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**PDEMODO-Software for Controls-Structures Optimization**

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and  
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**4th NASA Workshop on Computational Control of  
Flexible Aerospace Systems**

## **PDEMOD - Software for Control/Structures Optimization**

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### **ABSTRACT**

Because of the possibility of adverse interaction between the control system and the structural dynamics of large, flexible spacecraft, great care must be exercised to ensure stability and system performance. Because of the high cost of insertion of mass into low earth orbit, it is prudent to optimize the roles of structure and control systems simultaneously. Because of the difficulty and the computational burden in modeling and analyzing the control/structure system dynamics, the total problem is often split and treated iteratively. The awkwardness and inaccuracy of this approach can lead to poor designs.

It would aid design if the control/structure system dynamics could be represented in a single system of equations. Heretofore this has not been possible. With the availability of the software PDEMOD it is now possible to optimize structure and control systems simultaneously. The distributed parameter modeling approach enables embedding the control system dynamics into the same equations for the structural dynamics model. By doing this the current difficulties involved in model order reduction are avoided.

The NASA Mini-MAST truss is used as an example for studying integrated control/structure design. Comparisons are made for (1) structure without control, (2) controls only, and for optimal combinations of structure and control. Both proof-mass and torque-wheel actuators are considered. The results give insight with regard to the essential factors in trading structure and control for space applications.

### **INTRODUCTION**

Certain future space missions will be performed by large, flexible spacecraft. Because of the high cost of insertion of mass into low earth orbit, it is prudent to optimize the design of both the structure and control systems. The current practice is to create a finite element model of the structure and to use a reduced order modal model for control synthesis. Unfortunately, this disjoint procedure is inaccurate and is an impediment to integrated design. There is a need for a dynamics model

which includes both structural and control dynamics which is parameterized in terms of the design variables.

Distributed parameter modeling has been shown to be quite accurate for modelling (reference 1.) the dynamics of the first six modes of portal frames. The root-mean-square error in frequency was about 7/10 of one percent for the first six modes of two experimental configurations. The difficulty in modeling complex structures has been an obstacle to the use of distributed parameter modeling. The development of finite element software has resulted in the wide-spread practice of modeling flexible structures with finite element models. The availability of modeling software such as DISTEL (reference 2.) and BUNVIS-RG (reference 3.) offers the alternative of modeling complex configurations with distributed parameter models.

Another advantage of distributed parameter modeling is that control and sensor dynamics can be incorporated into the equations of motion of the structural dynamics. Again, the burden of assembling the necessary equations for complex configurations has become routine because of the capabilities of the software PDEMOM (reference 4). It is now possible to optimize structure and control systems simultaneously (references 5. and 6.) for complex spacecraft because of the incorporation of their dynamics into a single system of equations. The distributed parameter modeling embeds the control system dynamics into the same equations for the structural dynamics model so that model order reduction is not necessary. The resulting structures/controls model is particularly well suited for integrated design.

In this paper only preliminary comparisons are made for the Mini-MAST truss (references 7. and 8.) for (1) structure without control, (2) controls only, and for optimal combinations of structure and control. Both proof-mass and torque-wheel actuators are considered. The results give insight into the essential factors in trading structure and control for space applications. Subsequent, more in-depth study will consider the transient dynamics aspects using PDEMOM and will use the optimization techniques of Fogel and Holland (references 9. and 10.).

## DISCUSSION

The software PDEMOM enables the generation of models of complex structural configuration which include the dynamics of the control system as well as the structural dynamics. This is done using partial differential equations to describe the dynamics of flexible beam elements which together with rigid body elements form a connected network of components of the structure. The coefficients of the sinusoidal and hyperbolic functions for each flexible element give the mode shapes. The sensed motion and control forces and moment are expressed in terms of

the same parameters and the influence of control on the configuration is represented by terms added to the equations for the structural dynamics.

### **Structural Dynamics**

First, the structural configuration is viewed as an assembly of flexible and rigid elements. It is then necessary to indicate the connectivity of the elements. This is done by giving the identification of the rigid bodies at either end of each flexible beam element and the related points of attachment. The alignments of the flexible beam axes must also be given. The axis for each rigid body is that of a particular, attached beam.

The next input needed is the stiffness ( $EI_x$ ,  $EI_y$ ,  $EA_z$ ,  $EI_y$ ) and mass ( $m/L$ ,  $I_y/L$ ) characteristics and the length of each flexible beam element. The mass and inertia of each rigid body is needed to complete the information required for the structural dynamics model.

### **Control Dynamics**

The addition of feedback control does not increase the order of the system matrix unless the applied force or moment is applied to the interior of a flexible beam element. In such a case it is necessary to add a node or rigid body with negligible mass at that point. In other cases it is only necessary to add terms to the matrix elements which already exist. The dynamics of the sensors and actuators are inserted as transfer functions multiplying the additional terms. The additional terms are located in the rows which correspond to the body to which the control force and/or moment is applied, and in the columns which correspond to the beam elements which contain the location of the sensed motion.

### **Optimization**

In order to perform optimization for parameter estimation or for criteria involving structural dynamics, sensitivity functions are usually required. The sensitivity functions relate the change in the criterion to changes in the parameters involved in the optimization. Although it is possible to express in closed form these derivatives it is most unwieldy to evaluate the expressions. This is because derivatives must be taken of the determinant of the system matrix which can be quite large. It is more practical to approximate the derivative numerically by the ratio of the change in the criterion to the change in the parameter. This was the approach used in reference 8. for a parameter estimation application. Changes in the modal frequencies for each of the model parameters was generated numerically.

Although parameter estimation is an example of optimization, it is the optimal, integrated control/structural design that we now turn our attention. The selection of the design criterion and the corresponding conditions or constraints are of paramount importance. An ill-posed problem can easily result in nonsensical results which bear no relation to the actual design problem. Perhaps the most suitable design criterion for the integrated control/structure problem would be the life-cycle-cost of the entire system, subject to a set of system performance and structural specifications. In many cases it suffices to consider only the total structure and control system mass.

There are alternatives to optimization schemes which require the sensitivity functions mentioned earlier. Genetic algorithms (GA's), as introduced by Holland (reference 10), are one form of directed random search. The form of direction is based on Darwin's theory of the "survival of the fittest". In GA's a finite number of candidate solutions or designs are randomly (or heuristically) generated to create an initial population. This initial population is then allowed to evolve over generations to produce new, and hopefully better designs. The basic conjecture behind GA's is that evolution is the best compromise between determinism and pure chance. GA's have the capability to solve continuous, discrete, and mixed optimization problems.

