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Analysis of the Transient Calibration of Heat Flux Sensors – one Dimensional Case

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ABSTRACT

The purpose of this analysis is to determine the effect of transient heat flux on heat flux sensor response and calibration.

We have begun this work by studying a one dimensional case. This was done to elucidate the key parameters and trends for the problem. It has the added advantage that the solutions to the governing equations can be obtained by analytical means. The analytical results obtained to date indicate that the transient response of a heat flux sensor depends on the thermal boundary conditions, the geometry and the thermal properties of the sensor. In particular it was shown that if the thermal diffusivity of the sensor is small, say for a material such as Nichrome, then the transient behavior must be taken into account.

INTRODUCTION

The purpose of this analysis was to determine the effect of transient heat flux on sensor response and calibration.

Consider the case of a heat flux sensor mounted in a wall subjected to both radiative and convective heat transfer on the top side and purely convective heat transfer on the bottom side as shown in Fig. 1, where x and r are the axial and radial coordinates, respectively; L and ℓ, D and d are the thicknesses and diameters of the two layers; k is the thermal conductivity, α is the diffusivity, T is temperature, h_1 and h_2 are the convective heat transfer coefficients, q_0 is the radiant incident heat flux, and T_∞ is the ambient temperature. The subscripts $_1$ and $_2$ refer to the different materials of the sensor.

In this analysis we make the following simplifying assumptions

(1) neglect re–radiation;
(2) constant thermal properties;
(3) constant, uniform ambient temperatures;
(4) constant, uniform convection coefficients;
(5) constant, uniform radiant incident heat flux;
(6) perfect thermal contact between the regions of different k;
(7) the equations are one–dimensional;

Steady – State Solution To determine a base line case we first solve for the steady–state case.

For this case, the governing equations and corresponding boundary conditions are, under the above assumptions.

$$\frac{d^2T_1}{dx^2} = 0 \qquad 0 \leq x \geq \ell \tag{1}$$

$$\frac{d^2T_2}{dx^2} = 0 \qquad \ell \leq x \geq L \tag{2}$$

$$-k_1\frac{dT_1}{dx} + h_1(T_1 - T_{\infty 1}) - q_0 = 0 \quad \text{at } x = 0 \tag{3}$$

$$\left.\begin{aligned} T_1 &= T_2 \\ k_1 \frac{dT_1}{dx} &= k_2 \frac{dT_2}{dx} \end{aligned}\right] \quad \text{at } x = \ell \tag{4}$$

$$k_2 \frac{dT_2}{dx} = h_2 (T_2 - T_{\infty 2}) \quad \text{at } x = L \tag{5}$$

(3) and (5) are rearranged as:

$$-k_1 \frac{\partial T_1}{\partial x} + h_1 T_1 = h_1 f_1 \qquad \text{at } x = 0 \tag{3–a}$$

$$k_2 \frac{\partial T_2}{\partial x} + h_2 T_2 = h_2 f_2 \qquad \text{at } x = L \tag{5–a}$$

where $f_1 = T_{1_\infty} + \frac{q_0}{h_1}$

$f_2 = T_{2_\infty}$

The solutions to equs. (1) and (2) are

$T_1 = A_1 x + B_1$

$T_2 = A_2 x + B_2$

The coefficients A_1, B_1, A_2, B_2 can be determined in terms of the boundary conditions (3–a), (4), and (5–a).

The sensor output is based on the temperature difference between two fixed points where the two alumel wires are attached. Assume that the coordinates of these two points are x_1 and x_2 respectively (they are usually at different layers), then

$T_1 (x_1) = A_1 x_1 + B_1$

$T_2 (x_2) = A_2 x_2 + B_2$

Hence,

$$\Delta T^S = T_1 (x_1) - T_2 (x_2) \tag{6}$$

where

$$A_1 = \frac{k_2}{k_1} A_2 , \quad B_1 = (k_1 A_1 + h_1 f_1)/h_1$$
$$B_2 = B_1 + \ell (A_1 - A_2)$$
$$A_2 = \frac{k_1 h_1 h_2 \ (f_2 - f_1)}{k_1 k_2 h_1 + k_1 h_1 h_2 L + k_2 h_1 h_2 \ell + k_1 k_2 h_2 - k_1 h_1 h_2 \ell} \tag{7}$$

Note that ΔT^S is independent of time.

