N91-22163

Reactionless Propulsion Using Tethers

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Abstract

A orbiting tethered satellite can propel itself by reaction against the gravitational gradient, with expenditure of energy but with no use of on-board reaction mass. Energy can be added to the orbit by pumping the tether length in the same way as pumping a swing. Examples of tether propulsion in orbit without use of reaction mass are discussed, including: (1) using tether extension to reposition a satellite in orbit without fuel expenditure by extending a mass on the end of a tether; (2) using a tether for eccentricity pumping to add energy to the orbit for boosting and orbital transfer, and (3) length modulation of a spinning tether to transfer angular momentum between the orbit and tether spin, thus allowing changes in orbital angular momentum.

1. Introduction

A tether is a long, flexible cable which connects one part of a satellite with another. Although quite simple, many very interesting things can be done in space using tethers [1-3]. In the equilibrium configuration, as shown in figure 1, the tether is oriented radially outward, with a tension on the tether due to the gravitational gradient (or "tidal") force.

The effective acceleration due to the gravity gradient a distance x from the center of mass (CM) is, to first order:

$$a_{eff} = 3\, g_o r_e^2 x / r_o^3, \qquad (1)$$

where g_o is the gravity at the Earth's surface, r_o is the orbital radius and r_e is the radius of the earth.

Most analyses of tether orbits assume that the center of mass of a tethered satellite system remains in the original orbit; *i.e.*, that the angular velocity of the tethered satellite does not change as the tether is extended or retracted. This is true only to the first order approximation in tether length. Briefly, the mass that extends outward experiences an increase in centrifugal force that increases linearly with distance, but the mass that extends inward experiences gravity that increases faster than linearly. Thus, as the tether is unreeled, the center of mass of the orbit is pulled inward. To conserve angular momentum, the angular velocity of the orbit increases.

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The effect that a tethered satellite can extend across the gravity gradient can be used for propulsion. The following analysis (sections 2 and 3) follows my calculations from reference 4. Similar results to section 3 are also derived in reference 5.

2. Orbital Repositioning of a Satellite

In the following section we assume a tether of negligible mass in circular orbit. The extension of the analysis to tethers of non-negligible mass is straightforward.

Consider a satellite of mass m_i consisting of two pieces of mass $m_1=m_2=m_i/2$ connected by a tether. The initial orbit is assumed to be circular, with an angular velocity ω_o and an initial orbital radius (measured from the Earth's center) r_o. With the tether at initial length zero, the orbit has initial angular momentum

$$L_i = m_i \omega_o r_o^2. \quad (2)$$

Now assume that the tether is extended to length x in each direction from the CM, as shown in figure 1. The total length is $2x$. Note that energy decreases, since in deploying a tether work is done by the effective tidal force. Angular momentum is still conserved,

$$L = m_1 \omega r_1^2 + m_2 \omega r_2^2, \quad (3)$$

where r_{cm} is the orbital radius of the CM, and $r_1=r_{cm}-x$ and $r_2=r_{cm}+x$. The inward tension on the low end of the tether must equal the outward tension on the high end of the tether If we expand to second order in x, then set equation (2) equal to equation (3) to s e for ω and r_{cm} as a function of tether extension x, we find the center of mass dro ,

$$r_{cm} = r_o - 5\frac{x^2}{r_o} \quad (4)$$

and the orbital period P increases as the tether extends:

$$P = P_o[1 - 9(\frac{x}{r_c})^2]. \quad (5)$$

For example, a GEO satellite consisting of two equal masses on a 1000 km long tether will have a period faster than that of an untethered satellite by 0.44° per day.

Inclusion of higher order terms results in an increase in the effect.

If the two masses are allowed to differ, the orbital period change is proportional to $m_1 m_2/(m_1 + m_2)$, which is maximum when the two masses are equal.

