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A Scheme for Bandpass Filtering Magnetometer Measurements To Reconstruct Tethered Satellite Skiprope Motion

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

A SCHEME FOR BANDPASS FILTERING MAGNETOMETER MEASUREMENTS TO RECONSTRUCT TETHERED SATELLITE SKIPROPE MOTION

I. INTRODUCTION

The first flight of NASA's tethered satellite system is presently scheduled for 1992. It is to be carried onboard the space shuttle to a 300-km orbit with a 28.5° inclination [1]. There the satellite will be deployed outward along the orbit radius vector, to a tether length reaching 20 km, and later retrieved. It is well known that the tether may exhibit some very strange dynamics, one of which is the so-called skiprope motion [2]. This mode of vibration is of particular concern because it has very little damping and could pose a problem during satellite retrieval and docking. If this happens, the present plan is to have the shuttle flight crew execute an orbiter yaw maneuver that is properly phased relative to the rotating tether so as to cause the rotary motion to decay quickly [3]. However, to know when to maneuver, at what rate, and in what direction, requires knowing tether position as a function of time: and this means estimating it. The baseline approach is to do this on the ground, in near real-time, using a state observer. Originally, the observer used only satellite rate gyro measurements as inputs [4]. Afterwards, it was modified to use satellite magnetometer measurements also.

This paper describes a completely different approach to this problem. It uses the same magnetometer measurements, but filters them through bandpass filters tuned to the skiprope frequency. It is simple to implement, quite robust, and can be used in parallel with or as a backup to the observer. It is presented in this paper in the following manner. A theoretical development of its underlying equations is given in section II. These are utilized in the implementation scheme described in section III. Simulation results for it are given in section IV. Final comments are made in section V.

II. THEORETICAL DEVELOPMENT

Previous studies indicate a relationship between tether position caused by skiprope motion and satellite attitude, as shown in figure 1 [5]. Tether deflection at its midpoint, d, is related to the list of the satellite $\vec{i}_{S3}$ axis from the orbit radius vector, α_{SM}, by the relationship

$$d = \frac{\alpha_{SM} L}{\pi} \quad . \tag{1}$$

where L is tether length. As the tether skipropes, the satellite $\vec{i}_{S3}$ axis cones about the orbit radius vector, $\vec{i}_W$, at the skiprope frequency with the $\vec{i}_{S3}$ axis lying in the plane of the tether, as shown in figure 1. Hence, if the coning motion can be reconstructed, the tether motion can be inferred. The scheme proposed here seeks to do that based on measurements of the Earth's magnetic field with an onboard magnetometer.

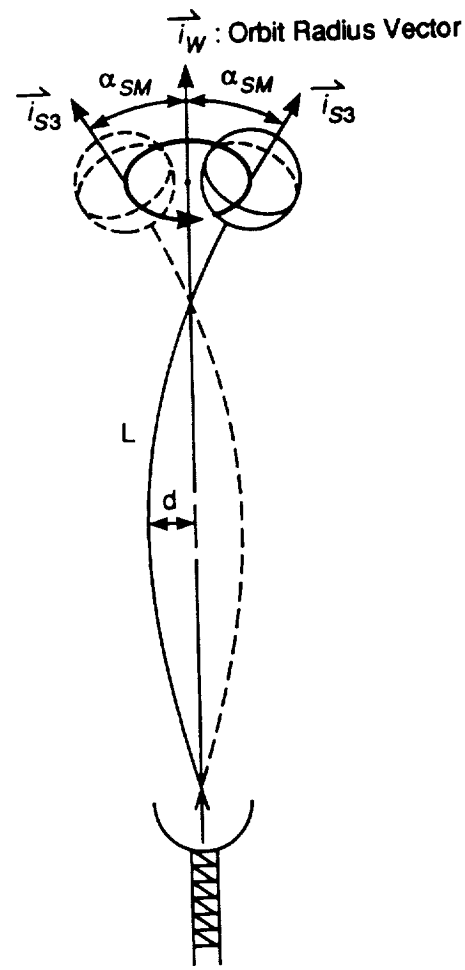

Figure 1. Tether skiprope motion.

To develop this scheme, two coordinate frames are utilized. One is the satellite body-fixed frame $(\vec{i}_{S1},\vec{i}_{S2},\vec{i}_{S3})$, and the other is the orbit-fixed frame $(\vec{i}_U,\vec{i}_V,\vec{i}_W)$, shown in figure 2. As noted there, $\vec{i}_U$ is aligned with the orbit velocity vector, $\vec{i}_V$ with the orbit north, and $\vec{i}_W$ with the orbit radius vector, directed away from the Earth. The $(\vec{i}_{S1},\vec{i}_{S2},\vec{i}_{S3})$ frame is referenced to the $(\vec{i}_U,\vec{i}_V,\vec{i}_W)$ frame by the Euler angles $(\alpha_{S3},\alpha_{S2},\alpha_{S1})$. The Earth's magnetic field flux density vector $\vec{B}$ is referenced to it by the parameters $(B, \beta_{EL},\beta_{AZ})$, as shown in figure 3, where B is its magnitude in Gauss. It is assumed that the steady-state attitude motion of the satellite due to tether skiprope can be described by

$$\left.\begin{aligned} \alpha_{S2} &= \alpha_{S2M} \sin(\omega_{SR} t) \\ \alpha_{S1} &= \alpha_{S1M} \cos(\omega_{SR} t) \ . \end{aligned}\right\} \qquad (2)$$

where ω_{SR} is the skiprope frequency and can be either positive or negative depending on the direction of coning. The angle α_{S3} is assumed constant and small enough, along with α_{S2} and α_{S1}, that $\cos(\alpha_{Si}) \doteq 1$ and $\sin(\alpha_{Si}) \doteq \alpha_{Si}$, for $i = 1,2,3$.