There are four main operations in a basic GA: evaluation, selection, crossover, and mutation. Evaluation is the process of assigning a fitness measure to each member of the current population. The fitness measure is typically chosen to be related to the objective function which is to be maximized. No gradient or auxiliary information is used. Therefore, GA's are more likely to converge to a global maximum than a hill climbing algorithm, although no algorithm can guarantee convergence to the global maximum. Selection is the operation of choosing members of the current population to be parents for producing the next generation. Selection is weighted towards the more fit members of the population. Therefore, designs which are better as viewed from the fitness function, and therefore the objective function, are more likely to be chosen as parents. Crossover is the process in which design information is transferred to the prodigy from the parents. Mutation is a low probability random operation which slightly perturbs the design represented by the prodigy. The mutation operation is used to retain design information over the entire domain of the design space during the evolutionary process.

### **INTEGRATED DESIGN PROBLEM**

The integrated control/structural design problem to be considered is to minimize the total system mass while limiting the response to a disturbance force at the tip of the Mini-MAST truss. This will be accomplished by the selection of the stiffness of the truss elements, the

use of a proof mass actuator and a torque wheel actuator, both located at the tip. The total system mass is increased when (1) the stiffness of truss elements is increased, (2) the capacity of the proof mass actuator is increased, and (3) the capacity of the torque wheel actuator is increased. Prior to involving the software PDEM0D to calculate the dynamic response of the actively controlled Mini-MAST to the disturbance force, it is prudent to investigate the best mix of structural stiffness and the use of active control in a more simple way.

The structural stiffness of a uniform Bernoulli beam is given by:

$$K = 3EI/L^3$$

Because the mass of the truss elements is proportional to their stiffness the truss mass is:

$$Mass_{truss} = Mass_{truss,0} [EI/EI_0] [L_0/L]^3$$

Note that to keep the same resistance to a disturbance force applied at the tip of the Mini-MAST the increased stiffness and corresponding mass increases by the length to the third power.

Because the 66-foot Mini-MAST truss has 18 bays it is possible to alter the stiffness of the truss elements as each bay to reduce the total mass while achieving the same stiffness or resistance to a disturbance force at the tip. The maximum saving that can be achieved appears to be about 17 percent.

A moving or proof mass actuator (PMA) can produce a force to oppose a constant disturbance force but only for a relatively short period of time which depends on the size of the proof mass and its stroke. Neglecting the stationary mass the mass for the PMA can be shown to be:

$$Mass_{PMA} = .5 * Force * Time^2 / Stroke$$

Note that the mass of the proof mass actuator is proportional to the time of application squared but does not depend on the truss length or stiffness.

A rotating mass or torque wheel actuator (TWA) can produce a moment which by reaction of the truss structure can oppose a constant disturbance force but only for a relatively short period. The mass of the TWA depends on the moment, maximum wheel speed and the time of application. Neglecting the stationary mass the mass of the TWA can be expressed as:

$$Mas_{TWA} = Moment * Time / (Wheel Speed * Radius^2)$$

A moment applied to the tip of the Mini-MAST produces a lateral deflection which can be used to oppose a disturbance force. The moment per deflection is given by:

$$\text{Moment/Deflection} = 2EI/L^2$$

The force per deflection is:

$$\text{Force/Deflection} = 3EI/L^3$$

The mass of the torque wheel actuator (TWA) capable of countering a particular force for a specific time becomes:

$$\text{Mass}_{\text{TWA}} = .667 * \text{Force} * \text{Time} * \text{Length} / (\text{Max Wheel Speed} * \text{Radius}^2)$$

Note that the mass of the torque wheel actuator is proportional to both the time of application of the disturbance force and the length of the Mini-MAST.

The problem now is to determine the combination of structural stiffness and the sizes of the proof mass and torque wheel actuators which minimizes the total mass.

$$\text{Mass}_{\text{Total}} = \text{Mass}_{\text{truss}} + \text{Mass}_{\text{PMA}} + \text{Mass}_{\text{TWA}}$$

The fundamental variables which determine the best mix is the time, T, for which the disturbance force is applied and the length, L, of the Mini-MAST truss. By examining different combinations of truss length, L and the time, T, for which the disturbance force is applied the regions for which the truss structure, the proof mass actuator, or the torque wheel actuator is best in countering the disturbance force can be determined. The result of such a study shows that the truss structure is best for large values of the time for which the disturbance force is applied. The proof mass actuator is best for long trusses. Under only limited conditions is the torque wheel actuator best for countering disturbance forces.

### CONCLUDING REMARKS

Distributed parameter models of structures have important advantages for problems involving the active control of flexible structures. This is especially true for repetitive lattice structures such as the Mini-MAST truss because its dynamics can be accurately represented by only a few parameters.

Until the advent of software for distributed parameter modeling of complex flexible structures it was not practical to model complex spacecraft configurations using distributed parameter models. Now the software PDEM0D enables modeling complex configurations and can also

include the control system dynamics in the same equations. The need to resort to model order reduction is eliminated because the closed loop stability and system performance can be determined without ignoring any of the modes.

This capability enables working integrated control/structures problems. An example problem is examined in which structure and active controls are used to counter the disturbance force at the tip of the NASA Mini-MAST truss. The regions are determined in which structure, proof mass actuators, and torque wheel actuators result in the minimum mass system. Variations of this tradeoff between control and structure considerations are being pursued.

The development of the PDEMODO software has been underway for about one year. The formulation and coding has been completed for modelling general three-dimensional configurations. Modal frequencies and mode shapes have been generated for the Spacecraft Control Laboratory Experiment (SCOLE) configuration and the Mini-MAST truss. Graphics for drawing wireframes of the deflected configurations has been the most recent addition. Transient response, transfer functions, Timoshenko beam option and improved root-finding algorithms will be added during the next year. Copies of the source code will be made available to anyone interested in modelling new configurations or contributing to the software development.

