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Formation of Planetesimals

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ABSTRACT

A widely accepted model for the formation of planetesimals is by gravitational instability of a dust layer in the central plane of the solar nebula. This mechanism does not eliminate the need for physical sticking of particles, despite published claims to that effect. Such a dust layer is extremely sensitive to turbulence, which would prevent gravitational instability unless coagulation forms bodies large enough to decouple from the gas (> meter-sized). Collisional accretion driven by differential motions due to gas drag may bypass gravitational instability completely. Previous models of coagulation assumed that aggregates were compact bodies with uniform density, but it is likely that early stages of grain coagulation produced fractal aggregates having densities that decreased with increasing size. Fractal structure, even if present only at sub-millimeter size, greatly slows the rate of coagulation due to differential settling and delays the concentration of solid matter to the central plane. Low-density aggregates also maintain higher opacity in the nebula than would result from compact particles. The size distribution of planetesimals and the time scale of their formation depend on poorly understood parameters, such as sticking mechanisms for individual grains, mechanical properties of aggregates, and the structure of the solar nebula.

INTRODUCTION

It is now generally accepted that the terrestrial planets and the cores of the giant planets were formed by accretion of smaller solid bodies

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(planetesimals). The principal alternative, production of giant protoplanets by large-scale gravitational instabilities of the gaseous component of the solar nebula, has been abandoned (Cameron 1988). Planetesimals, with initial sizes of the order of kilometers, must have formed from much smaller particles, perhaps consisting of a mixture of surviving presolar grains and condensates from the nebular gas. Such grains were small, probably sub-μm in size, and their motions were controlled by the drag of the surrounding gas, rather than by gravitational forces. It is necessary to understand the aerodynamic processes that affected these small bodies in order to understand how planetesimals formed.

The most common assumption is that planetesimals resulted from localized gravitational instabilities within a dust layer in the central plane of the nebular disk. Such a layer is assumed to form by settling of grains through the quiescent gas due to the vertical component of solar gravity. If the dust layer becomes sufficiently thin, and thereby sufficiently dense, it is unstable with respect to density perturbations. This instability causes the layer to fragment into self-gravitating clumps. These eventually collapse into solid bodies, i.e., planetesimals. This process was described qualitatively as early as 1949 (Wetherill 1980). Quantitative expressions for the critical density and wavelength were derived by Safronov (1969) and independently by Goldreich and Ward (1973). The critical density is approximately the Roche density at the heliocentric distance a,

$$\delta_c \simeq 3M_\odot/2\pi a^3, \tag{1}$$

where $M_\odot$ is the solar mass. Perturbations grow in amplitude if they are smaller in size than a critical wavelength

$$\lambda_c \simeq 4\pi^2 G\sigma_s/\Omega^2, \tag{2}$$

where σ_s is the surface density of the dust layer, G the gravitational constant, and $\Omega = (GM_\odot/a^3)^{1/2}$ is the Kepler frequency. The characteristic mass of a condensation is $m_c \sim \sigma_s\lambda_c^2$, and depends only on the two parameters σ and a. The assumption that σ_s equals the heavy element content of a planet spread over a zone surrounding its orbit (Weidenschilling 1977a) implies $\sigma_s \sim 10 g\ cm^{-2}$ and $m_c \sim 10^{18} g$ in the Earth's zone. Density perturbations grow rapidly, on the time scale of the orbital period, but condensations cannot collapse directly to solid bodies without first losing angular momentum. Goldreich and Ward (1973) estimated the contraction time to be $\sim 10^3$ years in Earth's zone, while Pechernikova and Vityazev (1988) estimate $\sim 10^5 - 10^6$ years.

The assumption that planetesimals formed in this manner has influenced the choice of starting conditions for numerical simulations of planetary accretion. Many workers (Greenberg *et al.* 1978; Nakagawa *et al.*

1983; Horedt 1985; Spaute *et al.* 1985; Wetherill and Stewart 1989) have assumed an initial swarm of roughly kilometer-sized bodies of uniform size or with a narrow size distribution. This assumption was also due to the lack of alternative models.

Another reason for popularity of the gravitational instability model was the belief that it requires no mechanism for physical sticking of grains, as was explicitly stated by Goldreich and Ward. Because the physical and chemical properties of grains in the solar nebula are poorly characterized, sticking mechanisms seem *ad hoc*. Still, there are reasons to believe that grains in the solar nebula did experience sticking. It is common experience in the laboratory (and Earth's atmosphere) that microscopic particles adhere on contact due to electrostatic or surface forces. Weidenschilling (1980) argued that for typical relative velocities due to settling in the solar nebula, van der Waals forces alone would allow aggregates to reach centimeter sizes. There are additional arguments that coagulation must have produced larger bodies, of meter size or larger, before gravitational instability could produce planetesimals. In order to understand these arguments, it is necessary to review the nature of the aerodynamic interactions between solid bodies and gas in the solar nebula.

