IN-37
19364
P11

NASA Technical Memorandum 104414

Comparison of Weibull Strength Parameters From Flexure and Spin Tests of Brittle Materials

(NASA-TM-104414) COMPARISON OF WEIBULL STRENGTH PARAMETERS FROM FLEXURE AND SPIN TESTS OF BRITTLE MATERIALS (NASA) 11 p CSCL 13H
N91-24593
Unclas
G3/37 0019364

Frederic A. Holland, Jr., and Erwin V. Zaretsky
Lewis Research Center
Cleveland, Ohio

Prepared for the
Ninth Biennial Conference on Reliability, Stress Analysis, and Failure Prevention
sponsored by the American Society of Mechanical Engineers
Miami, Florida, September 22–25, 1991

NASA

COMPARISON OF WEIBULL STRENGTH PARAMETERS FROM FLEXURE

AND SPIN TESTS OF BRITTLE MATERIALS

Frederic A. Holland, Jr.,* and Erwin V. Zaretsky**
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

Fracture data from five series of four-point bend tests of beams and spin tests of flat annular disks were reanalyzed. Silicon nitride and graphite were the test materials. The experimental fracture strengths of the disks were compared with the predicted strengths based on both volume flaw and surface flaw analyses of four-point bend data. Volume flaw analysis resulted in a better correlation between disks and beams in three of the five test series than did surface flaw analysis. The Weibull slopes (moduli) and characteristic gage strengths for the disks and beams were also compared. Differences in the experimental Weibull slopes were not statistically significant. It was shown that results from the beam tests can predict the fracture strength of rotating disks.

NOMENCLATURE

A_e effective area, m^2

h beam height, mm

L beam length, mm

L_1 length of outer span, mm

L_2 length of inner span, mm

m Weibull slope or modulus

n number of samples

P_f probability of failure

r radius, mm

r_i inner radius, mm

r_o outer radius, mm

t disk thickness, mm

V volume, m^3

V_e effective volume, m^3

w beam width, mm

ρ material density, kg/m^3

ν Poisson's ratio

σ failure stress, MPa

σ_{max} maximum stress, MPa

σ_o characteristic fracture strength, MPa

σ_{oa} characteristic gage fracture strength of a unit area, MPa (m^2)

σ_{ov} characteristic gage fracture strength of a unit volume, MPa (m^3)

σ_t tangential stress

ω rotational speed, rad/s

INTRODUCTION

Ceramics, which offer high-temperature strength, good oxidation and corrosion resistance, and low weight, are being considered in place of traditional metals in heat engine applications. Successful implementation of ceramic components into high-temperature propulsion system components, such as turbine disks and blades, promises both increased fuel efficiency (due to higher allowable operating temperatures and lower weight) and potential longer life (due to the material's resistance to chemical attack). A major concern in using ceramic materials for rotating components is the ability to accurately predict structural reliability. A first step toward achieving this objective is establishing a reliable data base.

A logical specimen choice for generating strength data for the purpose of predicting the failure of rotating components is a small rotating disk. A thin, flat annular disk could be regarded as a simple approximation of the more complex turbine disk. Performing a stress analysis of such a disk in rotation is much

*Associate Member, ASME.

**Fellow, ASME.

easier than analyzing an actual turbine geometry. In both the small disk and the turbine disk, the loading is by centrifugal force.

The most commonly used method of generating strength data for brittle materials is the flexure beam test. Flexure testing is relatively simple and inexpensive to perform. Because the bending of a beam creates a high stress gradient where the maximum stress is at the surface, bend tests are assumed to be most useful when the emphasis is on surface-initiated rather than volume-initiated failure. This implies that flexure testing is less desirable for volume flaw analysis.

Numerous experimental errors can take place in bending tests. These errors are due to the low compliance of the material typically used in these tests, which may cause specimen displacement during loading. Other errors include twisting, wedging, and friction imposed on the loaded specimen by the test fixture (Baratta et al., 1987; Hoagland et al., 1976).

Spin testing of disks places a larger volume of material under stress than does the typical flexure test. In theory, there is no test instrument reaction on the specimen. The only load is the centrifugal body force. However, any eccentricity about the axis of rotation may cause vibration and premature failure. Also, because rotating disks tend to disintegrate into many small pieces upon rupture, postmortem fractography can be extremely difficult.

Many attempts have been made to predict the reliability of brittle structures from four-point bend data (Anon., 1987; Cooper, 1988; Gyekenyesi, 1986; Paluszny and Wu, 1977; Salem et al., 1990; and Swank and Williams, 1981). In many of these cases, flexure data have been used to predict the failure of flat, rotating annular disks. It is reasoned that if simple disks can be successfully modeled from flexure beam data, more complicated geometries can be modeled as well. Test results from these investigators have varied. In view of the aforementioned, it was the objective of the work reported herein (1) to compare the strength characteristics of brittle materials resulting from four-point bend tests and from spin testing of flat annular disks, (2) to use the strength results from four-point bend tests to predict the failure strengths of flat, annular rotating disks, and (3) to compare the experimental and predicted strengths.

