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AN OVERVIEW OF THE ESSENTIAL DIFFERENCES
AND SIMILARITIES OF SYSTEM IDENTIFICATION
TECHNIQUES

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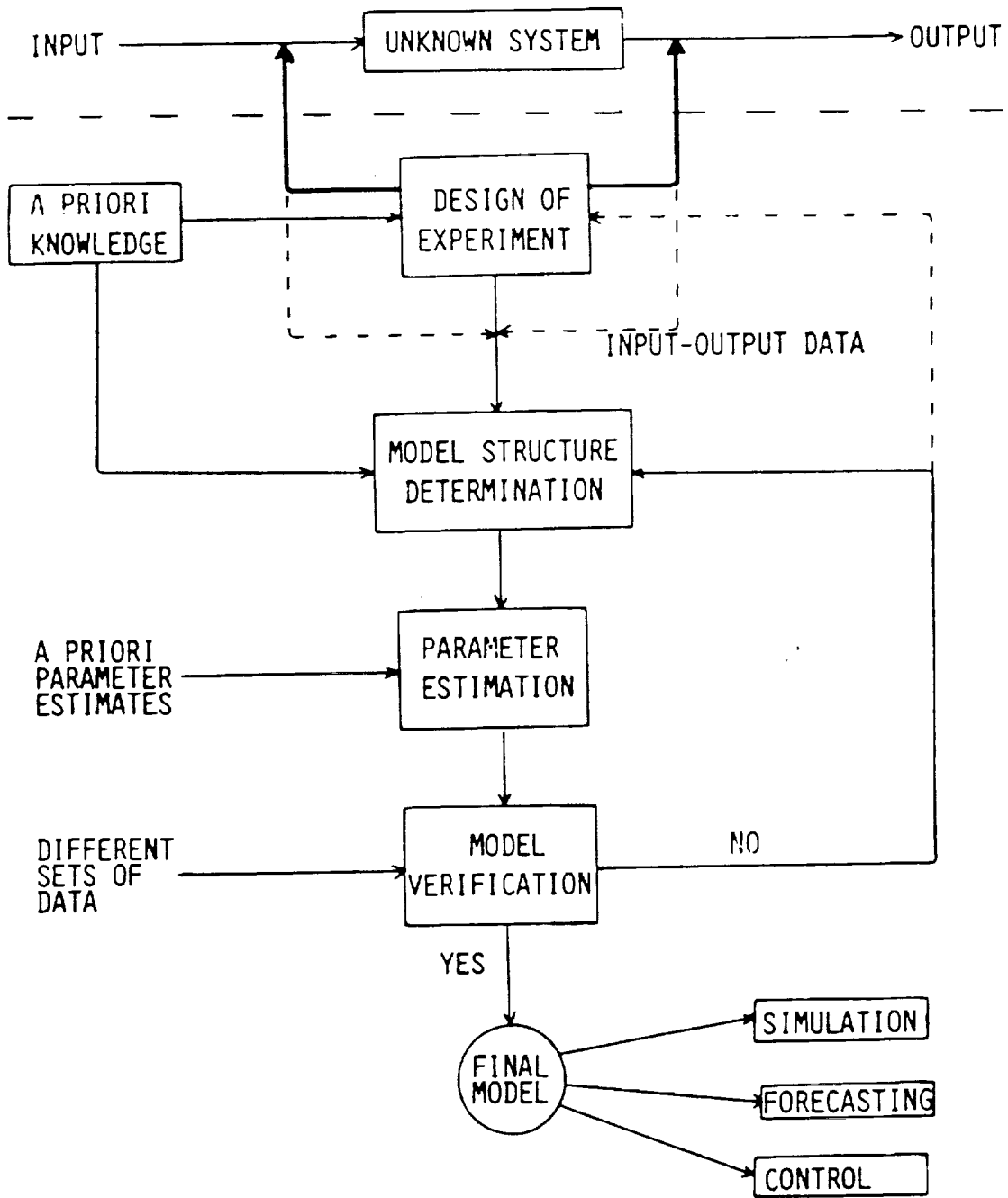
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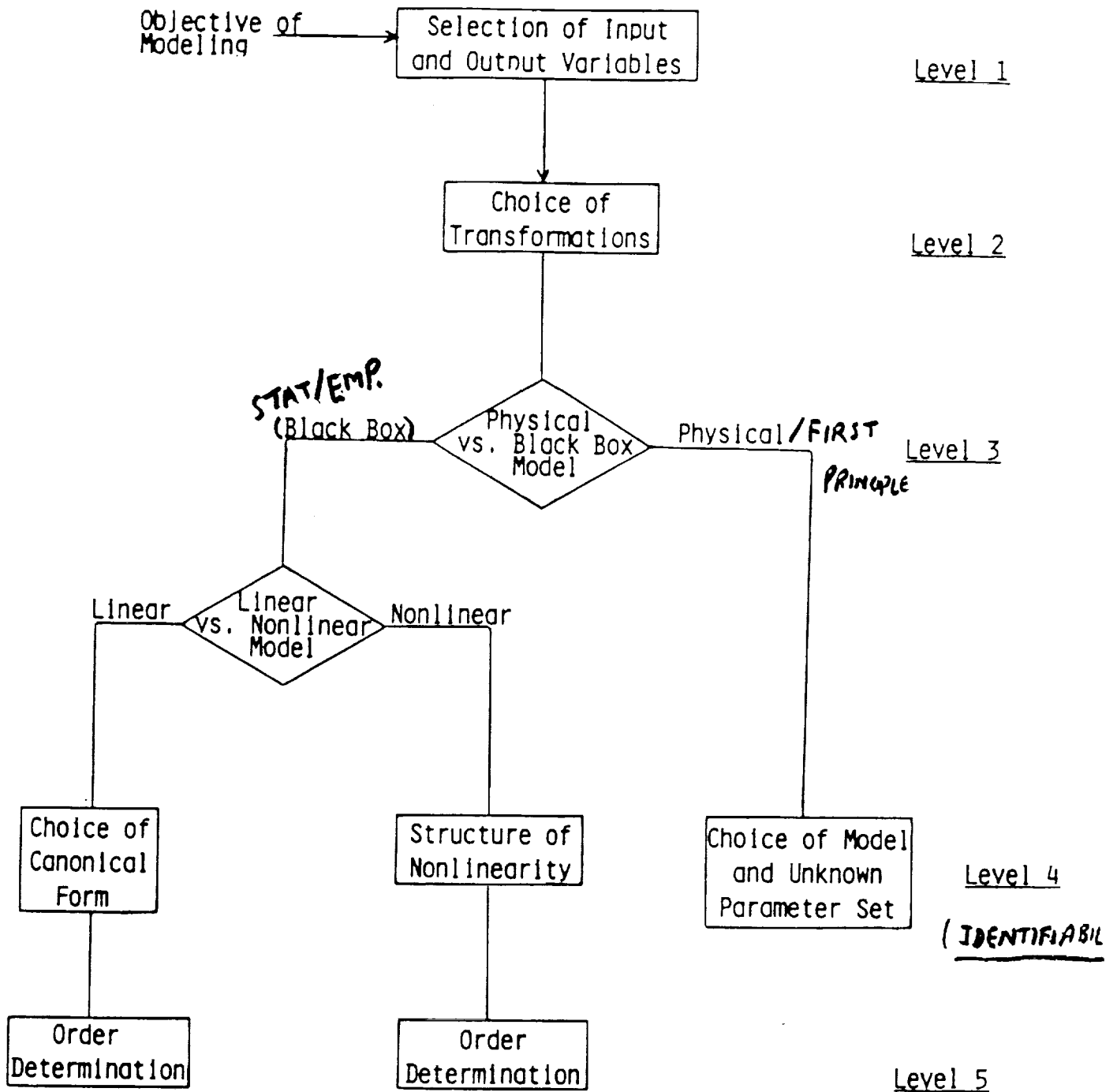
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Outline

1. System Identification - Four Basic Steps
2. Bayesian Statistical Decision Theory Framework
3. Maximum Likelihood Estimation and E-M algorithm
4. Similarities and Differences between Identification Methods
5. Minimal Realizations - Deterministic & Stochastic
6. Structural Mode Identification using Stochastic Realization Algorithm
7. Identification Results
 - o Membrane Simulations
 - o AL Grid
 - o X-29 Flutter Flight Test Data
8. Conclusions



STEPS IN SYSTEM MODELING AND IDENTIFICATION



Hierarchical Levels in Model Structure Determination

2. Bayesian Statistical Decision Theory Framework for System Identification

1. Unknown state of the world denoted by state vector $\{x(t)\}$.
2. A set of models M_1, M_2, \dots, M_ℓ with a priori probabilities $p(M_i)$, which describe the evolution of the state $x(t)$.
3. Unknown parameters θ_i associated with Model M_i and prior probabilities $p(\theta_i | M_i)$
4. Vector of observations $\{y(t)\}$ related to the state of the system $\{x(t)\}$, according to probability distributions. $p(\{y(t)\} | \{x(t)\}, M_i)$
5. Loss function $\mathcal{L}(M, \hat{M})$ which expresses the loss to the decision maker of choosing model \hat{M} when M is the true model.

Bayesian Solution:

Obtain posterior distributions $p(M_i | \{y(t)\})$ and select M_i which minimizes the expected value of the loss function.

