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A FAST ALGORITHM FOR CONTROL AND ESTIMATION USING A POLYNOMIAL STATE-SPACE STRUCTURE

James R. Shults and Thomas Brubaker
Colorado State University

Gordon K. F. Lee
North Carolina State University

ABSTRACT

One of the major problems associated with the control of flexible structures is the estimation of system states. Since the parameters of the structures are not constant under varying loads and conditions, conventional fixed parameter state-estimators can not be used to effectively estimate the states of the system. One alternative is to use a state-estimator which adapts to the condition of the system.

One such estimator is the Kalman filter. This filter is a time-varying recursive digital filter which is based upon a model of the system being measured. This filter adapts the model according to the output of the system. Previously, the Kalman filter has only been used in an off-line capacity due to the computation time required for implementation. With recent advances in computer technology, it is becoming a viable tool for use in the on-line environment. The following paper describes a distributed Kalman filter implementation for fast estimation of the state of a flexible arm. A key issue, is the sensor structure and initial work on a distributed sensor that could be used with the Kalman filter is presented.

INTRODUCTION

The parameters of flexible structure systems are generally dynamic. They change under varying load and environmental conditions. When there is a need to control such dynamic systems, these parameters must be measured or estimated. These systems are usually very complex and often more parameters are needed for control than can be measured. The parameters which cannot be measured must therefore be estimated in some manner.

With the rapid evolution of computers, the Kalman filter is becoming an excellent tool for estimation of system parameters. Previously, this filter could only be used in off-line applications such as filtering of laboratory data Brubaker. Now, it is becoming useful in on-line environments for state estimation.

The Kalman filter is a time-varying digital filter which is based upon a model of the system being studied. The filter uses signals from the system to adapt the model and estimate system parameters. These parameters along with the measured signals can then be used to control the system. A key issue in the use of parameter estimation is the sensor distribution and use of appropriate sensor types. Along with this is the data fusion issue from sensors to provide appropriate control information. Here we only describe the estimation procedure with comments on the sensor issue. A block diagram of the filter structure is shown in Fig. 1 for a single input system. For multiple sensors, multiple filters could be deployed and data fusion done with filter outputs.

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KALMAN FILTER THEORY

The Kalman filter is a time-varying recursive digital filter which estimates the states of a system from one or more sensor signals. The filter operates on time domain signals using linear least squares estimation that utilizes all of the past data from the output signals. This estimation can provide separation of signal components. These components can be used to determine the states of the system. The Kalman filter can also improve noise reduction on the signals under consideration [Brubaker].

The first step in the design of a Kalman filter is the choice of a linear or linearized signal model that describes the signal or serves as an approximation to the signal. One of the most flexible of these models is a polynomial. The input signal to the filter (output from the system), z(t), is represented by a polynomial of order m. At time t = nT, where T is the sampling period of the system, the state vector for the system is given by

$$z(nT) = \begin{bmatrix} z \\ \frac{dz}{dt} \\ \cdot \\ \cdot \\ \cdot \\ \frac{d^m z}{dt^m} \end{bmatrix}_{t=nT} \qquad (1)$$

Here, the system is being represented in canonical state-space form with the components of the state vector being the derivatives of the polynomial model. When a system is to be represented in a different state-space form, a linear transformation can be performed to change the state vector to the desired form.

To use the polynomial model for the Kalman filter, the state vector must be redefined using a Taylor Series representation for each element of z(nT). The resulting state vector is

$$x(nT) = \begin{bmatrix} z \\ T\frac{dz}{dt} \\ \cdot \\ \cdot \\ \cdot \\ \frac{T^m}{m!}\frac{dz^m}{dt^m} \end{bmatrix}_{t=nT} \qquad (2)$$

The state of the system at t = (n+h)T can now be described in terms of the state at t=nT by the relationship

$$x[(n+h)T] = \Phi(h)\,x[nT] \qquad (3)$$

where $\Phi[h]$ is the following (m+1) X (m+1) state transition matrix:

$$\Phi[h] = \begin{bmatrix} 1 & h & .. & h^m \\ 0 & 1 & .. & mh^m-1 \\ . & . & .. & . \\ . & . & .. & . \\ 0 & 0 & .. & 1 \end{bmatrix} \qquad (4)$$

The filter is implemented by first producing forecasts of the estimate

$$x_1(nT) = \Phi(1)x[(n-1)T] \qquad (5)$$

and covariance matrix

$$S_1(nT) = \Phi(1)S[(n-1)T]\Phi^T(1) + Q$$

at t = nT using the previous estimate, x[(n-1)T], and covariance matrix, S[(n-1)T]. These forecasts are obtained by using the state transition relationship given in (4). The matrix Q is the covariance of the driving noise. It allows the designer to "fade" the effects of past inputs. The covariance forecast is then used to calculate the Kalman Gain Matrix,

$$K(nT) = S_1(nT)M^T[\sigma^2 + MS_1(nT)M^T]^{-1} \qquad (7)$$

The term σ^2 is the variance of the measurement noise. The matrix M is a row matrix which relates the measurable state variables to the actual measurements. For this filter, the measurements are assumed to be components of the state-vector z(Nt) given in (1). Therefore, M relates the estimate vector x(Nt) given in (2) to the measured components of z(Nt). The Kalman Gain Matrix is then used to obtain the covariance estimate,

$$S(nT) = [I - K(nT)M]S_1(nT) \qquad (8)$$

and the state-vector estimate,

$$x(nT) = x_1(nT) + K(nT)[y(nT) - Mx_1(nT)] \qquad (9)$$

In (9), the term y(nT) is the data measurement vector at t=Nt which consists of the measured components of z(nT). Equations (1) through (9) are the basis for the Kalman filter. A complete derivation for these equations can be found in many texts for example, Liebelt and Meditch.