ANALYTICAL METHOD

The Weibull equation has been used to model the strength distribution of brittle materials. In its most basic form, for uniform uniaxial stress states, the function is expressed as

$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_o}\right)^m\right] \tag{1}$$

where P_f is the probability of failure, σ is the failure stress, and σ_0 is the characteristic fracture strength, or the stress at which 63.2 percent of the specimens fail. The Weibull slope, or modulus, m is a measure of the strength variability among identical samples. A high value of m indicates less strength variability.

If n number of samples are tested and ranked in order of increasing strength, the probability of failure associated with the i^{th} stress is often given by

$$P_f(\sigma_i) = \frac{i - 0.3}{n + 0.4} \tag{2}$$

For nonuniform stress distributions, assuming that the strength of the brittle material is volume dependent, the Weibull equation can be written in the integral form

$$P_f = 1 - \exp\left[\frac{-\int_v (\sigma/\sigma_o)^m \, dv}{V_e}\right] \tag{3}$$

where the integral is taken over the entire tensile-stressed volume of the structure and V_e is the effective volume associated with the characteristic fracture strength σ_o and the Weibull modulus m.

The effective volume V_e of a structure is defined as

$$V_e = \int_v \left(\frac{\sigma}{\sigma_{max}}\right)^m dv \tag{4}$$

where σ_{max} is the maximum stress of the structure. The effective volume defines the region where failure can occur. Substituting Eq. (4) into Eq. (3) for V_e gives

$$P_f = 1 - \exp\left[-\left(\frac{\sigma_{max}}{\sigma_o}\right)^m\right] \tag{5}$$

The effective volume conveniently allows the Weibull distribution to be written in terms of the maximum stress. The estimated failure distribution of a component with effective volume V_{e2} can then be computed with the following relation:

$$P_f = 1 - \exp\left[-\left(\frac{\sigma_{max}}{\sigma_o}\right)^m\right]^{V_{e2}/V_{e1}} \tag{6}$$

where V_{e1} is the effective volume whose characteristic strength is σ_o. The following relation is a corollary to Eq. (6) and can be used to scale the strength from one effective volume to another:

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{V_{e1}}{V_{e2}}\right)^{1/m} \tag{7}$$

It is desirable to normalize all strengths to a unit volume of material. This practice will allow all reported strengths to be on the same basis for direct comparison. This can be accomplished simply by setting V_{e2} in Eq. (7) to a convenient unit volume, say 1 mm^3 or 1 m^3. With the units of V_{e1} consistent with those of V_{e2}, the calculated value of σ_2 will then be for a unit volume of material. Because V_{e2} = 1 (unit), the characteristic strength for a unit volume of material derived from Eq. (7) is

$$\sigma_{ov} = \sigma_o V_e^{1/m} \tag{8}$$

where the units of σ_{ov} are those of stress because the volume dimensions cancel. In order to make clear the unit of volume upon which the reported strengths are based, the following practice is proposed and will be adopted in this report: All unit volume strength values will be followed by the unit of volume in parentheses. Therefore, if the calculated strength for one cubic meter (1 m^3) of material is 100 MPa, the unit volume strength will be denoted as 100 MPa (m^3).

Using a normalized strength in Eq. (3) gives

$$P_f = 1 - \exp\left[-\int\left(\frac{\sigma_{max}}{\sigma_{ov}}\right)^m dv\right] \qquad (9)$$

where σ_{ov} is the unit volume characteristic strength, or characteristic gage fracture strength, a normalizing constant. The unit volume characteristic strength is theoretically a material parameter. Substituting this normalized strength into Eq. (5) changes the probability of failure to

$$P_f = 1 - \exp\left[-V_e\left(\frac{\sigma_{max}}{\sigma_{ov}}\right)^m\right] \qquad (10)$$

where the effective volume V_e is that of the component whose reliability is required. Similar equations can be obtained for surface (area) analysis by substituting area for volume in the preceding equations. As a result Eq. (10) can be written for surface area as follows:

$$P_f = 1 - \exp\left[-A_e\left(\frac{\sigma_{max}}{\sigma_{oa}}\right)^m\right] \qquad (11)$$

where σ_{oa} is normalized for a gage or unit area.

RESULTS AND DISCUSSION

Comparison of Experimental and Predicted Fracture Strengths

Strength data for silicon nitride (Si_3N_4) and graphite were obtained from the open literature for four-point bend tests on beam specimens and for spin tests on flat annular disks. These data were reanalyzed. Although graphite is anisotropic, it was reported that the longitudinal axes of the beams and the planes of the flat disks were made to coincide with the isotropic plane of the material. Four different sources were used, representing U.S. (Swank and Williams, 1981), Japanese (Matsusue et al., 1981; Okamura et al., 1988), and British (Cooper, 1988) researchers. Table 1 gives the dimensions of the test beams and disks and their material properties. Figure 1 shows the configuration of a four-point bend test, and Fig. 2 illustrates a flat, annular disk specimen.

