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# ACTIVE VERSUS PASSIVE DAMPING IN LARGE FLEXIBLE STRUCTURES <sup>1</sup>

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## Abstract

Optimal passive and active damping control can be considered in the context of a general control/structure optimization problem. Using a mean square output response approach, it is shown that the weight sensitivity of the active and passive controllers can be used to determine an optimal mix of active and passive elements in a flexible structure.

## 1 Introduction

Because of the low inherent damping of the typical large flexible structure, some form of vibration control methodology is necessary to reduce the vibration response to an acceptable level. The control community has traditionally addressed this problem from an active control viewpoint, and has proposed a multitude of mathematical techniques for solving this difficult feedback control problem. While elegant in their mathematics, these analyses often show little consideration for the mass, cost, and reliability of the hardware required for these control strategies.

An alternate approach to active vibration control is to implement vibration suppression through some type of passive means. Passive schemes have a long history of use in the satellite business, having been used for suppression of rigid body motions for many years. The most typical of these are probably the viscous damper (e.g. a ball in a fluid filled tube), or the magnetic damper. For flexible structures the natural damping inherent in real materials will cause vibratory motions to damp out, although the time scale may be quite large. There is currently an effort to identify materials which can significantly increase the material damping with no other adverse effects.

A question which has seldom been addressed is whether there is a combination of active and passive techniques which is optimum for a structure to use? The entire question of optimization is one which must be considered carefully in the spacecraft design area. For the current study we define "optimal" to mean the minimum mass structure, while keeping all other constraints within specified limits. Optimal simultaneous structure/control design is an area of much recent interest in both the structures and the control communities. The answers obtained to any optimization problem are very much dependent on the initial assumptions, the mathematical framework, and the decision as to the definition of optimality. In previous works, the authors have proposed a stochastic approach to optimal control/structure design whereby the optimization problem is to find a minimum weight structure subject to fixed output constraints and to fixed control energy constraints (Refs. 1-3). The controller can be arbitrary, or the controller structure can be set in a specified way. For

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example, in a perfect information environment, a full state controller yields the best performance, and hence the minimum weight structure. If feedback is restricted to outputs, either with or without additional measurement noise, then the control structure can be modified to a direct output feedback, or filter feedback form. Using this approach one can, for example, determine the mass reduction possible through additional sensors.

Our approach to include passive control means is to consider the passive damper as a special type of output controller. For example, a linear dashpot is an element which generates a force opposing relative motion between two connected locations. This can be considered as an output controller, where the controller gain is the damping coefficient associated with that element. The task required to complete the optimization is a difficult one. If we are attempting to minimize the total mass of the structure, what is the mass penalty associated with the controller (either active or passive) and how does that mass penalty vary with control energy? In fact we are in no position to answer that question completely in this paper. Rather our approach will be to consider the problem parametrically, so that sensitivities of the optimal solution to component masses can be ascertained. The actual optimum can be obtained when appropriate information is available to the designer.

The outline of this paper is as follows. In Section 2 we give a short outline of the control/structure optimization approach required for this study. A short section on hardware implementations is included as Section 3 for general information only, as the authors' quantitative information in this area is incomplete. Finally, two simple examples are given which show of the benefits and possibilities for this type of analysis, followed by our conclusions.

## 2 Mathematical Framework for Control/Structure Optimization

The approach taken in this analysis is to consider the structure from a dynamic response point of view. We assume that the fundamental structural constraint is to support a load, or to hold various sub-system components (experiments, etc.) together to a required degree of accuracy in the presence of some form of excitation. The excitation specification may be either static or dynamic, deterministic or random, but to quantify the structure design we generally need to know the applied loads to ensure that stress or displacement constraints are met. The effect of the control system on this design problem is to effect a trade-off to reduce the effect of flexibility. As the structure is made lighter and more flexible, the control system can be used to reduce deflections and stresses to acceptable levels.

### Structure/Control Optimization

The mathematical approach taken to quantify the control-structure relationship is to initially regard the controller structure as fixed and satisfying certain control magnitude constraints, which for this analysis will be assumed to be a mean square control energy bound. Similarly the external force environment on the structure is known and is assumed to be stochastic with known mean square energy and spectrum. Within this framework the optimal structure-control design problem is to find the structural parameters and the control law to minimize a performance index while satisfying control energy and displacement constraints. This may be posed as a mathematical programming problem.