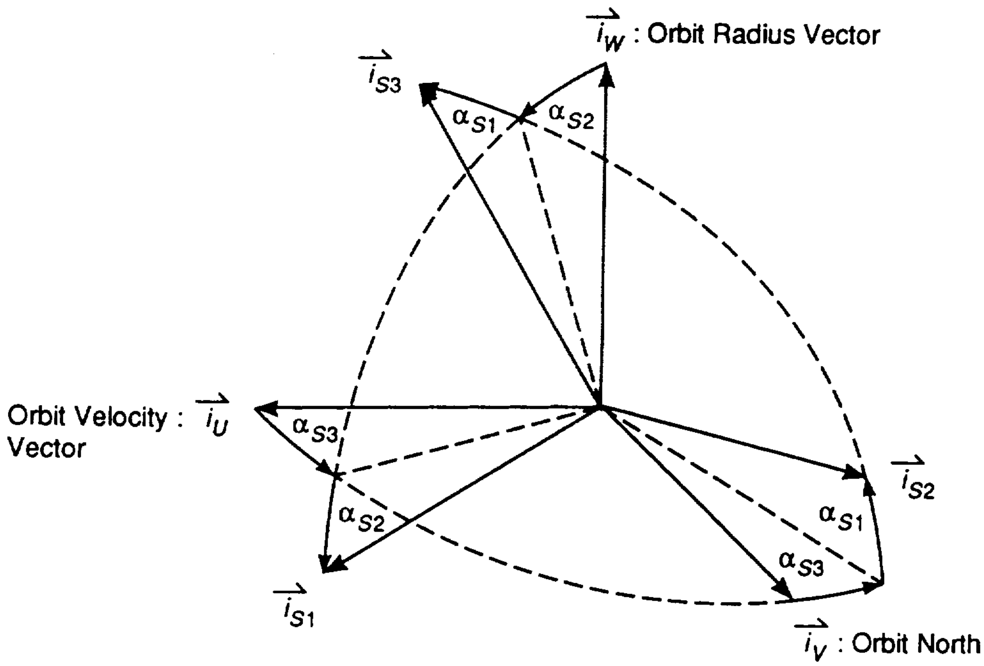

Figure 2. Satellite axes referenced to orbital coordinates.

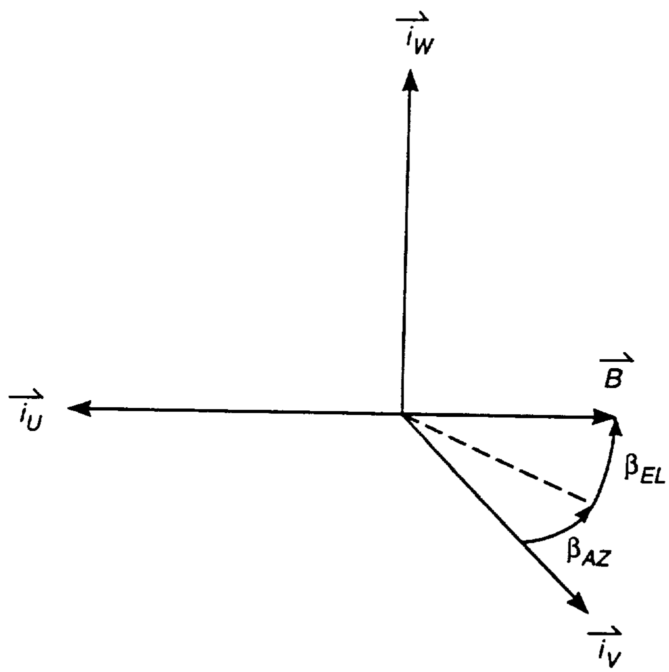

Figure 3. Earth's magnetic field referenced to orbital coordinates.

Given this model, it is straightforward to show that

$$\begin{Bmatrix} \vec{i}_{S1} \\ \vec{i}_{S2} \\ \vec{i}_{S3} \end{Bmatrix} = \begin{bmatrix} 1 & \alpha_{S3} & -\alpha_{S2} \\ -\alpha_{S3} & 1 & \alpha_{S1} \\ \alpha_{S2} & -\alpha_{S1} & 1 \end{bmatrix} \begin{Bmatrix} \vec{i}_U \\ \vec{i}_V \\ \vec{i}_W \end{Bmatrix} \tag{3}$$

and

$$\vec{B} = -B \cos \beta_{EL} \sin \beta_{AZ} \vec{i}_U + B \cos \beta_{EL} \cos \beta_{AZ} \vec{i}_V + B \sin \beta_{EL} \vec{i}_W \ . \tag{4}$$

From equations (3) and (4), the components of the Earth's magnetic field in satellite axes are given by

$$\left. \begin{aligned} B_{S1} &= -B \cos \beta_{EL} \sin \beta_{AZ} + \alpha_{S3} B \cos \beta_{EL} \cos \beta_{AZ} - (B \sin \beta_{EL}) \alpha_{S2} \\ B_{S2} &= B \cos \beta_{EL} \cos \beta_{AZ} + \alpha_{S3} B \cos \beta_{EL} \sin \beta_{AZ} + (B \sin \beta_{EL}) \alpha_{S1} \\ B_{S3} &= B \sin \beta_{EL} - (B \cos \beta_{EL} \cos \beta_{AZ}) \alpha_{S1} - (B \cos \beta_{EL} \sin \beta_{AZ}) \alpha_{S2} \ . \end{aligned} \right\} \tag{5}$$