KALMAN FILTER DESIGN

With the choice of a polynomial model for the input signal, the design of a Kalman filter involves the determination of a few key parameters. These parameters are set according to the system and design specifications.

The first group of parameters which must be determined are associated directly with the properties of the system. The first is the sampling period, T. This period is usually set according to the nyquist rate,

$$f_s = \frac{1}{T} \geq 2f_m \tag{10}$$

where f_m is the maximum frequency of the bandlimited input signal and f_s is the sampling frequency.

The second parameter is the variance of the sensor noise, σ^2. This parameter is a property only of the sensors which are employed. Another term which is determined from the sensors which are utilized is the Data Measurement Vector. This is the number of terms in the state vector of equation 1 which can be directly measured.

The remaining filter parameters are determined using both design and system specifications. The order of the polynomial model must be set according to the sampling period and the angular velocity of the input signal oscillations. The calculation time of the filter is larger for higher orders. The order must be small enough to allow the calculation time to be smaller than the sampling period. On the other hand, the order must be large enough to allow the filter to track the input signal well.

The key design parameter of the Kalman filter is the covariance matrix of the driving noise, Q. For this paper, the driving noise is assumed to be uncorrelated white noise. This simplifies the matrix Q to a diagonal matrix. After testing different forms for the Q matrix, little difference was found. Therefore, the matrix Q was taken to be the identity matrix times a constant, f.

The constant f is known as the fading factor. This term determines how the filter will handle past data. When $f = 0$, the filter is simply an expanding memory filter. All past data is used evenly to calculate the present estimate. This will cause the variance of the estimate and its covariance matrix to decrease to zero as time increases, but deterministic errors will become large. If the fading factor is greater than zero, more emphasis is placed on the present sample than on past samples. Thus, the past samples are faded from the filter's memory. Larger values of f cause past samples to fade more quickly. The fading of past samples causes the Kalman filter to have a much smaller deterministic error, but the variance of the estimate and its covariance approach the values at the input.

One way to minimize the deterministic error associated with a small fading factor is to periodically reinitialize the Kalman filter. This requires establishing a relationship between the frequency of initialization and the fading factor. The fading factor should be the minimum value for which the deterministic error just prior to reinitialization is within specifications. This value will cause the Kalman filter to provide maximum noise reduction while meeting deterministic error specifications.

To reinitialize the Kalman filter, a nonrecursive filter is used. This filter utilizes a window of the past samples to estimate the state vector and its covariance. These parameters are then passed to the Kalman filter.

DESIGN EXAMPLE

The Kalman filter was tested in two separate operating conditions. The first was with data acquired from the hub of the flexible arm at CSU. The second test was with an eighth order model of the flexible arm. This test included the simulation of the system with control in a closed-loop environment. A program has been developed which allows the design and testing of these and other filter structures. The results of these experiments are discussed in the following paragraphs.

The data acquired from the hub of the flexible arm was position information sampled at a rate of 100 samples per second. A polynomial model of order 3 was required to accurately represent the position of the hub. Since velocity information was not available, a Data Measurement Vector of one was used. The variance of the sensor noise was arbitrarily set at 10^{-4}. The filter was reinitialized every 100 samples, 1 second, and a fading factor of 10^{-9} was used.

The outputs of the filter are shown in Figs. 2 through 5. Figure 2 demonstrates the ability of the filter to estimate the position signal. Figure 3 displays the estimate of the velocity. Figures 4 and 5 show the estimates of the second and third derivatives of the position signal. No actual data was available for the second and third derivatives. Therefore, the accuracy of these estimates could not be determined.

The second test of the filter involved an eighth order simulation of the flexible arm with the filter outputs used as control feedback. The simulation provided the position of the tip as input to the filter. A third order polynomial model was used for the Kalman filter. The variance of the sensor noise was taken to be 10^{-4} and the fading factor was 5×10^{-12}. The feedback control was implemented as simple proportional position only negative feedback. The feedback gain was set to 2.5

The results of this simulation are shown in Figs. 6 through 9. Figure 6 shows a comparison of the position signal before and after the Kalman filter. Figures 7 through 9 show the first, second and third derivatives of position.

FUTURE WORK

Future work on the Kalman filter consists of implementation issues. A good design method has been set forward, but many implementation issues have not been addressed. The major implementation problem comes with the selection of hardware to run the Kalman filter. This hardware must consist of a microprocessor which is fast enough to meet the sampling rate, but inexpensive enough to make its use feasible. Another major problem comes in the sensors which will provide state information for the Kalman filter. These sensors must be selected for each application such that they provide accurate information with a low noise level. A final problem which must be addressed is a method to download the Kalman filter program to the hardware. This downloading could be done by the design program with the appropriate interfaces. After these problems have been resolved, the Kalman filter will provide a very effective state estimator in many applications.

A NEW SENSOR

Within the past few months our group has designed and performed initial tests on a distributed fiber optic sensor. Here, the fiber is connected to a flexible structure over a one meter length. The fiber is excited with a milliwatt laser and the defraction pattern out of the end is used to provide an estimate of displacement. Figures 10 and 11 illustrate the results via change in the output pattern. For implementation a CCD memory could be used to store the pattern and subsequently the information is driven into a computer where basic pattern recognition techniques are used to generate good estimates of displacement. A well designed system can also be used to estimate velocity. Within the context of this paper, a two-dimensional Kalman filter can be used to estimate parameters. Note that in Figs. 10 and 11, black and white images are shown. In a physical system color will be used.