Bayes Rule:

$$p(M_i | \{y(t)\}) = \frac{p(\{y(t)\} | M_i) p(M_i)}{p(\{y(t)\})}$$

$$p(\{y(t)\} | M_i) = \int_{\Theta} p(\{y(t)\} | M_i, \theta_i) p(\theta_i) d\theta_i$$

Likelihood Function:

$$L(M_i, \theta_i) = p(\{y(t)\} | M_i, \theta_i)$$

Maximum Likelihood Estimation

- o $\hat{\theta}_{ML}$ maximizes $p(\{y(t)\}|\theta)$ or its logarithm, $LL(\theta) = \log p(\{y(t)\}|\theta)$

- o Cramer-Rao Lower Bound:

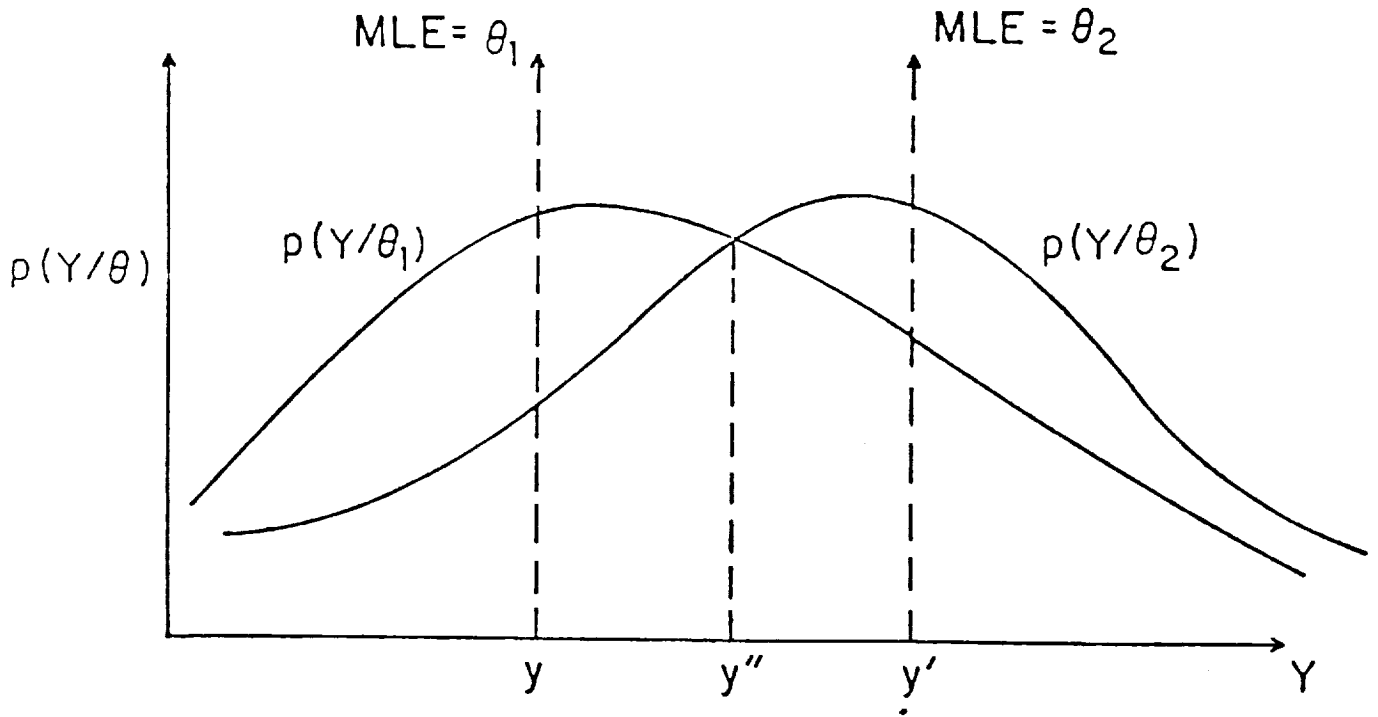
$$\text{cov}(\hat{\theta}) \geq \left[E \frac{\partial LL(\theta)}{\partial \theta} \left(\frac{\partial LL(\theta)}{\partial \theta} \right)^T \right]^{-1}$$

The quantity in brackets is the Fisher Information Matrix which is very useful for determining Identifiability and for Input Design.

- o when $p(\{y(t)\}|\theta)$ is Gaussian and θ effects the conditional mean linearly, $LL(\theta)$ is quadratic in θ and MLE is same as Weighted Least Squares.
- o $\hat{\theta}_{ML}$ can be obtained by a sequence of Expectation & Maximization steps (E-M Algorithm), each one of which is simpler than direct maximization of the Likelihood Function.

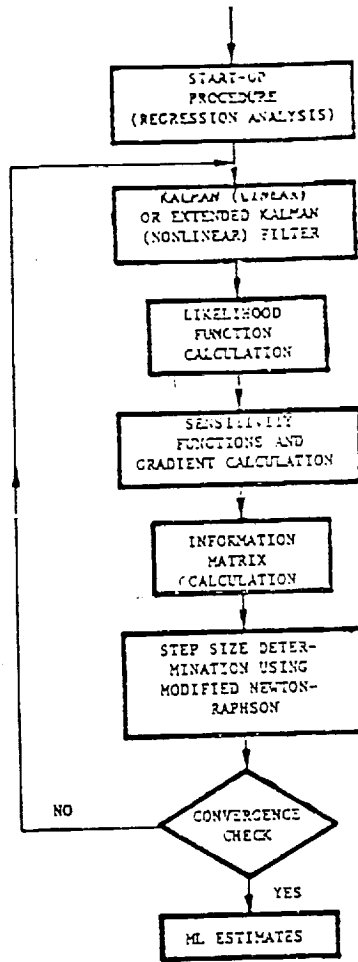
E-step: Estimate state given parameters

E-step: Estimate parameters given state statistics.

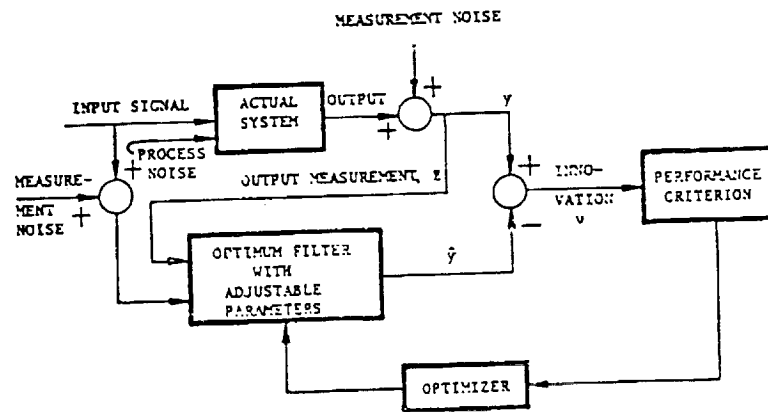


MAXIMUM LIKELIHOOD ESTIMATION

INPUT-OUTPUT DATA, A PRIORI BOUNDS ON PARAMETERS



Steps in Maximum Likelihood Estimation



Block Diagram for Maximum Likelihood

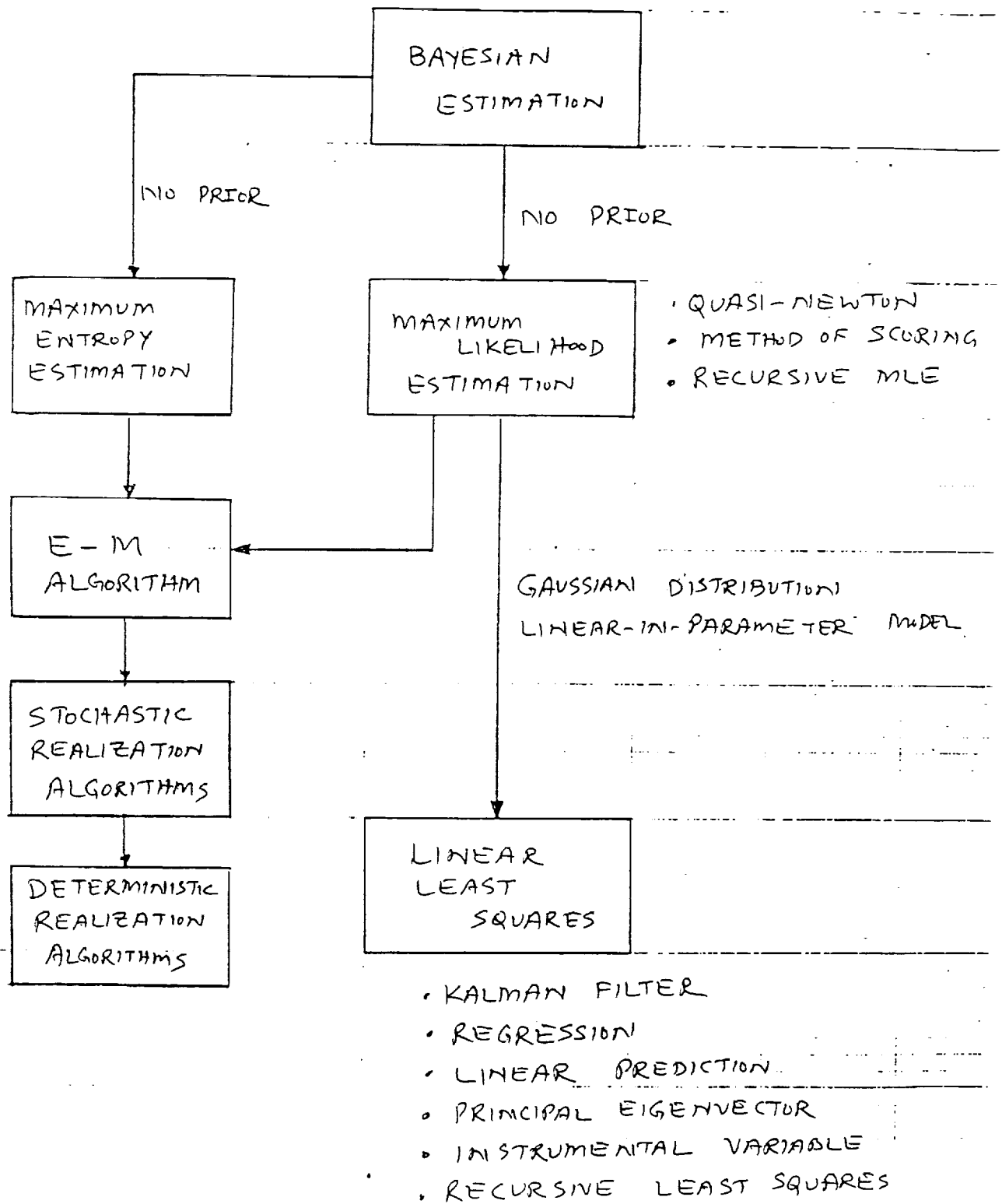


FIG: RELATIONSHIP OF DIFFERENT TECHNIQUES FOR PARAMETER ESTIMATION

**Combined Model Structure Determination
and Parameter Estimation for
Linear Systems**

$$x_{t+1} = A x_t + B u_t + K e_t$$

$$y_t = Cx_t + D u_t + e_t$$

$$t = 1, \dots, N$$

Stochastic Realization:

Given $\{y_t, u_t\}$, identify system order n and matrices A, B, C, D, K and $\text{Cov}(e_t)$.

Deterministic Realization:

Given impulse response parameters $CA^{k-1}B$, identify A, B, C and n

Solutions:

- o Deterministic (Ho-Kalman, Balanced Realization, ERA)
- o Stochastic (Akaike, Mehra, Aoki, QMARKOV, CVA, SRA)

Stochastic Realization Algorithm

DEFINE:

$$p_t = \text{column} \left[y_{t-1}, u_t, y_{t-2}, u_{t-1}, \dots \right], \quad \text{Past}$$

$$f_t = \text{column} \left[y_t, y_{t+1}, y_{t+2}, \dots \right], \quad \text{Future}$$

Correlation of Past & Future:

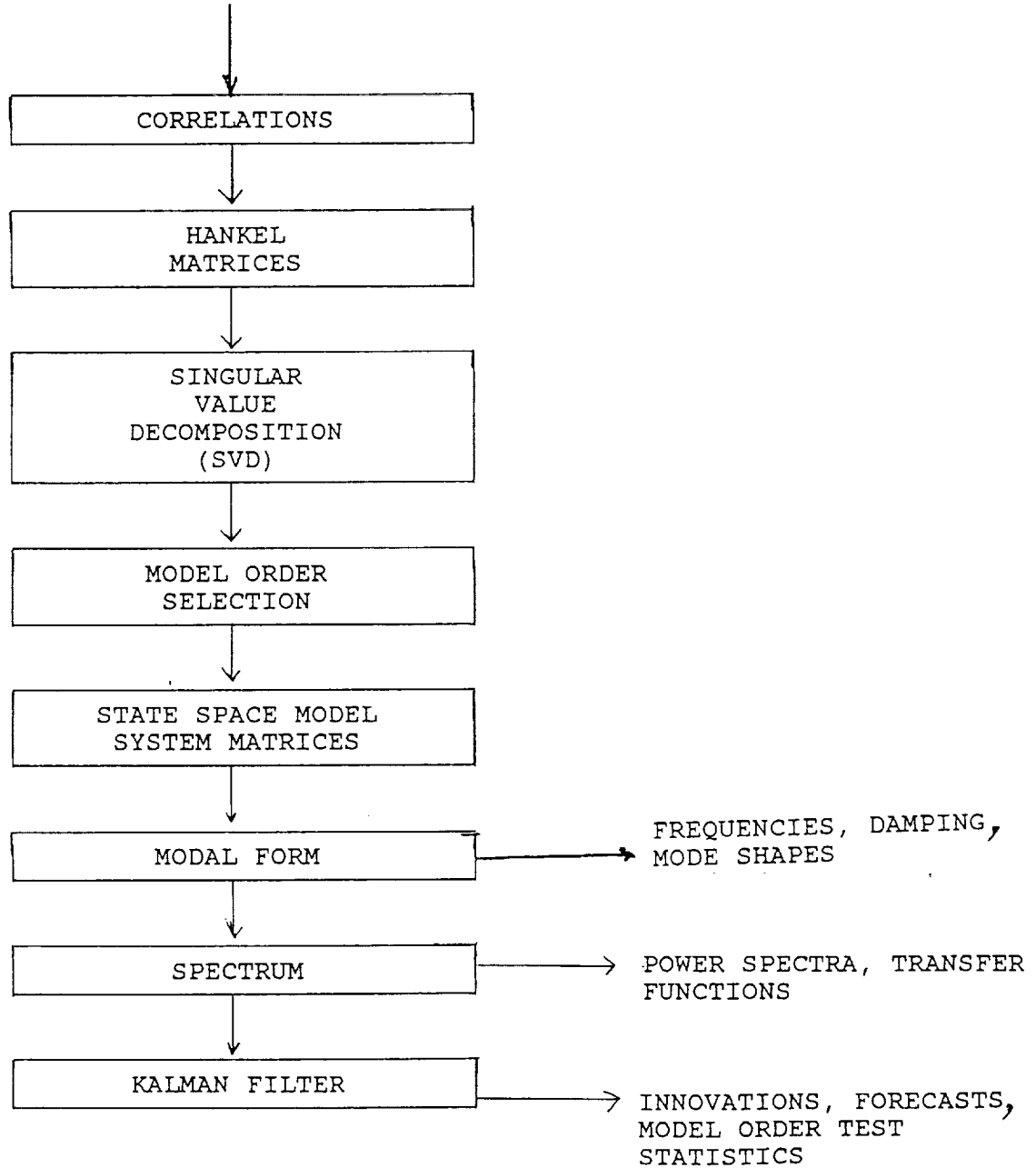
$$H = E (f_t p_t'), \quad \text{Hankel Matrix}$$

Singular Value Decomposition:

$$H = U \Sigma V'$$

STEPS IN SS-SRA

TIME SERIES DATA



Identification Results

1. Membrane Simulation:
 - o Data provided by Mark Norris, AFAL
 - o Two lightly damped modes at 0.5Hz
 - o 5 velocity measurements contaminated with different levels of multiplicative noise (1% to 1,000%).
 - o SRA results are satisfactory for 300% noise!

FREE-FREE MEMBRANE SIMULATION

DATA GENERATED BY MARK MORRIS (AFAL) WITH NOISE RANGING FROM 0% TO 1000% AND 2 DIFFERENT INITIAL CONDITIONS (VEL 1 AND VEL 2).

TRUE MODEL:

NATURAL FREQUENCIES

DAMPING

0 Hz	0
0.5 Hz	0.01
0.5 Hz	0.01

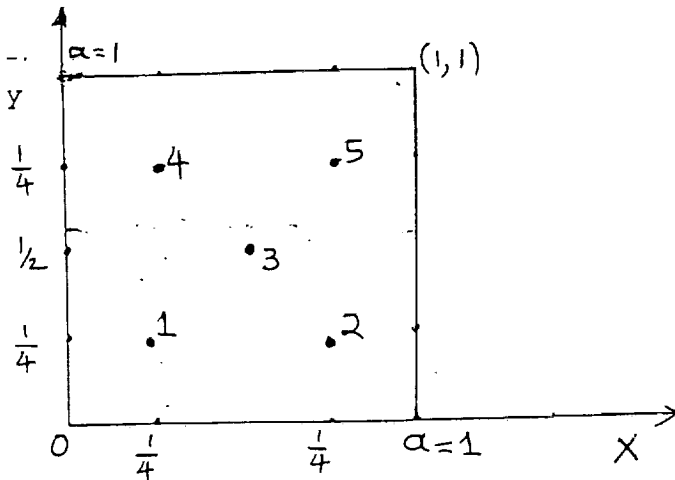
(REPEATED EIGENVALUE)

MODE SHAPES:

$$\phi_1(X, Y) = 1$$

$$\phi_2(X, Y) = \cos(\pi X)$$

$$\phi_3(X, Y) = \cos(\pi Y)$$



TENSION $T=1$ N/
DENSITY $\rho=1$ kg/
 $a=1$ m

MEMBRANE SENSOR LOCATIONS

859

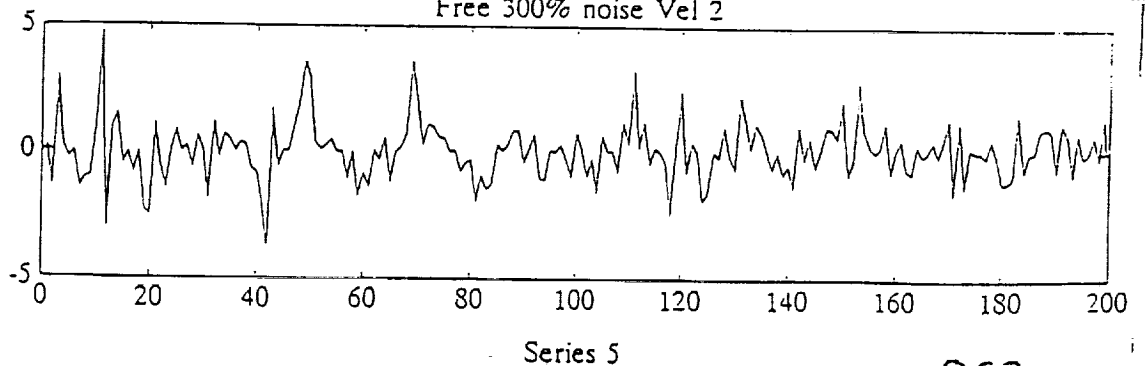
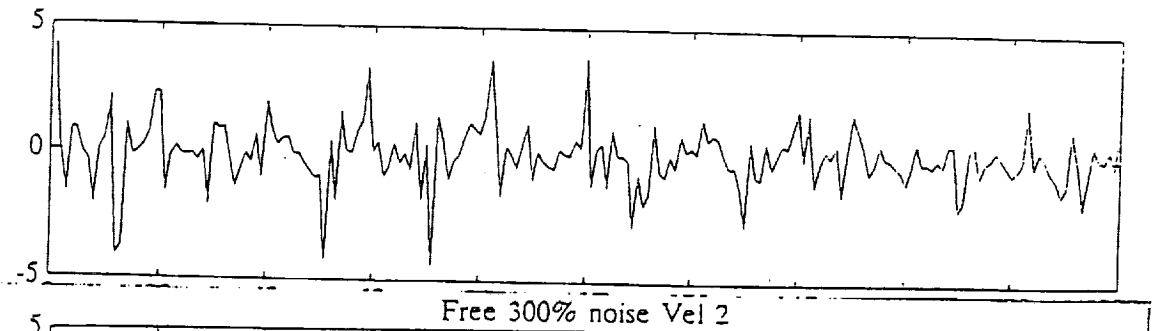
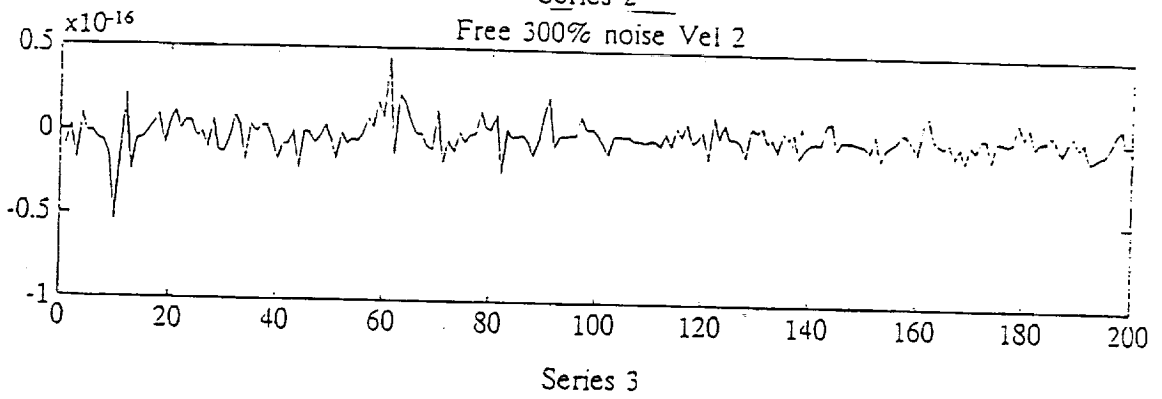
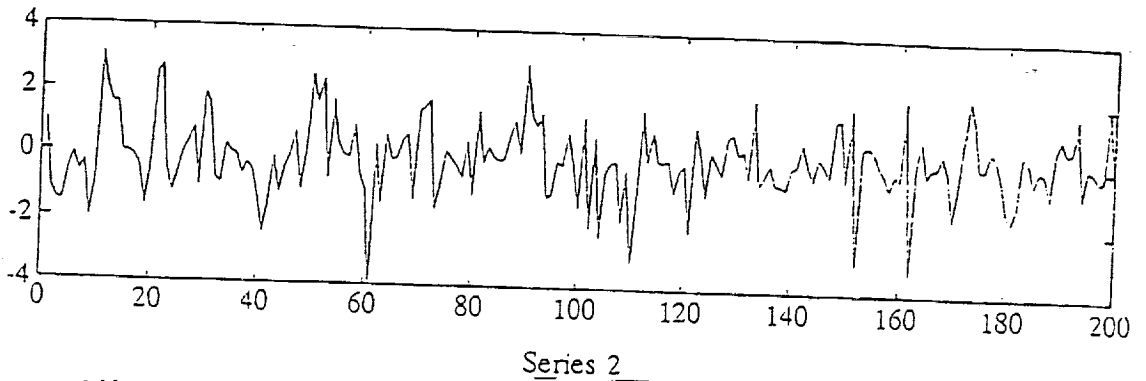
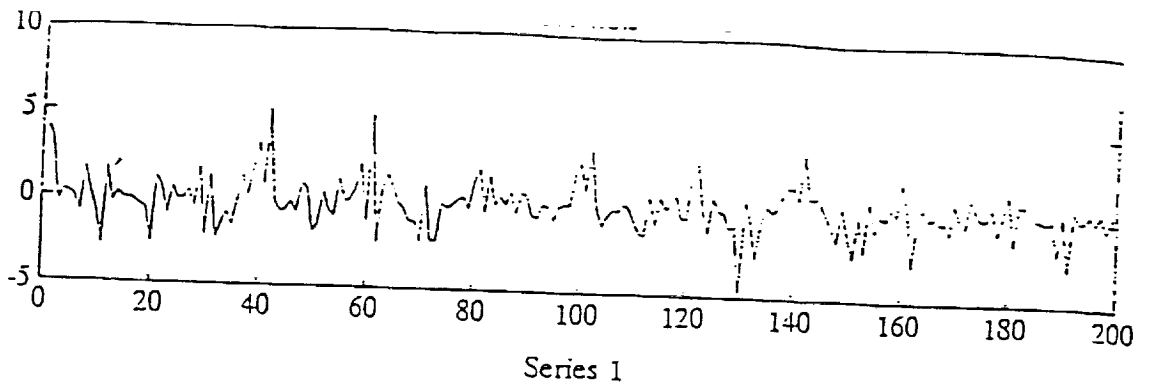
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IDENTIFICATION RUNS BY SCIENTIFIC SYSTEMS ON

MEMBRANE SIMULATION DATA USING SS-SRA

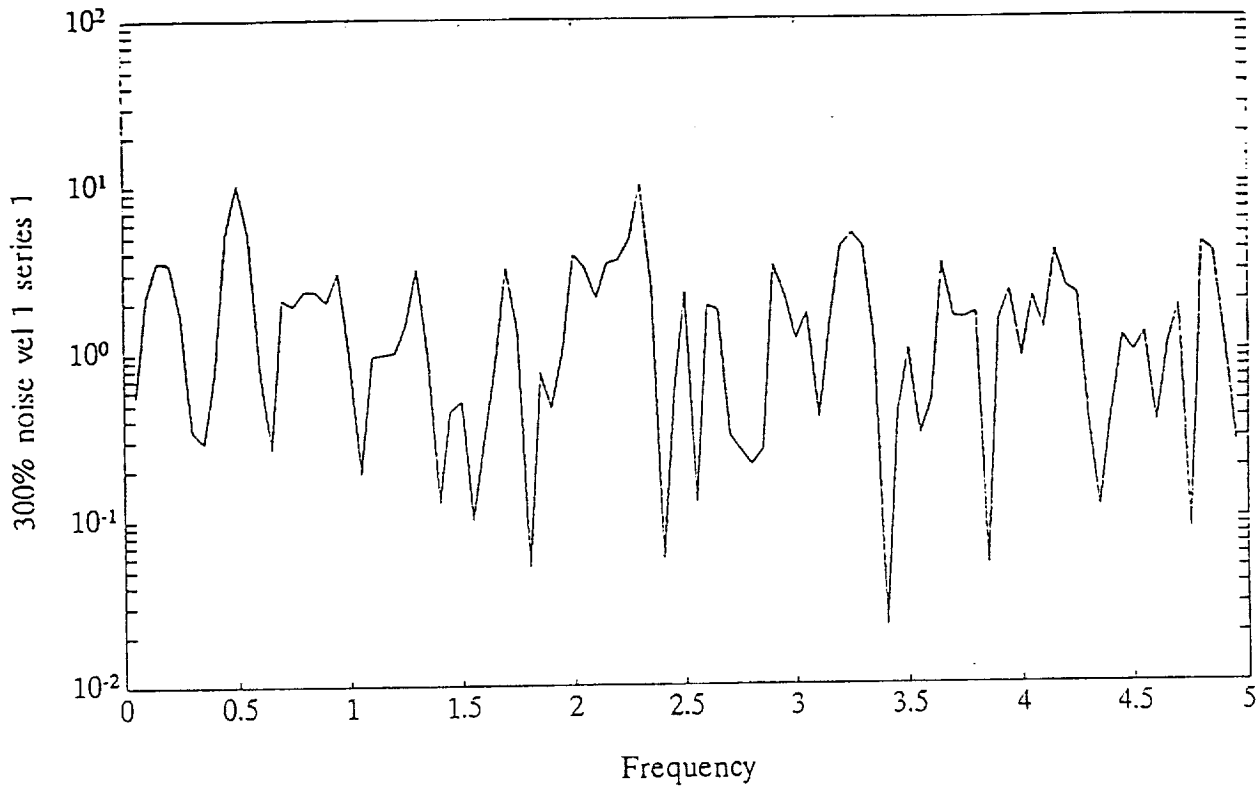
1. IDENTIFICATION DONE USING BOTH FREE AND FORCED RESPONSE DATA WITH 200 POINTS FOR 5 AND 10 OUTPUTS AT 0.1 HZ.
2. FREE RESPONSE RESULTS ARE BETTER DUE TO POOR INPUT EXCITATION FOR THE FORCED RESPONSE DATA
3. FREE RESPONSE RESULTS WITH VEL2 INITIAL CONDITION (IC) ARE BETTER THAN VEL1 IC RESULTS DUE TO BETTER OUTPUT EXCITATION
4. BEST RESULTS ARE OBTAINED BY AVERAGING THE CORRELATIONS FROM VEL1 AND VEL2 OUTPUTS, PARTICULARLY IN THE IDENTIFICATION OF MODE SHAPES. THESE RESULTS ARE SUPERIOR TO RESULTS USING 10 OUTPUTS AND CONCATENATED DATA (400 POINTS).
5. SATISFACTORY RESULTS ARE OBTAINED BY SS-SRA FOR ALL NOISE CASES INCLUDING 300% NOISE, AS OPPOSED TO ERA RESULTS WHICH DEGRADE GREATLY AFTER 50% NOISE. THE ACCURACY OF THE RESULTS IS SURPRISING CONSIDERING THE NOISE IS MULTIPLICATIVE.