By the same token, these are the satellite magnetometer outputs resolved into satellite axes. Substituting equation (2) into equation (5) and assuming for the remainder of the development that

$$\alpha_{SM} = \alpha_{S1M} \doteq \alpha_{S2M} \ ,$$

yields, after some manipulation,

$$\left. \begin{aligned} B_{S1C} &= \frac{B_{S1} + B \cos \beta_{EL} (\sin \beta_{AZ} - \alpha_{S3} \cos \beta_{AZ})}{B} = \alpha_{SM} \sin \beta_{EL} \cos (\omega_{SR} t + \pi/2) \\ B_{S2C} &= \frac{B_{S2} - B \cos \beta_{EL} (\cos \beta_{AZ} + \alpha_{S3} \sin \beta_{AZ})}{B} = \alpha_{SM} \sin \beta_{EL} \cos (\omega_{SR} t) \\ B_{S3C} &= \frac{B_{S3} - B \sin \beta_{EL}}{B \cos \beta_{EL}} = \alpha_{SM} \cos [\omega_{SR} t - (\beta_{AZ} + \pi)] \ . \end{aligned} \right\} \tag{6}$$

Hence, if the magnetometer outputs in satellite axes, (B_{S1}, B_{S2}, B_{S3}), are modified according to equation (6), the results are, in theory, three harmonics from which the following inferences can be made about satellite attitude motion, and hence tether motion, due to the skiprope phenomenon:

(1) α_{SM} is determined by the amplitude of B_{S3C}. Tether deflection at the midpoint of the tether can then be determined using equation (1) and knowing the tether length L.

(2) The period of B_{S3C}, T_{SR}, determines the magnitude of ω_{SR} by the relationship

$$|\omega_{SR}| = 2\pi/T_{SR} .$$

If B_{S1C} leads B_{S2C}, then $\omega_{SR} > 0$. If B_{S1C} lags B_{S2C}, then $\omega_{SR} < 0$.

(3) When B_{S3C} is a maximum, then $\omega_{SR}\, t = \beta_{AZ} + \pi$. At this point, $\vec{i}_{S3}$ lies in the plane formed by $\vec{i}_W$ and $\vec{B}$, tilted toward $\vec{B}$ from $\vec{i}_W$ by the angle α_{SM}. Then $\vec{i}_{S3}$ at any other point in time is readily determined, assuming β_{AZ} and ω_{SR} are known. Figure 4 helps to clarify this. There, $\vec{B}_P$ and $\vec{i}_{S3P}$ are the projections of $\vec{B}$ and $\vec{i}_{S3}$, respectively, onto the $\vec{i}_U - \vec{i}_V$ plane. Tether position is π rad out of phase with $\vec{i}_{S3P}$.

Hence, equation (6) and these associated inferences form the theoretical basis for the scheme proposed in this paper. A practical implementation of it is presented in section III.

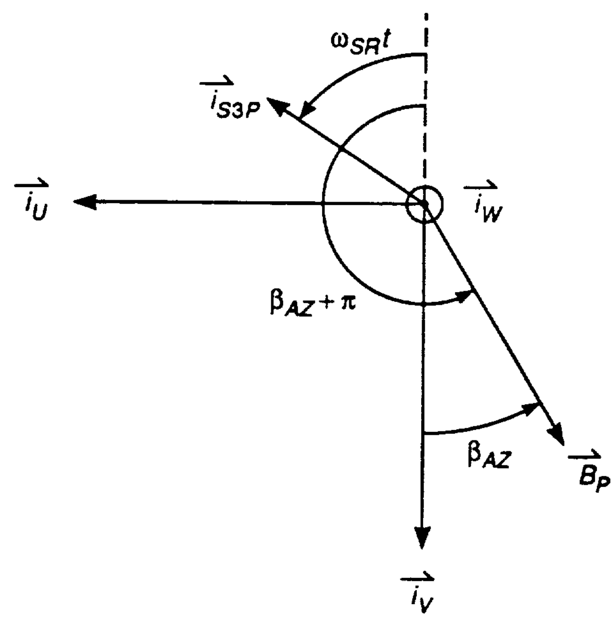

Figure 4. Projection of $\vec{i}_{S3}$ and $\vec{B}$ onto $\vec{i}_U - \vec{i}_V$ plane.

III. IMPLEMENTATION SCHEME

In implementing equation (6), it is assumed that the satellite magnetometer outputs are telemetered to the ground and resolved into satellite axes, if necessary, to generate (B_{S1}, B_{S2}, B_{S3}). The Earth's magnetic field parameters $(B, \beta_{EL}, \beta_{AZ})$ are estimated on the ground via a math model of the Earth's magnetic field and knowledge of the satellite orbit. Since α_{S3} is small, the terms in it are discarded to obviate the need to estimate it and thus simplify the scheme. Consequently, equation (6) is implemented as

$$\left.\begin{aligned}
\hat{B}_{S1C} &= \frac{B_{S1} + \hat{B}\cos\hat{\beta}_{EL}\sin\hat{\beta}_{AZ}}{\hat{B}} \doteq \alpha_{SM}\sin\beta_{EL}\cos(\omega_{SR}t + \pi/2) \\
\hat{B}_{S2C} &= \frac{B_{S2} - \hat{B}\cos\hat{\beta}_{EL}\cos\hat{\beta}_{AZ}}{\hat{B}} \doteq \alpha_{SM}\sin\beta_{EL}\cos(\omega_{SR}t) \\
\hat{B}_{S3C} &= \frac{B_{S3} - \hat{B}\sin\hat{\beta}_{EL}}{\hat{B}\cos\hat{\beta}_{EL}} \doteq \alpha_{SM}\cos[\omega_{SR}t - (\beta_{AZ} + \pi)] \ .
\end{aligned}\right\} \quad (7)$$