Unsteady – State Solution

For unsteady state case, we have

$$\alpha_1 \frac{\partial^2 T_1}{\partial x^2} = \frac{\partial T_1}{\partial t} \qquad 0 \leq x \leq \ell, \ t > 0$$
$$\alpha_2 \frac{\partial^2 T_1}{\partial x^2} = \frac{\partial T_2}{\partial t} \qquad 0 \leq x \leq L, \ t > 0 \tag{8}$$

with

$$-k_1 \frac{\partial T_1}{\partial x} + h_1 T_1 = h_1 f_1 \quad \text{at } x = 0, \quad t > 0$$
$$T_1 = T_2 \qquad \text{at } x = \ell, \quad t > 0$$
$$k_1 \frac{\partial T_1}{\partial x_1} = k_2 \frac{\partial T_2}{\partial x_2} \quad \text{at } x = \ell, \quad t > 0 \tag{9}$$
$$k_2 \frac{\partial T_2}{\partial x} + h_2 T_2 = h_2 f_2 \quad \text{at } x = L, \ t > 0$$

and

$$T_1 (x,0) = T_2 (x,0) = T_0 \tag{10}$$

The solutions to equs. (7) and (8) with nonhomogeneous boundary conditions (9) and (10) can be expressed in the form

$$T_i (x,t) = \theta_i (x,t) + \varphi_i (x) f_1 + \psi_i (x) f_2 \tag{11}$$
$$i = 1, 2$$

where the functions $\varphi_i(x)$, $\psi_i(x)$, and $\theta_i(x,t)$ are the solutions of the following subproblems:

(a) $\frac{d^2\varphi_1}{dx^2} = 0$

$$\frac{d^2\varphi_2}{dx^2} = 0$$

with $\quad -k_1 \frac{d\varphi_1}{dx} + h_1\varphi_1 = h_1 f_1 \qquad$ at $x = 0$

$\varphi_1 = \varphi_2 \qquad$ at $x = \ell$

$k_1 \frac{d\varphi_1}{dx} = k_2 \frac{d\varphi_2}{dx} \qquad$ at $x = \ell$

$k_2 \frac{d\varphi_2}{dx} + h_2 T_2 = 0 \qquad$ at $x = L$

Therefore,

$$\varphi_1 (x) = C_1 x + D_1$$

$$\varphi_2 (x) = C_2 x + D_2$$

(b) $\frac{d^2\psi_1}{d\ x^2} = 0$

$$\frac{d^2\psi_2}{dx^2} = 0$$

with $\quad -k_1 \frac{d\psi_1}{dx} + h_1\psi_1 = 0 \qquad$ at $x = 0$

$\psi_1 = \psi_2 \qquad$ at $x = \ell$

$k_1 \frac{d\psi_1}{dx} = k_2 \frac{d\psi_2}{dx} \qquad$ at $x = \ell$

$k_2 \frac{d\psi_1}{dx} + h_2 x_2 = h_2 f_2 \qquad$ at $x = L$

we have

$$\psi_1 (x) = E_1\, x + F_1$$

$$\psi_2 (x) = E_2\, x + F_2$$

In this case, the sensor output will be dependent on time. To determine the sensor output we calculate the temperature difference to be

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$$\Delta T^{u}(t) = T_1(x,t) - T_2(x_2,t)$$
$$= \theta_1(x_1,t) - \theta_2(x_2,t) + [\varphi_1(x_1) - \varphi_2(x_2)]f_1$$
$$+ [\psi_1(x_1) - \psi_2(x_2)]f_2$$