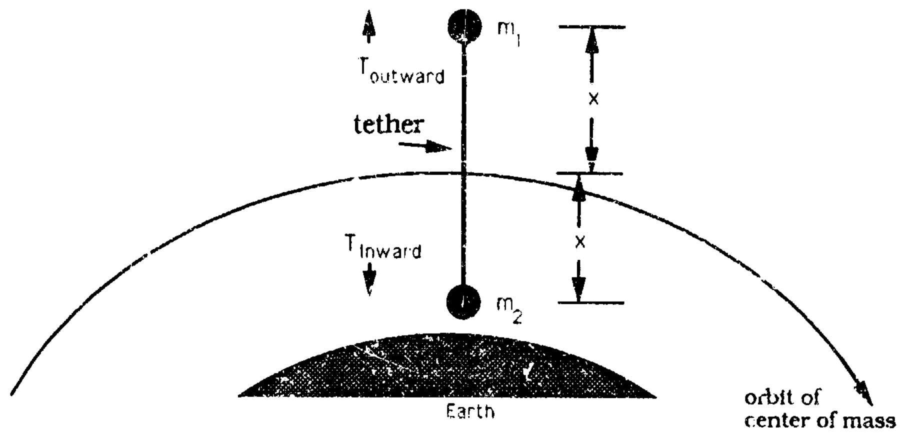


Figure 1. **Tether orbit and definitions**

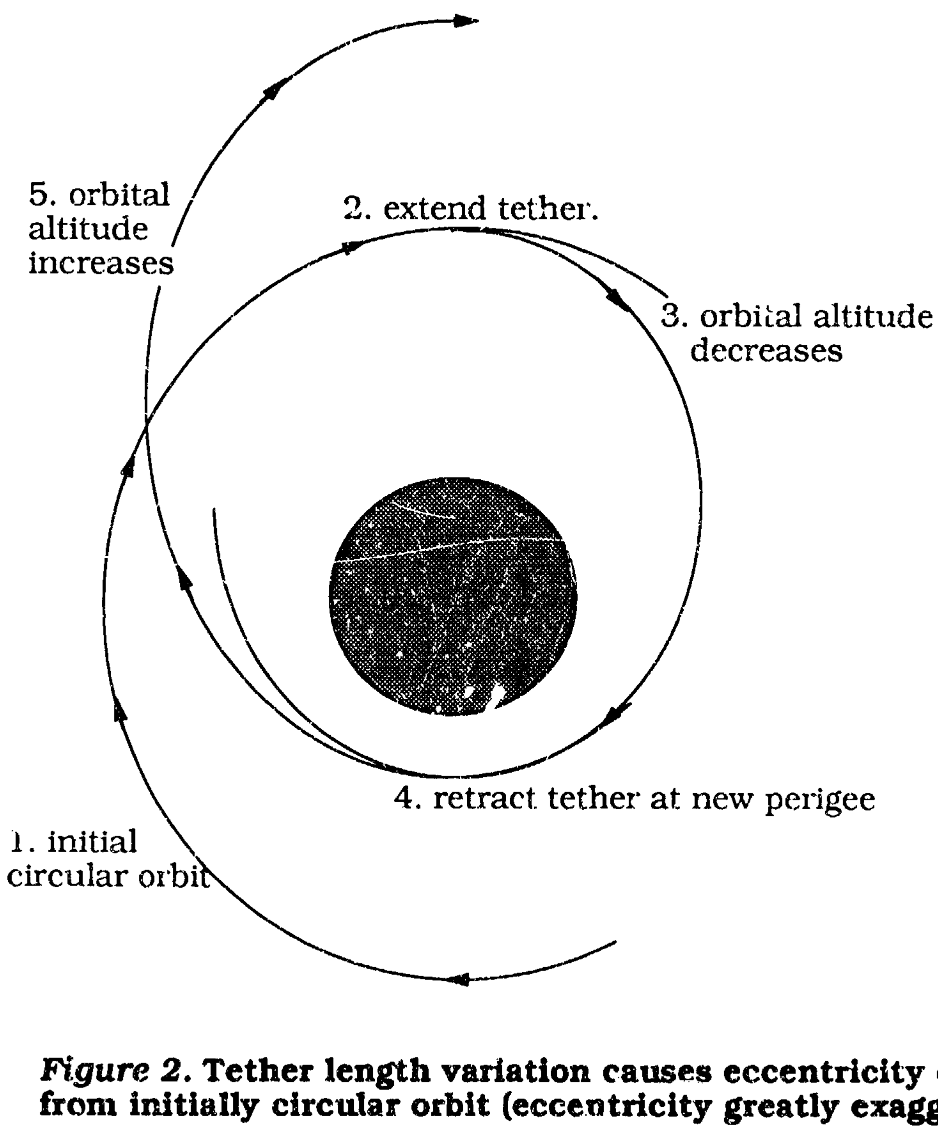


***Figure 2.* Tether length variation causes eccentricity change from initially circular orbit (eccentricity greatly exaggerated)**