861

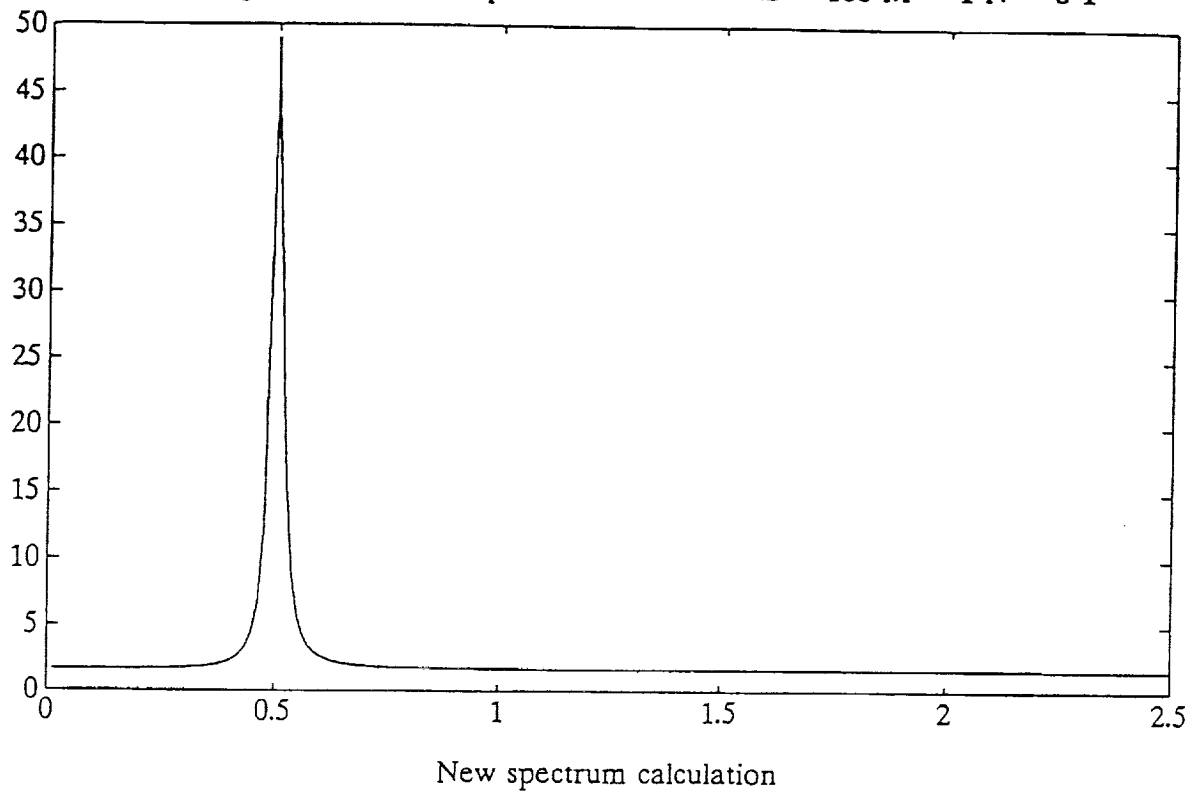


862

Pxx - X Power Spectral Density



Free avg 300% noise 150 pts J = 40 K = 20 L = 100 M = 1 N = 6 1



864

ans =

Free avg 300% noise 150 pts J = 40 K = 20 L = 100 M = 1 N = 6

ans =

Magnitude	Phase	Freq	Damping	Air Force Values	
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ans =

0.9808	2.3658	3.7655	0.0082	0	0
0.9808	-2.3658	3.7655	0.0082	0.5000	0.0100
0.9920	0.3120	0.4968	0.0256	0.5000	0.0100
0.9920	-0.3120	0.4968	0.0256	0	0
0.9899	0.3206	0.5105	0.0318	0	0
0.9899	-0.3206	0.5105	0.0318	0	0

ans =

Singular Values

ans =

1.0000	7.4279
2.0000	7.4144
3.0000	2.8451
4.0000	2.8444
5.0000	2.2254
6.0000	2.2090
7.0000	1.6678
8.0000	1.6350
9.0000	1.6062
10.0000	1.5315
11.0000	1.5038
12.0000	1.4938
13.0000	1.4624
14.0000	1.4458
15.0000	1.4098
16.0000	1.3894

cyy0 =

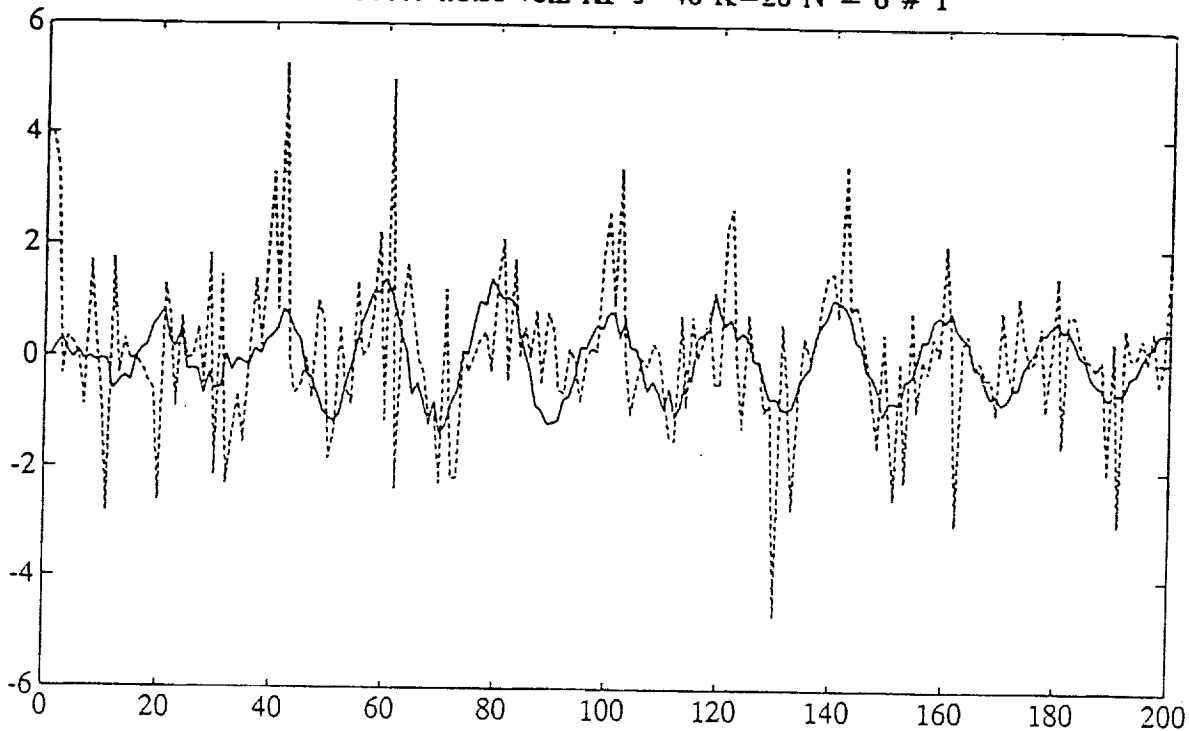
Columns 1 through 3

1.9543676887000000e+00	-9.804237999999993e-03	1.222204885150001e-03
-9.804237999999993e-03	1.533202479950000e+00	-1.735176744450000e-03
1.222204885150001e-03	-1.735176744450000e-03	4.806341140050000e-04
1.715561279090000e-01	-1.606591878000000e-01	1.414021973600000e-03
-4.222937015000000e-01	2.431914045500003e-02	3.260060618799988e-04