Computational results show that

$$[\varphi_1(x_1) - \varphi_2(x_2)]f_1 + (\psi_1(x_1) - \psi_2(x_2)]f_2 = \Delta T^{S}$$

Therefore,

$$\Delta T^{u}(t) = \theta_1(x_1,t) - \theta_2(x_2 t) + \Delta T^{S} \qquad (12)$$

(c) $\theta_1(x,t)$ and $\theta_2(x,t)$ are determined from the following homogeneous problem:

$$\alpha_1 \frac{\partial^2 \theta_1}{\partial x^2} = \frac{\partial \theta_1}{\partial t}$$
$$\alpha_2 \frac{\partial^2 \theta_2}{\partial x^2} = \frac{\partial \theta_2}{\partial t}$$

Subject to $-k_1 \frac{\partial \theta_1}{\partial x} + h_1\theta_1 = 0$ at $x = 0$

$\theta_1 = \theta_2$ at $x = \ell$

$k_1 \frac{\partial \theta_1}{\partial x} = k_2 \frac{\partial \theta_2}{\partial x}$ at $x = \ell$

$k_2 \frac{\partial \theta_2}{\partial x} + h_2\theta_2 = 0$ at $x = L$

and $\theta_i(x,0) = T_0 - \varphi_i(x)f_1 - \psi_i(x)f_2$

let $\theta_i(x,t) = X_i(x)\,\Gamma(t)$

$\Rightarrow \Gamma(t) = e^{-\beta_n^2 t}$

where β_n are eigenvalues which will be determined later.

$$X_{1n}(x) = A_{1n} \operatorname{Sin}\left(\frac{\beta_n}{\sqrt{\alpha_1}} x\right) + B_{1n} \operatorname{Cos}\left(\frac{\beta_n}{\sqrt{\alpha_1}} x\right)$$

$$X_{2n}(x) = A_{2n} \operatorname{Sin}\left(\frac{\beta_n}{\sqrt{\alpha_2}} x\right) + B_{2n} \operatorname{Cos}\left(\frac{\beta_n}{\sqrt{\alpha_2}} x\right)$$

Without loss of generality, we choose $A_{1n} = 1$, B_{1n}, A_{2n}, and B_{2n} can be determined with the corresponding boundary conditions(and several pages of algebra).

Finally,

$$\Delta T^u(t) = \sum_{n=1}^{\infty} C_n [X_{1n}(x_1) - X_{2n}(x_2)] e^{-\beta_n^2 t} + \Delta T^S \tag{13}$$

where C_n is determined by the initial conditions. The equation for the determination of the eigenvalues β_n is

$$\begin{vmatrix} \sin\gamma + H_1\gamma\sin\gamma & -\sin\eta & -\cos\eta \\ K\ \cos\gamma - KH_1\gamma\sin\gamma & \cos\eta & -\sin\eta \\ 0 \quad H_2\eta\cos(\frac{L}{\ell}\eta) + \sin(\frac{L}{\ell}\eta) & \cos(\frac{L}{\ell}\eta) - H_2\eta\sin(\frac{L}{\ell}\eta) & \end{vmatrix} = 0$$

where $\gamma = \dfrac{\beta_n}{\sqrt{\alpha_1}}\ell$, $\eta = \dfrac{\beta_b}{\sqrt{\alpha_2}}\ell$, $H_1 = \dfrac{k_1}{h_1\ell}$

$$K = \frac{k_1}{k_2}\sqrt{\frac{\alpha_1}{\alpha_2}}, \quad H_2 = \frac{k_2}{h_2 L}$$

Some additional alegebra will lead to the following transcendtal equation for the determination of the eigenvalues β_n:

$$tg\left(\frac{\ell}{\sqrt{\alpha_1}}\beta n\right) = K\frac{h_2}{h_1}\frac{\ell}{L}\,tg\left(\frac{L-\ell}{\sqrt{\alpha_2}}\beta n\right) \quad (14)$$