NEBULAR STRUCTURE

We assume that the solar nebula is disk-shaped, has approximately Keplerian rotation, and is in hydrostatic equilibrium. If the mass of the disk is much less than the solar mass ($< 0.1 M_\odot$), then its self-gravity can be neglected compared with the vertical component of the Sun's attraction, $g_z = GM_\odot z/a^3 = \Omega^2 z$, where z is the distance from the central plane. The condition of hydrostatic equilibrium implies that the pressure at $z = 0$ is

$$P_c = \Omega \Sigma c/4, \qquad (3)$$

where Σ is the surface density of the disk and c is the mean thermal velocity of the gas molecules. Equation (3) is strictly true only if the temperature is independent of z, but it is a good approximation even if the vertical structure is adiabatic. It can be shown that

$$P(z) = P_c \exp\left(-z^2/H^2\right), \qquad (4)$$

where $H = \pi c/2\Omega$ is the characteristic half-thickness or scale height.

There is also a radial pressure gradient in the disk. It is plausible to assume that the temperature and density decrease with increasing heliocentric distance. Some accretion disk models of the nebula have Σ approximately constant, but equation (3) shows that even these will have a strong pressure gradient, because Ω is proportional to $a^{-3/2}$ for Keplerian

motion. The decrease in pressure is due primarily to the weakening of solar gravity at larger distances. Because the gas is partially supported by the pressure gradient in addition to the rotation of the disk, hydrostatic equilibrium requires that its velocity be less than Keplerian;

$$V_g^2 = V_k^2 + \frac{a}{\rho}\frac{\partial P}{\partial a}, \tag{5}$$

where ρ is the gas density. We define $\Delta V = V_k - V_g$ as the difference between the Kepler velocity and the gas velocity. One can show (Weidenschilling, 1977b) that if one assumes a power law for the pressure gradient, so that $P \propto a^{-n}$, then the fractional deviation of the gas from V_k is

$$\frac{\Delta V}{V_k} = \frac{nRT/\mu}{2GM_\odot/a}, \tag{6}$$

where R is the gas constant and μ the molecular weight. We see that $\Delta V/V_k$ is approximately the ratio of thermal and gravitational potential energies of the gas. For most nebular models, this quantity is only a few times 10^{-3}, but this is enough to have a significant effect on the dynamics of solid bodies embedded in the gas.

AERODYNAMICS OF THE SOLID BODIES IN THE NEBULA

The effects of gas on the motions of solid bodies in the solar nebula have been described in detail by Adachi et al. (1976) and Weidenschilling (1977b); here we summarize the most important points. The fundamental parameter that characterizes a particle is its response time to drag,

$$t_e = mV/F_D, \tag{7}$$

where m is the particle mass, V its velocity relative to the gas, and F_D is the drag force. The functional form of F_D depends on the Knudsen number (ratio of mean free path of gas molecules to particle radius) and Reynolds number (ratio of inertial to viscous forces). In the solar nebula, the mean free path is typically a few centimeters, so dust particles are in the free-molecular regime. In that case, a spherical particle of radius s, bulk density ρ_s, has

$$t_e = s\rho_s/\rho c. \tag{8}$$

The dynamical behavior of a particle depends on the ratio of t_e to its orbital period, or, more precisely, to the inverse of the Kepler frequency. A "small" particle, for which $\Omega t_e \ll 1$, is coupled to the gas, i.e., the drag force dominates over solar gravity. It tends to move at the angular velocity

of the gas. The residual radial component of solar gravity causes inward radial drift at a terminal velocity given by

$$V_r = -2\Omega \Delta V t_e. \tag{9}$$

Similarly, there is a drift velocity due to the vertical component of solar gravity,

$$V_z = -\Omega^2 z t_e. \tag{10}$$

For a "large" body with $\Omega t_e \gg 1$ gas drag is small compared with solar gravity. Such a body pursues a Kepler orbit. Because the gas moves more slowly, the body experiences a "headwind" of velocity ΔV. The drag force causes a gradual decay of its orbit at a rate

$$V_r = da/dt = -2\Delta V / \Omega t_e. \tag{11}$$

For plausible nebular parameters, and particle densities of a few g cm^{-3}, the transition between "small" and "large" regimes occurs at sizes of the order of one meter. The peak radial velocity is equal to ΔV when $\Omega t_e = 1$. The dependence of a radial and transverse velocities on particle size are shown for a typical case in Figure 1.