Columns 4 through 5

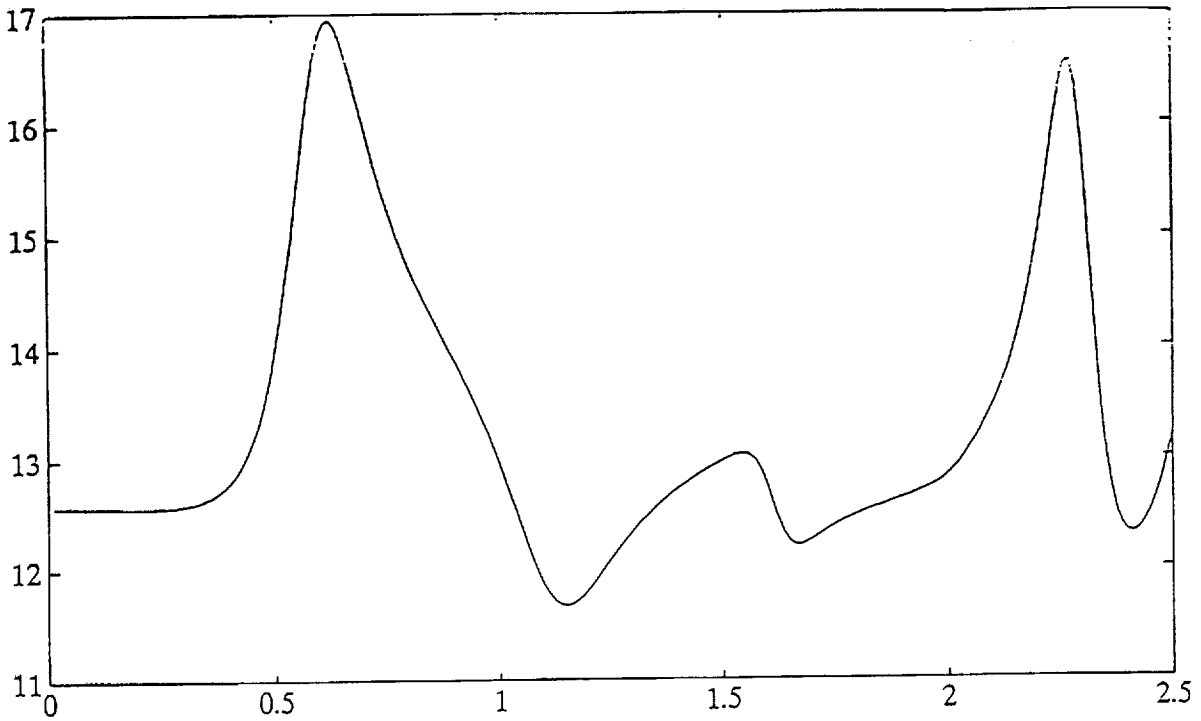
1.715561279090000e-01	-4.222937015000000e-01
-1.606591878000000e-01	2.431914045500003e-02

Free 300% noise vel2 KF J=40 K=20 N = 6 # 1



866

free avg 10Hz 1000% noise J = 40 K = 20 L = 100 M = 1 N = 29 2

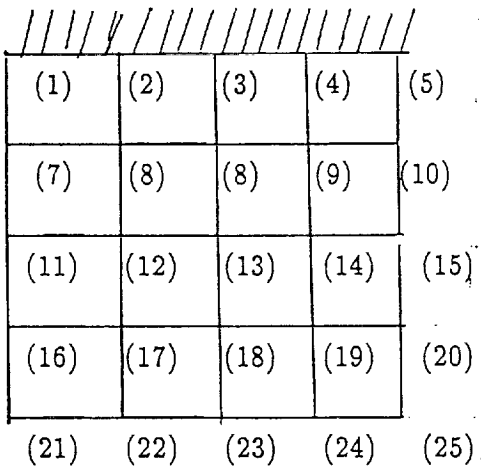


New spectrum calculation

AFAL Grid Data

- o Data provided by Mark Norris, AF Astronautics Lab
- o Three sets of data at 100hz.
 - (i) PMASS Input (2000 pts)
 - (ii) X-TORQUE Input (24,000 pts)
 - (iii) Y-TORQUE Input (24,000 pts)
- o SRA identifies all 11 modes in all cases and provides good estimates of freq., dampings and mode shapes.
- o ERA identifies all 11 modes only for combined X and Y Torque inputs
- o Damping estimates from SRA are satisfactory with 2000 pts of P-MASS Input.
- o SRA gives satisfactory identification results with single output.

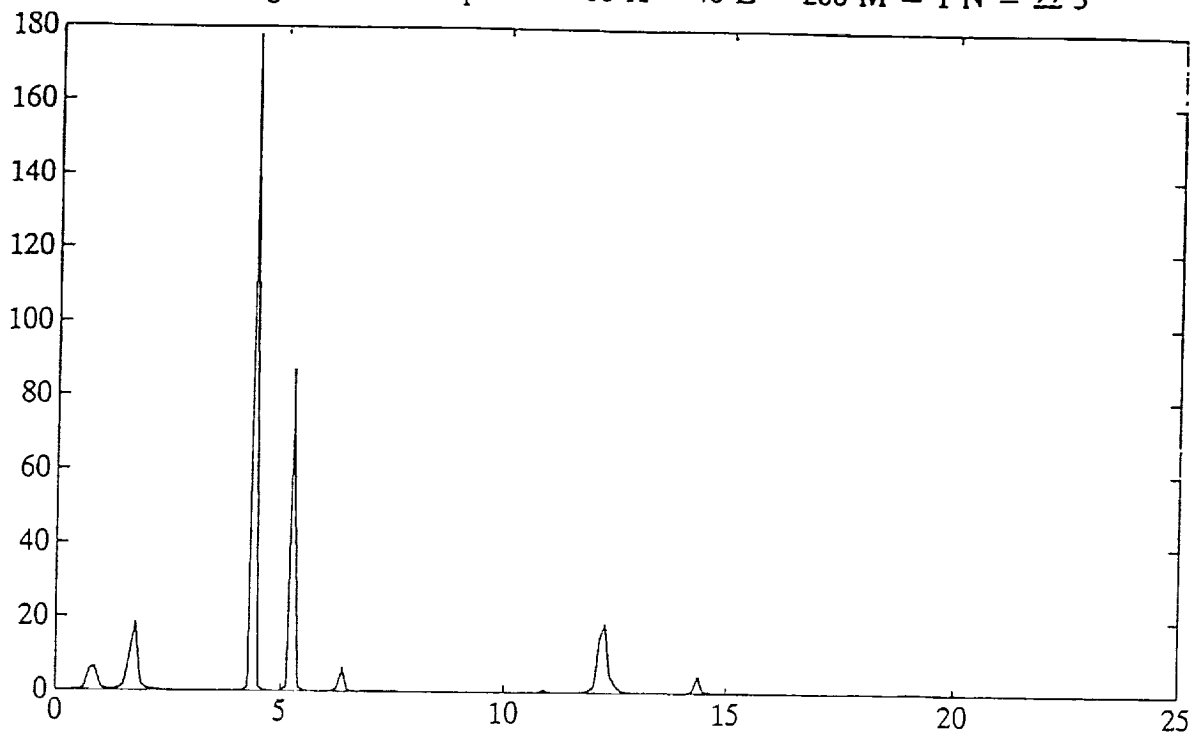
AFAL GRID



↑
 → X Z - OUT OF PAPER (NORMAL TO GRID)

NODES (1) - (5) HAVE NO DISPLACEMENT AND NO ROTATION.

Grid avg 100Hz 24000pts J = 60 K = 40 L = 200 M = 1 N = 22 3



New spectrum calculation

870

ans =

Grid avg 100Hz 24000pts J = 60 K = 40 L = 200 M = 1 N = 22

ans =

Magnitude	Phase	Freq	Damping	Air Force Values	
-----------	-------	------	---------	------------------	--

ans =

0.9978	1.1919	18.9699	0.0019	0.8100	0.0023
0.9978	-1.1919	18.9699	0.0019	1.6610	0.0016
0.9987	1.1350	18.0648	0.0011	4.3550	0.0013
0.9987	-1.1350	18.0648	0.0011	5.2120	0.0017
0.9980	0.9013	14.3439	0.0023	6.2910	0.0013
0.9980	-0.9013	14.3439	0.0023	10.7550	0.0011
0.9986	0.7822	12.4486	0.0018	12.1530	0.0016
0.9986	-0.7822	12.4486	0.0018	12.3840	0.0016
0.9986	0.7659	12.1900	0.0018	14.2540	0.0019
0.9986	-0.7659	12.1900	0.0018	17.9550	0.0015
0.9991	0.6813	10.8428	0.0013	18.7720	0.0010
0.9991	-0.6813	10.8428	0.0013	0	0
0.9994	0.1066	1.6967	0.0053	0	0
0.9994	-0.1066	1.6967	0.0053	0	0
0.9993	0.0512	0.8142	0.0132	0	0
0.9993	-0.0512	0.8142	0.0132	0	0
0.9991	0.3986	6.3441	0.0022	0	0
0.9991	-0.3986	6.3441	0.0022	0	0
0.9994	0.3292	5.2401	0.0019	0	0
0.9994	-0.3292	5.2401	0.0019	0	0
0.9994	0.2750	4.3775	0.0021	0	0
0.9994	-0.2750	4.3775	0.0021	0	0