In analyzing the data the following assumptions were made:

(1) The material had a uniform distribution of flaws.

(2) Spinning disk fracture was dominated by the tangential stress (radial stress was neglected).

(3) The flexure beam and disk specimens failed in the same manner (either by volume flaws or surface flaws).

The second assumption is believed to be plausible because the maximum tangential stress was significantly higher than the maximum radial stress in the rotating disks under study. The differences between the tangential and radial stress distributions in a typical disk analyzed herein (Swank and Williams, 1981) can be seen in Fig. 3. In this disk the maximum tangential stress was nearly three times the maximum radial stress. The ratios of the maximum tangential stress to the maximum radial stress for the disks analyzed herein are shown in Table 2. Where the tangential stress was maximum, the radial stress was zero. However, only knowledge of the fracture origins can indicate whether the radial stress may have contributed to fracture. This information was not available. In addition, it was assumed that the disks were sufficiently thin that no significant stress variation occurred through the thickness.

The third assumption was also necessary because no information was available on the failure origins of the disks or the beams. It was not known whether specimen fracture was dominated by surface flaws or by subsurface (volume) flaws or if both failure modes were present. Surface analysis is an important consideration because the maximum stress occurred at the surface in both the disks and the beams. Because of this, both types of analysis were performed, one assuming that the probability of failure was a function of stressed volume and the other assuming that the probability of failure was dependent upon the stressed surface area.

The reported fracture data from the reference sources were ranked according to Eq. (2), and the Weibull parameters (characteristic strength and Weibull slope) were determined by linear regression analysis. These results are shown in Table 3. Because fracture data were not given for the beam specimens in reference sources A and B, the reported Weibull parameters were used.

The effective volume of a beam in four-point bending was calculated by integrating Eq. (4) over the volume in tension to obtain

$$V_e = \frac{wh}{2}\left[\frac{L_1 + mL_2}{(m + 1)^2}\right] \qquad (12)$$

where the L_1 is the length of the outer span, L_2 is the length of the inner span, w is the beam width, h is the beam height, and m is the Weibull slope associated with a volume flaw population. Similarly, the effective area A_e is defined as

$$A_e = \left[\frac{\left(\frac{L_2}{L_1}\right)m + 1}{(m + 1)^2}\right]\left[\left(\frac{mw}{w + h}\right) + 1\right](w + h)L_1 \qquad (13)$$

where m is the Weibull slope resulting from surface analysis.

The effective volume of the disks was obtained by numerical integration of Eq. (4). The stress σ in Eq. (4) was replaced by the tangential stress σ_t of the disk, where

$$\sigma_t = \frac{3 + \nu}{8}\rho\omega^2\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1 + 3\nu}{3 + \nu}r^2\right) \qquad (14)$$

and r_i and r_o designate the inner and outer radii, respectively, r is the variable radius, ν is Poisson's ratio, ρ is the material density, and ω is the rotational speed. The effective area of the disk was found by substituting area for volume in Eq. (4).

The characteristic strength based on gage volume (unit volume, m^3) and gage area (unit area, m^2) for the four-point bend tests and the rotating disk tests are given in Table 4. The confidence numbers for these tests are also given. These numbers indicate the percentage of time that the characteristic strengths from the disk and the beam will have the same relation to each other (Johnson, 1959). As an example, a confidence number of 90 percent means that in 90 out of 100 tests the relationship of the characteristic strengths of the beams and disks will be the same. A confidence number of 95 percent is equivalent to a 2σ (standard deviation) confidence limit.

Figure 4 shows the statistical distribution of strength for the disks derived from experiments and from predictions based on the four-point bend data. Volume analysis of the flexure data resulted in a better correlation with the experimental disk data for reference sources A and B. Area analysis resulted in better agreement between the disks and the beams for reference sources C and D.

The graphite (reference source D (Cooper, 1988)) showed little scatter in strength (Fig. 4(d)) as indicated by its high Weibull slope (m = 20). A body under nonuniform stress that is composed of material with a small variation in strength is more likely to fail at the maximum stress than a material with a large variation in strength (low Weibull slope). Because the maximum stress occurs at the surface for both a rotating disk and a four-point bend specimen, the fracture probability for graphite beams and disks can be expected to be more sensitive to surface area than to volume.

For reference source C (Matsusue et al., 1981) the correlation between the experimental and predicted disk strengths based on surface analysis of the beam data was exceptionally good. Note that the variables affecting specimen strength were well controlled in this reference source. The surface roughnesses of all specimens were fixed at 1 μm. The disk was actually a ring and the beam specimens were taken from the center of the ring. This procedure minimized any strength differences that may have occurred from nonuniformity of flaws within the batch.

The comparison between experiment and prediction for the Si_3N_4 material from reference sources B (Okamura et al., 1988) and C (Matsusue et al., 1981) was reasonably good. However, for the Si_3N_4 from source A (Swank and Williams, 1981) the beam data predicted a statistically higher strength for the disk than was experimentally obtained. The opposite was true for the graphite material from reference source D (Cooper, 1988), where experiment gave the higher value. These trends were consistent for both the volume and surface analyses.