PROBLEMS WITH GRAVITATIONAL INSTABILITY

Could the gravitational instability mechanism completely eliminate the need for particle coagulation? We first consider the case in which there is no turbulence in the gas. The time scale for a particle to settle toward the central plane of the nebula is $\tau_z = z/V_z$. From equations (8) and (10),

$$\tau_z = \rho c / s \rho_s \Omega^2. \tag{12}$$

If we take $\rho \sim 10^{-10}$g cm^{-3}, $\tau_z \sim (10^2/s)$ years, where s is in cm, so if s $= 1$ μm, $\tau_z \sim 10^6$ years. This is merely the e-folding time for z to decrease. For an overall solids/gas mass ratio f $= 3 \times 10^{-3}$, corresponding to the cosmic abundance of metal plus silicates, and an initial scale height H $\sim c/\Omega$, the dust layer requires ~ 10 $\tau_z \sim 10^7$ years to become thin enough to be gravitationally unstable. This exceeds the probable lifetime of a circumstellar disk. Undifferentiated meteorites and interplanetary dust particles include sub-μm sized components. These presumably had to be incorporated into their parent bodies not as separate grains, but as larger aggregates, which settled more rapidly.

In addition to the problem of settling time scale, there is another argument for growth of particles by sticking. A very slight amount of turbulence in the gas would suffice to prevent gravitational instability.
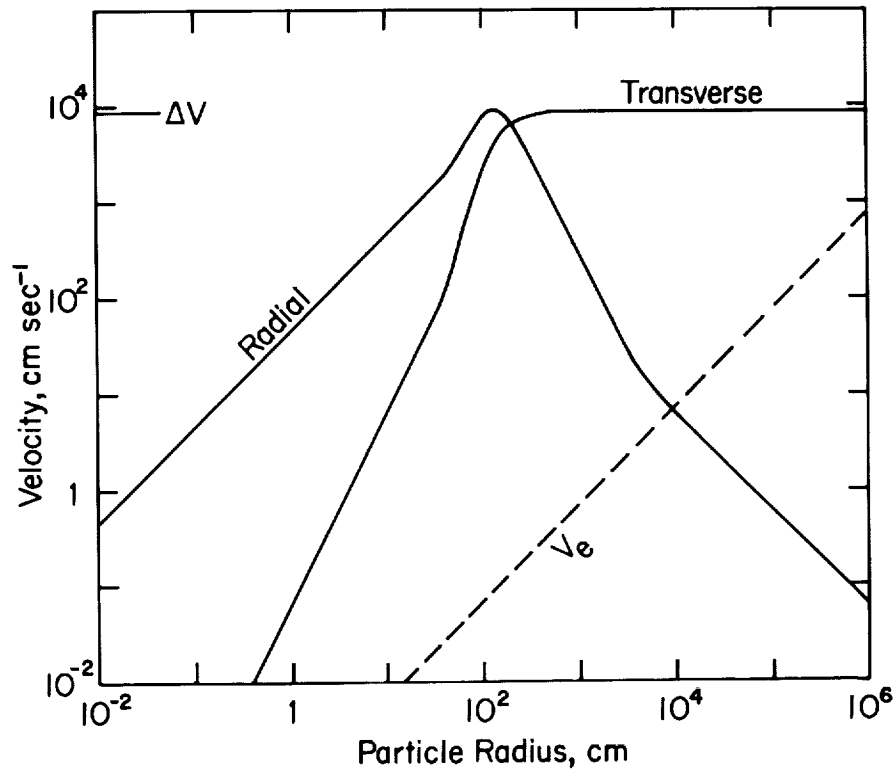
FIGURE 1   Radial and transverse velocities relative to the surrounding gas for a spherical body with density 1 g/cm^3. Also shown is the escape velocity from the body's surface, V_e. Numerical values are for the asteroid zone (Weidenschilling 1988) but behavior is similar for other parts of the nebula.

Particles respond to turbulent eddies that have lifetimes longer than $\sim t_e$. The largest eddies in a rotating system generally have timescales $\sim 1/\Omega$, so bodies that are "small" in the dynamical sense of $\Omega t_e < 1$, or less than about a meter in size, are coupled to turbulence. A particle would tend to settle toward the central plane until systematic settling velocity is of the same order as the turbulent velocity, V_t. From this condition, we can estimate the turbulent velocity that allows the dust layer to reach a particular density (Weidenschilling 1988). In order to reach a dust/gas ratio of unity

$$V_t \sim f\Omega s \rho_s/\rho. \tag{13}$$

Typical parameters give $V_t \sim (s\rho_s)$ cm sec^{-1} when s and ρ_s are in cgs units, i.e., if particles have density of order unity, then the turbulent velocity of the gas must be no greater than about one particle diameter per second in order

for the space density of solids to exceed that of the gas. In order to reach the critical density for gravitational instability, the dust density must exceed that of the gas by about two orders of magnitude, with correspondingly smaller V_t, $\sim 10^{-2}$ particle diameters per second. For μm-sized grains, this would imply turbulent velocities of the order of a few meters *per year*, which seems implausible for any nebular model.