where (^) denotes an estimated variable. If the only motion of the satellite was that caused by tether skiprope, then equation (7) could be utilized directly. However, there will be satellite motion due to other effects that create unwanted signals in (B_{S1}, B_{S2}, B_{S3}) and hence $(\hat{B}_{S1C}, \hat{B}_{S2C}, \hat{B}_{S3C})$. An example of this is pendulous motion, which is satellite oscillation about its center of mass. This is expected to have amplitudes on the order of several degrees at a frequency in the neighborhood of 0.03125 Hz. Motion due to tether skiprope is expected to be on the order of degrees or fractions of a degree, depending on the tether length [5]. Its frequency is around 0.005 Hz for tether lengths greater than 2 km. To eliminate unwanted signals like this, $(\hat{B}_{S1C}, \hat{B}_{S2C}, \hat{B}_{S3C})$ in equation (7) are each filtered by a bandpass filter whose transfer function has the form

$$G(s) = \frac{2\xi\,(s/\omega_n)}{(s/\omega_n)^2 + 2\xi(s/\omega_n) + 1} \ .$$

where $\xi << 1$ and $\omega_n = \omega_{SR}$ ideally. At $\omega = \omega_n = \omega_{SR}$, the filter passes the skiprope frequency with unity gain and zero phase shift while attenuating all other frequencies. Hence, it filters higher frequency signals like those due to pendulous motion, magnetometer electronic noise, and A/D converter quantization. In addition, it desensitizes the scheme to bias errors and low frequency errors, like those occurring at orbital frequency (0.00018 Hz). These are bound to exist in estimating the Earth's magnetic field. Hence, the steady-state outputs of the bandpass filters should better reflect the right hand sides in equation (7) than the inputs. Inferences about satellite skiprope motion will be drawn from them.

Two final comments about implementation are appropriate. First, the smaller the ξ of the bandpass filters, the better they attenuate unwanted signals, but the longer it takes their outputs to reach steady state. The latter can be improved by choosing the initial states of each filter so that one equals the initial input to the filter and the other equals zero. Doing this, $\xi = 0.1$ was found by simulation to give a good compromise between filtering and speed. The other point to make is that the scheme will normally require two iterations, since the skiprope frequency can only be estimated a priori. For the first iteration, ω_n should be set to the best guess for ω_{SR}. At steady state, the filtered output for $\hat{B}_{S3C}$ in equation (7) will reveal ω_{SR} more precisely. Then, ω_n can be tuned to it and the process repeated for the second iteration.

IV. SIMULATION RESULTS

A computer simulation was developed to test the scheme described in section III, subject to the following conditions. (See the appendix for a listing of the program.) The tethered satellite was assumed to be in a 300-km orbit with a 28.5° inclination. Its motion relative to the orbital coordinate frame described in section II was given by

$$\alpha_{S3} = \alpha_{S3B}$$

$$\alpha_{S2} = \alpha_{S2B} + \alpha_{SR2M} \sin(\omega_{SR}t) + \alpha_{P2M} \sin(\omega_P t + \pi/4)$$

$$\alpha_{S1} = \alpha_{S1B} + \alpha_{SR1M} \cos(\omega_{SR}t) + \alpha_{P1M} \sin(\omega_P t + \pi/4) \ ,$$

where

$$\alpha_{S3B} = 5^\circ$$

$$\alpha_{S2B} = 1^\circ$$

$$\alpha_{S1B} = 1^\circ$$

$$\alpha_{SR2M} = 0.5^\circ$$

$$\alpha_{SR1M} = 0.75^\circ$$

$$\alpha_{P2M} = 2^\circ$$

$$\alpha_{P1M} = 2^\circ$$

$$\omega_{SR} = 2\pi(0.005) \text{ rad/s}$$

$$\omega_P = 2\pi(0.03125) \text{ rad/s} \ .$$

Hence, the satellite showed skiprope and pendulous motion, plus bias errors. The bandpass filters were set so that

$$\omega_n = \omega_{SR} = 2\pi(0.005\text{Hz})$$

$$\xi = 0.1 \ ,$$

which implies that the filters were already tuned to the skiprope frequency. The Earth's magnetic field was modeled by a six-displaced-dipole model. The estimated field was given by

$$\hat{B} = 1.1B$$

$$\hat{\beta}_{EL} = \beta_{EL} + 5^\circ$$

$$\hat{\beta}_{AZ} = \beta_{AZ} + 5^\circ \ .$$

where $(B, \beta_{EL}, \beta_{AZ})$ are its true values. The magnetometer axes were assumed to be aligned with the satellite axes with measurements made 16 times per second and quantized to an LSB = 0.02 milli-Gauss [6]. This corresponds to a 16-bit A/D converter scaled to a range of ± 0.65536 Gauss.

For these conditions, the simulation results are shown in figures 5, 6, and 7. Using the rules outlined in section II for interpreting the plots in figure 7, the following conclusions are drawn about the skiprope motion. In figure 7, observe that $\hat{B}_{S1C}$ leads $\hat{B}_{S2C}$ by $\pi/2$ rad; hence, $\omega_{SR} > 0$. From $\hat{B}_{S3C}$, it can be seen that $T_{SR} \doteq 200$s and so $\omega_{SR} \doteq 2\pi(0.005)$ rad/s. The amplitude of $\hat{B}_{S3C}$ shows that $\hat{\alpha}_{SM} \doteq 0.0125$ rad $\equiv 0.72^\circ$. Hence, the deflection of the tether at its midpoint due to skiprope is $d = 0.0125\ L/\pi$ by virtue of equation (1). At $t \doteq 100, 300, \ldots$ s, the $\vec{i}_{S3}$ axis lies in the plane of $\vec{i}_W$ and $\vec{B}$, tilted toward $\vec{B}$. At $t \doteq 200, 400, \ldots$ s, it lies in the same plane, but tilted away from $\vec{B}$. The tether is π rad out of phase with the projection of $\vec{i}_{S3}$ onto the $\vec{i}_U - \vec{i}_V$ plane.

