N91-22968

Formation of the Terrestrial Planets from Planetesimals

GEORGE W. WETHERILL
Carnegie Institution of Washington

ABSTRACT

Previous work on the formation of the terrestrial planets (e.g. Safronov 1969; Nakagawa *et al.* 1983; Wetherill 1980) involved a stage in which on a time scale of $\sim 10^6$ years, about 1000 embryos of approximately uniform size ($\sim 10^{25}$ g) formed, and then merged on a $10^7 - 10^8$ year time scale to form the final planets. Numerical simulations of this final merger showed that this stage of accumulation was marked by giant impacts ($10^{27} - 10^{28}$ g) that could be responsible for providing the angular momentum of the Earth-Moon system, removal of Mercury's silicate mantle, and the removal of primordial planetary atmospheres (Hartmann and Davis 1975; Cameron and Ward 1976; Wetherill 1985). Requirements of conservation of angular momentum, energy, and mass required that these embryos be confined to a narrow zone between about 0.7 and 1.0 AU. Failure of embryos to form at 1.5 – 2.0 AU could be attributed to the longer ($\sim 10^7$ years) time scale for their initial stage of growth and the opportunity of effects associated with the growth of the giant planets to forestall that growth.

More recent work (Wetherill and Stewart 1988) indicates that the first stage of growth of embryos at 1 AU occurs by a rapid runaway on a much shorter $\sim 3 \times 10^4$ year time scale, as a consequence of dynamical friction, whereby equipartition of energy lowers the random velocities and thus increases the gravitational cross-section of the larger bodies. Formation of embryos at ~ 2 AU would occur in $< 10^6$ years, and it is more difficult to understand how their growth could be truncated by events in the outer solar system alone. Those physical processes included in this earlier work are not capable of removing the necessary mass, energy, and angular momentum

from the region between the Earth and the asteroid belt, at least on such a short time scale.

An investigation has been made of augmentation of outer solar system effects by spiral density waves produced by terrestrial planet embryos in the presence of nebular gas, as discussed by Ward (1986). This can cause removal of angular momentum and mass from the inner solar system. The theoretical numerical coefficients associated with the radial migration and eccentricity damping caused by this effect are at present uncertain. It is found that large values of these coefficients, compression of the planetesimal swarm by density wave drag, followed by resonance effects following the formation of Jupiter and Saturn, "clears" the region between Earth and the asteroid belt, and also leads to the formation of Earth and Venus with approximately their observed sizes and heliocentric distances. For smaller, and probably more plausible values of the coefficients, this mechanism will not solve the angular-momentum-energy problem. The final growth of the Earth on a $\sim 10^8$ year time scale is punctuated by giant impacts, up to twice the mass of Mars. Smaller bodies similar to Mercury and the Moon are vulnerable to collisional fragmentation. Other possibly important physical phenomena, such as gravitational resonances between the terrestrial planet embryos have not yet been considered.

INTRODUCTION

This article will describe recent and current development of theories in which the terrestrial planets formed by the accumulation of much smaller (one- to 10-kilometer diameter) planetesimals. The alternative of forming these planets from massive gaseous instabilities in the solar nebula has not received much attention during the past decade, has been discussed by Cameron *et al.* (1982), and will not be reviewed here.

In its qualitative form, the planetesimal, or "meteoric" theory of planet formation dates back at least to Chladni (1794) and was supported by numerous subsequent workers, among the most prominent of which were Chamberlain and Moulton (Chamberlain 1904). Its modern development into a quantitative theory began with the work of O.Yu. Schmidt and his followers, most notably V.S. Safronov. The publication in 1969 of his book "Evolutionary of the Protoplanetary Swarm" (Safronov 1969) and its publication in English translation in 1972 were milestones in the development of this subject, and most work since that time has consisted of extension of problems posed in that work.

The formation of the terrestrial planets from planetesimals can be conveniently divided into three stages:

(1) The formation of the planetesimals themselves from the dust of the solar nebula. The current status of this difficult question has been reviewed by Weidenschilling *et al.* (1988).

(2) The local accumulation of these one- to 10-kilometer planetesimals into $\sim 10^{25} - 10^{26}$g "planetary embryos" revolving about the sun in orbits of low eccentricity and inclination. Recent work on this problem has been summarized by Wetherill (1989a), and will be briefly reviewed in this article.

(3) The final merger of these embryos into the planets observed today. Fairly recent discussions of this stage of accumulation have been given by Wetherill (1986, 1988). This work needs to be updated in order to be consistent with progress in our understanding of stage (2). Particular attention will be given to that need in the present article.