Rewrite equation (13) as:

$$\Delta T^{u}(t) = \sum_{n=1}^{\infty} C^{*}_{n}\, e^{-\beta n^2 t} + \Delta T^{s} \quad (15)$$

where

$$C^{*}_{n} = C_n \left[X_{1_n}(x_1) - X_{2_n}{}'(x) \right]$$

Numerical example shows that $\beta_1^2 << \beta_2^2 << \beta_3^2 <<$....., so we can just take the first term to approximate the result. In other words, we have:

$$\Delta T^{u} = c_1 e^{-\beta_1^2 t} + \Delta T^{s}$$

but $\quad \Delta T^{u} = 0 \quad$ at $t = 0$

Therefore, $\quad c_1 = -\Delta T^{s}$

Then, $$\frac{\Delta T^{u}}{\Delta T^{s}} = 1 - e^{-\beta_1^2 t} \quad (16)$$

where β_1 can be determined from Equation (14). The following observations can be made by examining Eqs. (14) and (16):

(1) $\Delta T^{u}(t) = \Delta T^{s}$ as $t \to \infty$;

(2) The transient process depends on the boundary conditions (h_1, h_2), the geometry (ℓ,L) and the properties ($k_1, k_2, \alpha_1, \alpha_2$) of the sensor.

(3) The transient term ($e^{-\beta_1^2 t}$) may be very important if β_1 is small.

Typical Example

To illustrate the impact of these results consider a typical example. Assume that D = 0.8 cm, ℓ = 0.02 cm, L = 0.11 cm, choose $h_2/h_1 = 1.0$. See Fig. 1. The material of the sublayer is alumel whose properties are $k_2 = 37$ w/m·k, $\alpha_2 = 9.7 \times 10^{-5}$ m^2/s. If the materials of the foil are pure nickel and nichrome (80% Ni, 20% Cr), respectively. The results are shown in Table 1.

Table 1

Foil Material	α_1 (m^2/s)	β_1	$e^{-\beta^2 t}$ (at t = 0.05 second)
Nickel (pure)	2.3×10^{-5}	16.745	8.15×10^{-7}
Nichrome (80% Ni, 20% Cr)	3.4×10^{-6}	6.438	1.26×10^{-1}

As can be seen from Table 1, if we choose Ni–Cr alloy as the foil material, the transient term "$e^{-\beta_1^2 t}$" would be important.

SUMMARY

We have initiated a study of the transient heat flux response of heat flux sensors. This paper presents a one dimensional analysis of the problem. This was done to elucidate the key parameters and trend for the problem. The results of a heat flux sensor depends on the thermal boundary conditions, the geometry and the thermal properties of the sensor. In particular, it was shown that if the thermal diffusivity of the sensor is small, say for a material like Nichrome, then the transient behavior must be taken into account.

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Schematic of Transient Analysis

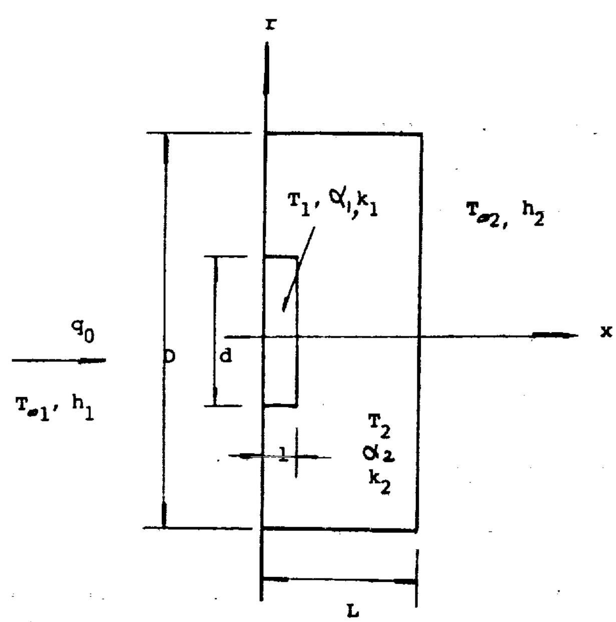

Fig. 1