CONCLUSIONS

An investigation has been started into the usefulness of the Kalman filter as a state estimator in an on-line environment. Previously, the filter has been used strictly in an off-line capacity to do data analysis. With the advances in computing speed, the filter is now becoming feasible as a real-time state estimator.

A Kalman filter based on a polynomial state-space model has been tested on flexible structure data. This filter has proven to give excellent state-estimation and noise reduction in such systems. The polynomial model provides for a very easy design of the filter. Do to the ease of design, a program has been written to provide assistance in the design process.

The Kalman filter design program provides a very straight forward design methodology with an interactive graphics approach. This approach allows the designer to see how well the filter works and the effects of changes in the design parameters. When the program is combined with the appropriate hardware, a very effective state estimation tool will become available for use in the real-time environment.

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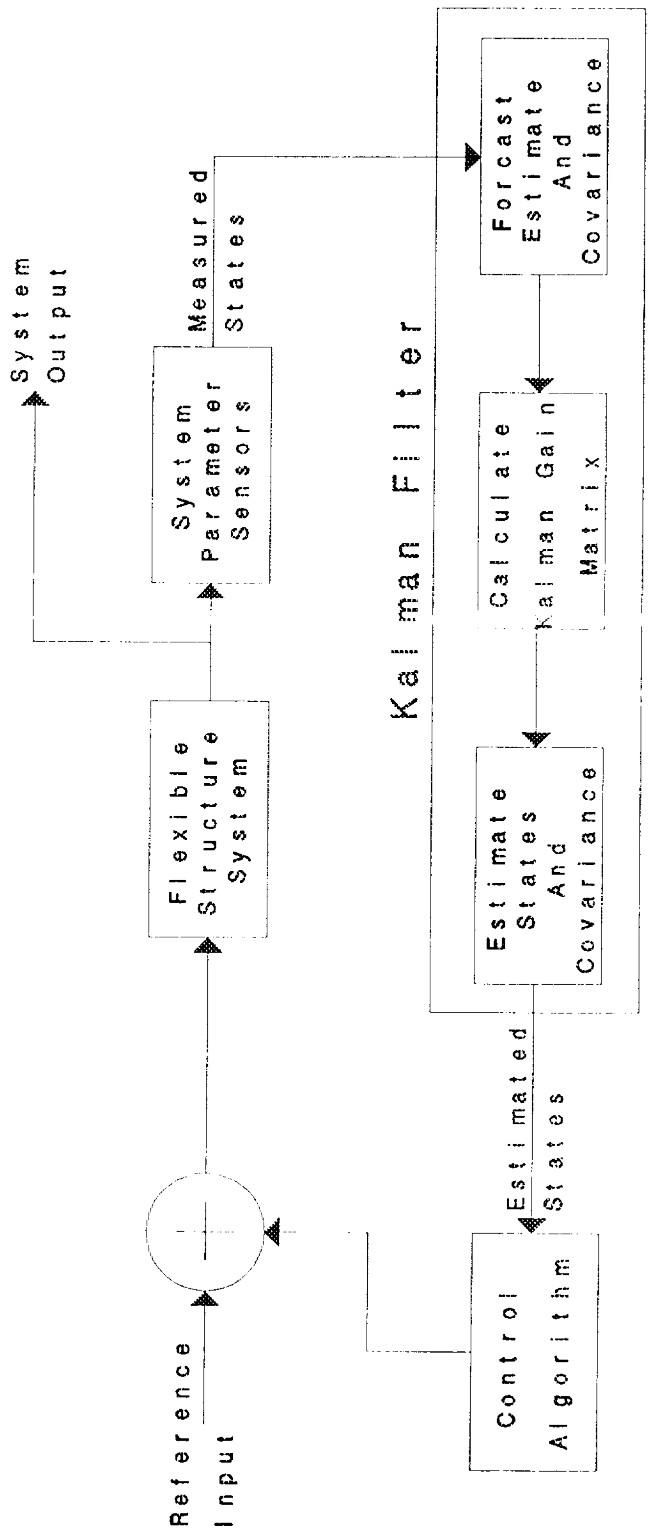


Figure 1. Block Diagram of Kalman Filter System.
Note: For multiple sensors, multiple distributed Kalman Filters could be employed with data fusion a key, important and still a research issue.

