 to 14 and improves the final solution. Thomas *et al.* (1990) used modal participation factors as intermediate responses in their approximation of dynamic displacements near resonance.

Design Variable Linking and Reduced Basis Method Schmit and Miura (1976) proposed a two-step approach to reducing the number of design variables in an optimization problem by combining design variable linking and reduced basis vectors. *Design variable linking* is an approach introduced initially when optimizing structures modelled by finite element models. In this approach, many finite elements may be controlled by one or several design variables. The choice of what elements are linked is based on considerations of symmetry, manufacturability or even some preexisting optimization results which show that some areas of the structures appear to converge to similar designs. In effect, design variable linking amounts to writing

$$\mathrm{X} = [\mathrm{L}]\mathrm{Y} \tag{17}$$

where X is the original unlinked variable vector, Y is the new linked variable vector (of size smaller than X) and $[\mathrm{L}]$ the linking matrix. It should be noted that, for a minimization problem, the design obtained after variable linking is an upper bound to that obtained with the full set of design variables.

A further reduction in the number of design variables may be achieved by the introduction of *reduced basis vectors*. The linked variable vector may now be written

$$\mathbf{Y} = [\mathbf{B}]\mathbf{Z} \tag{18}$$

where [B] is a matrix whose columns are the basis vectors. Z is now the final variable vector, much smaller in size than X. Picket *et al.* (1973) introduced the reduced basis concept. They recommended to include among the basis vectors i) quasi-fully stressed designs generated for each load case of the initial problem using a few cycles of fully stress design, ii) quasi-fully displaced designs generated similarly with a few cycles of fully displaced design, iii) the vector of minimum gauge for the variables. For design problems for trusses with up to 200 members, they showed computer time reductions by factors of at least 7-8 with little weight penalty. Rajamaran and Schmit (1981) suggested a different type of basis vectors. They recommended to create one auxiliary design problem for each type of behavior (stresses, displacements, buckling) constrained in the initial problem. Then, the basis vectors are generated with a few iterations in each of the auxiliary subproblems. When designing a truss with 132 members, they showed a 33% time reduction with a 26% increase in optimum objective value. A reduced basis representation is an excellent approach to modelling shapes for optimization, although it is highly dependent on a proper choice of basis vectors. The initial implementation of this idea is due to Vanderplaats (1979) who used it to model an airfoil shape by superposition of shapes of basis airfoils. The same ideas has since been used in structural optimization; Barthelemy *et al.* (1991), for example, compare different trigonometric functions as basis for the representation for the shape of a hole in a plate.

Envelope Function The use of envelope functions is one approach to reducing the number of functions in an optimization problem. It essentially reduces the number of constraints handled by the optimizer and makes it more efficient. It also makes it easier to get global understanding of the problem. It can be combined with conventional constraint deletion and regionalization techniques and can reduce the number of gradients to calculate if and adjoint method is used for sensitivity analysis. One such envelope function was introduced by Kreisselmeier and Steinhauser (1979). It replace the constraints $g_j(\mathbf{X}), j = 1, n_g$ by the function

$$K(\mathbf{X}) = \frac{1}{\rho} ln \left(\sum_{j=1}^{n_g} exp(-\rho g_j(\mathbf{X})) \right) \tag{19}$$

where ρ is a user-specified parameter which controls how close the envelope is to the original constraints. Hajela (1982) first proposed this approach to fold all the constraints of a problem in a single envelope. In the shape optimization of a three-dimensional solid, Barthelemy *et al.* (1988) used this formulation to reduce the total number of constraints handled by the optimizer from 421 to 14.

MID-RANGE APPROXIMATIONS

Mid-range approximations are an attempt to endow local function approximations with a wider range of applicability. Two general devices are used for this purpose. The first is the use of information at several points, and the second is the combination of a local approximation and a global approximation. There are no mid-range problem approximations.

Multipoint Approximations Because the optimization process requires the calculation of constraints and their derivatives at more than one point, it makes sense to try and use the information and construct approximations based on that data, that would have a wider range of accuracy than approximations based on information at a single point. Early work in this area was limited to using the values of the constraint functions generated in a line search to construct a polynomial approximation along that line (e.g., Kirsch and Toledano, 1983). More recent work investigates the use of data generated during several optimization iterations for the purpose of generating approximations in an entire region of the design space. Haftka *et al.* (1989) examined approximations based on two and three points. One approach that they employed was based on the modified reciprocal approximation, Eq. (4), where the information on the derivatives at a second point was used to estimate the best values of the x_{mi}'s. However, the results indicated that while the approximation was good when it represented interpolation (for example, at points inside the triangle formed by three data points in a three-point approximation), the improvement in accuracy was marginal when it represented extrapolation.