FORMATION OF THE ORIGINAL PLANETESIMALS

The original solid material in the solar nebula was most likely concentrated in the micron size range, either as relic interstellar dust grains, as condensates from a cooling solar nebula, or a mixture of these types of material. The fundamental problem with the growth of larger bodies from such dust grains is their fragility with regard to collisional fragmentation, not only at the approximate kilometers per second sound speed velocities of a turbulent gaseous nebula, but even at the more modest $\sim$ 60 m/sec differential velocities associated with the difference between the gas velocity and the Keplerian velocity of a non-turbulent nebula (Whipple 1973; Adachi *et al.* 1976; Weidenschilling 1977). Agglomeration under these conditions requires processes such as physical "stickiness," the imbedding of high-velocity projectiles into porous targets, or physical coherence of splash products following impact. Despite serious efforts to experimentally or theoretically treat this stage of planetary growth, our poor understanding of physical conditions in the solar nebula and other physical properties of these primordial aggregates make it very difficult.

Because of these difficulties, many workers have been attracted to the possibility that growth of bodies to one- to 10-kilometer diameters could be accomplished by gravitational instabilities in a central dust layer of the solar nebula (Edgeworth 1949; Safronov 1960; Goldreich and Ward 1973). Once bodies reach that size, it is plausible that their subsequent growth would be dominated by their gravitational interactions. Weidenschilling (1984) however has pointed out serious difficulties that are likely to preclude the development of the necessary high concentration and low relative velocity (approximately 10 centimeters per second) in a central dust layer. Therefore the question of how the earliest stage of planetesimal growth took place remains an open one that requires close attention.

GROWTH OF PLANETESIMALS INTO PLANETARY EMBRYOS

If somehow the primordial dust grains can agglomerate into one- to 10-kilometer diameter planetesimals, it is then necessary to understand the processes that govern their accumulation into larger bodies.

The present mass of the terrestrial planets is $\sim 10^{28}$g, therefore about 10^{10} 10km ($\sim 10^{18}$g) bodies are required for their formation. It is completely out of the question to consider the gravitationally controlled orbital evolution of such a large swarm of bodies by either the conventional methods of celestial mechanics, or by Monte Carlo approximations to these methods. Therefore all workers have in one way or another treated this second stage of planetary growth by methods based on gas dynamics, particularly by the molecular theory of gases, in which the planetesimals assume the role of the molecules in gas dynamics theory. This approach is similar to that taken by Chandrasekhar (1942) in stellar dynamics. Nevertheless, the fact that the planetesimals are moving in Keplerian orbits rather than in free space requires some modification of Chandrasekhar's theory.

The most simple approach to such a "gas dynamics" theory of planetesimals is to simply assume that a planetesimal grows in mass (M) by sweep up of smaller bodies in accordance with a simple growth equation:

$$\frac{dM}{dt} = \pi R^2 \rho_s V F_g, \qquad (1)$$

where R is the physical radius of the growing planetesimal, ρ_s is the surface mass density of the material being swept up, V is their relative velocity, and F_g represents the enhancement of the physical cross-section by "gravitational focussing," given in the two-body approximation by

$$F_g = (1 + 2\theta), \qquad (2)$$

where θ is the Safronov number, $\theta = \frac{V_e^2}{V^2}$, and V_e is the escape velocity of the growing body.

Although it is possible to gain considerable insight into planetesimal growth by simple use of equation (1), its dependence on velocity limits its usefulness unless a way is found to calculate the relative velocity. Safronov (1962) made a major contribution to this problem by recognition that this relative velocity is not a free parameter, but is determined by the mass distribution of bodies. The mass distribution is in turn determined by the growth of the bodies, which in turn is dependent on the relative velocities by equation (1). Thus the mass and velocity evolution are coupled.

Safronov made use of Chandrasekhar's relaxation time theory to develop expressions for the coupled growth of mass and velocity. He showed that a steady-state velocity distribution in the swarm was established as a result of the balance between "gravitational stirring" that on the average increased the relative velocity, and collisional damping, that decreased their relative velocity. The result was that the velocity and mass evolution were coupled in such a way that the relative velocity of the bodies was self-regulated to remain in the proper range, i.e. neither too high to prevent growth by fragmentation, nor too low to cause premature isolation of the growing bodies as a result of the eccentricity becoming too low. In Safronov's work the effect of gas drag on the bodies was not included. Hayashi and his coworkers (Nakagawa *et al.* 1983) complemented the work of Safronov and his colleagues by including the effects of gas drag, but did not include collisional damping. Despite these differences, their results are similar. The growth of the planetesimals to bodies of $\sim 10^{25} - 10^{26}$g begins with a steep initial distribution of bodies of nearly equal mass. With the passage of time, the larger bodies of the swarm remain of similar size and constitute a "marching front" that diminishes in number as the mass of the bodies increases. Masses of $\sim 10^{25}$g are achieved in $\sim 10^6$ years.