In all aspects, the estimated skiprope motion matches the actual reasonably well, in spite of the demanding test conditions. This verifies the scheme and demonstrates its robustness.

V. FINAL COMMENTS

This paper has presented a unique scheme for reconstructing tethered satellite skiprope motion by bandpass filtering satellite magnetometer outputs telemetered to the ground. It was tested in a computer simulation with a demanding set of test conditions. This showed it to be quite robust.

As a final remark, this scheme is not just limited to the tethered satellite skiprope problem posed here. Indeed, it has potential application wherever:

1. A body cones about a known axis while measuring a known vector in body-fixed axes.

2. There is a need to know the time history, or perhaps just the fundamental characteristics of, the coning motion.

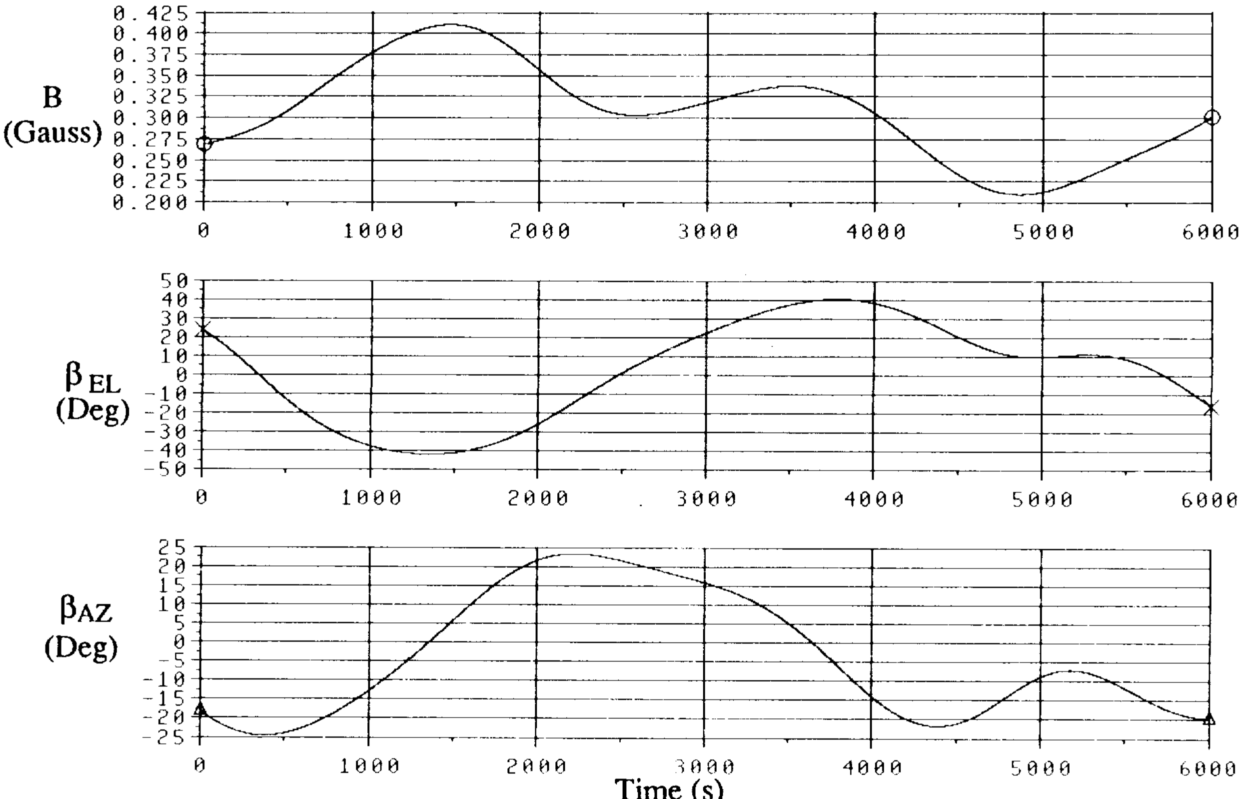

Figure 5. Earth's magnetic field versus time.