Even if the nebula as a whole was perfectly laminar, formation of a dense layer of particles would create turbulence. If the solids/gas ratio exceeds unity, and the particles are strongly coupled to the gas by drag forces, then the layer behaves as a unit, with gas and dust tending to move at the local Kepler velocity. There is then a velocity difference of magnitude ΔV between the dust layer and the gas on either side. Goldreich and Ward (1973) showed that density stratification in the region of shear would not suffice to stabilize it, and this boundary layer would be turbulent. Weidenschilling (1980) applied a similar analysis to the dust layer itself, and showed that the shear would make it turbulent as well. Empirical data on turbulence within boundary layers suggest that the eddy velocities within the dust layer would be a few percent of the shear velocity ΔV, or several meters per second. The preceding analysis argues that gravitational instability would be possible only if the effective particle size were of the order of a meter or larger. Bodies of this size must form by coagulation of the initial population of small dust grains.

PARTICLE GROWTH BY COAGULATION

If it is assumed that particles stick upon contact, then their rate of growth can be calculated. The rate of mass gain is proportional to the product of the number of particles per unit volume, their masses and relative velocities, and some collisional cross-section. We assume that the latter is simply the geometric cross-section, $\pi(s_1 + s_2)^2$ (although electrostatic or aerodynamic effects may alter this in some cases). For small particles (less than a few tens of μm), thermal motion dominates their relative velocities. The mean thermal velocity is $\overline{v} = (3kT/m)^{1/2}$, where T is the temperature and k is Boltzmann's constant. If we assume that all particles have the same radius s (a reasonable approximation, as thermal coagulation tends to produce a narrow size distribution), the number of particles per unit volume is $N = 3f\rho/4\pi\rho_s s^3$. The mean particle size increases with time as

$$s(t) = s_o + \left[15f\rho \left(kT/8\pi p_s^3\right)^{1/2} t\right]^{2/5} \tag{14}$$

As a particle grows its thermal motion decreases, while its settling rate increases. When settling dominates, a particle may grow by sweeping up

smaller ones (particles of the same size, or t_e, have the same settling rate, and hence do not collide). If it is much larger than its neighbors, then the relative velocity is approximately the settling rate of the larger particle. It grows at the rate

$$\frac{ds}{dt} = f\rho V_z/4\rho_s = f\Omega^2 zs/4c, \tag{15}$$

giving

$$s(t) = s_o \exp\left(f\Omega^2 zt/4c\right). \tag{16}$$

A particle growing by this mechanism increases in size exponentially on a time scale $\tau_g = 4c/f\Omega^2 z$, or $\sim 4/f\Omega$ at $z \sim c/\Omega$. This time scale is a few hundred years at $a = 1$ AU, and is independent of the particle density or the gas density. The lack of dependence on particle density is due to the fact that a denser particle settles faster, but has a smaller cross-section, and can sweep up fewer grains; the two effects exactly cancel one another. Likewise, if the gas density is increased, the settling velocity decreases, but the number of accretable particles increases, provided f is constant. The time scale increases with heliocentric distance; $\tau_g \propto a^{3/2}$. A particle settling vertically from an initial height z_o and sweeping up all grains that it encounters can grow to a size $s(\text{max}) = f\rho z_o/4\pi_s$, typically a few cm for $z_o \sim c/\Omega$. Actually, vertical settling is accompanied by radial drift, so growth to larger sizes is possible.

The growth rates and settling rates mentioned above have been used (with considerable elaboration) to construct numerical models of particle evolution in a laminar nebula (Weidenschilling 1980; Nakagawa *et al.* 1981). The disk is divided into a series of discrete levels. In each level the rate of collisions between particles of different sizes is evaluated, and the changes in the size distribution during a timestep Δt is computed. Then particles are distributed to the next lower level at rates proportional to their settling velocities. A typical result of those simulations shows particle growth dominated by differential settling. Because the growth rate increases with z, large particles form in the higher levels first, and "rain out" toward the central plane through the lower levels. The size distribution remains broad, with the largest particles much larger than the mean size, justifying the assumptions used in deriving equation (16). After $\sim 10 \; \tau_g$, typically a few thousand orbital periods, the largest bodies exceed one meter in size and the solids/gas ratio in the central plane exceeds unity. The settling is non-homologous, with a thin dense layer of large bodies containing $\sim$ 1-10% of the total surface density of solids, and the rest in the form of small aggregates distributed through the thickness of the disk.