An alternative mode of growth was proposed by Greenberg *et al.* (1978). They found that instead of the orderly "marching front," runaway growth caused a single body to grow to $\sim 10^{23}$g in 10^4 years, at which time almost all the mass of the system remained in the form of the original 10^{16}g planetesimals. It is now known (Patterson and Spaute 1988) that the runaway growth found by Greenberg *et al.* were the result of an inaccurate numerical procedure. Nevertheless, as discussed below, it now appears likely that similar runaways are expected when the problem is treated using a more complete physical theory and sufficiently accurate numerical procedures.

This recent development emerged from the work of Stewart and Kaula (1980) who applied Boltzmann and Fokker-Planck equations to the problem of the velocity distribution of a swarm of planetesimals, as determined by their mutual gravitational and collisional evolution. This work was extended by Stewart and Wetherill (1988) to develop equations describing the rate of change of the velocity of a body of mass m_1 and velocity V_1 as a result of collisional and gravitational interaction with a swarm of bodies with masses m_2 and velocities V_2. In contrast with earlier work, these equations for the gravitational interactions contain dynamical friction terms of the form

$$\frac{dV_1}{dt} \propto (m_2 V_2^2 - m_1 V_1^2). \tag{3}$$

These terms tend to equipartition energy between the larger and smaller members of the swarm. For equal values of V_1 and V_2, they cause the velocity of a larger mass m_1 to *decrease* with time. In earlier work, the gravitationally induced "stirring" was always positive-definite, as a result of using relaxation time expressions that ensured this result.

The dV/dt equations of Stewart and Wetherill have been used to develop a numerical procedure for studying the evolution of the mass and velocity distribution of a growing swarm of planetesimals at a given heliocentric distance, including gas drag, as well as gravitational and collisional interactions (Wetherill and Stewart 1988).

When approached in this way it becomes clear that the coupled nonlinear equations describing the velocity and size distribution of the swarm bifurcate into two general types of solutions. The first, orderly growth, was described by the Moscow and Kyoto workers. The second is "runaway" growth whereby within a local zone of the solar nebula (e.g. 02 AU in width) a single body grows much faster than its neighbors and causes the mass distribution to become discontinuous at its upper end. Whether or not the runaway branch is entered depends on the physical parameters assumed for the planetesimals. More important however, are the physical processes included in the equations. In particular, inclusion of the equipartition of energy terms causes the solutions to enter the runaway branch for a very broad range of physical parameters and initial conditions. When these terms are not included, the results of Safronov, Hayashi, and their coworkers are confirmed (see Figure 1). On the other hand, when these terms are included, runaway solutions represent the normal outcome of the calculations.

The origin of the runaway can be easily understood. For an initial swarm of planetesimals of equal or nearly equal mass, the mass distribution will quickly disperse as a result of stochastic differences in the collision rate and thereby the growth rate of a large number of small bodies. As a result of the equipartition of energy terms, this will quickly lead to a velocity dispersion, whereby the larger bodies have velocities, relative to a circular orbit, significantly lower than that of the more numerous smaller bodies of the swarm. The velocity of the smaller bodies is actually accelerated by the same equipartition terms that decrease the velocity of the larger bodies.

A simplified illustration of this effect is shown in Figure 2. This calculation is simplified in that effects associated with failure of the two-body approximation at low velocities and with fragmentation are not included.

After only $\sim 3 \times 10^4$ years, the velocities of the largest bodies relative to those of the smaller bodies has dropped by an order of magnitude. These lower velocities increased the gravitational cross-section of the larger bodies sufficiently to cause them to grow approximately 100 times larger than those bodies in which most of the mass of the swarm is located. This "midpoint mass" (m_p), defined by being the mass below which half the mass of the swarm is located, is indicated on Figure 2. For bodies of this mass the Safronov number θ is 0.6 when defined as

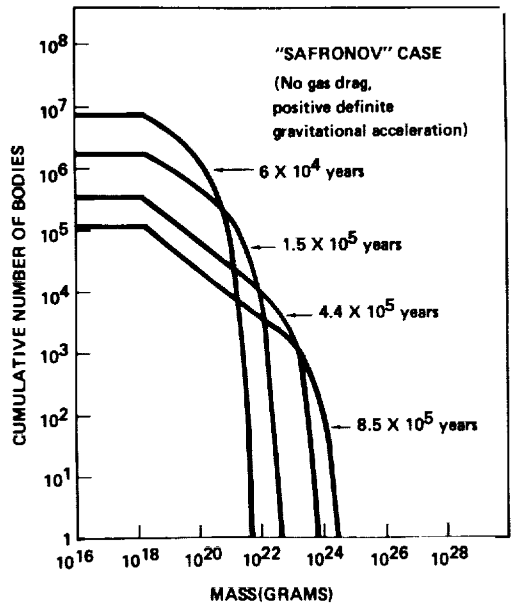

FIGURE 1a Evolution of the mass distribution of a swarm of planetesimals distributed between 0.99 and 1.01 AU for which the velocity distribution is determined entirely by the balance between positive-definite gravitational "pumping up" of velocity and collisional damping. The growth is "orderly," i.e., it does not lead to a runaway, but rather to a mass distribution in which most of the mass is concentrated in $10^{24} - 10^{25}$g bodies at the upper end of the mass distribution.