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# INTRODUCTION

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- Difficulties of Current Practice
- Advantages of Distributed Parameter Modeling
- Difficulties of Modeling Complex Structures
- Capabilities of PDEMODO Software
- Integrated Design Objective Functions
- Necessary Additions to PDEMODO
- Example Controls-Structures Design Problem
- Insights Offered by Particular Tradeoffs
- Concluding Remarks

## Issues in Modeling Complex Structures

### Finite Element Modeling

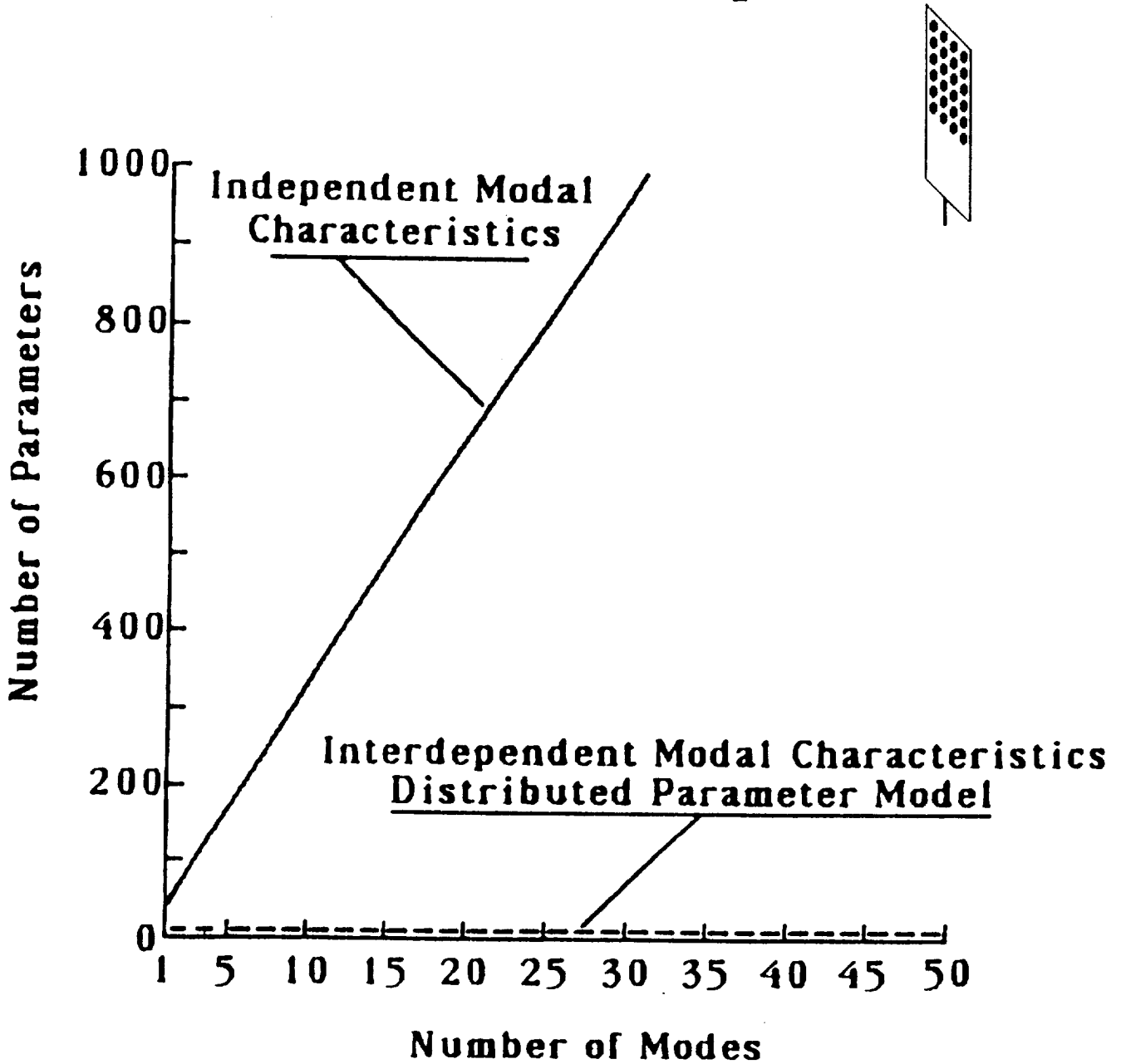
- Excessive Complexity
- Parameter Estimation is Difficult
- Model Order Reduction Required for Control Analysis

### Distributed Parameter Modeling

- Fewer Model Parameters
- Parameter Estimation Straightforward
- Closed-Loop Stability Analysis does not Require Order Reduction

# Number of Model Parameters

Solar Array Example:



# Modeling Software

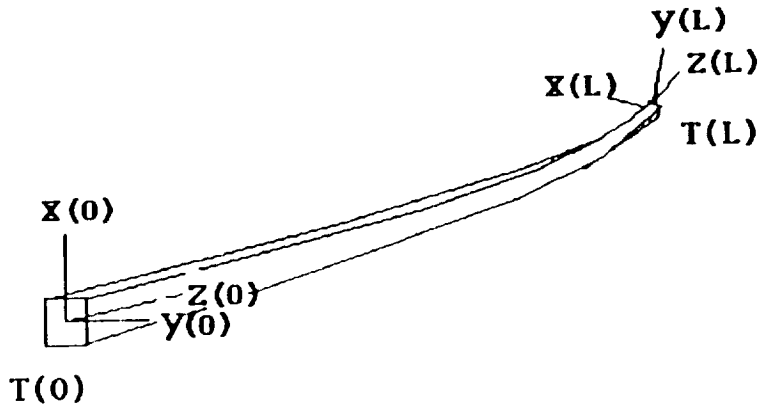
<u>Name</u>	<u>Structural Dynamics</u>	<u>Optimization*</u>
DISTEL <i>by Poelaert</i>	✓	
BUNVIS <i>by Anderson</i>	✓	
PDEMOD <i>by Taylor</i>	✓	✓

\* — Parameter Estimation,  
Control-Synthesis, and  
Structural Design

# **PDEMODO**

- **Distributed Parameter Modeling**
- **Flex Beam Elements/Rigid Bodies  
(Bend-x, Bend-y, Twist, Stretch)**
- **Parameter Estimation**
- **Structural Damping**
- **Control System Dynamics**
- **Parametric Studies**