3. Orbital Propulsion by Eccentricity Pumping

The preceding analysis has assumed equilibrium conditions, *i.e.*, that the orbit remains circular during the extension and deployment of the tether. This assumption is true only if the tether is deployed or retracted over a time greater than an orbital period. Faster deployment will result in dynamic changes to the orbital eccentricity. This is shown in figure 2, where an initially circular orbit is altered to an eccentric orbit by modulation of the tether length. Continuing the length modulation allows eccentricity to be continuously increased (or decreased). This effect can be used as a means for orbital propulsion that does not require expenditure of reaction mass. In the process energy is added to the orbit (from a power source on board the spacecraft), while the orbital angular momentum is constant. Particular applications are injection of a spacecraft into an escape orbit from an initially circular orbit, and use of the process for transfer orbits, *e.g.*, LEO to GEO.

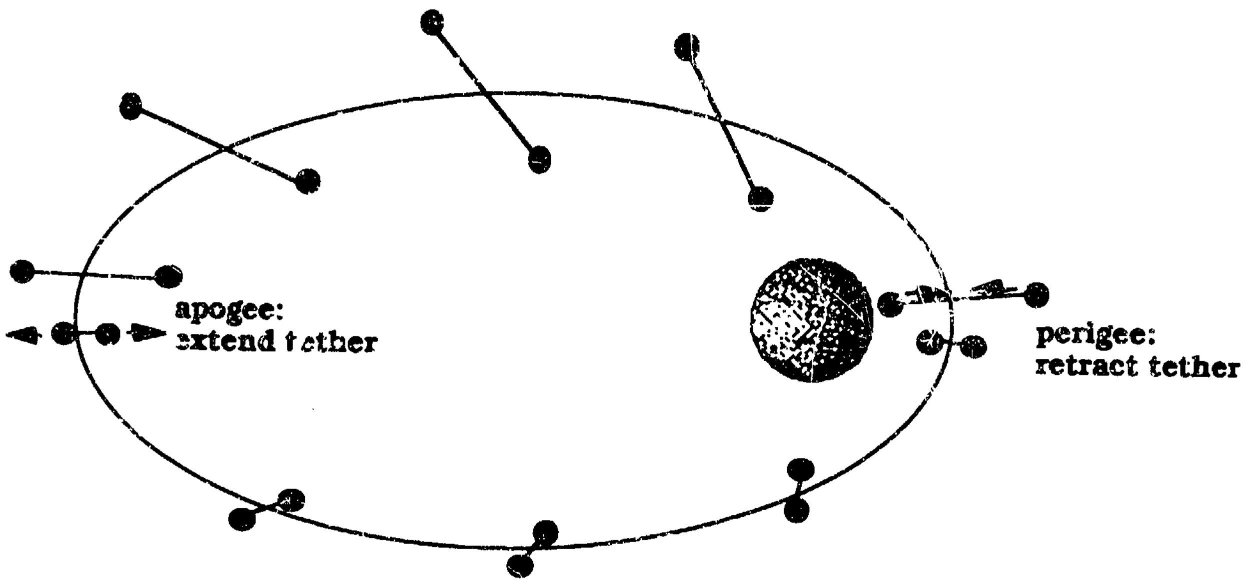
The method is straightforward. A mass is deployed away from the spacecraft on the end of a tether. The stable configuration is with the tether oriented radially from the central body. The tether is mounted on a reel with a motor which can pull it in or let it out. The method of orbit pumping consists of pulling the tether in at perigee (more generally, periapsis) and letting the tether out at apogee. Since gravitational gradient (tidal) forces are to first order proportional to the inverse radial distance cubed, more mechanical work is done against the tidal force in pulling the tether in than is returned when the tether is let back out. Thus, energy is added to the orbit. Since the orbital angular momentum is unchanged, the eccentricity ϵ of the orbit increases. This is shown in schematic in figure 3.

As an aside, it may be noted that this process is essentially the same as the process of adding energy to a playground swing by "pumping" [6].

Contrary to expectations, the eccentricity pumping process is most effective when the orbit is nearly circular. Although the amount of energy available per orbit decreases as the orbit becomes nearly circular, the sensitivity of the eccentricity ϵ to small changes in energy increases as $1/\epsilon$, and this factor dominates over the decrease in energy. For a perfectly circular orbit, higher order terms contribute as well.