$$\theta = \frac{V_e^2(m_p)}{2V^2(m_p)} \tag{4}$$

i.e. its relative velocity is similar to its own escape velocity. In contrast, the value of θ_L calculated using the velocity of this body and that of the *largest* body of the swarm has a quite high value of 21.

At this early stage of evolution the growth is still orderly and continuous (Figure 3). However, by 1.3×10^5 years, the velocities of the largest bodies have become much lower than their escape velocities (Figure 2), and a bulge has developed at the upper end of the swarm as a result of their growing much faster than the smaller bodies in the swarm. At 2.6×10^5 years, a single discontinuously distributed body with a mass $\sim 10^{26}$ is found. At this time it has accumulated 13% of the swarm, and the next largest bodies are more than 100 times smaller. This runaway body will quickly capture all the residual material in the original accumulation zone, specified in this case to be 0.02 AU in width.

The orbit of the runaway body will be nearly circular, and it will be able

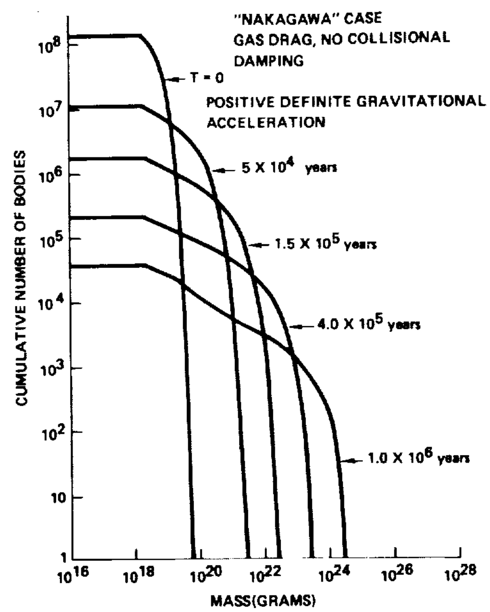

FIGURE 1b Evolution of the mass distribution of a swarm in which the velocity damping is provided by gas drag, rather than by collisional damping. The resulting distribution is similar to that of Figure 1a.

to capture bodies approaching within several Hill sphere radii (Hill sphere radius = distance to colinear Lagrangian points). Even in the absence of competitors in neighboring zones, the runaway growth will probably self-terminate because additions to its mass (Δm) will be proportional to $(\Delta D)^2$, where ΔD is the change in planetesimal diameter, whereas the material available to be accumulated will be proportional to ΔD. Depending on the initial surface density, runaway growth of this kind can be expected to produce approximately 30 to 200 bodies in the terrestrial planet region with sizes ranging from that of the Moon to that of Mars.

There are a number of important physical processes that have not been included in this simplified model. These include the fragmentation of the smaller bodies of the swarm, the failure of the two-body approximation at low velocities, and the failure of the runaway body to be an effective perturber of small bodies that cross the orbit of only one runaway. These conditions are more difficult to model, but those calculations that have been made indicate that they all operate in the direction of increasing the rate of the runaway.

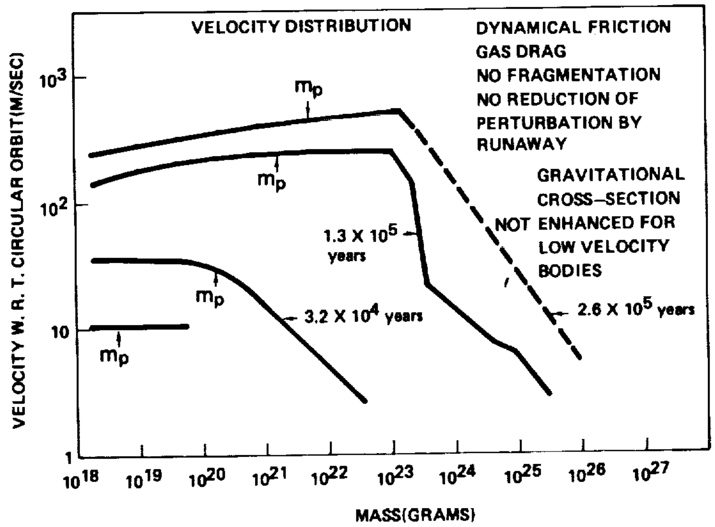

FIGURE 2 Velocity distribution corresponding to inclusion of equipartition of energy terms. After 3×10^4 years, the velocities of the largest bodies drop well below that of the midpoint mass m_p. This leads to a rapid growth of the largest bodies, and ultimately to a runaway, as described in the text.