Weibull Slope Variation

There were some differences in the experimental Weibull slopes obtained from flexure testing and spin testing. However, these differences are not considered significant. They may be explained by the relatively few samples tested. Ninety-percent confidence limits on the Weibull slope showed significant overlap between the beams and the disks in all but one case. This case was the reference source A material (Table 3), where the Weibull slope for the beams was 7.65 whereas a slope of 4.86 was obtained for the fractured disks. For the 85 flexure specimens tested, the confidence limits on the Weibull slope were 6.56 and 8.68. For the seven disks tested, the confidence limits were 2.27 and 6.98. The overlap in Weibull slope between the beams and the disks is therefore in the very narrow range from 6.56 to 6.98. This suggests that even if more disk specimens had been tested, it is likely that there would be no overlap in the Weibull slope. However, if the specimens and the disks were from the same batch of material, the slopes would be expected to be the same. For the reference source A material it was reported that the billets used to make the beams and the disks were fabricated at different times (Swank and Williams, 1981). Any difference in fabrication that may have resulted might account for some of the disparity between the Weibull slopes.

Disk Strength Prediction

For aerospace components as well as for critical components in heat engines, the prediction of early failure is of primary importance. Generally, the fracture strength at a 99-percent probability of survival, or a 1-percent probability of failure, is used for comparison purposes. It is also generally considered by some investigators that an experimental strength of ±20 percent of that predicted by analysis is an acceptable correlation between experiment and theory. This criterion was used to compare the predicted strengths for the rotating disks from the reference sources.

For reference source B good correlation was obtained between the volume analysis predictions and the experimental results. For reference source C the experimental strength was between those values predicted by the volume and surface analyses. Hence, it may be reasonably concluded from these experiments that four-point bend tests of beams can predict with reasonable engineering certainty the experimental results obtained from a rotating disk.

For the Si_3N_4 material of reference source A the experimental characteristic strengths were 54 and 44 percent of the results predicted by using volume and surface analyses, respectively. For the graphite material from reference source D the experimental strengths were 3.2 and 2.1 times the values predicted by using volume and surface analyses, respectively. According to the previously mentioned criterion, these two sets of experiments by themselves would suggest that the results from flexure beam specimens may not always reflect those obtained with a rotating disk. Unfortunately, the material and physical variances between the disk and beam tests were not sufficiently defined within the reference sources to explain the difference in results. However, as previously discussed, for reference source A the billets used to make the beams and the disks were fabricated at different times. Any differences in fabrication may account for the differences in strength.

An issue remains whether obtaining gage fracture strengths from rotating disks would be a better predictor of fracture strength of another rotating body than using four-point bend specimens of the same material. Figure 5 shows the experimental fracture strength distribution of disk 2 from reference source B (Okamura et al., 1988). Disk 2 was the larger of the two disks tested by Okamura. The dimensions of the disks are given in Table 1. The fracture strength distribution of disk 2 was predicted by using the data of Tables 3 and 4 for disk 1 and both volume flaw and surface flaw analyses. The predicted fracture strengths were identical for each method. This distribution is shown in Fig. 5 and is compared with the experimental results. As can be seen, the prediction was lower than the experimental results except at the lowest probabilities of failure.

Figure 4(b) shows the distributions for disk 2 predicted by using beam specimens. The experimental characteristic strength of disk 2 was 688 MPa. The predicted characteristic strength of disk 2 based on disk 1 data was 607 MPa, whereas the predicted characteristic strength of disk 2 based on four-point bend data was 660 MPa, assuming that fracture was due to volume defects. Thus, a closer correlation was actually obtained between disk 2 and the beams than between disk 2 and disk 1.

An engineering approach to the problem of fracture prediction is to predict with reasonable engineering certainty the speed at which a rotating body will fail or, conversely, the probability of a rotating body failing at a certain speed. The prediction of early failures is important for most engineering applications. By using Eq. (14) the speed at a 1-percent probability of failure (i.e., 99 percent of a population distribution will exceed this speed without failure) was determined by both volume flaw and surface flaw analyses. These results are compared with the experimental results in Table 5. Whether the correlation between prediction and experiment is reasonably close is left to the reader. However, it appears that where high reliability is required the predictions may in some instances not be sufficiently conservative without some correlation or safety factor that can be used by a design engineer to ensure product reliability.