# Beam Model



The Moments and Forces at (0) in Beam Axes are:

$$M_x = EI_y u''_{y'}(0)$$

$$F_x = EI_y u'''_{y'}(0)$$

$$M_y = -EI_x u''_{x'}(0)$$

$$F_y = -EI_x u'''_{x'}(0)$$

$$M_z = EI_{\psi} u'_{\psi}(0)$$

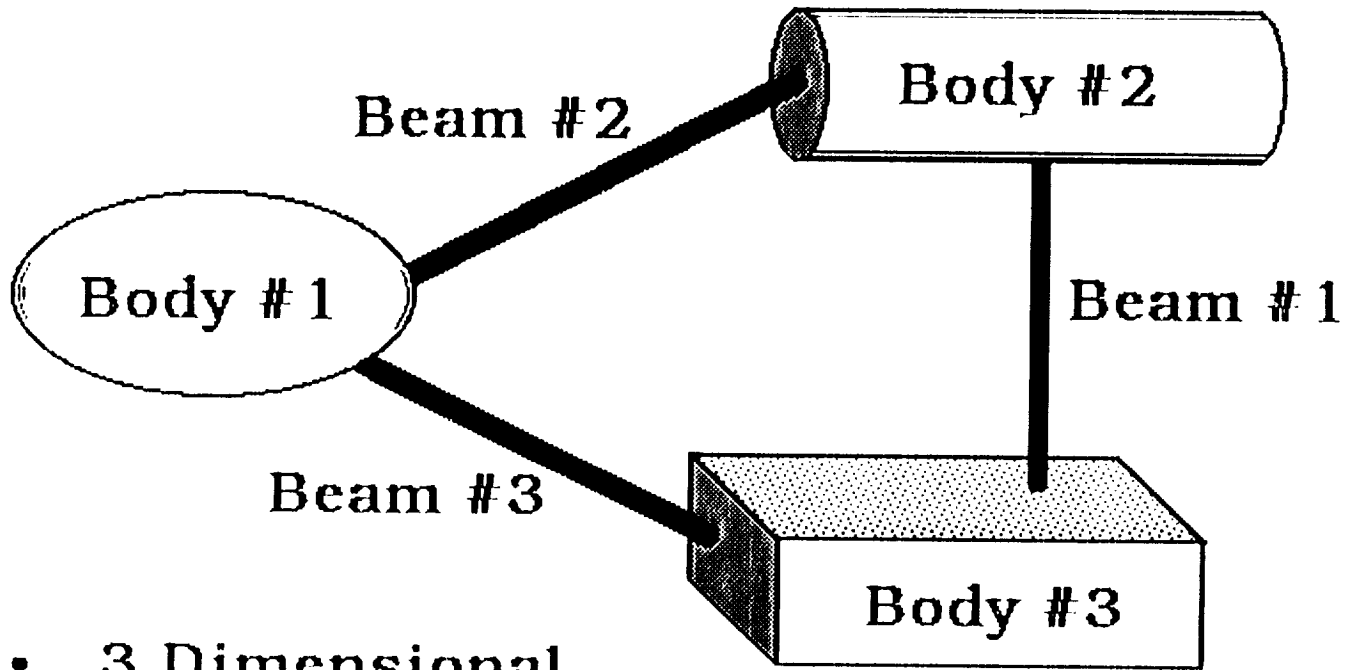
$$F_z = EA_z u'_z(0)$$

$$M_{\text{Beam}} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$F_{\text{Beam}} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

The sign of the forces and moments at the outboard end of the beam are opposite to those of the inboard end.

## Generic Configuration



- 3 Dimensional
- Flexible Beams (Bend, Twist, Elongate)
- Rigid Bodies (Full Inertia Matrices)



## Partial Differential Equations

Bernoulli-Euler Equations will be used for bending

$$m\ddot{u}_x + EI_x u_x'''' = 0$$

$$m\ddot{u}_y + EI_y u_y'''' = 0$$

String Equations are used for elongation and torsion.

$$m\ddot{u}_z + EA_z u_z'' = 0$$

$$\rho I_\psi \ddot{\psi} + EI_\psi \psi'' = 0$$

Examination of each equation will establish the relationship between the "b" parameters and frequency,  $\omega$ .

From  $(m\ddot{u}_x + EI_x u_x'''' = 0)$  we get:

$$m\omega^2 u_x = b_x^4 EI_x u_x$$

It follows that

$$(b_x L)^2 = \frac{\omega}{\sqrt{\frac{EI_x}{m L^4}}}$$

## Forces and Moments

The forces and moments in body axes are:

$$F_{\text{beam}} = P_F \begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ A_y \\ B_y \\ C_y \\ D_y \\ A_z \\ B_z \\ A_\psi \\ B_\psi \end{bmatrix}$$

$$M_{\text{beam}} = P_M \begin{bmatrix} A_x \\ B_x \\ C_x \\ D_x \\ A_y \\ B_y \\ C_y \\ D_y \\ A_z \\ B_z \\ A_\psi \\ B_\psi \end{bmatrix}$$

## Equations of Motion

**Force Equations:**

$$A_j = Q_{u_j} + T_j^T R_j T_j Q_{u'_j} + \frac{1}{m_j \omega^2} \sum T_j^T T_{\text{beam } i}^T T_{\text{body } j} P_{F_i}$$

**Moment Equations:**

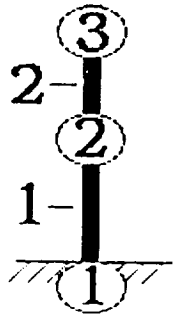
$$A_j = Q_{u'_j} + \frac{T_j^T I_j^{-1}}{\omega^2} \sum \{ T_{\text{beam } i}^T P_M + R_{\text{beam } i}^T T_{\text{beam } i}^T P_F \}$$

**Constraint Equations:**

$$T_2 u_2^{(0)} - R_2 T_2 u_2'^{(0)} = T_1 u_1^{(0)} - R_1 T_1 u_1'^{(0)}$$

$$T_2 u_2'^{(0)} = T_1 u_1'^{(0)}$$

# System Equations



	Beam 1	Beam 2
Body 1	$\begin{matrix} \text{Force}(\omega) \\ \text{Moment}(\omega) \end{matrix}$	0
Body 2	$\begin{matrix} \text{Force}(\omega) \\ \text{Moment}(\omega) \end{matrix}$	$\begin{matrix} \text{Force}(\omega) \\ \text{Moment}(\omega) \end{matrix}$
Body 3	0	$\begin{matrix} \text{Force}(\omega) \\ \text{Moment}(\omega) \end{matrix}$
Constraint	$\begin{matrix} u(L) \\ u'(L) \end{matrix}$	$\begin{matrix} u(0) \\ u'(0) \end{matrix}$

Characteristic Equation:

$$|A(\omega)| = 0$$

## Mode Shape Functions

$$u_x = A_x \sin(\beta z) + B_x \cos(\beta z) + C_x \sinh(\beta z) + D_x \cosh(\beta z)$$

$$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \mathbf{Q} \mathbf{u}^\Theta$$

12 Coefficients  
form State Vector

Trigonometric and  
Hyperbolic Functions  
of  $\omega$

Similarly for  $u'$ , Force, Moment, etc.

## Control System

Output feedback control systems can be modeled directly using the structural model.