GROWTH OF RUNAWAY PLANETARY EMBRYOS INTO TERRESTRIAL PLANETS

Because of the depletion of material in their vicinity, it seems most likely that the runaway bodies described above will only grow to masses in the range of 6×10^{25}g to 6×10^{26}g, and further accumulation of a number of these "planetary embryos" will be required to form bodies of the size of Earth and Venus.

Both two-dimensional and three-dimensional numerical simulations of this final accumulation of embryos into terrestrial planets have been reported. All of these simulations are in some sense "Monte Carlo" calculations, because even in the less demanding two-dimensional case, a complete numerical integration of several hundred bodies for the required number of orbital periods is computationally prohibitive. Even if such calculations were possible, the intrinsically chaotic nature of orbital evolution dominated by close encounters causes the final outcome to be so exquisitely sensitive to the initial conditions that the final outcome is essentially stochastic. Two-dimensional calculations have been reported by Cox and Lewis (1980); Wetherill (1980); Lecar and Aarseth (1986); and Ipatov (1981a). In some of these two-dimensional cases numerical integration was carried out during the close encounter.

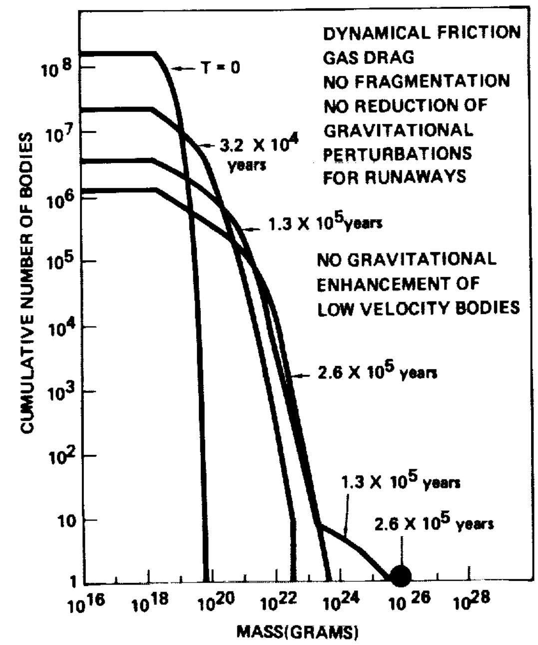

FIGURE 3 Effect of introducing equipartition of energy terms on the mass distribution. The tendency toward equipartition of energy results in a velocity dispersion (Figure 2) in which the velocity (with respect to a circular orbit) of the massive bodies falls below that of the swarm. After $\sim 10^5$ years, a "multiple runaway" appears as a bulge in the mass distribution in the mass range $10^{24} - 10^{25}$g. After 2.6×10^5 years, the largest body has swept up these larger bodies, leading to a runaway in which the mass distribution is discontinuous. The largest body has a mass of $\sim 10^{26}$g, whereas the other remaining bodies have masses $< 10^{24}$g.

The three-dimensional calculations (Wetherill 1978, 1980, 1985, 1986, 1988) make use of a Monte Carlo technique based on the work of Öpik (1951) and Arnold (1965). In both the two- and three-dimensional calculations, the physical processes considered are mutual gravitational perturbations, physical collisions, and mergers, and in some cases collisional fragmentations and tidal disruption (Wetherill 1986, 1988).

In the work cited above it was necessary to initially confine the initial swarm to a region smaller than the space presently occupied by the observed terrestrial planets. This is necessary because a system of this kind nearly conserves mass, energy, and angular momentum. The terrestrial planets are so deep in the Sun's gravitational well that very little (< 5%) of the material is perturbed into hyperbolic solar system escape orbits. The loss of mass, energy, and angular momentum by this route is therefore small.

Angular momentum is strictly conserved by gravitational perturbations and physical collisions. Some energy is radiated away as heat during collision and merger of planetesimals. A closed system of bodies, such as a stellar accretion disk, that conserves angular momentum and loses energy can only spread, not contract. Therefore the initial system of planetesimals must occupy a narrower range of heliocentric distance than the range of the present terrestrial planets. In particular (Wetherill 1978), it can be shown by simple calculations that a swarm that can evolve into the present system of the terrestrial planets must be initially confined to a narrow band extending from about 0.7 to 1.1 AU.