The reverse process, circularizing an eccentric orbit by removal of energy by a viscoelastic tether, has been discussed in detail by Columbo *et al.* [7]. This is equivalent to tidal damping, a natural phenomenon that accounts for the fact that most of the moons in the solar system have nearly circular orbits.

Eccentricity pumping can only be done if the initial orbit is high enough that the minimum perigee does not impact the primary. For pumping from an initial circular orbit at distance a to escape this implies that $a_o \geq 2\ r_e$. In general, the minimum perigee must also be high enough not to intersect the atmosphere. This corresponds to an initial orbital radius of ~13,150 km, or a minimum initial orbital altitude of 6,575 km.



Tether detail

Figure 3. **Eccentricity pumping (schematic). Tether is retracted at perigee of orbit, extended at apogee. Since the gravity gradient is higher at perigee than at apogee, more energy is input to the system during retraction than is recovered during extension of the tether, and so net work is done. This work results in an increase of orbital energy.**

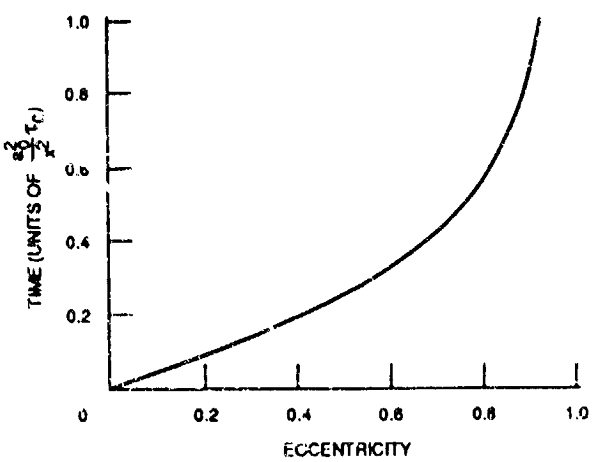


Figure 4. Time required to reach a given eccentricity.

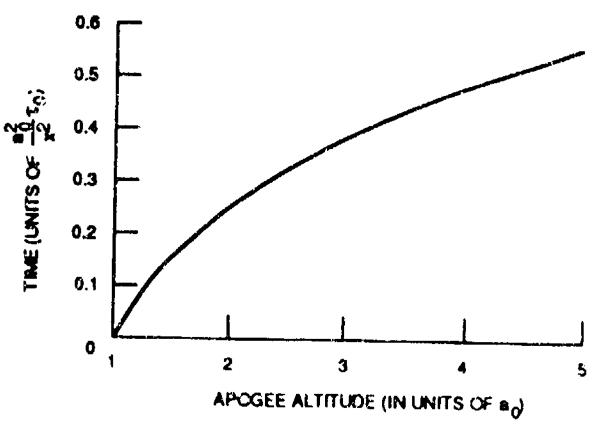


Figure 5. Time required to reach a given apogee altitude (in units of initial altitude

Again, assume that the tether itself is of negligible mass. The case of a massive tether can be straightforwardly calculated by integration over the mass distribution. We also assume that the masses on each end of the tether are equal, the case which for fixed tether length maximizes the effect. Extrapolation to unequal masses is straightforward. Assume equal masses m/2 extended on the ends of a tether of full length d (*i.e.*, half-length x). The mechanical energy stored in the tether is:

$$E = \frac{3}{8}\, m g_o \frac{r_e^2}{r_o^2} d^2 \tag{6}$$

Now assume the orbit is elliptical, with eccentricity ϵ. The orbital energy is

$$E = -\, m g_o \, \frac{r_e^2(1-\epsilon^2)}{2a_o} \tag{7}$$

The amount of energy required to retract the tether at perigee minus the amount recovered in extending the tether at perigee is:

$$\frac{\Delta E}{orbit} = \frac{3}{8}\, m g_o d^2 \, \frac{r_e^2}{a_o^3}\, \epsilon(6+\epsilon^2) \tag{8}$$

In the real case, the tether length d will not reeled in all the way to zero length. An effective value of d can be used, $d_{eff}^2 = d_{max}^2 - d_{min}^2$.