A two-point approximation that shows more promise was proposed by Fadel *et al.* (1990). The approximation is a linear approximation in the variables

$$y_i = x_i^{p_i}\,, \tag{20}$$

as suggested by Prasad (1983). However, while Prasad suggested the choice of arbitrary exponentials, here the exponentials p_i are selected to match the data at a second point. The linear approximation in terms of y_i may be written in terms of the original variables x_i as

$$g_{tp}(\mathbf{X}) = g(\mathbf{X}_0) + \sum_{i=1}^{n} \left[\left(\frac{x_i}{x_{0i}} \right)^{p_i} - 1 \right] \left(\frac{x_{0i}}{p_i} \right) \left(\frac{\partial g}{\partial x_i} \right)_{\mathbf{X}_0} . \tag{21}$$

If we have the value of the derivatives at another design point, $\mathbf{X}_1$, we can now impose the condition that

$$\left(\frac{\partial g_{tp}}{\partial x_i} \right)_{\mathbf{X}_1} = \left(\frac{\partial g}{\partial x_i} \right)_{\mathbf{X}_1} = \left(\frac{x_i}{x_{0i}} \right)^{p_i - 1} \left(\frac{\partial g}{\partial x_i} \right)_{\mathbf{X}_0} . \tag{22}$$

From this equation p_i can be extracted

$$p_i = 1 + log \left\{ \left(\frac{\partial g}{\partial x_i} \right)_{\mathbf{X}_1} \Big/ \left(\frac{\partial g}{\partial x_i} \right)_{\mathbf{X}_0} \right\} \Big/ log(x_{1i}/x_{0i}) . \tag{23}$$

Belegundu *et al.* (1990) developed a two-point posynomial approximation (Eq. 8). They used a least-square approach to find the approximation parameters, matching not only the value of the function and its gradient at the current point but also the value of the function at a second point. Except for one set of frequency approximations, they showed significantly improved approximations for small conventional problems when comparing their results with linear, quadratic, reciprocal and one-point posynomial approximations.

Another multipoint approximation has been proposed by Rasmussen (1990). The so called cumulative approximation assumes that values of the constraint function g are available at m points $\mathbf{X}_1, \ldots, \mathbf{X}_m$, and seeks to improve the linear approximation at $\mathbf{X}_0$ based on this data. The influence of $\mathbf{X}_p$ on the approximation is determined by an exponentially decaying influence function, ϕ_p, given as

$$\phi_p(\mathbf{X}) = e^{-||\mathbf{X}-\mathbf{X}_p||^2/s_p}, \tag{24}$$

where s_p is a positive number that defines the range of the influence, and the Euclidean norm is used. It is suggested that a good choice for s_p is

$$s_p = \alpha||\mathbf{X}_0 - \mathbf{X}_p||^2, \tag{25}$$

where α is a constant. The cumulative approximation g_c is given as

$$g_c(\mathbf{X}) = \frac{g_L(\mathbf{X})\prod_{p=1}^{m}\left[1-\phi_p(\mathbf{X})\right] + \left[1-\phi_0(\mathbf{X})\right]\sum_{p=1}^{m}\phi_p(\mathbf{X})g(\mathbf{X}_p)}{\prod_{p=1}^{m}\left[1-\phi_p(\mathbf{X})\right] + \left[1-\phi_0(\mathbf{X})\right]\sum_{p=1}^{m}\phi_p(\mathbf{X})}. \tag{26}$$

where $g_L(x)$ is the linear approximation (Eq. 1) based on data at $\mathbf{X}_p$. The exponential decay is an attractive feature of the approximation because it limits the influence of far away points. However, the cumulative approximation fails to take advantage of the derivatives at the other points, which are typically available.

Finally, it is worth noting that in many cases in optimization, it maybe unnecessary to combine the data from several point into a single approximation. Instead a constraint $g \leq 0$ is replaced not only by its most recent linearization, but by several of its previous linearizations (see, for example, Mistree *et al.* (1981)).