The further evolution of such a model population has not yet been calculated. The main difficulty is the change in the nature of the interaction

between particles and gas when the solids/gas ratio exceeds unity. As mentioned previously, the particle layer begins to drag the gas with it. The relative velocities due to drag no longer depend only on t_e, as in equations (7-11), but on the local concentration of solids: there is also shear between different levels. Work is presently under way to account for these effects, at least approximately. A complete treatment may require the use of large computers using computational fluid dynamics codes.

PROPERTIES OF FRACTAL AGGREGATES

The earlier numerical simulations mentioned above assumed that all particles are spherical and have the same density, regardless of size. This is a good assumption for coagulation of the liquid drops, but it does not apply to solid particles. When two grains stick together, they retain their identities, and the combined particle is not spherical. Aggregates containing many grains have porous, fluffy structures. It has been shown (Mandelbrot 1982; Meakin 1984) that such aggregates have fractal structures. A characteristic of fractal aggregates is that the average number of particles found within a distance s of any arbitrary point inside the aggregate varies as $n(s) \propto s^D$, where D is the fractal dimension. The density varies with size according to the relation $\rho \propto s^{D-3}$. "Normal" objects have $D = 3$ and uniform density. Aggregates of particles generally have $D \simeq 2$, so that their density varies approximately inversely with size, i.e., they become more porous as they grow larger. For the most simple aggregation processes (Jullien and Botet 1986; Meakin 1988a,b) the fractal dimension lies in the range $1.7 < D < 2.2$, but more complex mechanisms can lead to values of D lying outside of this range. A hierarchical ballistic accretion in which clusters of similar size stick at their point of contact yields $D < 2$. Building up a cluster of successive accretion of single grains or small groups, or allowing compaction after contact, leads to $D > 2$. An example of this type is shown in Figure 2. This is a computer-generated model, but it corresponds closely to soot particles observed in the laboratory (Meakin and Donn 1988).

The structure of aggregates, which are very unlike uniform-density spheres, affects their aerodynamic behavior in the solar nebula. For a compact sphere, the response time is proportional to the size (equation 8). For a fractal aggregate, t_e increases much more slowly with size. Meakin has developed computer modeling procedures to determine the mean projected area of an aggregate, as viewed from a randomly selected direction (Meakin and Donn 1988; Meakin, unpublished). The variation of the projected area with the number of grains in the aggregate depends on the fractal dimension. If $D < 2$, aggregates become more open in structure at larger sizes, and are asymptotically "transparent," i.e., the ratio of mass to projected area never exceeds a certain limit. For $D > 2$, large aggregates
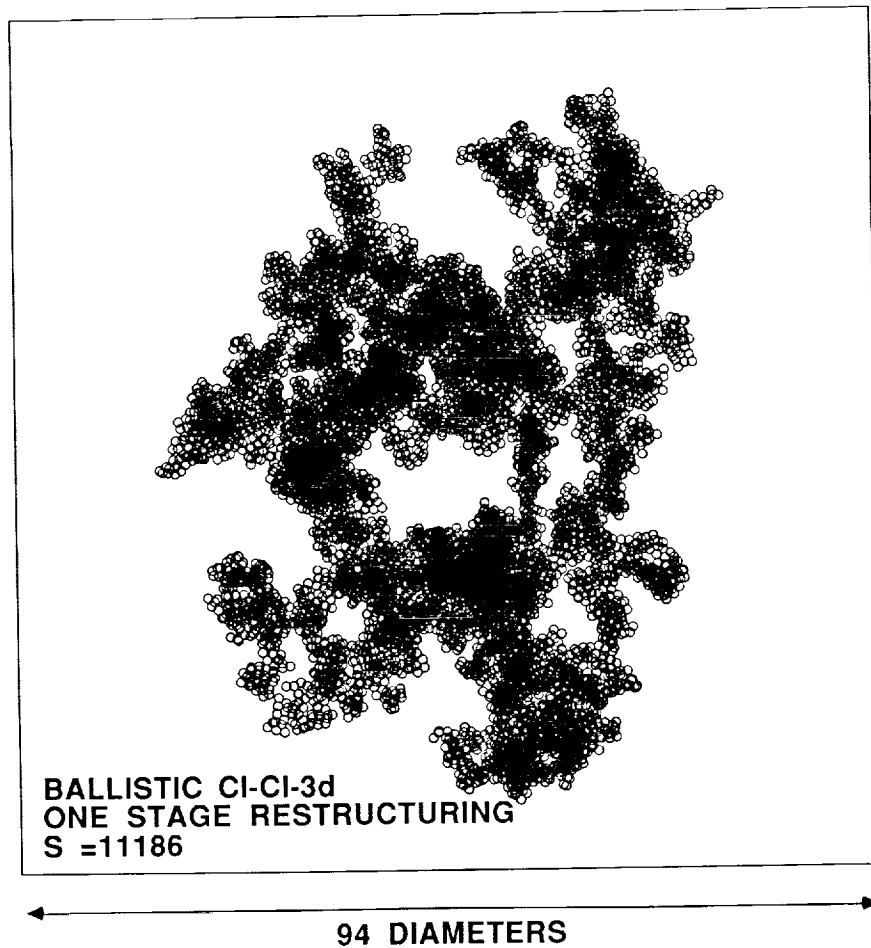
BALLISTIC CI-CI-3d
ONE STAGE RESTRUCTURING
S =11186