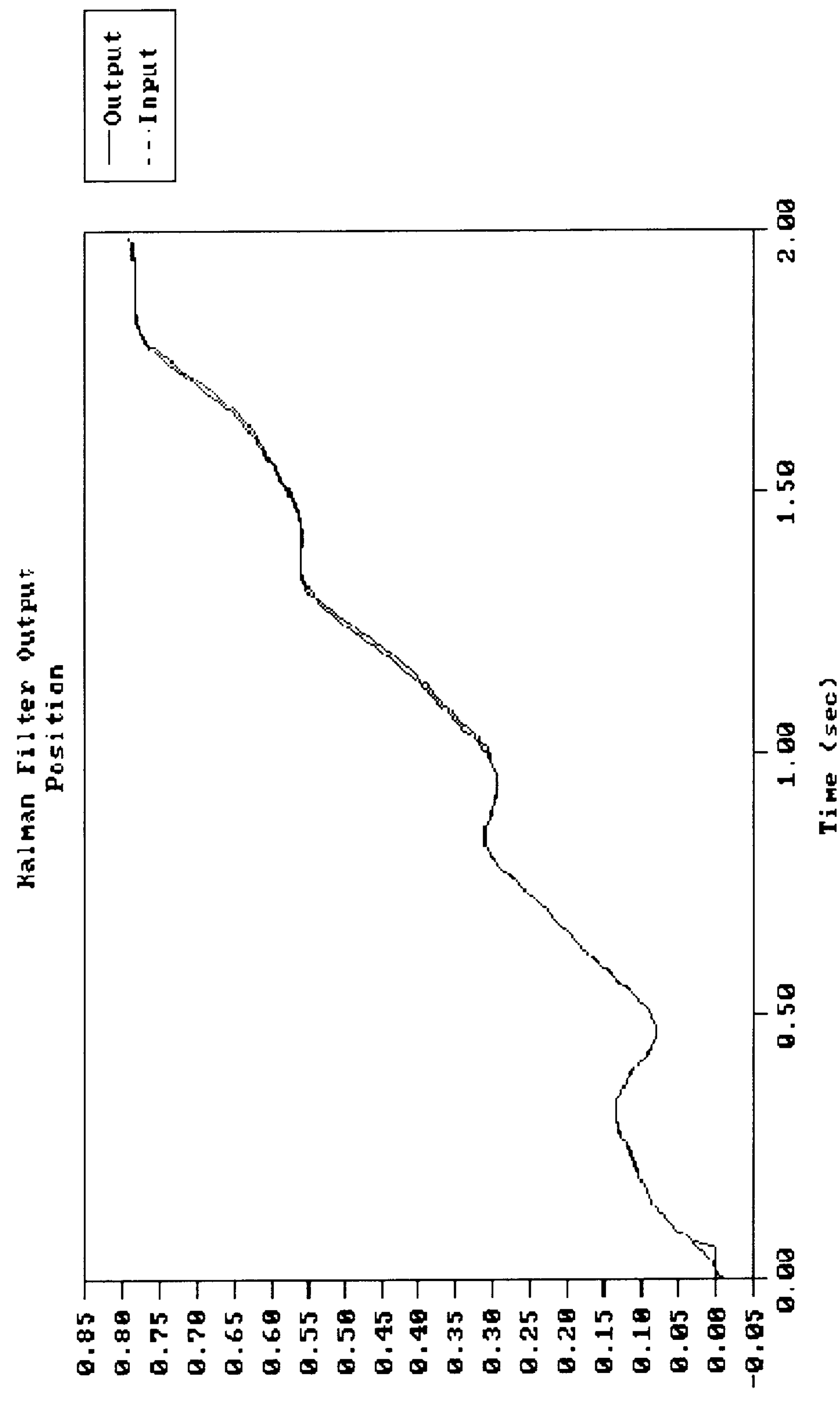


Figure 2. Position Output of Kalman Filter.