Assume the system is given as

$$\begin{aligned} \dot{x} &= Ax + Bu + Gd \\ y &= Cx \\ z &= Hx + v \end{aligned} \tag{1}$$

where we have the conventional definitions of the state( $\mathbf{x}$ ), control( $\mathbf{u}$ ), performance output( $\mathbf{y}$ ), measured output( $\mathbf{z}$ ), and input and output disturbances ( $\mathbf{d}$ ,  $\mathbf{v}$ , respectively).

The disturbance  $\mathbf{d}$  is taken as the specified load. Generally we have assumed that  $\mathbf{d}$  is Gaussian white noise,  $\mathbf{d} \sim N(0, D)$ , although other forms (e.g. harmonic disturbance) could be used. For this system we pose the following optimization problem:

### Optimization Problem

Minimize the function  $J(p_1, p_2, \dots)$ , where the  $p_i$  are structural parameters such as mass, stiffness, area, etc., and find the feedback law

$$\mathbf{u} = f(\mathbf{z}) \quad (2)$$

where  $f(\cdot)$  is a specified functional form based on the controller type desired. For example, if  $\mathbf{z} \equiv \mathbf{x}$ , then the problem is full state feedback and  $f(\cdot)$  becomes naturally a gain matrix. The resulting optimization becomes a special form of the linear quadratic regulator. The functional  $f(\cdot)$  can specify an (unknown) dynamical system in the general output feedback problem. For the problem considered in this paper we consider the controller to consist of two parts: The first is the passive controller which consists of unknown damping coefficients of the specified form. The second part is a "conventional" full state active controller. The active controls must satisfy the control energy constraints

$$E[\mathbf{u}^T R \mathbf{u}] = \beta^2 \quad (3)$$

while the outputs satisfy

$$E[\mathbf{y}^T W_i \mathbf{y}] \leq w_i \quad (\text{Output disturbance Inequalities, } i = 1, \dots, n) \quad (4)$$

The rationale for a fixed control energy constraint is that for an active control implementation, the desired control should utilize the full control capability to reduce the structural loading. The output inequalities may be several, in which case one or more constraints may be equality constraints, but others will be strict inequalities.

Using the Gaussian disturbance case, the expectations can be converted to simple operations on the covariance matrix, which is determined by a Lyapunov equation (assuming here linear controls). For details on this see the references. Using this approach the optimization framework is quite flexible and can be adjusted to a variety of special types of constraints and controllers. Numerically the resulting optimization problem can be solved by a variety of general non-linear optimization software. For the full state feedback case, linear regulator software can be incorporated also.

## 3 Active and Passive Control Implementation

There are a number of technologies possible for the active and passive control of the damping of flexible space structures. The simplest in concept are the linear and rotational momentum exchange devices. Linear momentum devices (LMED's) are extremely simple in concept, yet have proven remarkably difficult to construct

(for a real space environment). For the LMED there is a trade-off between the magnitude of the proof mass and the total track length. For the low frequencies contemplated in most large space structures, the LMED's will probably have a mass which is a large fraction of the structure mass, a generally undesirable feature! For a fixed geometry track, the force obtained from the LMED is directly proportional to the proof mass itself, hence we may expect the mass contributed by this controller to be roughly proportional to the square root of the control energy  $\beta^2$ .

Torque wheel actuators or momentum wheels, are similar in concept to the LMED, absorbing angular momentum, rather than linear momentum. Track length here however is not a problem; the primary limitation is wheel speed. Maximum wheel speed is rarely encountered, although some means of dumping stored angular momentum must also be considered. Note that for a free-free structure every non-zero frequency vibration mode has zero linear and angular momentum. Hence both these devices need only worry about the saturation due to vibration transients, not in the steady state. It seems very likely that momentum wheels used for rigid body control, could easily be accommodated for control of flexible motions also.

Linear thrusters are generally used on satellites for attitude control, and could easily be adapted to the vibration suppression role. Note that for the rocket, thrust is proportional to mass flow, so that again average energy consumption would make thruster mass proportional to  $\beta^2$ . In reality for the common reaction jet, specific impulse is relatively low, and fuel required for a long duration mission could be a significant problem. This may be no worse than the LMED mass problem, and seems to be considerably more reliable.