94 DIAMETERS

FIGURE 2   View of a computer-generated aggregate of fractal dimension $\simeq$ 2.11 and containing $\simeq$ 10^4 individual grains.

are opaque; the mass/area ratio increases without limit, although more slowly than for a uniform density object.

We assume that in the free molecular regime, when aggregates are smaller than the mean free path of a gas molecule (> cm in typical nebular models), t_e is proportional to the mass per unit projected area (m/A). From the case of a spherical particle in this regime, as in equation (8), we infer

$$t_e = 3(m/A)/4\rho c. \tag{17}$$

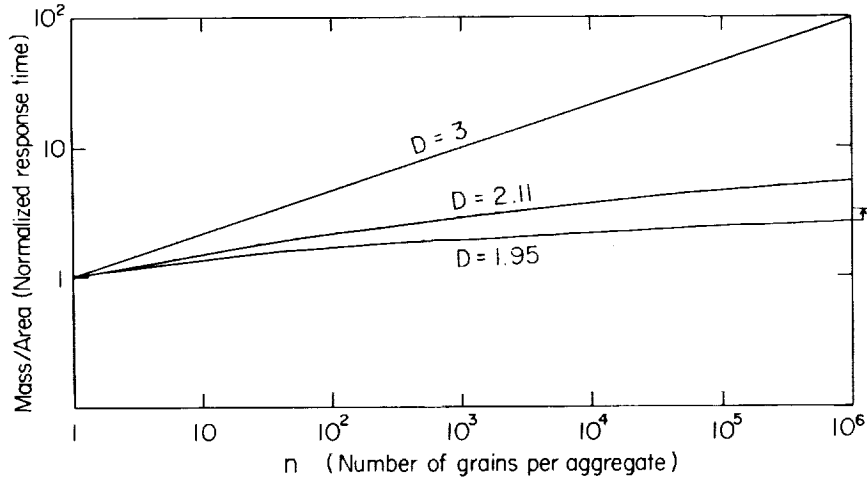We can express t_e for aggregates conveniently in terms of the value

FIGURE 3 Mass/area ratio vs. number of grains for aggregates of different dimensions. $D = 3$ assumes coagulation produces a spherical body with the density of the separate components (liquid drop coalescence). For $D > 2$, $m/A \propto m^{1-2/D}$, or $\propto m^{1/3}$ for $D = 3$, and $\propto m^{0.052}$ for $D = 2.11$. For $D < 2$, m/A approaches an asymptotic value, shown by the arrow.

for an individual grain, of size s_o and density ρ_o, $t_{eo} = s_o \rho_o / \rho c$. Fits to Meakin's data give

$$t_e = t_{eo}(0.343n^{-0.052} + 0.684n^{-0.262}) \qquad (18)$$

for $D = 2.11$, where n is the number of grains in the aggregate. This relation is plotted in Figure 3. For large aggregates of 10^6 grains, the mass/area ratio and t_e are $\sim$ 20-30 times smaller than for a compact particle of equal mass.

Fractal properties of aggregates also have implications for the opacity of the solar nebula. The dominant source of opacity is solid grains (Pollack *et al.* 1985). The usual assumption in computing opacity is that particles are spherical (Weidenschilling 1984). Their shapes are unimportant if they are smaller than the wavelength considered (Rayleigh limit). However, when sizes are comparable to the wavelength, the use of Mie scattering theory for spherical particles is inappropriate. In the limit of geometrical optics when particles are large compared with the wavelength, the opacity varies inversely with the mass/area ratio. From Figure 3 we see that if aggregates of 10^6 grains are in the geometrical optics regime, then the opacity is $\sim$ 20-30 times greater than for compact spherical particles of equal mass. Actually, if individual grains are below the Rayleigh limit, aggregates may not be in the geometrical optics regime, even if they are larger than the

wavelength. The optical properties of fractal aggregates need further study, but it is apparent that coagulation of grains is less effective for lowering the nebula's opacity than has been generally assumed.