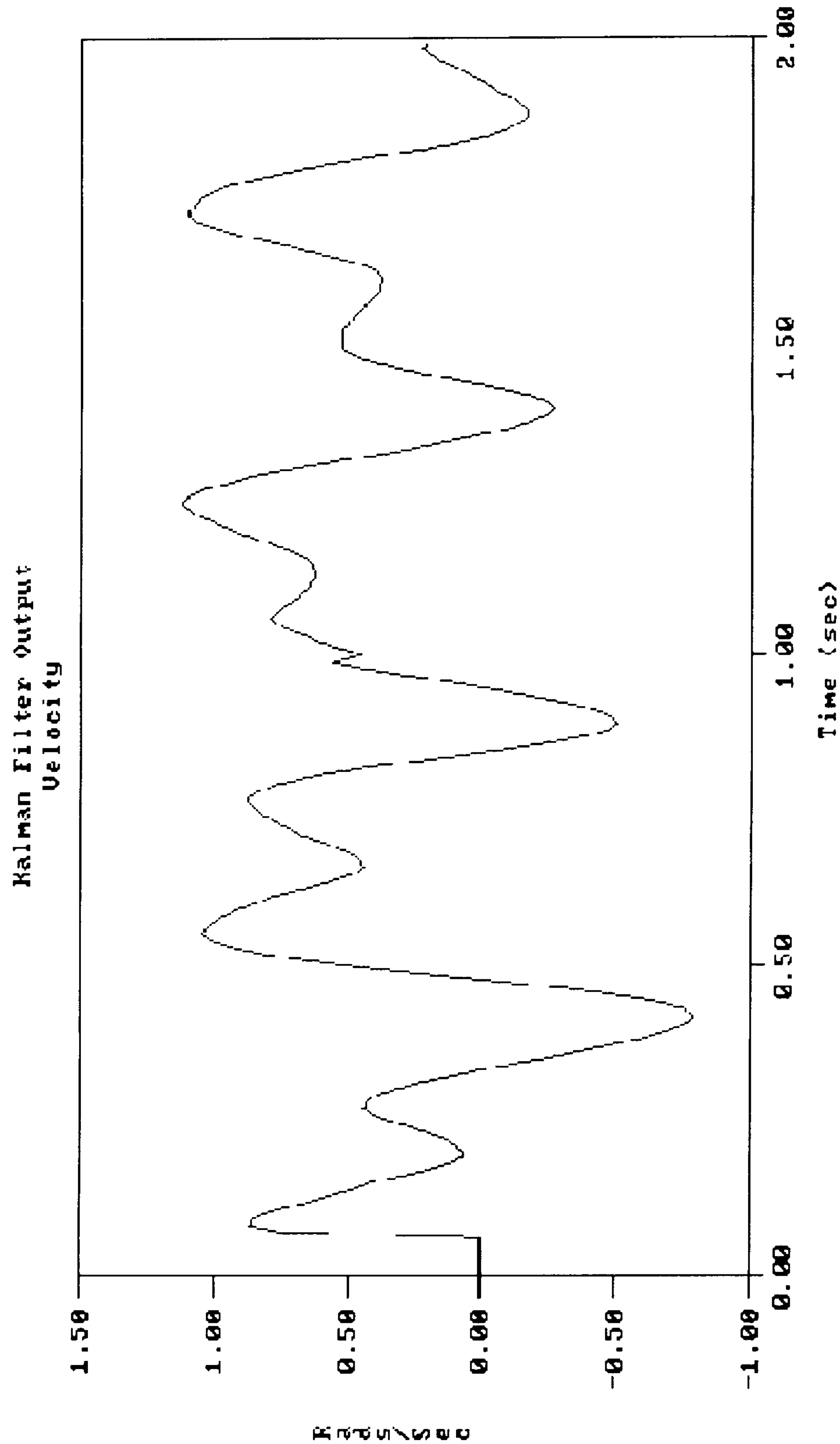


Figure 3. Velocity Output of Kalman Filter.

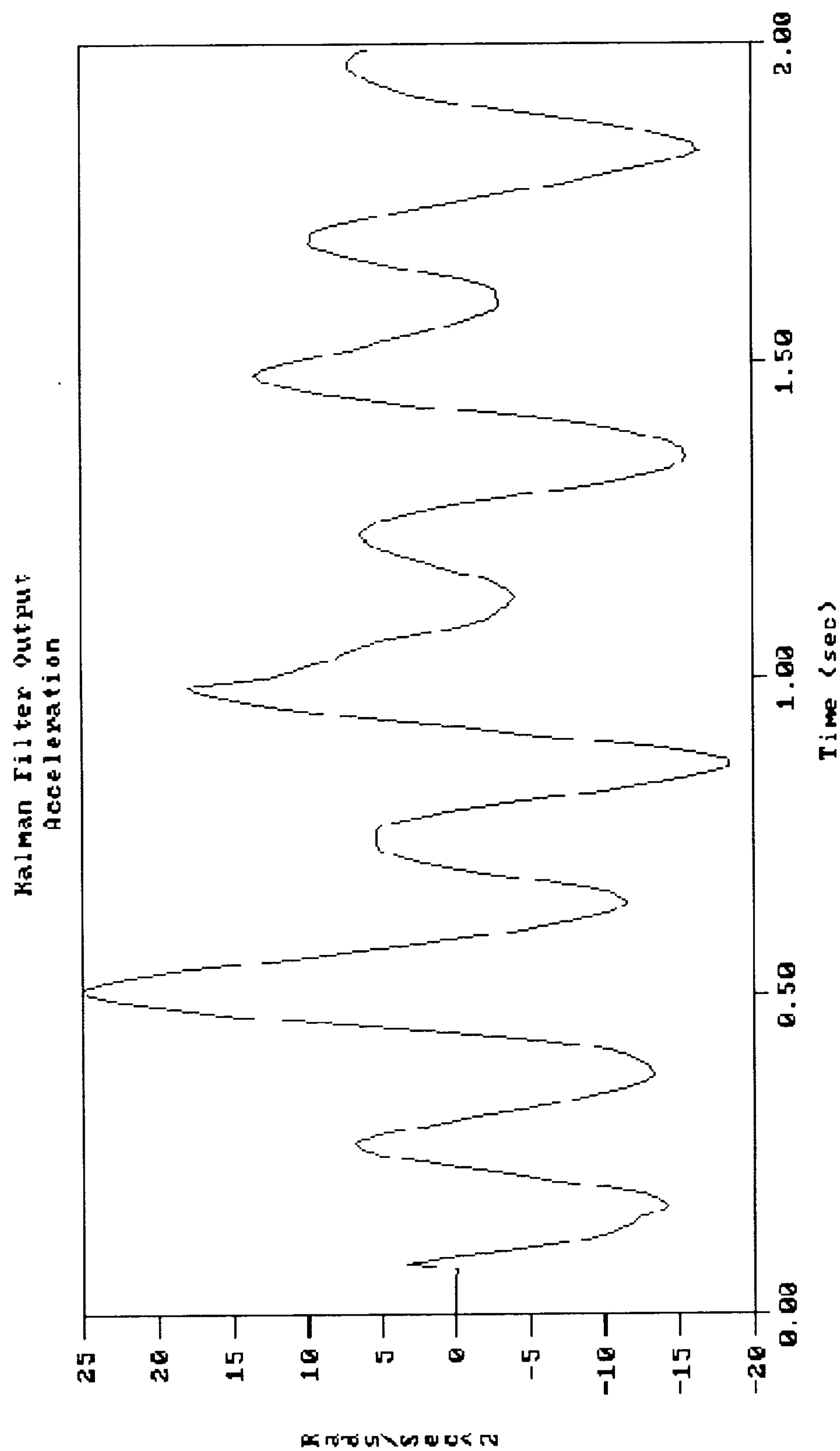


Figure 4. Acceleration Output of Kalman Filter.