The sensitivity of eccentricity to changes in energy is

$$\frac{d\epsilon}{dE} = \frac{a_o}{m g_o r_e^2 \epsilon} \tag{9}$$

The orbital period, expressed in terms of the orbital period of the initial circular orbit, is

$$\tau = \tau_o(1-\epsilon^2)^{-\frac{3}{2}} \tag{10}$$

The average power required is:

$$\frac{dE}{dt} = \frac{3}{8} m g_o \, \frac{r_e^2}{a_o^3 \tau_o}\, d^2 \, \epsilon(6+\epsilon^2)(1-\epsilon^2) \tag{11}$$

The function of ϵ has a maximum of 2.45 at ϵ=0.6. In practical units, this is an average specific power of

$$\dot{P} = (280 w/kg)\ a_o^{-9/2} L^2\ \epsilon(2.45 + 0.41\epsilon^2)(1 - \epsilon^2) \qquad (12)$$

where a_o is the initial semimajor axis in multiples of the earth radius r_e and L is the tether length in thousands of kilometers. Note that the function of ϵ has been normalized to a maximum value of 1. Since for most applications d will be << 1000 km and a_o will be >~1.5 r_e, this specific power is well within achievable levels. The peak power levels required will be higher, and depend on how fast the tether is reeled in.

Then the rate of change of eccentricity is

$$\frac{d\epsilon}{dt} = \frac{d\epsilon}{dE}\frac{dE}{dt} = \frac{3}{8\tau_o}\frac{d^2}{a_o^2}\ (6 + \epsilon^2)(1 - \epsilon^2)^{\frac{3}{2}} \qquad (13)$$

To find the time required to reach a given eccentricity, this expression is inverted and integrated. The integral can be done exactly,

$$t(\epsilon) = \frac{8}{3}\tau_o\frac{a_o^2}{d^2}\ \frac{1}{7\sqrt{42}}\ tan^{-1}\left[\frac{7\epsilon}{\sqrt{42(1 - \epsilon^2)}}\right] + \frac{\epsilon}{7\sqrt{1 - \epsilon^2}} \qquad (14)$$

Figure 4 shows the time to reach a given eccentricity. (Here time is plotted in the units of $a_o^2/d^2\tau_o$, which is simply the initial orbital period scaled by the squared tether length. Initial orbital period is 90 minutes for LEO). Escape is approached asymptotically.

It is also of interest to look at the time required to reach a given apogee altitude: this is shown in figure 5. Again time is in units of the scaled orbital period, and altitude is given as a multiple of the initial orbital radius (measured from the center of the Earth). At long times the altitude increases almost linearly with time.

As an example, consider the case where eccentricity pumping is used to move from LEO to geosynchronous transfer orbit. a_{GEO}=6.63 r_e, so $a_{LEO-GEO}$ = 3.82 r_e, and $\epsilon_{LEO-GEO}$ = 0.738. The minimum perigee requirement leads to a minimum initial orbital radius of a_o = 1.74 r_e, *i.e.*, initial orbital altitude 4700 km and initial orbital period $\tau_o \sim$ 200 min. From figure 6, the time needed is 0.43 $\tau_o(\frac{a_o}{d})^2$.

For a 500 km tether length, $(\frac{a_o}{d})^2$ ~500, and the orbital pumping process takes 725 hours, or about 31 days.

An efficient technique for the apogee kick would be to continue eccentricity pumping until the apogee is well past GEO, perform the apogee kick, then use eccentricity pumping in reverse to circularize the orbit. The amount of velocity change ΔV required to give the orbit sufficient angular momentum to attain circular orbit at GEO is inversely proportional to the distance. Thus, in theory, the ΔV needed for apogee kick could be made arbitrarily low by pumping the apogee to a high enough initial value, although this would take a long time.

The decrease in eccentricity rate with increasing eccentricity is due to several factors. one of them being that the energy transfer per orbit is proportional to the square of the tether length over the orbital semimajor axis. As the eccentricity, and thus the semimajor axis, increases, the rate decreases. Since the tether is retracted at perigee, the effect can be eliminated by increasing the tether length as the apogee distance increases. (As tether length increases, this will require initiating the retraction slightly before perigee.) Note that since the total stress due to tidal force goes as d^2/r^3, the maximum stress at apogee decreases despite the increased tether length.

The requirement for a minimum initial altitude comes from the necessity that the minimum perigee of the orbit not intercept the atmosphere. This requirement can be alleviated if the pumping maneuver is combined with an incremental ΔV at each apogee to increase the angular momentum just sufficiently to keep the perigee from decreasing.

4. Propulsion Using a Spinning Tether

The calculations in the preceeding section have all been done assuming that the tether is not spinning. This is not required by physics, but is a practical consideration due to the limits of real material strength. For a spinning tether, centrifigal stresses can very rapidly become extremely large. The constraint to avoid spinning the tether during retraction will put a limit on the maximum modulation d_{eff}/d possible.