SUMMARY OF RESULTS

Fracture data from four reference sources and five series of beam four-point bend tests and disk spin tests were reanalyzed. Two brittle materials, silicon nitride (Si_3N_4) and graphite, were evaluated. The Weibull slopes (moduli) and characteristic fracture strengths of the beams and the disks were compared. The characteristic gage fracture strength was determined from volume flaw and surface flaw analyses. The characteristic gage strength of the beams was used to predict the strength distribution of the rotating disks. The following results were obtained:

1. Four-point bend (flexure) tests of beams can predict with reasonable engineering certainty the experimental fracture strength obtained from a rotating disk.
2. In the five test series presented, a closer correlation between experimental disk strength and predicted strength was obtained in three of the test series by using a volume flaw analysis and in two of the test series by using a surface flaw analysis of the four-point bend data.
3. The difference in Weibull slopes between the disks and the beams that were obtained for each test series were not statistically significant.
4. Experimental rotating disk data may not be a better predictor of rotating body strength than four-point bend tests of beams.

REFERENCES

Anon., 1987, "Advanced Gas Turbine (AGT) Technology Development Project," REPT-31-3725(12), Garrett Corp., Phoenix, AZ, NASA CR-180891.

Baratta, F.I., Matthews, W.T., and Quinn, G.D., 1987, "Errors Associated With Flexure Testing of Brittle Materials," MTL-TR-87-35, U.S. Army Materials Technology Laboratory, Watertown, MA. (Avail. NTIS, AD-A187470.)

Cooper, N.R., 1988, "Probabilistic Failure Prediction of Rocket Motor Components," Ph.D. Thesis, Royal Military College of Science, England. (Avail. UMI, BRDX83446.)

Gyekenyesi, J.P., 1986, "SCARE: A Postprocessor Program to MSC/NASTRAN for the Reliability Analysis of Structural Ceramic Components," Journal of Engineering for Gas Turbines and Power, Vol. 108, No. 3, pp. 540-546. (Also, NASA TM-87188.)

Hoagland, R.G., Marschall, C.W., and Duckworth, W.H., 1976, "Reduction of Errors in Ceramic Bend Tests," Journal of the American Ceramic Society, Vol. 59, No. 5-6, pp. 189-192.

Johnson, L.G., 1959, "The Statistical Treatment of Fatigue Experiments," Rept. No. GMR-202, General Motors Corp., Detroit, MI.

Matsusue, K., Takahara, K., and Hashimoto, R., 1981, "Strength Evaluation Test of Hot-Pressed Silicon Nitride at Room Temperature," NAL-TR-676, National Aerospace Lab., Tokyo, Japan.

Okamura, J., Katohno, K., Matui, M., and Turuta, H., 1988, "High Strength Ceramic Model Tests," Fine Ceramics," pp. 177-190. (Also NASA TT-20889.)

Paluszny, A., and Wu, W., 1977, "Probabilistic Aspects of Designing With Ceramics," Journal of Engineering for Power, Vol. 99, No. 4, pp. 617-630.

Salem, J.A., Manderscheid, J.M., Freedman, M.R., and Gyekenyesi, J.P., 1990, "Reliability Analysis of a Structural Ceramic Combustion Chamber," NASA TM-103264.

Swank, L.R. and Williams, R.M., 1981, "Correlation of Static Strengths and Speeds of Rotational Failure of Structural Ceramics," American Ceramic Society Bulletin, Vol. 60, No. 8, pp. 830-834.

TABLE 1.—MATERIAL PROPERTIES AND SPECIMEN DIMENSIONS FOR FOUR-POINT BEND TESTS AND SPIN TESTS

Reference source[a]	Density, ρ, kg/m^3	Poisson's ratio, ν	Beam dimensions, mm					Disk dimensions, mm		
			Height, h	Width, w	Length, L	Inner load span, L_2	Outer load span, L_1	Inner radius, r_i	Outer radius, r_o	Thickness, t
					Si_3N_4					
A	3250	0.219	3.2	6.4	31.8	9.5	19.0	6.4	41.3	3.8
B (disk 1)	3260	.270	3.0	4.0	40.0	10.0	30.0	15.0	60.0	3.0
B (disk 2)	3260	.270	3.0	4.0	40.0	10.0	30.0	15.0	75.0	3.0
C	3270	.240	5.0	5.0	110.0	50.0	100.0	30.0	55.0	3.0
					Graphite					
D	1840	.100	5.0	5.0	110.0	50.0	100.0	6.4	38.1	3.2

[a]A—Swank and Williams (1981).
B—Okamura et al. (1988).
C—Matsusue et al. (1981).
D—Cooper (1988).

TABLE 2.—RATIOS OF MAXIMUM TANGENTIAL STRESS TO MAXIMUM RADIAL STRESS IN FLAT, ROTATING ANNULAR DISKS

Reference source[a]	Stress ratio
A	2.81
B (disk 1)	3.60
B (disk 2)	3.15
C	10.35
D	2.90

[a]A—Swank and Williams (1981).
B—Okamura et al. (1988).
C—Matsusue et al. (1981).
D—Cooper (1988).