Passive control elements could be one of a variety of viscoelastic materials used as coatings, or internal strut material. For such materials, the damping coefficient is proportional to the amount, hence the mass, of the damping material added. Other devices are "passively active" (or is it "actively passive"?) such as piezoelectric materials (coating or embedded). For most materials considered then the mass added by damping elements can be considered to be proportional to the mean square energy ( $\beta^2$ ), or more directly for the passive damping elements to the damping element " $c$ ". The exact proportionality constant is extremely important in establishing an optimum, as is any fixed mass components not considered here.

## 4 Two Simple Examples

### (a) Longitudinal vibration of a rod

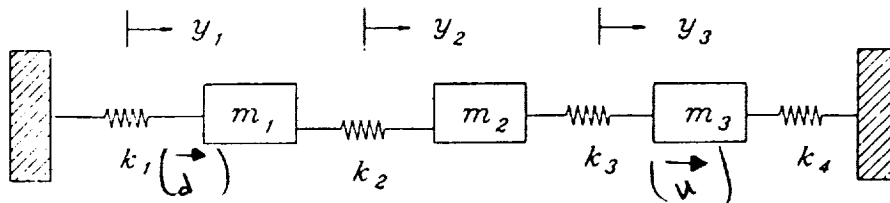


Figure 1: Discretized model for Example (a)

To demonstrate the approach to evaluate the effectiveness of active or passive control, consider the longitudinal vibrations of a rod, where the rod is discretized to the simple mass-spring-damper system shown in Figure 1. For this case we will consider only the trade-off between the active control element, assumed to be acting at mass 3, and the damping element, which is assumed to be inserted between masses 2 and 3. For this problem,

we assume the rod structural elements are themselves fixed, although in the general optimization problem, the segment areas (hence spring constants) would be design variables also. For this problem Figure 2 shows the amount of active control energy required to meet the displacement constraint  $E[x_2^2] = \alpha^2$  for variable passive damping. Note that this curve has a minimum at a fairly low value of damping, meaning that as passive damping goes up, the active controller must work harder to meet the output displacement constraint. This curve is typical, although for different constraint levels the minimum will shift and may, in fact, disappear. For this example the system mass is considered to be fixed but we may propose a cost function then to be