COAGULATION AND SETTLING OF FRACTAL AGGREGATES

We have modeled numerically the evolution of a population of particles in the solar nebula with fractal dimension of 2.11, using the response time of equation (18). The modeling program is based on that of Weidenschilling (1980). The calculations assumed a laminar nebula with surface density of gas 3×10^3g cm^{-2}, surface density of solids 10 g cm^{-2}, and temperature of 500K at a heliocentric distance of 1 AU. At t = 0 the dust was in the form of individual grains of diameter 1μm, uniformly mixed with the gas. With the assumption that coagulation produced spherical particles of dimension D = 3, or constant density (the actual value is unimportant; compare the discussion of equation (16)), "raining out" with growth of approximately 10-meter bodies in the central plane occurs in a few times 10^3 years.

For the case of fractal aggregates with D = 2.11, we assumed that an individual μm-sized grain (s$_o$ = 0.5 μm) has a density $\rho_o = 3$ g cm^{-3}. Aggregates of such grains have densities that decrease with size according to

$$\rho_s = \rho_o(s/s_o)^{D-3} \tag{19}$$

until s = 0.5 mm, at which size $\rho_s \simeq 0.01$ g cm^{-3} (densities of this order are achieved by some aggregates under terrestrial conditions; Donn and Meakin 1988). The density is assumed constant at this value until s = 1 cm, and then increases approximately as s^2 to a final density of 2 g cm^{-3} for s $\geq$ 10 cm. This variation is arbitrary, but reflects the plausible assumption that fractal structure eventually gives way to uniform density for sufficiently large bodies due to collisional compaction. In a laminar nebula, relative velocities may be low enough to allow fractal structure at larger sizes than assumed here, so this assumption may be conservative. Experimental data on the mechanical behavior of fractal aggregates are sorely needed.

Even the limited range of fractal behavior assumed here has a strong effect on the evolution of the particles in the nebula. Growth and settling are slowed greatly. After a model time of 2 × 10^4 years, the largest aggregates are < one millimeter in size. The highest levels of the disk, above one scale height, are slightly depleted in solids due to the assumption that the gas is laminar. The largest aggregates at this time have settling velocities ~ 1 cm sec^{-1} , so there would be no concentration toward the central plane if turbulence in the gas exceeded this value. Continuation of this calculation results in more rapid growth by differential settling beginning at about 2.5
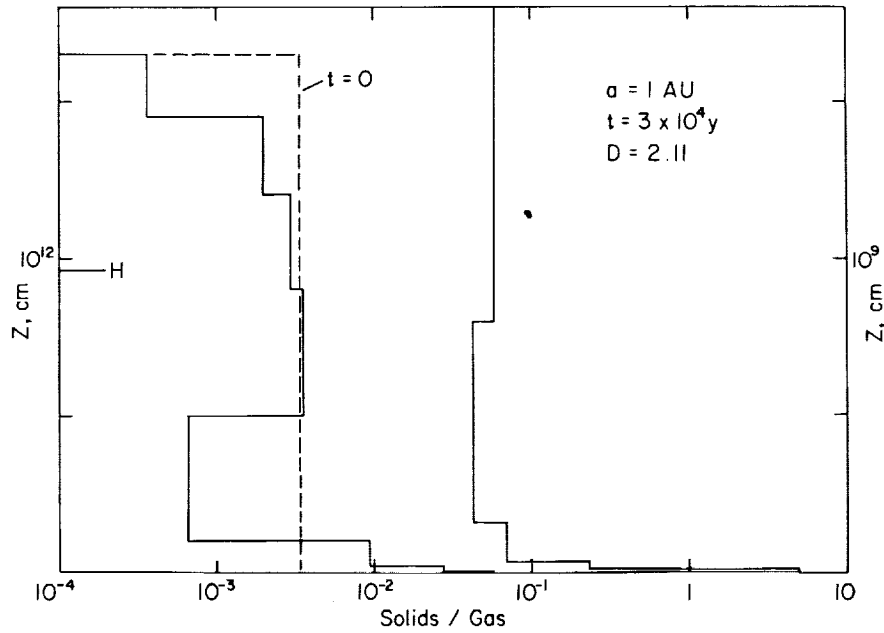
FIGURE 4 Outcome of a numerical simulation of coagulation and settling in the solar nebula at a $= 1$ AU. Aggregates are assumed to have fractal dimension 2.11 at sizes $<$ 0.1 cm. AT t $= 0$, dust/gas ratio is uniform at 0.0034, corresponding to cosmic abundance of metal and silicates. At t $= 3 \times 10^4$y, dust/gas exceeds unity in a narrow region near the central plane (expanded scale at right).