TABLE 3.—EXPERIMENTAL RESULTS FROM FOUR-POINT BEND TESTS AND SPIN TESTS OF FLAT ANNULAR DISKS

Reference source[a]	Material	Type of test specimen	Number of tests, n	Weibull slope, m	Characteristic strength, σ_o, MPa
A	Si_3N_4	Disk	7	4.86	428.3
		Beam	85	7.65	808
B	Si_3N_4	Disk 1	9	13.5	607.9
		Disk 2	9	10.2	687.7
		Beam	(b)	14	906.3
C	Si_3N_4	Disk	9	6.44	481.5
		Beam	15	7.05	613.5
D	Graphite	Disk	28	20.1	37.3
		Beam	41	17	17.6

[a]A—Swank and Williams (1981).
B—Okamura et al. (1988).
C—Matsusue et al. (1981).
D—Cooper (1988).
[b]Unknown.

TABLE 4.—CHARACTERISTIC GAGE STRENGTHS FROM VOLUME AND SURFACE FLAW ANALYSES

Reference source[a]	Material	Type of test specimen	Volume analysis			Surface analysis		
			Effective volume, m^3	Characteristic gage fracture strength, σ_{ov}, MPa (m^3)	Confidence number, percent[b]	Effective area, m^2	Characteristic gage fracture strength, σ_{os}, MPa (m^2)	Confidence number, percent[b]
A	Si_3N_4	Disk	$0.681X10^6$	23.0	>99	$0.486X10^3$	89.1	>99
		Beam	.0124	74.8		.0714	231.9	
B	Si_3N_4	Disk 1	.397	204	50	.548	348.5	70
		Disk 2	.610	169.1	>60	.689	336.8	>85
		Beam	.00453	229.8		.0476	445.2	
C	Si_3N_4	Disk	3.29	67.9	50	2.77	192.9	50
		Beam	.0873	61.2		.527	210.3	
D	Graphite	Disk	.0474	16.1	>99	.156	24.1	>99
		Beam	.0386	6.42		.513	11.3	

[a]A—Swank and Williams (1981).
B—Okamura et al. (1988).
C—Matsusue et al. (1981).
D—Cooper (1988).
[b]Percentage of occurrence that characteristic gage strengths will have the same relation to each other.

TABLE 5.—COMPARISON OF EXPERIMENTAL AND PREDICTED DISK SPEEDS AT A 1-PERCENT PROBABILITY OF FAILURE BASED UPON VOLUME AND SURFACE FLAW ANALYSES

Reference source[a]	Experimental	Volume flaw analysis	Surface flaw analysis
		Predicted	
	Speed at failure, rpm		
A	58 088	78 858	87 176
B (disk 1)	63 637	66 730	71 734
B (disk 2)	51 499	53 629	57 659
C	50 105	45 311	52 177
D	35 590	19 833	25 601

[a]A—Swank and Williams (1981).
B—Okamura et al. (1988).
C—Matsusue et al. (1981).
D—Cooper (1988).

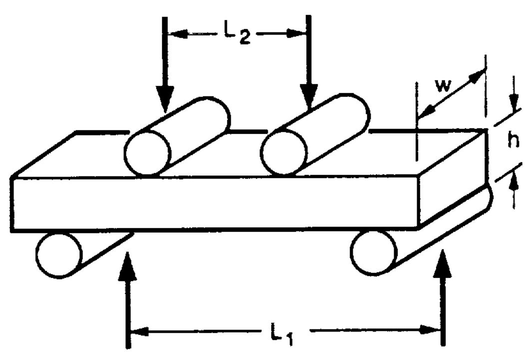


Fig. 1.—Four-point bend test.

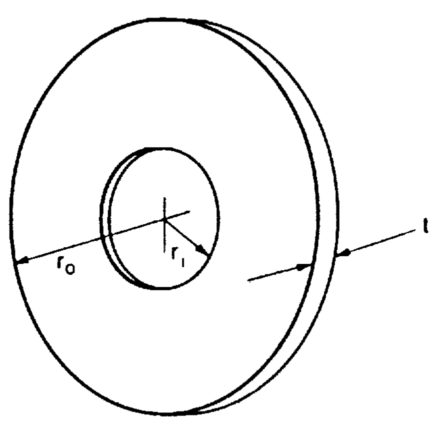


Fig. 2.—Rotating annular disk.

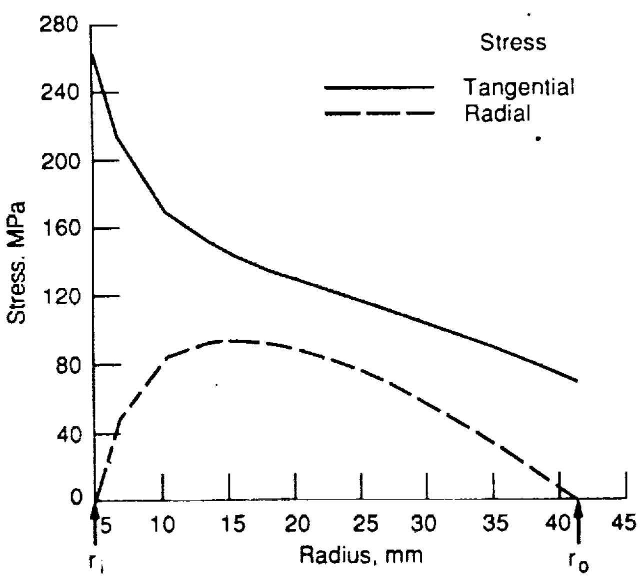


Fig. 3.—Stress distribution in rotating annular disk. Speed, 73 700 rpm; inner radius, r_i, 6.35 mm; outer radius, r_o, 41.275 mm; thickness, t, 3.8 mm; material, Si_3N_4.