If the tether is allowed to spin, however, a vastly more effective propulsion method which is not limited by orbital angular momentum is possible. This is shown in schematic in figure 6. For example, to increase the orbital altitude, the tether is extended while horizontal. and retracted when vertical. Since in retraction work is done against the gravitational gradient, the orbital energy increases. The orbital angular momentum also increases, and the spin of the tether increases (if the spin is opposite to the orbital direction) or decreases (if the spin is the same as the orbit). Effectively, angular momentum is transferred from the orbit to the tether.

The amount of energy transferred per spin is nearly independent of the spin rate, and thus the higher the spin rate of the tether, the faster (in principle) the orbital energy can be changed. The rate of orbital change is limited only by the power source and the materials strength. If the energy source had sufficiently high power, it would even be possible to propel past escape velocity.

Orbital plane changes are also possible (although slightly more difficult to illustrate). For plane changes the tether spin axis is optimally in the plane of the orbit, and again the tether length is modulated in phase with the orbit.

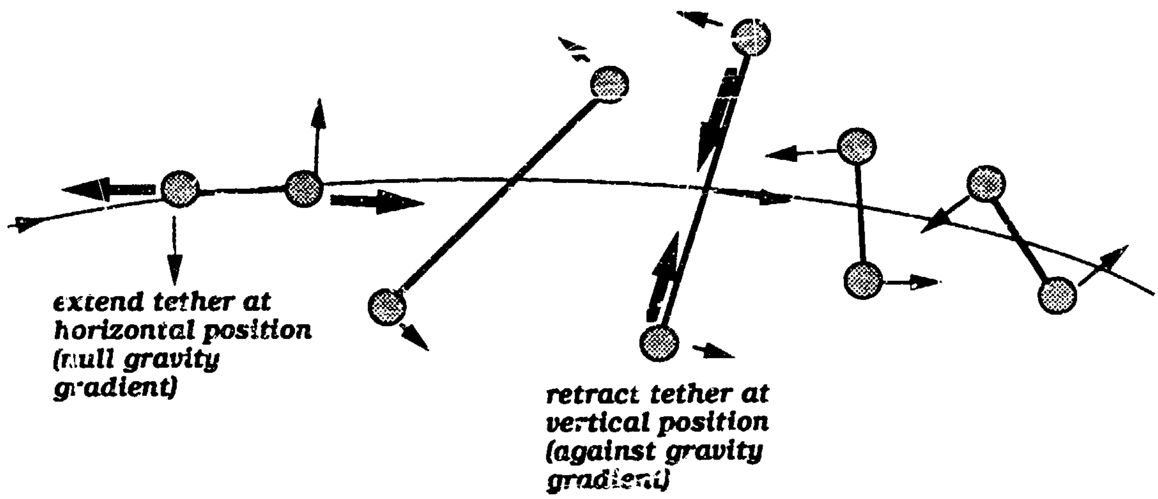


***Figure* 6. Orbital propulsion using a spinning tether (orbital motion here is clockwise; tether spin counterclockwise).**