### Collocated Force Control

$$F_{\text{body}_i} = K_u u_i + K_{\dot{u}} \dot{u}_i$$

This controller would add to the appropriate term:

$$\Delta A_{i,i} = \frac{1}{m_i} \{K_u + j\omega K_{\dot{u}}\} Q_{u_i}$$

### Collocated Moment Control

$$M_{\text{body}_i} = K_{u'} u'_i + K_{\dot{u}'} \dot{u}'_i$$

This controller would add to the appropriate term:

$$\Delta A_{i,i} = \bar{I}_i^{-1} \{K_{u'} + j\omega K_{\dot{u}'}\} Q_{u'_i}$$

Higher order compensation for controller can be included as complex functions of  $\omega$ .

Uncollocated controllers would have the same form but would have different indices to reflect the locations of sensors and actuators.

## Dimensionality

Because 12 modal parameters are involved for each beam, A like number of equations are involved in the eigen value solution. The SCOLE configuration is an example of such a simple configuration.

12



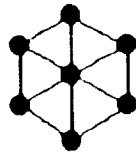
24



The Mini-MAST, a cantilevered beam with lumped masses at the tip and bay 10 will involve 24 equations.

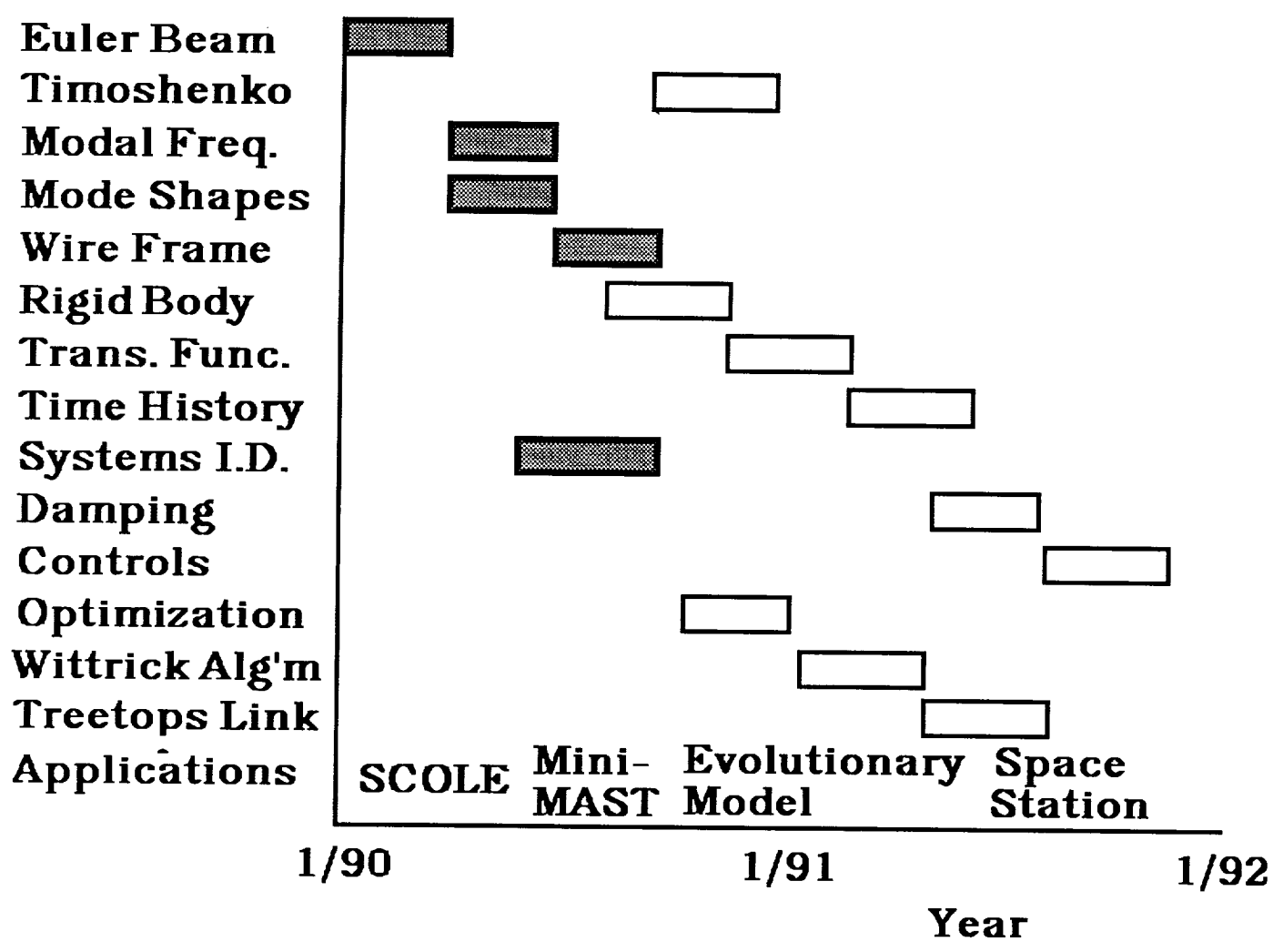
A hexagonal shape with bodies at each point and at its center will involve 144 equations.

144



Clearly, numerical difficulties can be expected to limit the complexity of a configuration that can be handled.