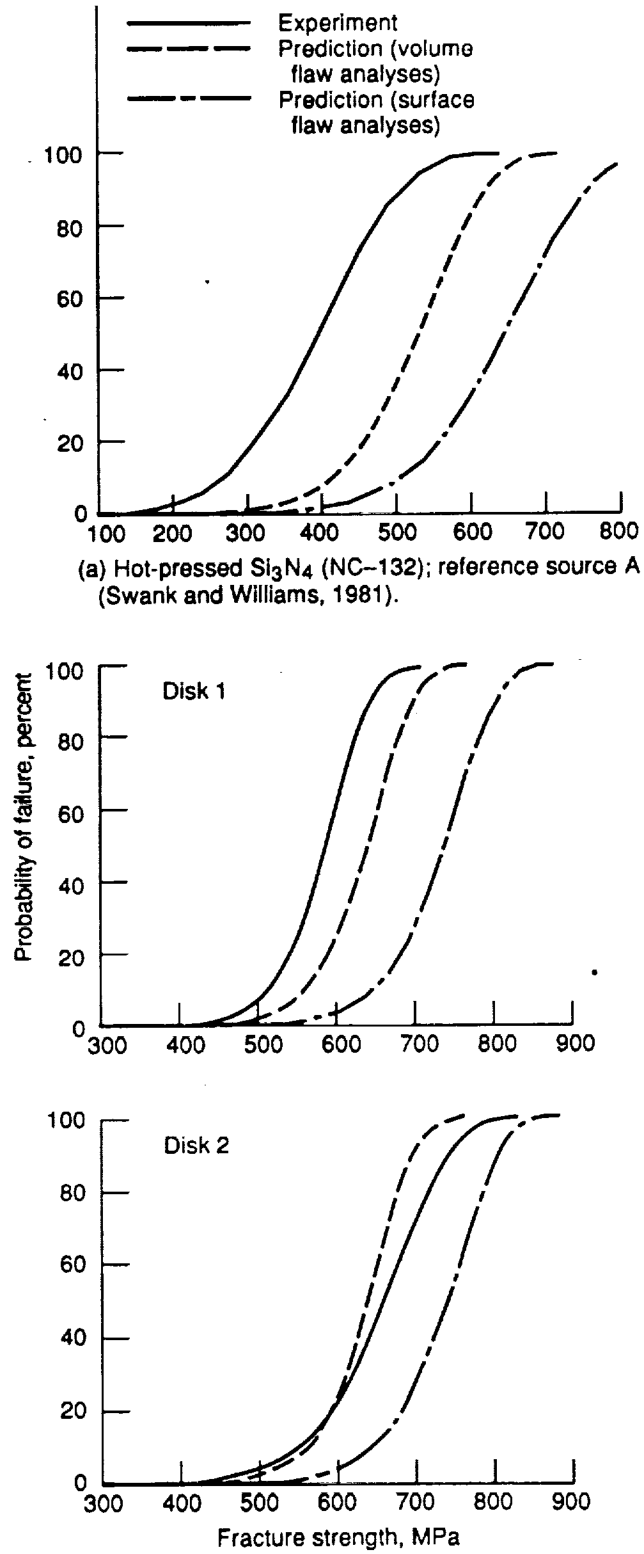
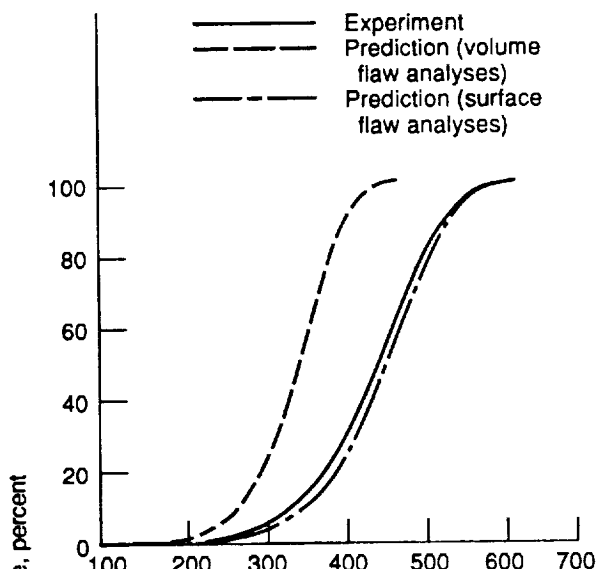
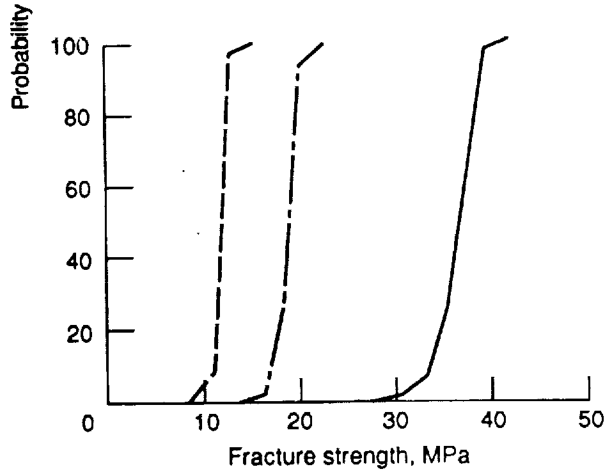