In theories in which planetesimals grow into embryos via the *orderly* branch of the bifurcation of the coupled velocity-size distribution equations, the time scale for growth of $\sim 10^{26}$g embryos at 1 AU is 1 to 2×10^6 years. If the surface density of material falls off as $a^{-3/2}$ beyond 1 AU, the time scale for similar growth at larger heliocentric distances will vary as a^{-3}, the additional $a^{-3/2}$ arising from the variation of orbital encounter frequency with orbital period. Thus at 2 AU, the comparable time scale for the growth of planetesimals into embryos would be 10-20 million years. Jupiter and Saturn must have formed while nebular gas was still abundant. Observations of pre-main-sequence stars, and theoretical calculations (Lissauer 1987; Wetherill 1989b) permits one to plausibly hypothesize that Jupiter and Saturn had already formed by the time terrestrial-type "rocky" planetesimals formed much beyond 1 AU. In some rather uncertain way it is usually supposed that the existence of these giant planets then not only cleared out the asteroid belt, aborted the growth of Mars, and also prevented the growth of planetesimals into embryos much beyond 1 AU. Interior to 0.7 AU, it can be hypothesized that high temperatures associated with proximity to the Sun restrained the formation or growth of planetesimals.

Subject to uncertainties associated with hypotheses of the kind discussed above, the published simulations of the final stages of planetary growth, show that an initial collection of several hundred embryos spontaneously evolve into two to five bodies in the general mass range of the present terrestrial planets. In some cases the size and distribution of the final bodies resemble rather remarkably those observed in the present solar system (Wetherill 1985). The process is highly stochastic, however, and more often an unfamiliar assemblage of final planets is found, e.g. $\sim$ three bodies, $\sim 4 \times 10^{27}$g of mass at 0.55, 1.0, and 1.4 AU.

Even when the initial embryos are quite small (i.e. as small as 1/6 lunar mass), it is found that the growth of these bodies into planets is characterized by giant impacts at rather high velocities (> 10 kilometers per second). In the case of Earth and Venus, these impacting bodies may exceed the mass of the present planet Mars. These models of planetary accumulation thereby fit in well with theories of the formation of the Moon accumulation thereby fit in well with theories of the formation of the Moon by "giant splashes" (Hartmann and Davis 1975; Cameron and Ward 1976), removal of Earth's atmosphere by giant impacts (Cameron 1983); and impact removal of Mercury's silicate mantle (Wetherill 1988; Benz *et al.* 1988).

Some rethinking of this discussion is required by the results of Wetherill and Stewart (1988). Now that it seems more likely that runaway growth of embryos at 1 AU took place on a much faster time scale, possibly as short as 3×10^4 years, it seems much less plausible that the formation of Jupiter and Saturn can explain the absence of one or more Earth-size planets beyond 1 AU. Growth rates in the asteroid belt may still be slow enough to be controlled by giant planet formation, but this will be more difficult interior to 2 AU.

The studies of terrestrial planet growth discussed earlier in which the only physical processes included are collisions and merger, are clearly inadequate to cause an extended swarm of embryos to evolve into the present compact group of terrestrial planets. Accomplishment of this will require inclusion of additional physical processes.

Three such processes are known, but their significance requires much better quantitative evaluation and understanding. These are:

(1) Loss of total and specific negative gravitational binding energy as well as angular momentum by exchange of these quantities to the gaseous nebula via spiral density waves (Ward 1986, 1988).

(2) Loss of material, with its associated energy and angular momentum from the complex of resonances in the vicinity of 2 AU (Figure 4). All of these resonances will not be present until after the formation of Jupiter and Saturn, and therefore are not likely to be able to prevent the formation of runaway bodies in this region. After approximately one million years however, they can facilitate removal of material from this region of the solar system by increasing the eccentricity of planetesimals into terrestrial planets and Jupiter-crossing orbits. Because bodies with larger semi-major axes are more vulnerable to being lost in this way, the effect will be to decrease the specific angular momentum of the swarm as well as cause the energy of the swarm to become less negative.

(3) Resonant interactions between the growing embryos. Studies of the orbital evolution of Earth-crossing asteroids (Milani *et al.* 1988) show that although the long-term orbital evolution of these bodies is likely to be dominated by close planetary encounters, more distant resonant interactions are also prominent. Similar phenomena are to be expected during the growth of embryos into planets. These have not been considered in a detailed way in the context of the mode of planetary growth outlined here, but relevant studies of resonant phenomena during planetary growth have