$$J = \gamma_1 c_3 + \gamma_2 \beta^2$$

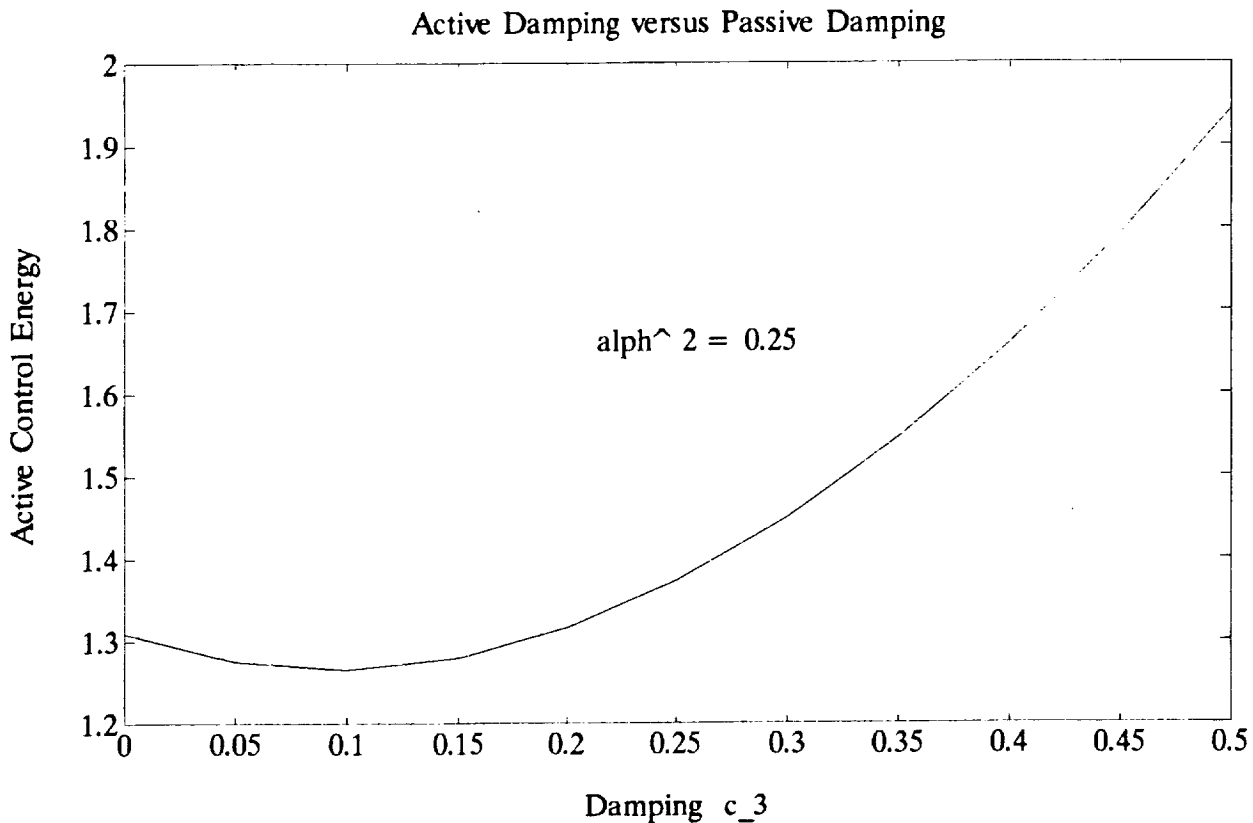


Figure 2: Active control energy required to achieve mean square response constraint

Constants  $\gamma_1$  and  $\gamma_2$  reflect the relative cost (either mass or dollars) of the active and passive control components. A line of constant cost then is a straight line on Figure 2 with a slope  $m = -(\gamma_1/\gamma_2)$  meaning that the optimal mix of active and passive damping is somewhere to the left of the absolute minimum shown on Figure 2, and is found from meeting a tangency condition between the curve and the straight line. If the passive damping cost is much less than the active damping cost (a common assumption in most discussions), then the optimal passive damping is that value at the minimum of Figure 2.

### (b) DRAPER I optimal truss

In solving an optimal structure/controller problem McLaren and Slater [Ref. 3] determined the optimal mass of the tetrahedral truss model model known as "Draper I" (see Figure 3), for various types of controller

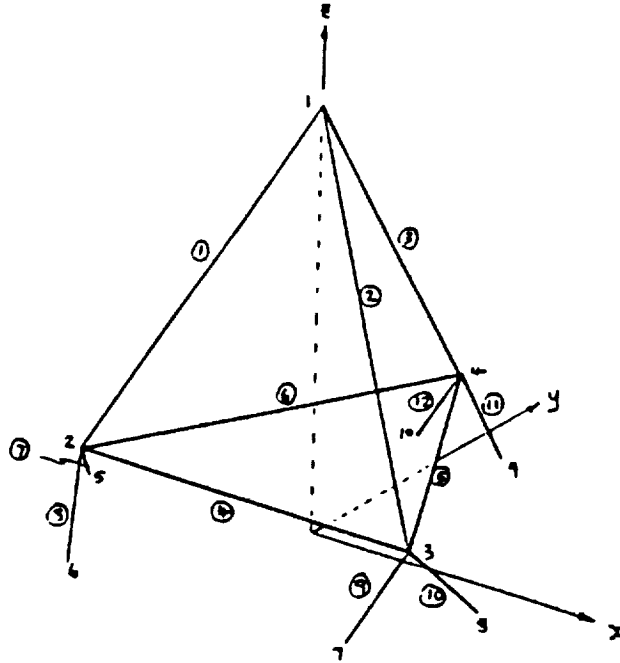


Figure 3: The DRAPER I truss model

implementations. For this structure there are six control actuators, situated in each of the base legs of the truss, and six collocated velocity sensors, giving the longitudinal velocity of each truss leg. The comparison between full state feedback and direct output feedback then is almost a direct comparison of the active versus passive control analysis done for the previous simple model. (The comparison is not exact as the results in Ref. 3 determined the optimal general feedback matrix. For a passive damping study we need to go back and additionally restrict the gain matrix to be diagonal. This is straight-forward and hopefully will be done soon.)

For this case the problem was to design the optimal structural elements to minimize the mass and to simultaneously design the controller. The controller mass was not considered part of the performance index, nor was a fixed controller mass part of the system. The output constraint is to keep the vertex of the truss within specified limits. For these two cases the final masses are shown in Figure 4. The results indicate that generally full state designs may achieve almost 50% less mass than the optimum velocity feedback designs. Ideally we should go back and re-run with these two controllers in parallel, and with relative weights associated with each. The resultant family of controllers, combined with mass information on the controller implementation, could then be used to determine an optimal control implementation. Based on the large mass reduction from full state feedback, it seems reasonable that for this, and probably for most structures, the advantage of an active feedback scheme can be quantified explicitly.