Fig. 4.—Comparison of experimental and predicted fracture strength distributions for flat, rotating annular disks. Predictions based on four-point bend data using volume and surface flaw analyses.



(c) Hot-pressed Si_3N_4; reference source C (Matsusue et al., 1981).



(d) Graphite; reference source D (Cooper, 1988).

Fig. 4.—Concluded.

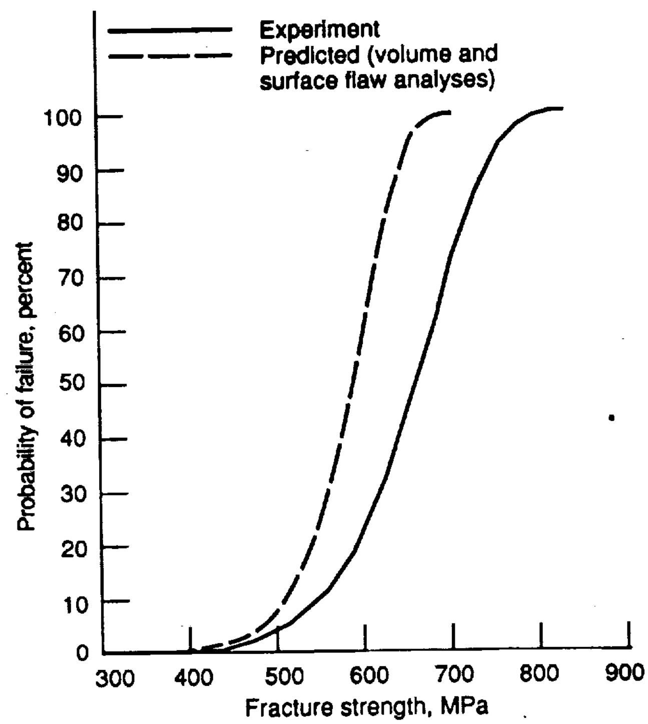


Fig. 5.—Comparison of experimental and predicted fracture strength distributions for a rotating annular disk (disk 2). Predictions based on spin test data from smaller disk (disk 1). Disk 1: inner radius, r_i, 15 mm; outer radius, r_o, 60 mm. Disk 2: r_i, 15 mm; r_o, 75 mm; material, sintered Si_3N_4. Reference source B (Okamura et al., 1988).

NASA
National Aeronautics and Space Administration

Report Documentation Page

1. Report No. NASA TM-104414	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Comparison of Weibull Strength Parameters From Flexure and Spin Tests of Brittle Materials		5. Report Date
		6. Performing Organization Code
7. Author(s) Frederic A. Holland, Jr., and Erwin V. Zaretsky		8. Performing Organization Report No. E-6245
		10. Work Unit No. 505-63-1B
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001		14. Sponsoring Agency Code

15. Supplementary Notes
Prepared for the Ninth Biennial Conference on Reliability, Stress Analysis, and Failure Prevention sponsored by the American Society of Mechanical Engineers, September 22-25, 1991. Responsible person, Frederic A. Holland, Jr., (216) 433-8367.

16. Abstract
Fracture data from five series of four-point bend tests of beams and spin tests of flat annular disks were reanalyzed. Silicon nitride and graphite were the test materials. The experimental fracture strengths of the disks were compared with the predicted strengths based on both volume flaw and surface flaw analyses of four-point bend data. Volume flaw analysis resulted in a better correlation between disks and beams in three of the five test series than did surface flaw analysis. The Weibull slopes (moduli) and characteristic gage strengths for the disks and beams were also compared. Differences in the experimental Weibull slopes were not statistically significant. It was shown that results from the beam tests can predict the fracture strength of rotating disks.

17. Key Words (Suggested by Author(s)) Spin tests; Bend tests; Silicon nitride; Graphite; Brittle materials; Ceramics; Fracture strength; Probability theory; Statistics	18. Distribution Statement Unclassified - Unlimited Subject Category 37

19. Security Classif. (of the report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified	10	A02

NASA FORM 1626 OCT 86

*For sale by the National Technical Information Service, Springfield, Virginia 22161