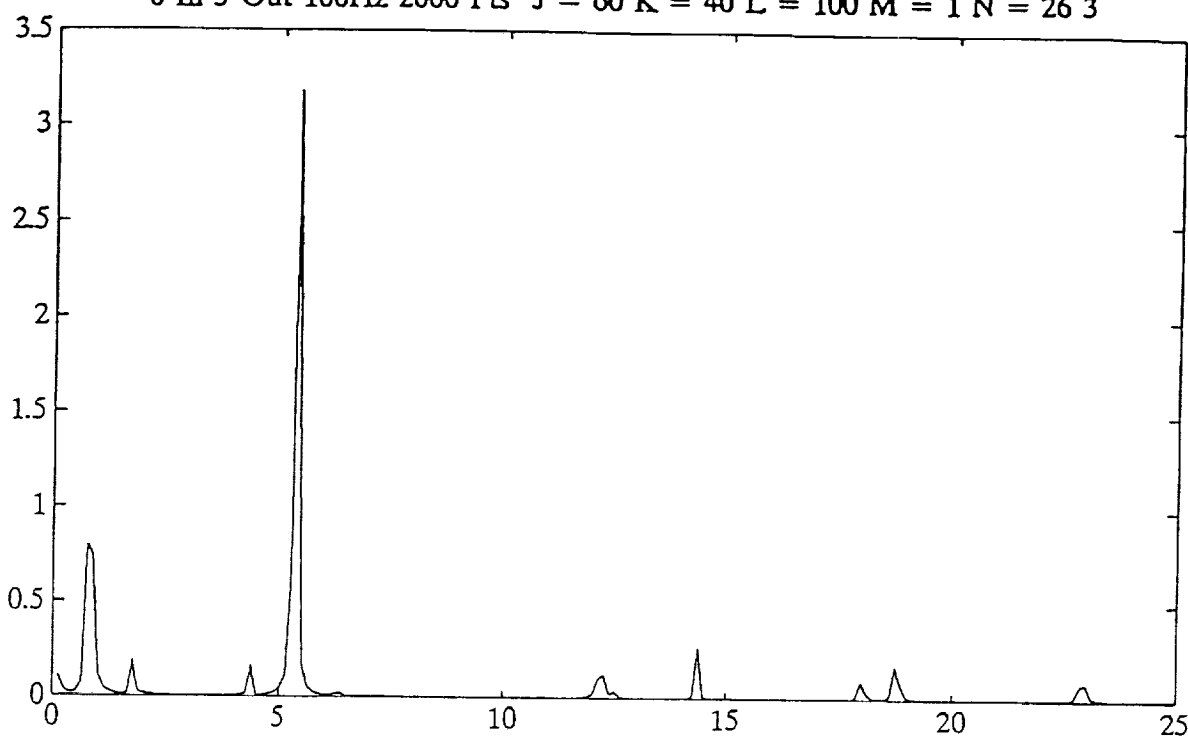
ans =

Singular Values

ans =

1.0000	7.4654
2.0000	6.0473
3.0000	4.9555
4.0000	3.1561
5.0000	2.9364
6.0000	2.8078
7.0000	2.4539
8.0000	2.3086
9.0000	1.0343
10.0000	0.9902
11.0000	0.7768
12.0000	0.7767
13.0000	0.7255
14.0000	0.7051
15.0000	0.1895
16.0000	0.1849

0 In 5 Out 100Hz 2000 Pts J = 60 K = 40 L = 100 M = 1 N = 26 3



ans =

0 In 5 Out 100Hz 2000 Pts J = 40 K = 20 L = 100 M = 1 N = 26

ans =

Magnitude	Phase	Freq	Damping	Air Force Values	Computed Values
-----------	-------	------	---------	------------------	-----------------

ans =

0.9853	1.4770	23.5089	0.0100	0.8100	0.0023	.0019
0.9853	-1.4770	23.5089	0.0100	1.6610	0.0016	.0078
0.0163	3.1416	82.3967	0.7948	4.3550	0.0013	.0024
0.9984	1.1810	18.7954	0.0013	5.2120	0.0017	.0023
0.9984	-1.1810	18.7954	0.0013	6.2910	0.0013	.0044
0.9985	1.1336	18.0411	0.0014	10.7550	0.0011	.0022
0.9985	-1.1336	18.0411	0.0014	12.1530	0.0016	.0036
0.9989	0.9029	14.3699	0.0013	12.3840	0.0016	.0017
0.9989	-0.9029	14.3699	0.0013	14.2540	0.0019	.0013
0.9986	0.7821	12.4471	0.0017	17.9550	0.0015	.0014
0.9986	-0.7821	12.4471	0.0017	18.7720	0.0010	.0013
0.9972	0.7675	12.2151	0.0036	0	0	
0.9972	-0.7675	12.2151	0.0036	0	0	
0.9985	0.6840	10.8855	0.0022	0	0	
0.9985	-0.6840	10.8855	0.0022	0	0	
0.9982	0.3981	6.3367	0.0044	0	0	
0.9982	-0.3981	6.3367	0.0044	0	0	
0.9992	0.3355	5.3394	0.0023	0	0	
0.9992	-0.3355	5.3394	0.0023	0	0	
0.9993	0.2752	4.3797	0.0024	0	0	
0.9993	-0.2752	4.3797	0.0024	0	0	
0.9992	0.1078	1.7151	0.0078	0	0	
0.9992	-0.1078	1.7151	0.0078	0	0	
0.9994	0	0.0099	1.0000	0	0	
0.9990	0.0508	0.8082	0.0190	0	0	
0.9990	-0.0508	0.8082	0.0190	0	0	

ans =

Singular Values

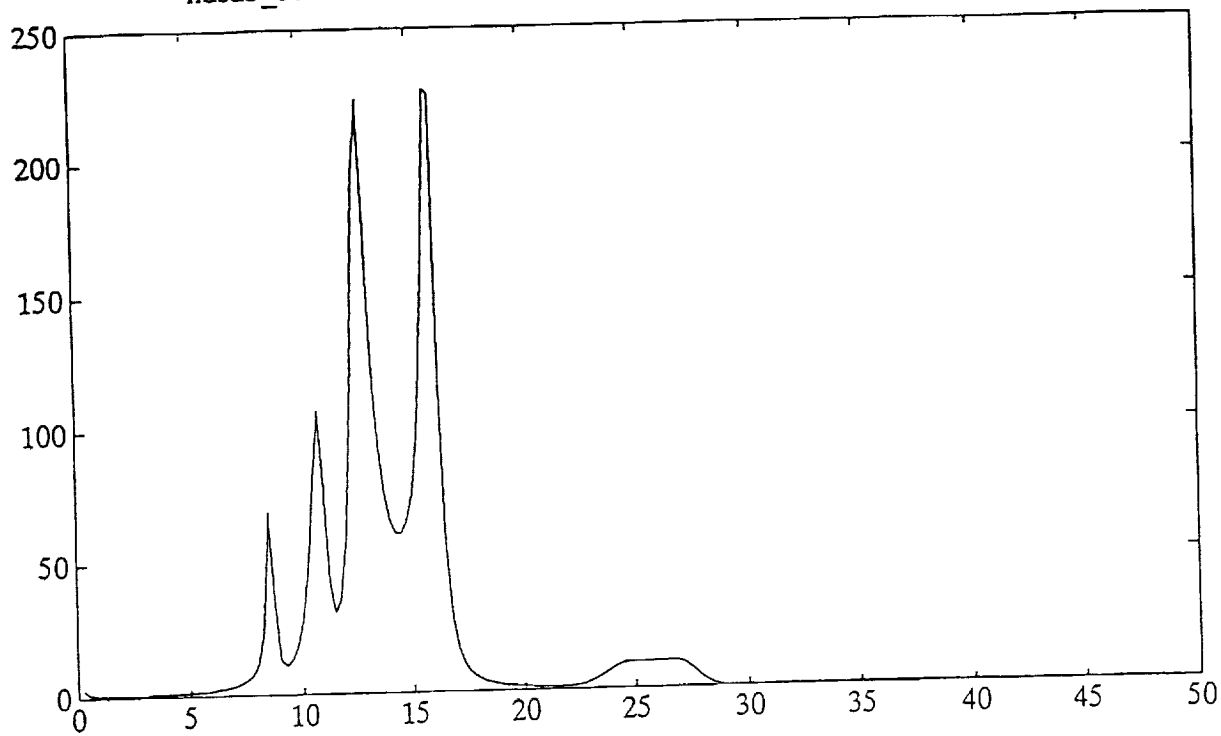
ans =

1.0000	2.5860
2.0000	2.3951
3.0000	2.3470
4.0000	2.0355
5.0000	1.9648
6.0000	0.2399
7.0000	0.2144
8.0000	0.2021
9.0000	0.1929
10.0000	0.1778
11.0000	0.0959
12.0000	0.0919

X-29 Flutter Flight Test Data

- o Data provided by Mike Kehoe, NASA Dryden (33,000 points, 400hz).
- o X-29 Flutter modes are excited by natural turbulence
- o SRA identifies all the modes from each data channel.
- o Damping and frequency estimates in good agreement with NASA values.
- o SRA parameter estimates based on 2,000 points are satisfactory.

nasa3_33000 rwtfl s0 J = 130 K = 70 L = 100 M = 1 N = 14 1



New spectrum calculation

ans =

nasa3_33000 rwtfl s0 J = 130 K = 70 L = 100 M = 1 N = 14

ans =

Magnitude	Phase	Freq	Damping	Nasa Values	
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ans =

0.6474	0	27.6811	1.0000	8.5900	.0182
0.9845	0.4328	27.5681	0.0361	10.6400	.0370
0.9845	-0.4328	27.5681	0.0361	12.7900	.0224
0.9774	0.3760	23.9821	0.0606	15.7200	.0286
0.9774	-0.3760	23.9821	0.0606	0	0
1.0000	0	0.0023	1.0000	0	0
0.9941	0.2502	15.9314	0.0236	0	0
0.9941	-0.2502	15.9314	0.0236	0	0
0.9924	0.1979	12.6095	0.0387	0	0
0.9924	-0.1979	12.6095	0.0387	0	0
0.9973	0.1348	8.5838	0.0197	0	0
0.9973	-0.1348	8.5838	0.0197	0	0
0.9936	0.1694	10.7909	0.0377	0	0
0.9936	-0.1694	10.7909	0.0377	0	0