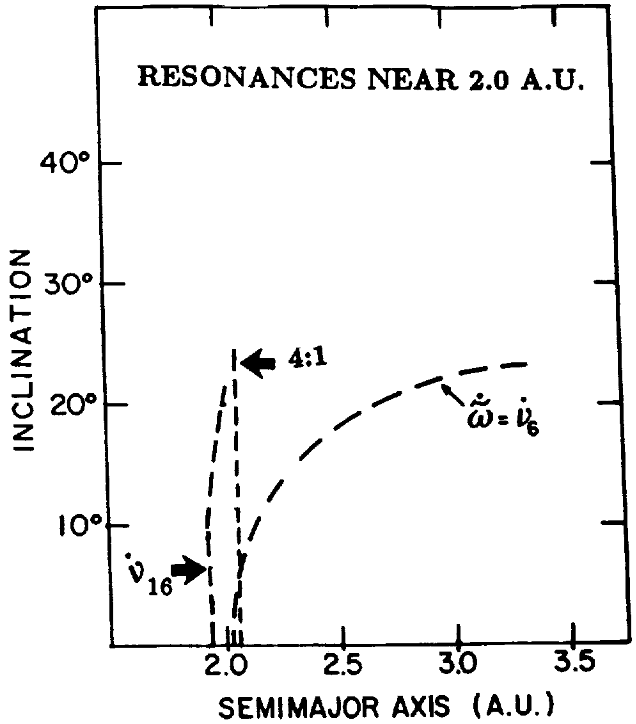

FIGURE 4 Complex of resonances in the present solar system in the vicinity of 2 AU. These resonances are likely responsible for forming a chaotic "giant Kirkwood gap" that defines the inner edge of the main asteroid belt.

been published (Ipatov 1981b; Weidenschilling and Davis 1985; Patterson 1987).

All of these phenomena represent real effects that undoubtedly were present in the early solar system and must be taken into consideration in any complete theory of terrestrial planet formation. Quantitative evaluation of their effect however, is difficult at present, and there is no good reason to believe they are adequate to the task.

A preliminary evaluation of the effect of the first two phenomena, spiral density waves and Jupiter-Saturn resonances near 2 AU have been carried out. Some of the result of this investigation are shown in Figures 5 and 6.

In Figure 5 the point marked "initial swarm" corresponds to the specific energy and angular momentum of an extended swarm of runaway planetesimals extending from 0.45 to 2.35 AU, with a surface density falling off as 1/a. The size of the runaway embryos is prescribed by the condition

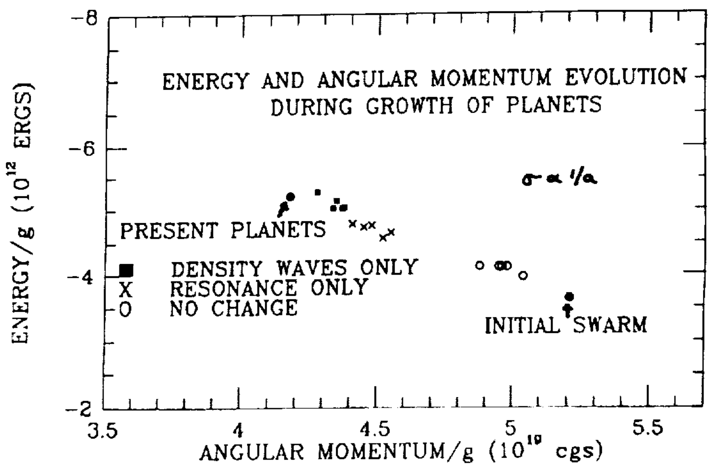

FIGURE 5 Energy and angular momentum evolution of an initial swarm with runaway planetesimal and gas surface density as 1/a between the 0.7 and 2.20 AU. Interior of 0.7 AU, it is assumed that the gas density is greatly reduced, possibly in an association bipolar outflow producing a "hole" in the center of the solar nebula. The surface density falls off exponentially between 2.20 and 2.35 AU and between 0.45 and 0.7 AU. The total initial mass of the swarm is 1.407 10^{28}g. The open circles represent simulations in which only gravitational perturbations and collisional damping are included. The crosses are simulations in which mass, angular momentum, and negative energy are lost by means of the resonances shown in Figure 3. The solid squares represent simulations in which angular momentum and losses result from inclusion of spiral density wave damping, as described by Ward (1986, 1988).

that they be separated by 4 Hill sphere radii from one another. As a result, the mass of the embryos is 2.3×10^{26}g at 2 AU, 0.8×10^{26}g at 1 AU, and 0.5×10^{26}g at 0.7 AU. The specific energy and angular momentum of the present solar system is indicated by the point so marked near the upper left of the Figure. The question posed here is whether inclusion of 2 AU resonances and Ward's equations for changes in semi-major axis and eccentricity of the swarm can cause the system to evolve from the initial point to the "goal" representing the present solar system. It is found that this is possible for sufficiently large values of the relevant parameters. More recent work (Ward, private communication 1989) indicates that the published parameters may be an order of magnitude too large. If so, this will greatly diminish the importance of this mechanism for angular momentum removal.