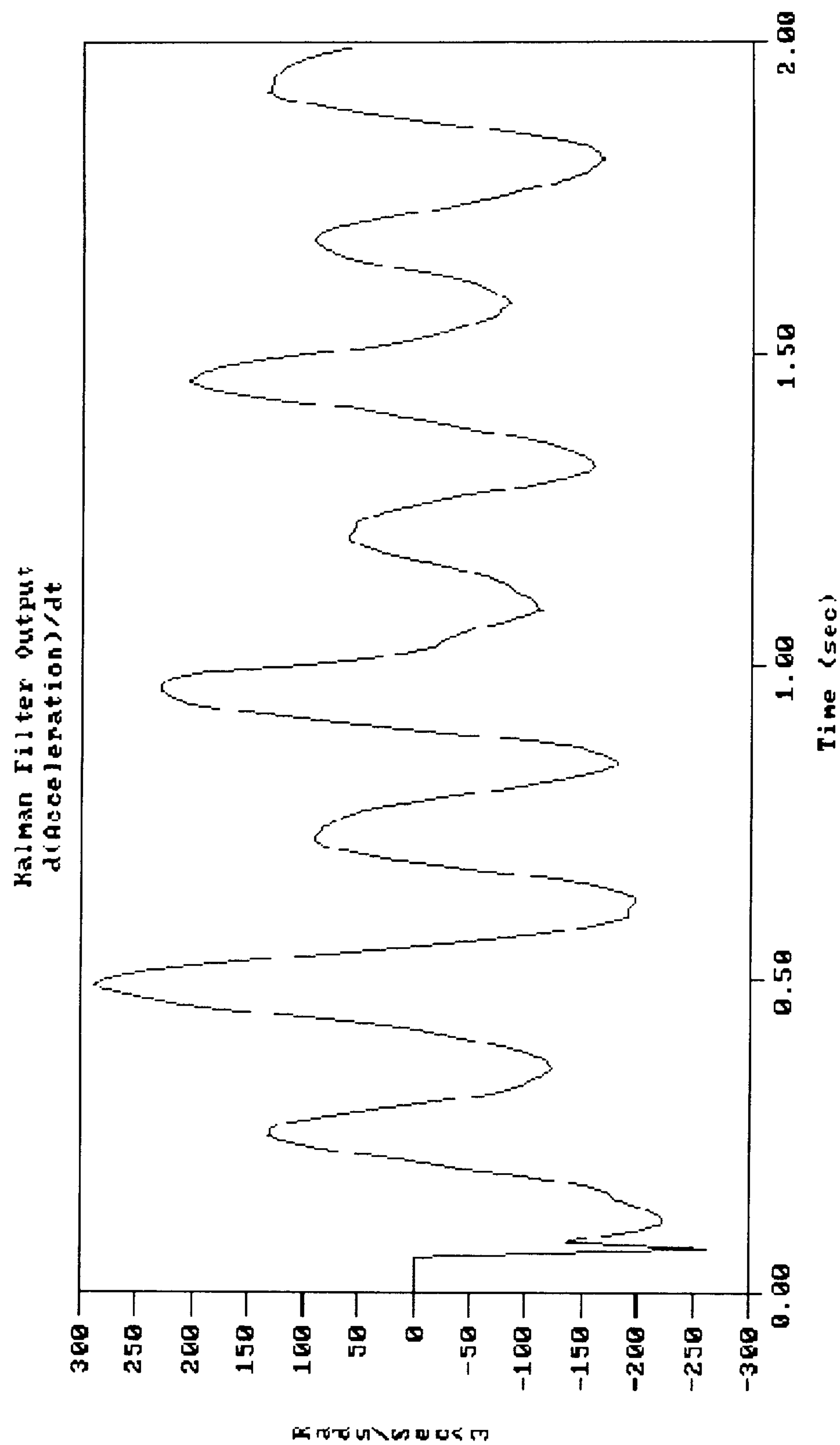
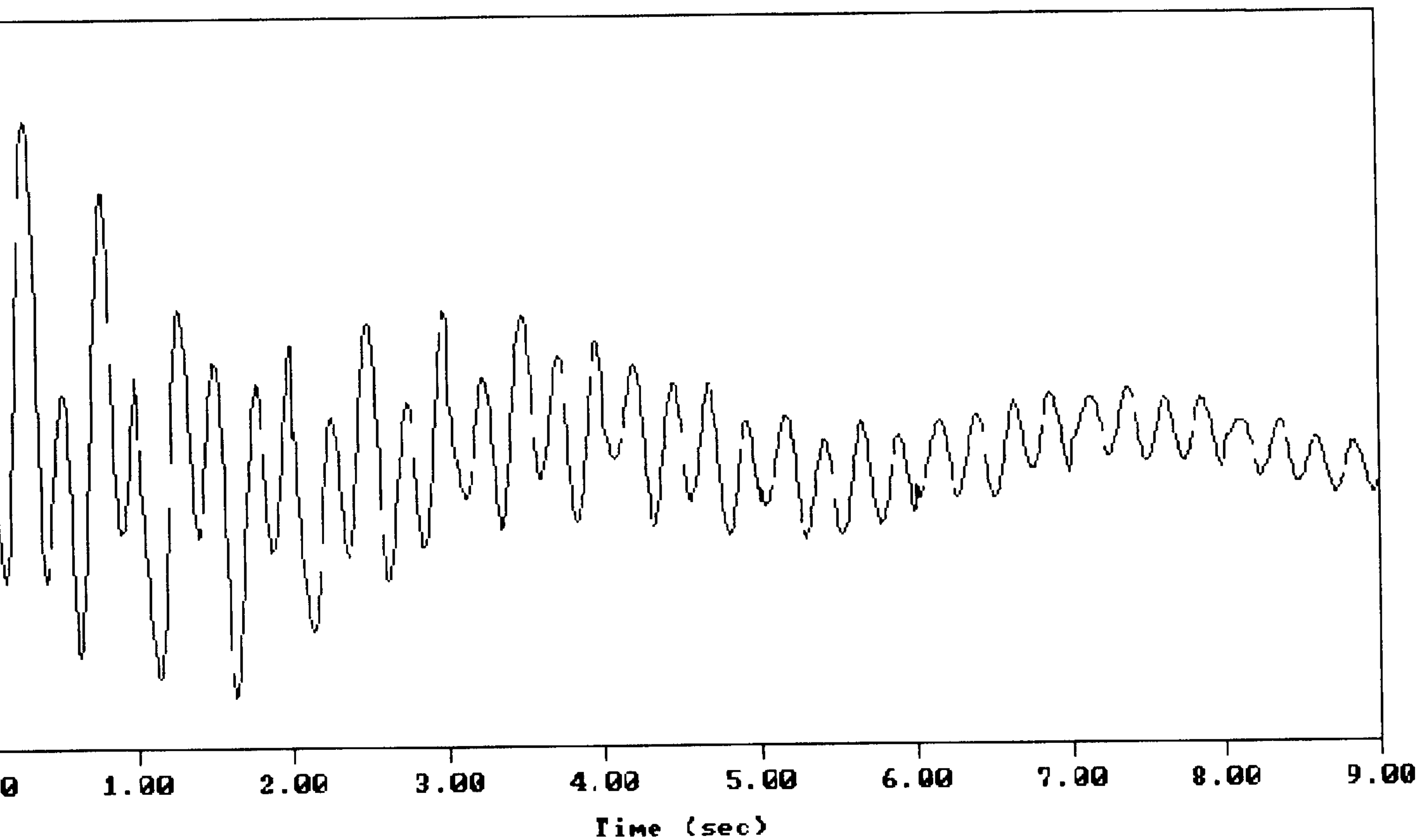
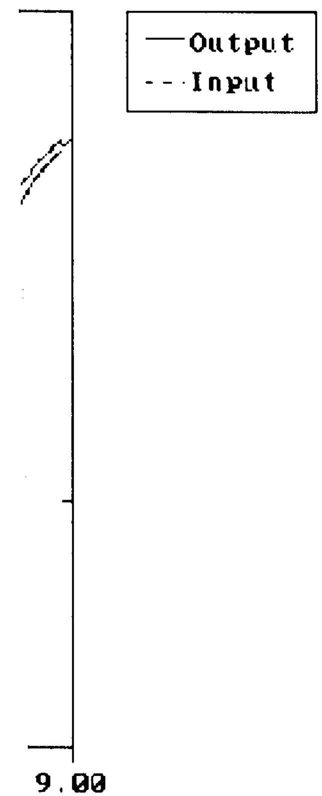