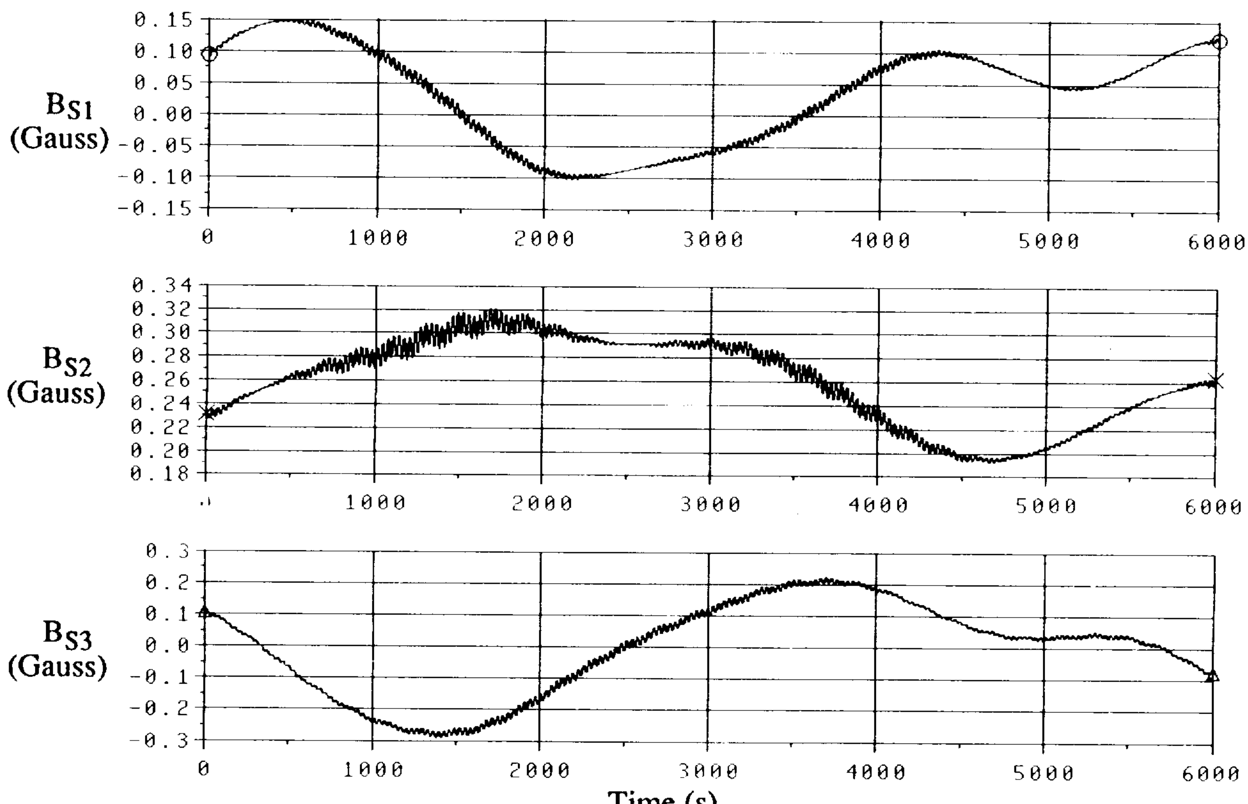

Figure 6. Magnetometer outputs versus time.

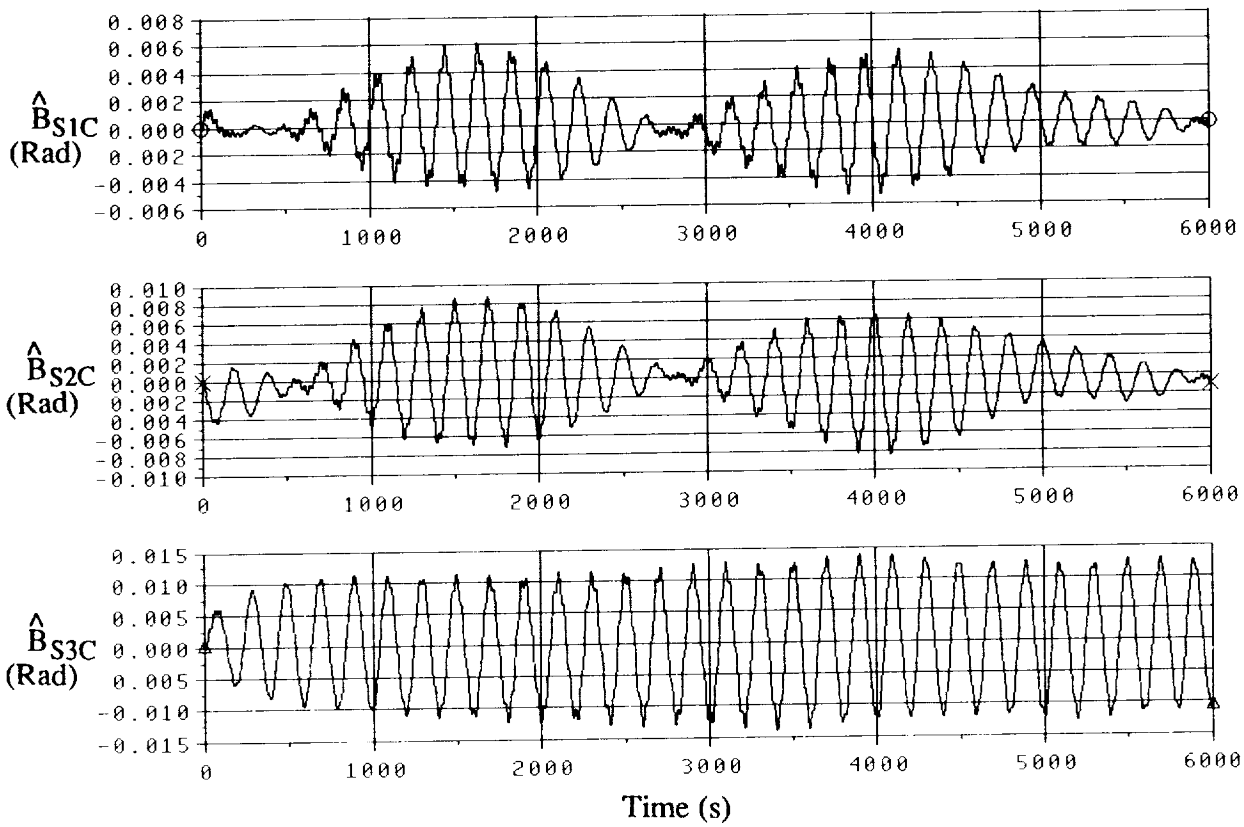

Figure 7. Bandpass filter outputs versus time.