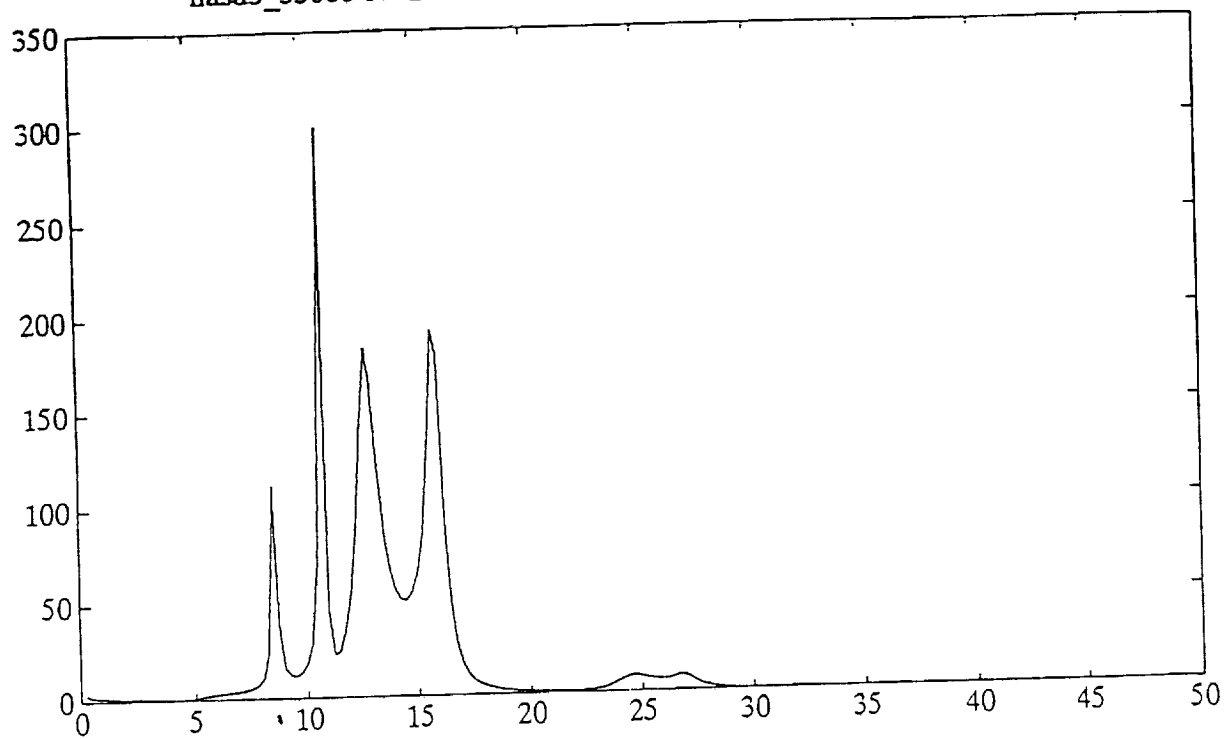
ans =

Singular Values

ans =

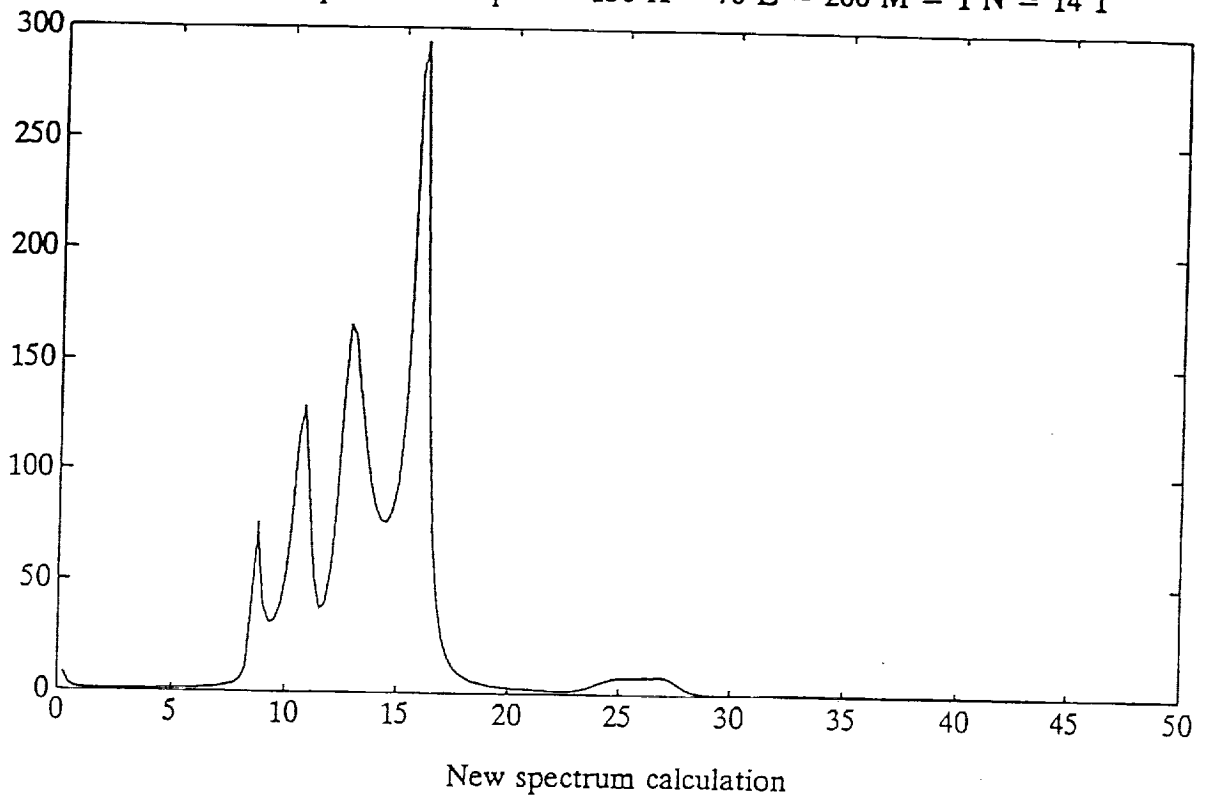
1.0000	60.7314
2.0000	41.1338
3.0000	38.8540
4.0000	23.9111
5.0000	23.5237
6.0000	5.4193
7.0000	5.0483
8.0000	4.6103
9.0000	4.4074
10.0000	3.6840
11.0000	3.4194
12.0000	1.2080
13.0000	1.0580
14.0000	0.6707
15.0000	0.6336
16.0000	0.4258
17.0000	0.3790
18.0000	0.3534
19.0000	0.3174
20.0000	0.1659
21.0000	0.1548
22.0000	0.1426
23.0000	0.1332
24.0000	0.1189

nasa3_33000 rwtfl s0 J = 130 K = 70 L = 100 M = 1 N = 20 1



New spectrum calculation

rwtafl 11000pts 400Hz skip J = 130 K = 70 L = 200 M = 1 N = 14 1



CONCLUSIONS

1. BAYESIAN STATISTICAL DECISION THEORY PROVIDES A UNIFYING FRAMEWORK FOR SYSTEM IDENTIFICATION TECHNIQUES. FUNDAMENTAL SIMILARITIES ALSO EXIST BETWEEN STRUCTURAL MODE IDENTIFICATION AND EXTRACTION OF SINUSOIDS FROM NOISE. (APPLICATION IN RADAR AND SONAR SIGNAL PROCESSING)
2. STOCHASTIC REALIZATION ALGORITHM (SRA) IS HIGHLY ROBUST AGAINST NOISE. IT PROVIDES COMPLETE IDENTIFICATION OF MULTI-INPUT MULTI-OUTPUT SYSTEMS IN A FULLY STOCHASTIC SETTING.
3. SRA OUTPERFORMS ERA UNDER CONDITIONS OF
 - o LOW SIGNAL TO NOISE RATIO
 - o SHORT DATA LENGTHS
 - o POOR INPUT EXCITATION
 - o SINGLE OUTPUT MODELING
4. SRA CAN IDENTIFY SYSTEMS WITHOUT INPUT MEASUREMENTS. NO PRIOR KNOWLEDGE OF SYSTEM MODEL STRUCTURE AND ORDER REQUIRED.
5. SRA STATE SPACE MODELS ARE IN NESTED KALMAN FILTER FORM. LOWER ORDER MODELS ARE OBTAINED FROM HIGHER ORDER MODELS BY SIMPLE DELETION OF ROWS AND COLUMNS. NOISE COVARIANCES ARE IDENTIFIED ALONG WITH SYSTEM MATRICES.
6. MODEL ORDER DETERMINATION IS DONE EASILY USING BREAKS IN SINGULAR VALUES AND AIC (AKAIKE INFORMATION CRITERION).

7. AS SYSTEM ORDER INCREASED, FREQUENCIES WITH LARGE SPECTRAL POWER ARE IDENTIFIED FIRST.
8. SEPARATION OF CLOSELY SPACED FREQUENCIES REQUIRES INCREASE IN THE DIMENSION OF THE HANKEL MATRIX, BUT NOT THE SYSTEM DIMENSION.
9. SURPRISING RESULTS:
 - o SATISFACTORY FREQUENCY AND DAMPING IDENTIFICATION CAN BE DONE FROM SINGLE OUTPUT DATA SERIES.
 - o FREQUENCY AND DAMPING PARAMETERS IDENTIFIED FOR THE 300% NOISE CASE (MEMBRANE SIMULATION). THE NOISE CAN BE MULTIPLICATIVE!
 - o THE IMPROVEMENT IN IDENTIFICATION RESULTS USING INPUT DATA IS MINOR.
10. IMPORTANCE FOR CONTROL DESIGN:

ADAPTIVE CONTROL IS FEASIBLE SINCE MODES WITH DISCERNIBLE SPECTRAL POWER CAN BE IDENTIFIED USING SRA. SINCE IT IS NOT POSSIBLE TO DESIGN CONTROLLERS, WHICH ARE ROBUST AGAINST ALL POSSIBLE UNCERTAINTIES, ADAPTIVE CONTROL OF STRUCTURAL MODES USING ON-LINE IDENTIFICATION IS A MORE PRACTICAL APPROACH.

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