$\times$ 10^4 years. By 3×10^4 years there is a high concentration of solids in a narrow zone at the central plane of the disk (see Figure 4). This layer contains approximately 1% of the total mass of solids, in the form of bodies several meters in diameter.

Evidently, the fractal nature of small aggregates greatly prolongs the stage of well-mixed gas and dust, before "rainout" to the central plane. The reason for this behavior is subtle. If we assume that the density varies as equation (19), then a generalization of the thermal coagulation growth rate of equation (14) gives

$$s(t) = s_o + \left[(3D/2 - 2)3f\rho(kT/2\pi\rho_0^3)^{1/2} s_o^{3(D-3)/2} t \right]^{1/(3D/2-2)} \tag{20}$$

For D $= 2.11$, this gives s $\propto t^{0.86}$, vs. s $\propto t^{0.4}$ for D $= 3$. Thus, particle sizes increase *more* rapidly for thermal coagulation of fractal aggregates, due to their larger collisional cross-sections. For aggregates with D > 2, the behavior of t_e at large sizes is $t_e \simeq t_{eo}(s/s_o)^{D-2}$. Using this relation

in equation (15), D appears in both the numerator and denominator, leaving the growth rate unchanged. If thermal coagulation is faster for fractal aggregates and the growth rate due to settling is independent of D, then why is the evolution of the particle population slower? The answer is that the derivation of equation (15) assumes that the mass available to the larger aggregate is in much smaller particles, so that the relative velocity is essentially equal to the larger body's settling rate. Thermal coagulation tends to deplete the smallest particles most rapidly, creating a narrowly peaked size distribution, and rendering differential settling ineffective. Inserting the parameters used in our simulation into equation (20) predicts $s \simeq 0.1$ cm at $t = 2 \times 10^4$ years, in good agreement with the numerical results.

The qualitative behavior of this simulation, a long period of slow coagulation followed by rapid growth and "rainout," is to some degree an artifact of our assumption for the variation of particle density with size. A transition from fractal behavior to compact bodies without abrupt changes in slope would probably yield a more gradual onset of settling. It is likely that fractal structure could persist to sizes larger than the one millimeter we have assumed here, with correspondingly longer evolution time scales.

We have not yet modeled the evolution of the particle population after the solids/gas ratio exceeds unity in the central plane and can only speculate about possible outcomes. At the end of our simulations most of the mass in this level is in bodies approximately 10 meters in size. These should undergo further collisional growth, and in the absence of turbulence will settle into an extremely thin layer. Because these bodies are large enough to be nearly decoupled from the gas, it is conceivable that gravitational instability could occur in this layer. However, the surface density represented by these bodies is small, approximately 1% of the total surface density of solids. The critical wavelength, as in equation (2), is correspondingly smaller. If they grow to sizes >0.1 km before the critical density is reached, then their mean spacing is $\sim \lambda_c$, and gravitational instability is bypassed completely. In any case, the first-formed planetesimals would continue to accrete the smaller bodies that rain down to the central plane over a much longer time.

CONCLUSIONS

The settling of small particles to the central plane of the solar nebula is very sensitive to the presence of turbulence in the gaseous disk. It appears that a particle layer sufficiently dense to become gravitationally unstable cannot form unless the particles are large enough to decouple from the gas, i.e. > meter-sized. Thus, the simple model of formation of planetesimals directly from dust grains is not realistic; there must be an intermediate stage of particle coagulation into macroscopic aggregate bodies.

Coagulation of grains during settling alters the nature of the settling process. The central layer of particles forms by the "raining out" of larger aggregates that contain only a fraction of the total mass of solids. The surface density of this layer varies with time in a manner that depends on the nebular structure and particle properties. Thus, if planetesimals form by gravitational instability of the particle layer, the scale of instabilities and masses of planetesimals are not simply related to the total surface density of the nebula. It is possible that collisional coagulation due to drag-induced differential motions may be sufficiently rapid to prevent gravitational instability from occurring.

It is probable that dust aggregates in the solar nebula were low-density fractal structures. The time scale for settling to the central plane may have been one or two orders of magnitude greater than estimates which assumed compact particles. The inefficiency of settling by "raining out" suggests that a significant fraction of solids remained suspended in the form of small particles until the gas was dispersed; the solar nebula probably remained highly opaque. The mass of the nebula may have been greater than the value represented by the present masses of the planets.

ACKNOWLEDGEMENTS

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