Figure 5. d^3(Position)/dx^3 Output of Kalman Filter.

Kalman Filter Output
Acceleration





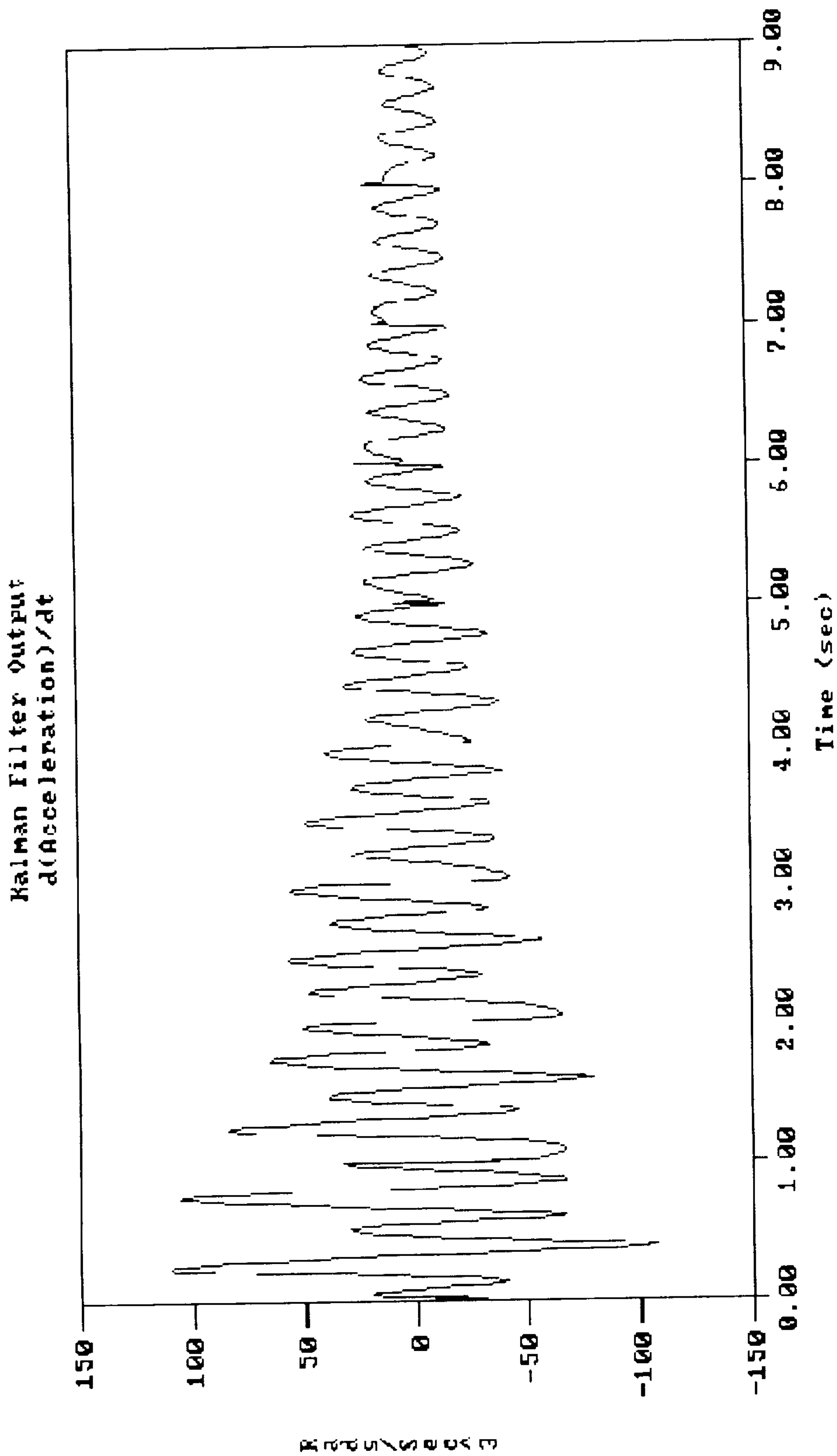


Figure 9. d^3(Position)/dx^3 Output of Kalman Filter with Flexible Arm Simulation.

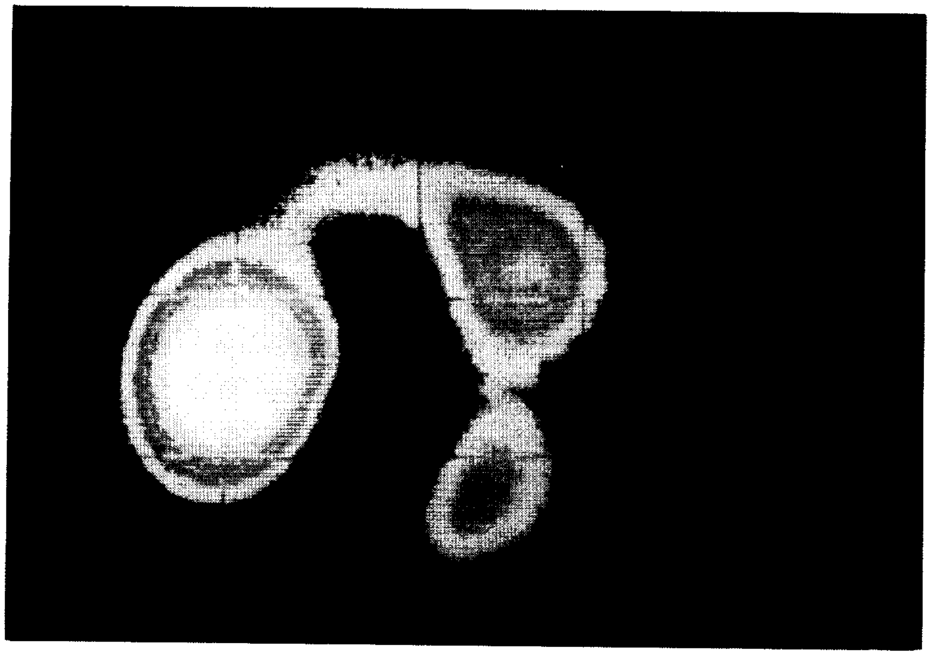


Figure 10. Fiber-Optic Output With Zero Beam Displacements

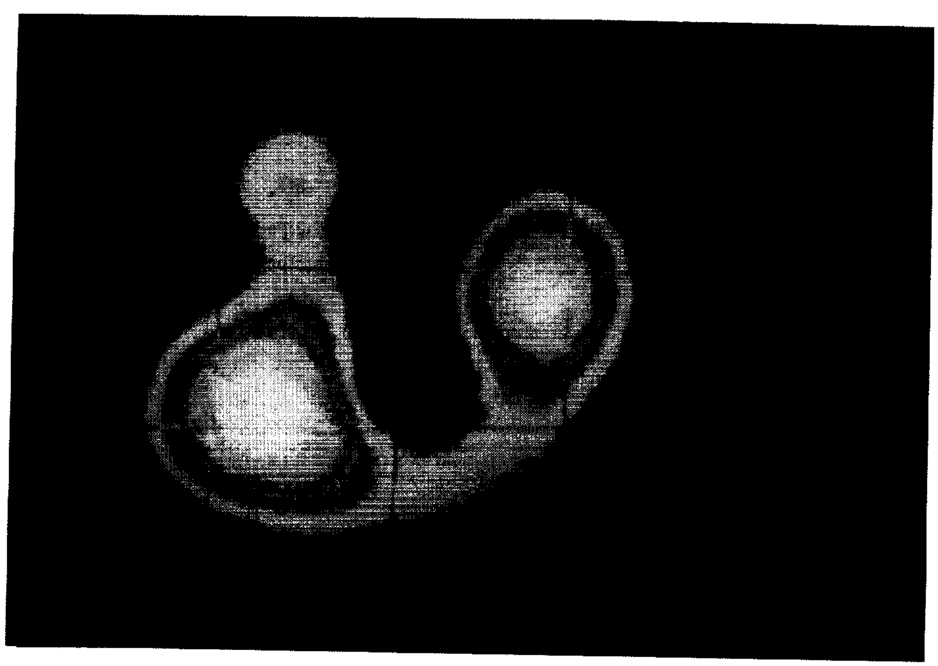


Figure 11. Fiber Optic Output With 5 cm Displacement