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APPENDIX

COMPUTER PROGRAM LISTING

```
C THIS PROGRAM IS NAMED TSSMAG.FORT(CRAY6). IT TESTS SCHEME TO
C RECONSTRUCT TETHERED SATELLITE ATTITUDE MOTION DUE TO TETHER
C SKIPROPE USING MAGNETOMETER OUTPUTS. BANDPASS FILTERS ARE USED TO
C RECONSTRUCT SKIPROPE FREQUENCY COMPONENTS. LOWPASS FILTER IS USED TO
C RECONSTRUCT AS3. FOURTH ORDER RUNGA-KUTTA IS USED TO SOLVE FILTER
C DIFF. EQS. THIS PROGRAM WAS LAST REVISED ON FEB 19,1991.
      REAL NU2,LA2,LA3,GA2
      DIMENSION RVG(3),BMAG(3),AA(38)
C SPECIFY AND COMPUTE CONSTANTS----------------------------------------
      READ(4,2)IDUM
    2 FORMAT(I4)
      PI=3.1415926
      TWOPI=2.0*PI
      PIO2=PI/2.0
      RFD=0.017453292
      DFR=1.0/RFD
C  ALT=ORBIT ALT IN KM
C  REA=RADIUS OF EARTH IN KM
      GME=398601.2
      ALT=300.0
      REA=6371.2
      RV=REA+ALT
C  OMO=ORBIT RATE IN RAD/SEC
      OMO=SQRT(GME/(RV**3))
      TO=TWOPI/OMO
C LA3=ORBIT ICLINATION IN RAD
C OMA=ORBIT REGRESSION RATE IN RAD/SEC (SEE THOMPSON'S "INTRO TO SPACE
C     DYNAMICS, P.99 FOR EQUATIONS GIVEN BELOW)
      LA3=RFD*28.5
      XJ=1.637E-03
      PSI=TWOPI*XJ*COS(LA3)*(REA/RV)**2
      OMA=-PSI/TO
C OMG=EARTH RATE IN RAD/SEC
      OMG=RFD*360.0/(24.0*3600.0)
      ASB3=RFD*5.0
      ASB2=RFD*1.0
      ASB1=RFD*1.0
      WSR=TWOPI*0.0050
      ASR2M=RFD*0.50
      ASR1M=RFD*0.75
      WP=TWOPI*0.03125
      AP2M=RFD*2.0
      AP1M=RFD*2.0
      PHIP=RFD*45.0
      WNB=TWOPI*0.0001
      ZETAB=0.707
      WNF=TWOPI*0.0050
      ZETAF=0.100
      DT=1.0/16.0
      NPMAX=16
      TMAX=6000.0
      PRINT*,OMO,OMA,OMG
C COMPUTE GAIN AND PHASE OF BANDPASS FILTER TUNED FOR
C SKIPROPE FREQUENCY
      RATIO=WSR/WNF
```

```
      XR=1.0-RATIO*RATIO
      XI=2.0*ZETAF*RATIO
      GM=RATIO/SQRT(XR*XR+XI*XI)
      GP=PIO2-ATAN2(XI,XR)
      PRINT*,ZETAF,WNF,WSR,GM,GP
C SPECIFY ERRORS IN ESTIMATING EARTH'S MAGNETIC FIELD
      BHEF=0.10
      BELHE=RFD*5.0
      BAZHE=RFD*5.0
C MAGNETOMETER OUTPUT QUANTIZATION PARAMETERS
      NBITS=16
      BSMAX=0.65536
      IQ=1
      IQFST1=1
      IQFST2=1
      IQFST3=1
C DUMMY VARIABLE FOR PLOT VARIABLES NOT USED
      DUM=0.0
C SPECIFY INITIAL CONDITIONS-------------------------------------------
      T=0.0
      NP=NPMAX
      START=1.0
C  GA2=ANGLE FROM VERNAL EQUINOX TO PRIME MERIDIAN, NORTH IS POSITIVE,
C      IN RAD
C  LA2=LONGITUDE OF ASCENDING NODE IN RAD
C  NU2=POSITION OF VEHICLE IN ORBIT WRT ASCENDING NODE IN RAD
      GA2=0.0
      LA2=0.0
      NU2=0.0
C  INITIAL STATES OF FILTERS:
      BS1PF=0.0
      BS1PFD=0.0
      BS2PF=0.0
      BS2PFD=0.0
      BS3PF=0.0
      BS3PFD=0.0
C  IFLGMF=0 GIVES CONSTANT EARTH MAG FIELD WRT LOCAL VERTICAL AXES
C  IFLGMF=1 GIVES 6-DISPLACED-DIPOLE MODEL OF EARTH MAG FIELD WITH
C          REALISTIC TETHERED SATELLITES ORBIT MECHANICS--------------
      IFLGMF=1
   10 CONTINUE
C DETERMINE EARTH'S MAG FIELD FLUX DENSITY VECTOR IN GAUSS IN LOCAL
C VERTICAL AXES---------------------------------------------------------
      OE11=COS(NU2)*COS(LA3)*COS(LA2)-SIN(NU2)*SIN(LA2)
      OE12=COS(NU2)*SIN(LA3)
      OE13=-COS(NU2)*COS(LA3)*SIN(LA2)-SIN(NU2)*COS(LA2)
      OE21=-SIN(LA3)*COS(LA2)
      OE22=COS(LA3)
      OE23=SIN(LA3)*SIN(LA2)
      OE31=SIN(NU2)*COS(LA3)*COS(LA2)+COS(NU2)*SIN(LA2)