## 5 Conclusions

The results shown indicate that there is an easy way to explicitly characterize the relative merits of an active versus a passive control scheme. No attempt here is made to quantify the exact trade-off due to the uncertainty in mass figures associated with the controller types. This, and further exploration of controller trade-offs, are subjects of continuing research.

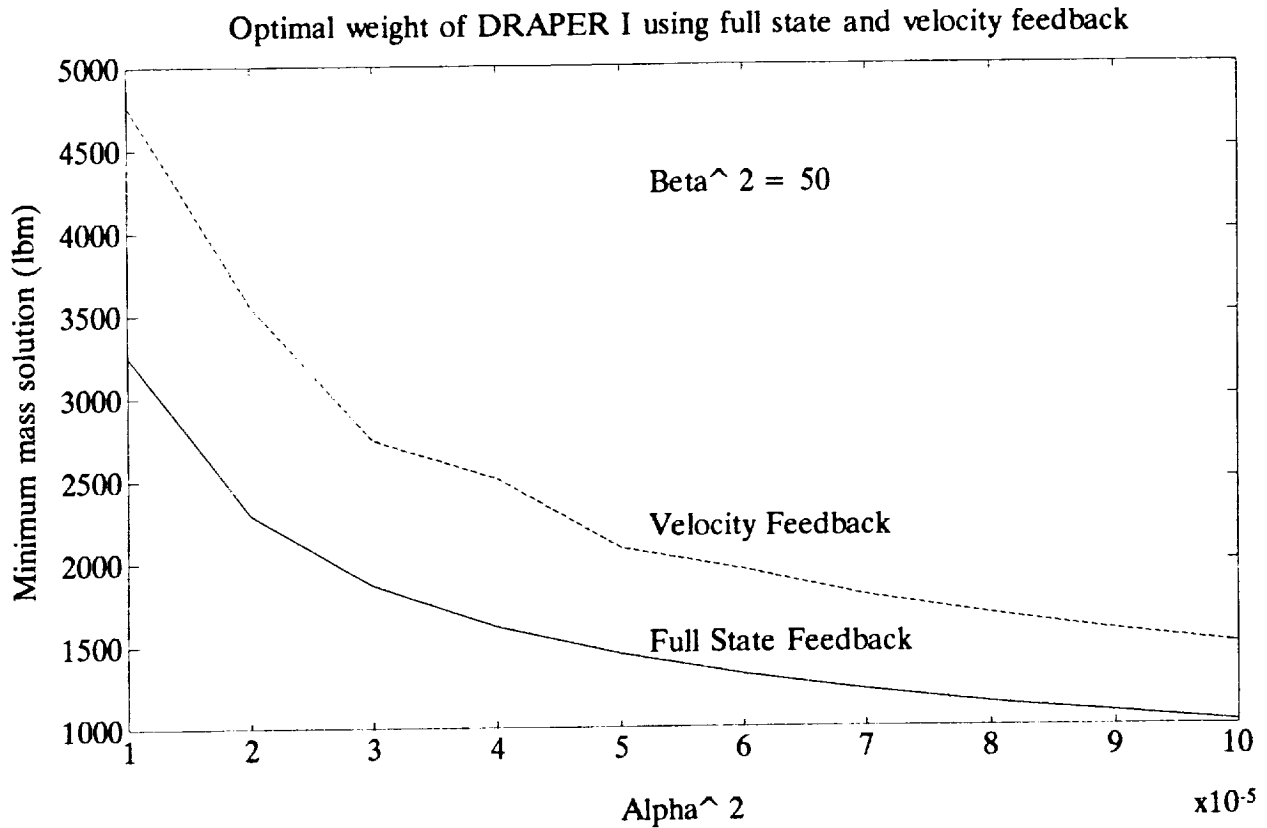


Figure 4: Comparison of optimal DRAPER I mass for full state vs. velocity feedback

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Due to time constraints, it was not possible to include a reference list for the many pertinent documents that are available in the literature. Only earlier material of the authors which fills in many of the missing details in this paper is listed below.

### References

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