# PDEMOD Schedule





## **PDEMOD Users/Contributors**

**Augenstine - U. of Maryland**

**Balakrishnan - U.C.L.A.**

**Huang/Shen - O.D.U.**

**Junkins - Texas A. & M.**

**Kakad - U. of N.C. Charlotte**

**Kaufman - R.P.I.**

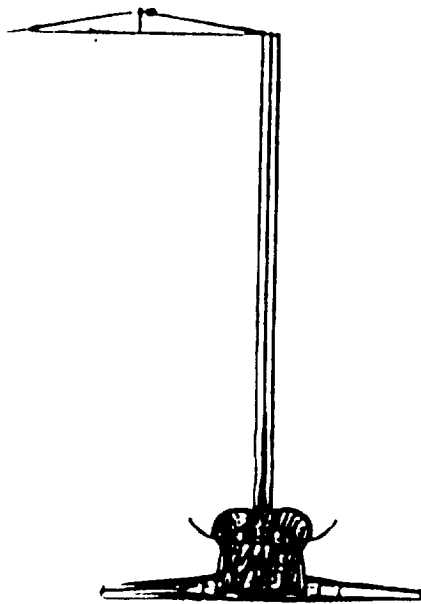
**Mitchell - V.P.I. & S.U.**

**Pitkin - Dynacs Engineering**

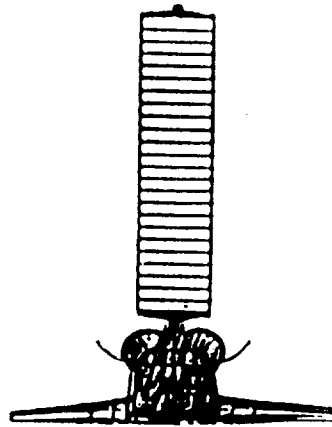
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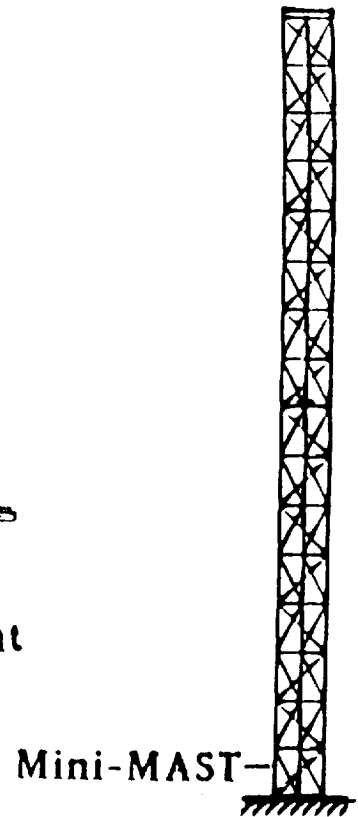
## Configurations Modeled



Spacecraft Control  
Laboratory Experiment



Solar Array  
Flight Experiment



Mini-MAST-

Sketches of the Three Spacecraft-Type Structures for which Distributed Parameter Models are Constructed.

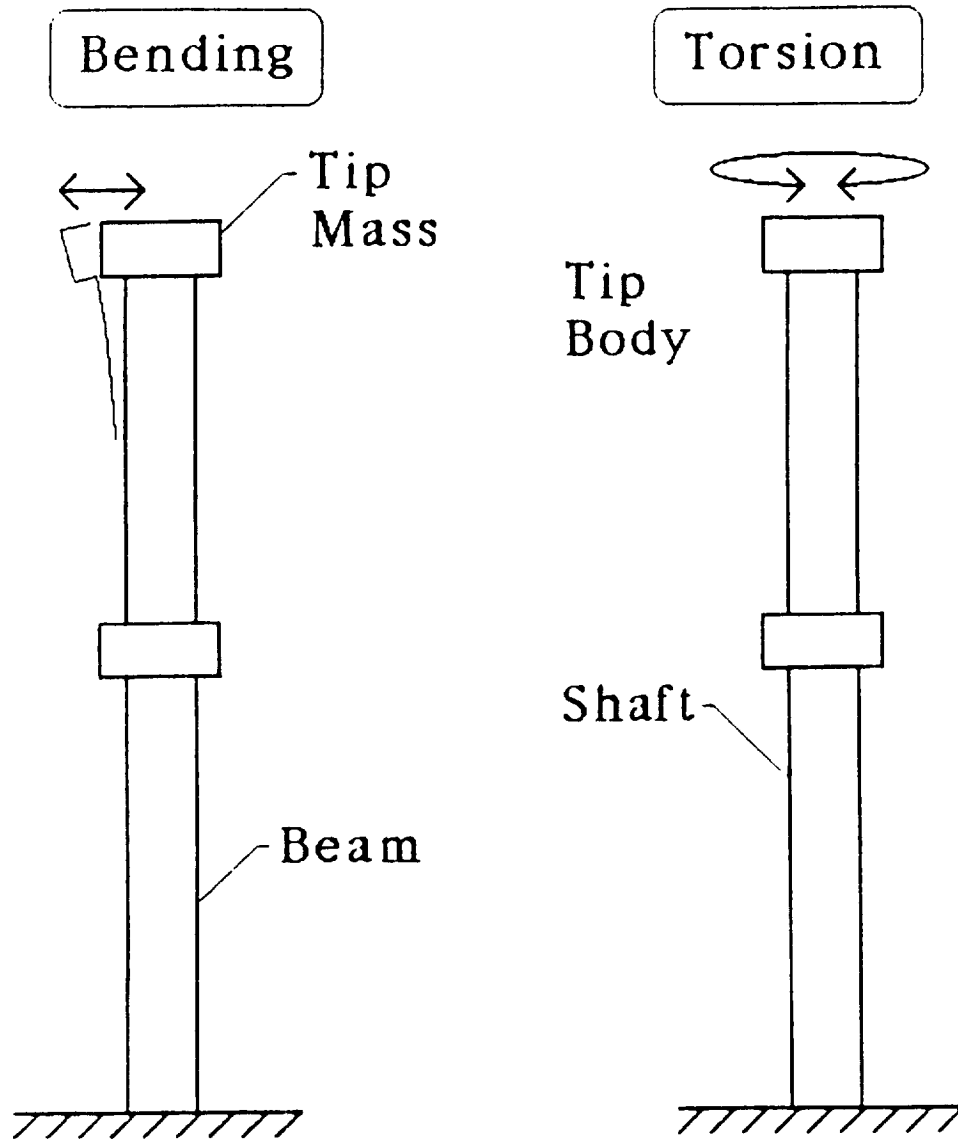


Figure 14. Schematics of Distributed Parameter Models for Bending and Torsion of the Mini-MAST Truss.

## Optimization\*

$$J = \delta(\Theta)^T \mathbf{M}^{-1} \delta(\Theta)$$

$$\frac{\partial J}{\partial \Theta} = 2 \frac{\partial \delta(\Theta)}{\partial \Theta}^T \mathbf{M}^{-1} \delta(\Theta) + 2 \frac{\partial \delta(\Theta)}{\partial \Theta}^T \mathbf{M}^{-1} \frac{\partial \delta(\Theta)}{\partial \Theta} \Delta \Theta = 0$$

$$\Delta \Theta = \left[ \frac{\partial \delta(\Theta)}{\partial \Theta}^T \mathbf{M}^{-1} \frac{\partial \delta(\Theta)}{\partial \Theta} \right]^{-1} \frac{\partial \delta(\Theta)}{\partial \Theta}^T \mathbf{M}^{-1} \delta(\Theta)$$

\* – Parameter Estimation,

Control Synthesis,

Structural Design

Genetic algorithms (GA's) are one form of directed random search. In GA's, a finite number of candidate solutions are randomly generated to create an initial population. This initial population is then allowed to evolve over generations to produce new, and hopefully better designs. The basic motivation behind the development of GA's is that they are robust problem solvers for a wide class of problems. The basic conjecture behind GA's is that evolution is the best compromise between determinism and chance.