The open circles near the initial point show the results of five simulations in which neither of these effects were included. Some evolution toward the goal is achieved, nevertheless. This results from the more

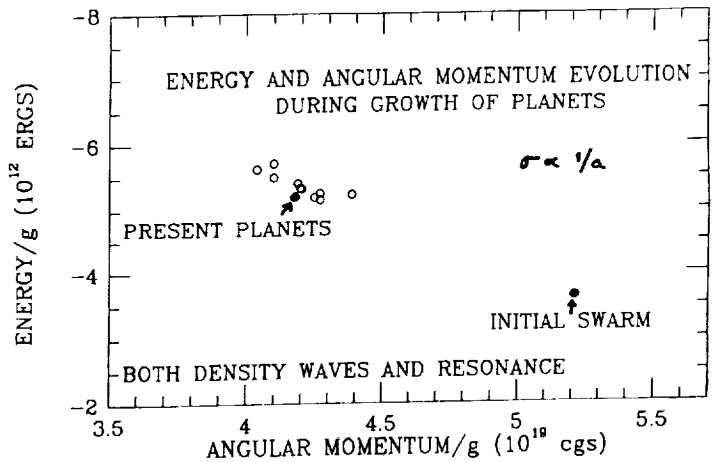

FIGURE 6 Energy and angular momentum loss when both the effects of resonances and spiral density are included. For the choice of parameters described in the text, the system evolves into a distribution matching that parameters described in the text, the system evolves into a distribution matching that observed.

distant members of extended swarm being more loosely bound than that employed in earlier studies, and consequently relatively more loss of bodies with higher angular momentum and with less negative energy.

This effect is enhanced when the resonances shown in Figure 4 are included (crosses in Figure 5). The effect of the resonances is introduced in a very approximate manner. If after a perturbation, the semi-major axis of a body is between 2.0 and 2.1 AU, its eccentricity is assigned a random value between 0.2 and 0.8. A similar displacement toward the specific angular momentum and energy of the present solar system is reached when da/dt and de/dt terms of the form given by Ward (1986, 1988) are included. The open squares in Figure 5 result from use of a coefficient having a value of 29 in Ward's equation for da/dt, and a value of 1 for de/dt. The value for da/dt is about twice that originally estimated by Ward (1986).

The effect of including both the resonances and the same values of spiral density wave damping are shown in Figure 6. The points lie quite near the values found for the terrestrial planets. Thus if one assumes appropriate values for these two phenomena, an initial swarm can evolve into one with specific energy and angular momentum about that found in the present solar system. The initial surface density can be adjusted to match the present total mass of the terrestrial planets, without disturbing the agreement with the observed energy and angular momentum. Similar agreement has been obtained in calculations in which both the gas and embryo surface densities varied as $a^{-3/2}$, instead of a^{-1} as used for the data of Figure 6. Satisfactory matching has also been found for a swarm in which the gas surface density fell off as $a^{-3/2}$ whereas the embryo surface density decreased as a^{-1}.

Matching the mass, angular momentum, and energy of the present terrestrial planet system is a necessary, but not sufficient condition, for a proper model for the formation of the terrestrial planets. It is also necessary that to some degree the configuration (i.e. the number, position, mass, eccentricity, and inclination) of the bodies resemble those observed. Because a model of this kind is highly stochastic, and we have only one terrestrial planet system to observe, it is hard to know how exactly the configuration should match. As in the earlier work "good" matches are sometimes found, more often the total number of final planets with masses > 1/4 Earth mass is three, rather than the two observed bodies. It is possible this is a stochastic effect, but the author suspects it more likely that the differences result from the model being too simple. Neglecting such factors as the resonant interactions between the embryos as well as less obvious phenomena may be important.

Like the previous models in which the swarm was initially much more localized, the final stage of accumulation of these planets from embryos involves giant impacts. Typically, at least one body larger than the present planet Mars impacts the simulated "Earth," and impacts twice as large are not uncommon. Therefore, all of the effects related to such giant impacts, formation of the Moon, fragmentation of smaller planets, and impact loss of atmospheres are to be expected for terrestrial planet systems arising from the more extended initial embryo swarms of the kind considered here. Furthermore, the inward radial migration associated with density wave drag, as well as the acceleration in eccentricity caused by the resonances, augment the tendency for a widespread provenance of the embryos responsible for the chemical composition of the final planets.

ACKNOWLEDGMENTS

The author wishes to thank W.S. Ward for helpful discussions of density wave damping, and Janice Dunlap for assistance in preparing this manuscript. This work was supported by NASA grant NSG 7347 and was part of a more general program at DTM supported by grant NAGW 398.

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