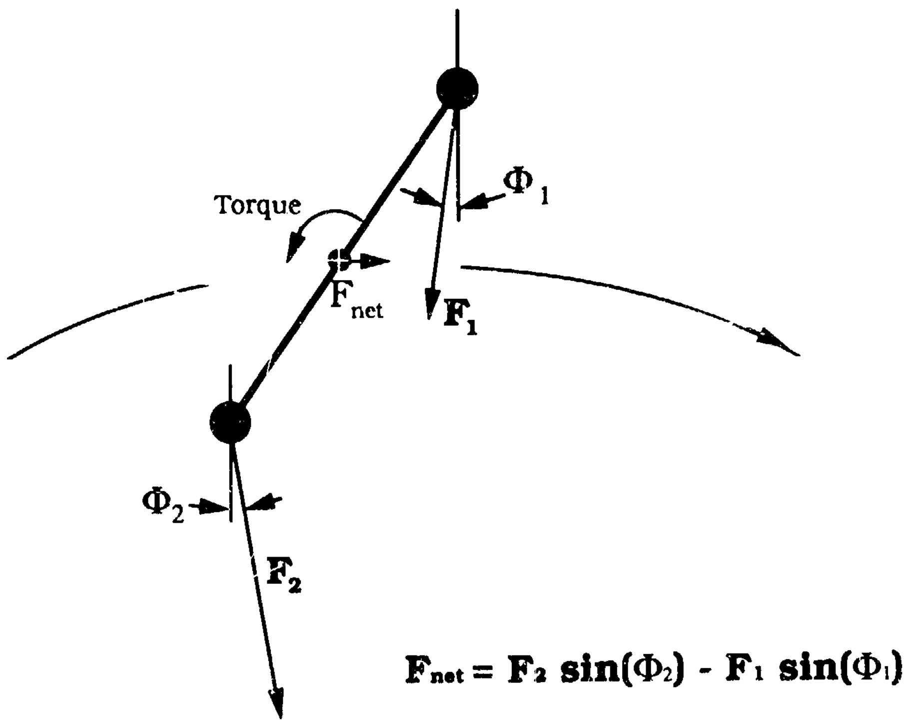


***Figure* 7. Forces on a tilted tether (shown in locally inertial reference frame). Since $F_2 > F_1$ and $\phi_2 > \phi_1$, there is a net side force on the center of mass of the tether as well as a torque.**

An alternative view of this propulsion system, showing the origin of the forces, is shown in figure 7. When the tether is not aligned vertically, there is a net side force on the tether due to the fact that the gravity on the two end masses is not the same. In general for a spinning tether this side force averages to zero. However, by (for example) increasing the tether length when the tether is angled to the left of vertical, and decreasing the tether length when it is to the right of vertical (*i.e.*, modulating the length) the average can be made non-zero. This side force is then used for propulsion.

For real materials the amount of angular momentum which can be stored in the tether is limited. An untapered material can rotate at a maximum tip speed which is characteristic of the material, $v_t = \sqrt{[(\textit{breaking stress})/\textit{density}]}$. This value v_t is slightly under 2 km/sec for the best currently existing fiber. Defect-free fibers of high-strength materials–silicon carbide, diamond–have theoretically much better values, and ten times this value, 20 km/sec, is not unreasonable to expect in the long term. For constant tip speed, angular momentum increases linearly with tether length, and so the effectiveness increases with tether length.

As an example, suppose a spinning tether is used for propulsion from LEO to escape. Angular momentum at LEO is (6500 km)(7.9 km/sec) or about 50,000 km^2/sec. Angular momentum at escape is km^2/sec. The difference, about 20,000 km^2/sec, must be taken up in tether spin. At a maximum V_t of 2 km/sec, the tether length required will be 10,000 km. If 20 km/sec V_t could be achieved, the required tether length is only 1000 km, a tether length which is not unreasonable to expect to be achievable in the long term.

Alternatively, if angular momentum can be transfered to some external sink, this may not be a limitation. The obvious choice is to transfer momentum to the Earth's magnetic field via a magnetic torquer, such as is used in many satellites for orientation control. This could be done by a method as simple as driving an alternating current along the length of the tether and using the v x B potential to drive energy through a load (appropriately this load would be the tether winch motor, allowing the energy put into tether spin to be recovered). This then becomes conceptually similar to electrodynamic tether propulsion (see, for example, discussions in references 1-3]), except that tether spin velocity is substituted for orbital velocity, and since required the current is AC, no return path along the space plasma is required.

5. Conclusions

A tethered satellite system can extend significant distances across the gravitational gradient of the body it is orbiting. This effect can be made use of, using the gravity gradient itself for propulsion. Several applications are discussed. These applications are noteworthy as examples of raising an orbit "by its bootstraps" by pulling against the gravity gradient.

Of course, these propulsion systems are not reactionless in the physics sense: Newton's law of conservation of momentum is not violated, since momentum is transferred to the Earth (or primary) by the gravitational attraction. However, they *are* reactionless in a real, engineering sense, in that no propellant is expended as reaction mass. If a limitless energy source is available, such as a solar power system, the tether system can maneuver completely in Earth orbit.

6. Acknowledgments

This work was done while the author was a NASA/NRC Resident Research Associate at NASA Lewis. I would like to acknowledge the discussions, suggestions, and support of Frank Hrach at NASA, and of Dr. Robert Forward at Forward, Unlimited, who contributed significantly to this work.

I would also like to note that use of tethers for propulsion by eccentricity change of an orbit has been proposed independently, and earlier, by Manuel Martinez-Sanchez and Sarah A. Gavit of MIT [5].

7. References

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