## GENETIC OPTIMIZATION

- o Darwins "Survival of the Fittest"
- o Related to Simulated Annealing
- o Requires Function Evaluations (no Gradient Information required)
- o Seeks to Maximize a "Fitness" Index - related to objective function
- o Works with Multiple Designs Simultaneously
  - o Identifies "Nearly Optimal" Alternatives
  - o Suitable for Parallel Processing

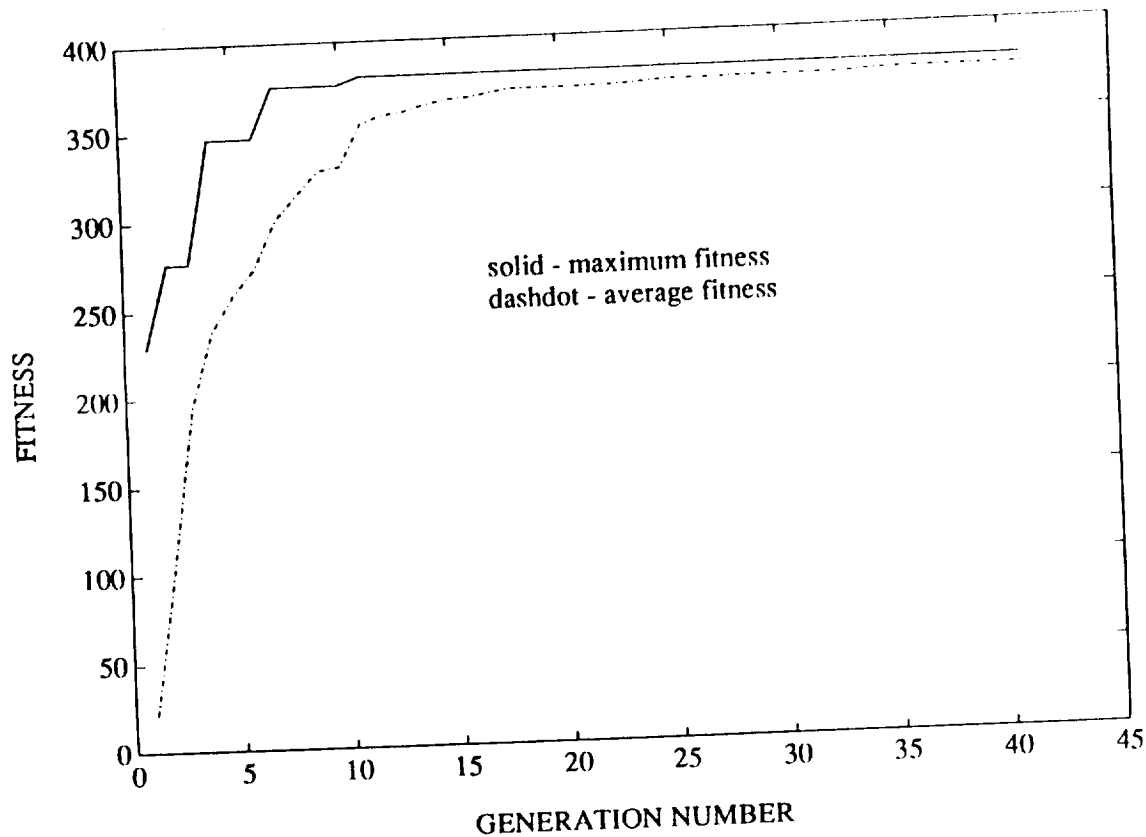
The genetic algorithm was used to determine the optimal continuum beam characteristics of the two sections of the NASA LaRC Mini-Mast for bending in one plane of motion. The objective of the design was to minimize the total structural mass subject to constraints on the tip displacement. As would be expected, the optimizer increases the stiffness of the lower section of the Mini-Mast while decreasing the stiffness of the upper section.

## DESIGN PROBLEM

- o FEM Representation of Continuum Model of Mini-Mast  
(PDEMODO soon to follow)
- o Design Variables -  $EI(z)$  of Two Sections  
(  $d * EI(z)_{\text{nominal}}$  )
- o Problem Statement - Minimize Total Mass Subject to  
Dynamic Displacement Constraints
- o Results
  - o Nominal -  $d(1) = d(2) = 1$   
  
Total Mass = 7.1208 slugs  
Constraint Violation = 5.3%
  - o Optimized Design -  $d(1) = 1.2059$      $d(2) = 0.4647$   
  
Total Mass = 6.2412 slugs  
Constraint Violation = None

The GA search was run for 40 generations with a population size of 20 members. Thus, the total number of function evaluations was 800. The convergence history of the GA is shown below. At a given generation number, the maximum fitness value represents the most fit member in the population whereas the average fitness is the mean fitness of the entire population. It can be seen that the average fitness increases with each new generation, which is a property of the particular GA used. It is also seen that the maximum fitness (i.e. optimal solution) is obtained in an early generation.