Scaling or Local-Global Approximations Because of computational constraints, optimization is often performed on the basis of a model of the structure that is simpler than the one which is used for analysis of the same structure. Such a simpler model is based on a simpler theory (e.g., beam theory versus shell analysis), or a coarser discretization of the numerical model associated with the same theory. This simpler model can be viewed as a global approximation, as discussed earlier. Here we consider the complex model to provide the exact value of the function $g(\mathbf{X})$, while the simple model is assumed to provide a global approximation $g_G(\mathbf{X})$. A local flavor can be injected into the approximation by calculating a scale factor associated with the simple and complex model at a point $\mathbf{X}_0$. That is the scale factor s_{c0} is given as

$$s_{c0} = g(\mathbf{X}_0)/g_G(\mathbf{X}_0). \tag{27}$$

Then the scaled global approximation, g_{s0}, is given as

$$g_{s0}(\mathbf{X}) = s_{c0} g_G(\mathbf{X}) . \tag{28}$$

The scaling is likely to improve the quality of the global approximation near $\mathbf{X}_0$, but it may increase the error far from $\mathbf{X}_0$ if the scale factor varies significantly with $\mathbf{X}$. Haftka (1991) suggested using a linear approximation to the scale factor. That is,

$$s_{c1} = s_{c0} + \sum_{i=1}^{n} (x_i - x_{i0}) \left(\frac{\partial s_c}{\partial x_i} \right)_{\mathbf{X}_0} . \tag{29}$$

and then the linear-scale-factor approximation is given as

$$g_{s1} = s_{c1} g_G(\mathbf{X}) . \tag{30}$$

where

$$\left(\frac{\partial s_c}{\partial x_i} \right)_{\mathbf{X}_0} = s_{c0} \left(\frac{\partial g}{\partial x_i} \Big/ g - \frac{\partial g_G}{\partial x_i} \Big/ g_G \right)_{\mathbf{X}_0} . \tag{31}$$

Chang *et al.*. (1991) have applied this approach to a plate theory approximation of a built up wing structure where the refined analysis is based on a finite element model. Their results indicate that the linear scale factor is better than either the constant scale factor or the linear approximation based on the refined model.

Other Mid-Range Approximations In a departure from the use of response surfaces as global approximations, Free *et al.* (1987) used the theory of experiments to develop an optimization algorithm which constructs local approximations in the vicinity of the current design point, solves the approximate design problem and then updates the current approximation, contracting or expanding its range as appropriate. Using this algorithm on standard nonlinear programming problems and a ten-bar truss design they showed performance comparable to that of a very efficient SQP/GRG algorithm for noise-free problems. When noise was present in the problem formulation, they showed significantly improved performance.

CONCLUDING REMARKS

This paper reviews the main approximation concepts in applications of nonlinear programming to structural optimization. It shows that approximation concepts can be classified according to their range into local, medium-range, and global approximations. Approximations can be designed to approximate the functional relationship between dependent variables (objective function and constraints) and independent variables. Also, they can aim at approximating the problem formulation by reducing the number of problem variables or constraints, effecting changes of variable or response definition to improve functional relationships or simplifying structural model.

There is not enough data in the open literature to establish a comprehensive comparison of the effectiveness of various approximation concepts. Often, a particular approximation is compared for accuracy only in extrapolations about a given design point. The question of overall performance, as indicated by the number of reanalyses required in reference test problems or that of overall cost, as indicated by comparison of CPU requirements are not always addressed. However, available results show that computational cost can generally be reduced by the use of approximation concepts, particularly if several of them are combined. Recently, structural optimization programs have begun to offer as options combinations of the approximation concepts described herein.

The more recent contributions found for this review focus on two specific areas: i) careful selection of intermediate variable and response quantity and ii) improvement of the traditional Taylor series expansion by inclusion of some higher-order terms or of information at different points in the design space.

However, newer methods may emerge and older approaches may be revisited in light of development of new computer hardware. For example, the construction of response surfaces may become cost effective even for large problems with the advent of massively parallel computers. On the other hand, neural networks may also provide a unique opportunity to build inexpensive approximations to expensive analytical models.

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16. Abstract

This paper reviews the basic approximation concepts used in structural optimization. It also discusses some of the most recent developments in that area since the introduction of the concept in the mid-seventies. The paper distinguishes between local, medium-range and global approximations; it covers functions approximations and problem approximations. It shows that, although the lack of comparative data established on reference test cases prevents an accurate assessment, there have been significant improvements. The largest number of developments have been in the areas of local function approximations and use of intermediate variable and response quantities. It appears also that some new methodologies emerge which could greatly benefit from the introduction of new computer architectures.

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