## CONVERGENCE HISTORY



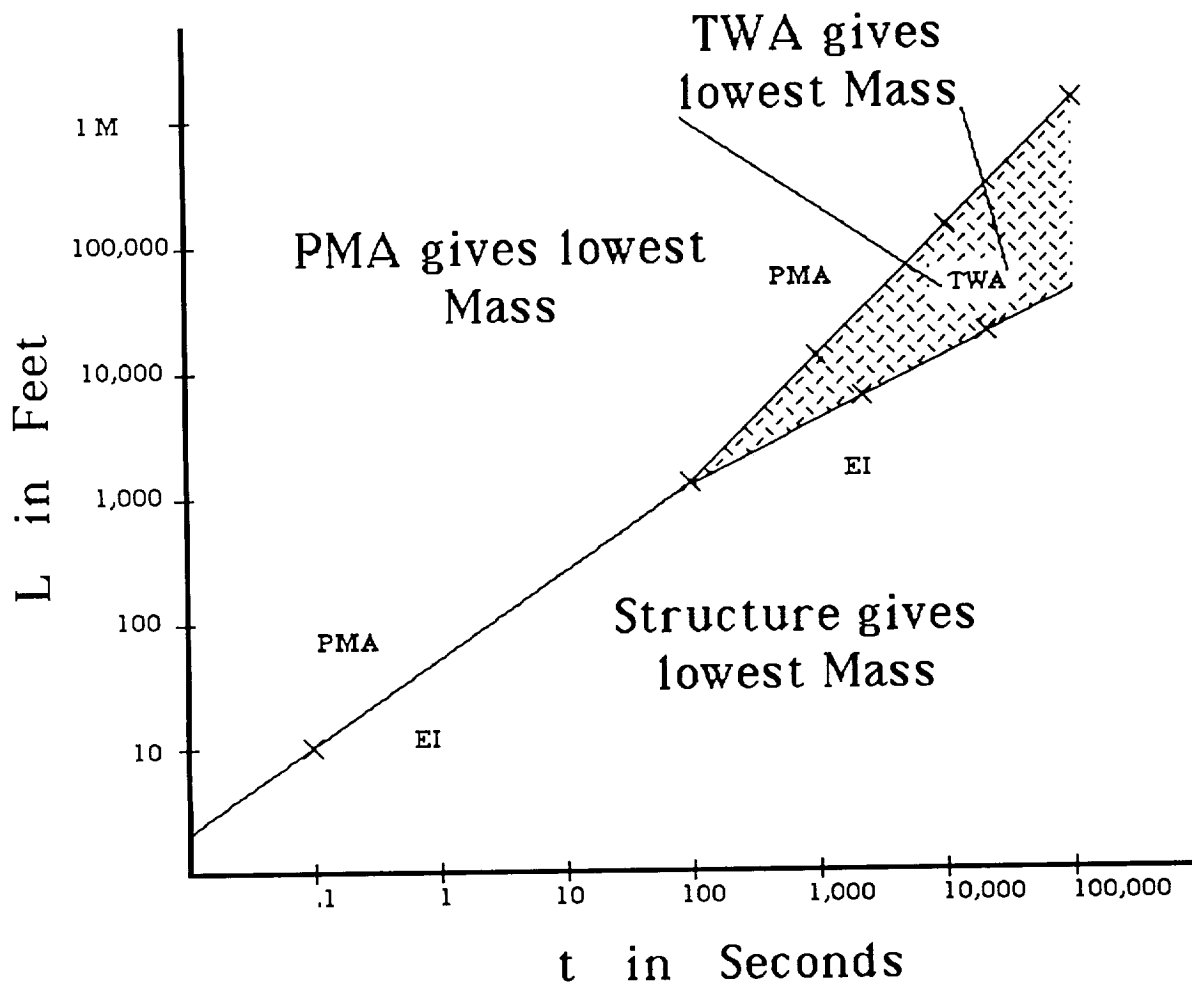
- o Each Generation Represents 20 Function Evaluations (corresponding to 20 Designs)
- o Average "Fitness" of the Multiple Designs Increase Quickly

## Example Design Problem

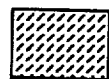
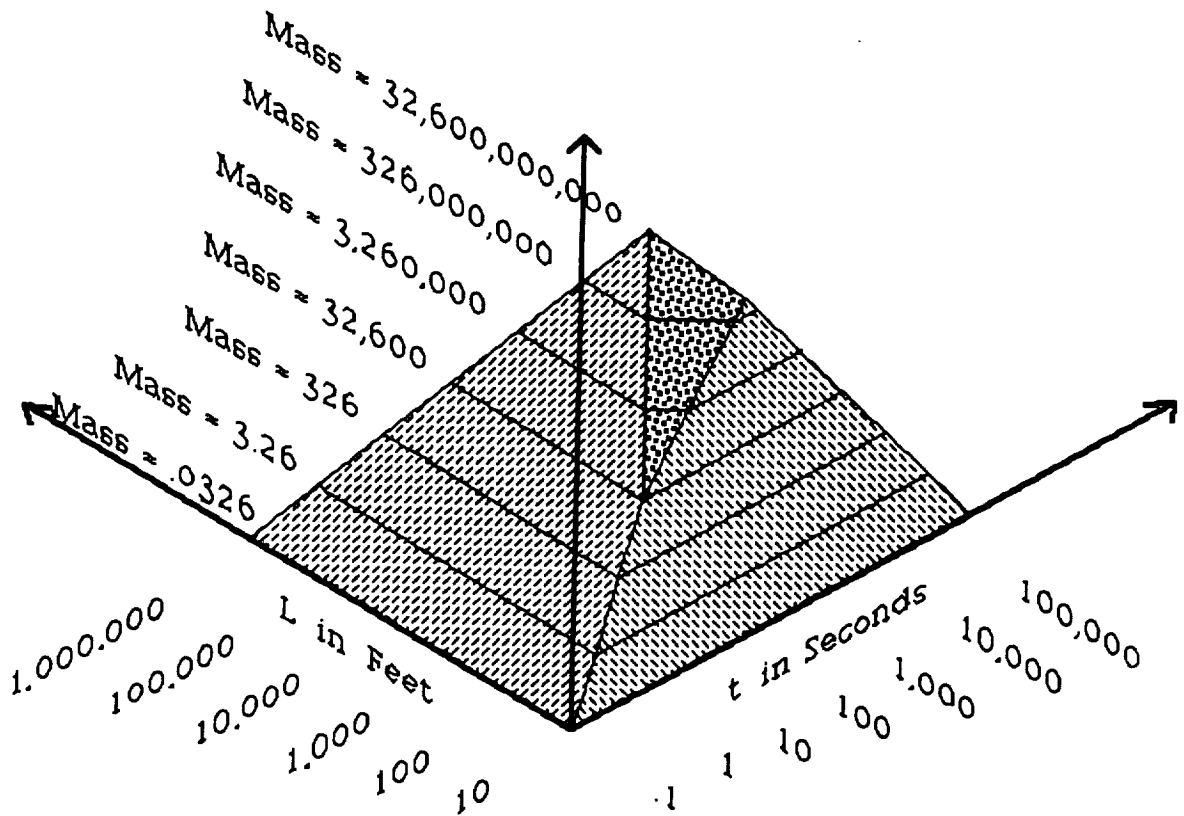
<u>Article:</u>	Mini-MAST Truss
<u>Disturbance:</u>	$F = F_{\max}$
<u>Specification:</u>	$u_{\text{tip}} < u_{\max}$
<u>Controls:</u>	Proof Mass Actuator Torque Wheel Actuator
<u>Objective:</u>	$J = \min\{m_{\text{truss}} + m_{\text{proof}} + m_{\text{torque}}\}$



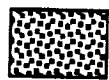
# Control/Structures Optimization



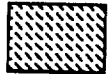
# Control / Structures Optimization



Proof Mass Actuator yields lowest Mass



Torque Wheel Actuator yields lowest Mass



Structure yields lowest Mass