      OE32=SIN(NU2)*SIN(LA3)
      OE33=-SIN(NU2)*COS(LA3)*SIN(LA2)+COS(NU2)*COS(LA2)
      SGA2=SIN(-GA2)
      CGA2=COS(-GA2)
C
```

```
      AEG11=CGA2
      AEG12=0.
      AEG13=-SGA2
      AEG21=0.
      AEG22=1.0
      AEG23=0.
      AEG31=SGA2
      AEG32=0.
      AEG33=CGA2
C     SUBROUTINE CALL STATMENTS
      TEMBF=GA2-LA2
      CTEM=COS(TEMBF)
      CLA3=COS(-LA3)
      SNU2=SIN(-NU2)
      STEM=SIN(TEMBF)
      CNU2=COS(-NU2)
      SLA3=SIN(-LA3)
      RVG(1)=(-SNU2*CTEM*CLA3-STEM*CNU2)*RV
      RVG(2)= (SNU2*SLA3)*RV
      RVG(3)=(-SNU2*STEM*CLA3+CTEM*CNU2)*RV
      CALL BFIELD(RVG,START,BMAG)
C
      BMAGT=SQRT(BMAG(1)**2+BMAG(2)**2+BMAG(3)**2)
C
      BEG1=AEG11*BMAG(1)+AEG12*BMAG(2)+AEG13*BMAG(3)
      BEG2=AEG21*BMAG(1)+AEG22*BMAG(2)+AEG23*BMAG(3)
      BEG3=AEG31*BMAG(1)+AEG32*BMAG(2)+AEG33*BMAG(3)
C
      BU=OE11*BEG1+OE12*BEG2+OE13*BEG3
      BV=OE21*BEG1+OE22*BEG2+OE23*BEG3
      BW=OE31*BEG1+OE32*BEG2+OE33*BEG3
C  IFLGMG=0 GIVES CONSTANT EARTH MAG FIELD IN LOCAL VERTICAL AXES
C  IFLGMG=1 GIVES VARIABLE EARTH MAG FIELD IN LOCAL VERTICAL AXES------
      IF(IFLGMF.EQ.1)GO TO 15
      B=0.3
      BEL=RFD*30.0
      BAZ=RFD*30.0
      BU=-B*COS(BEL)*SIN(BAZ)
      BV=B*COS(BEL)*COS(BAZ)
      BW=B*SIN(BEL)
   15 CONTINUE
      B=SQRT(BU**2+BV**2+BW**2)
      BP=SQRT(BU**2+BV**2)
      BUN=-BU
      BEL=ATAN2(BW,BP)
      BAZ=ATAN2(BUN,BV)
C DETERMINE EULER ANGLES RELATING SATELLITE AXES TO LOCAL VERTICAL
C AXES-----------------------------------------------------------------
      THSR=WSR*T
      THP=WP*T+PHIP
      AS3=ASB3
      ASR2=ASR2M*SIN(THSR)
      ASR1=ASR1M*COS(THSR)
      AP2=AP2M*SIN(THP)
      AP1=AP1M*SIN(THP)
```

```
      AS2=ASB2+ASR2+AP2
      AS1=ASB1+ASR1+AP1
C DETERMINE MAGNETOMETER OUTPUTS IN SATELLITE AXES
      S1=SIN(AS1)
      C1=COS(AS1)
      S2=SIN(AS2)
      C2=COS(AS2)
      S3=SIN(AS3)
      C3=COS(AS3)
      T11=C2*C3
      T12=C2*S3
      T13=-S2
      T21=S1*S2*C3-C1*S3
      T22=S1*S2*S3+C1*C3
      T23=S1*C2
      T31=C1*S2*C3+S1*S3
      T32=C1*S2*S3-S1*C3
      T33=C1*C2
      BS1=T11*BU+T12*BV+T13*BW
      BS2=T21*BU+T22*BV+T23*BW
      BS3=T31*BU+T32*BV+T33*BW
C MAGNETOMETER OUTPUT QUANTIZATION
      CALL QUANT(IQ,IQFST1,NBITS,BSMAX,BS1,BS1Q)
      CALL QUANT(IQ,IQFST2,NBITS,BSMAX,BS2,BS2Q)
      CALL QUANT(IQ,IQFST3,NBITS,BSMAX,BS3,BS3Q)
      BS1QE=BS1Q-BS1
      BS2QE=BS2Q-BS2
      BS3QE=BS3Q-BS3
C COMPUTE INPUTS TO FILTERS
      BH=(1.0+BHEF)*B
      BELH=BEL+BELHE
      BAZH=BAZ+BAZHE
      SBELH=SIN(BELH)
      CBELH=COS(BELH)
      SBAZH=SIN(BAZH)
      CBAZH=COS(BAZH)
      BS0P=(BS1Q*CBAZH+BS2*SBAZH)/(BH*CBELH)
      BS1P=(BS1Q+BH*CBELH*SBAZH)/BH
      BS2P=(BS2Q-BH*CBELH*CBAZH)/BH
      BS3P=(BS3Q-BH*SBELH)/(BH*CBELH)
C SPECIFY BETTER INITIAL CONDITIONS FOR FILTER OUTPUTS AT T=0.0--------
      IF(T.GT.0.0)GO TO 40
      BS0PF=BS0P
      BS1PF=BS1P
      BS2PF=BS2P
      BS3PF=BS3P
   40 CONTINUE
C TEST FOR TIME TO STOP OR STORE DATA IN ARRAYS FOR PLOTTING LATER-----
      IF(T.GT.TMAX)GO TO 100
   20 IF(NP.LT.NPMAX)GO TO 30
      BELU=DFR*BEL
      BAZU=DFR*BAZ
      BELHU=DFR*BELH
      BAZHU=DFR*BAZH
      AS3H=BS0PF
```

```
      BS1CH=(BS1PFD/WNF)/GM
      BS2CH=(BS2PFD/WNF)/GM
      BS3CH=(BS3PFD/WNF)/GM
      AA(01)=T
      AA(02)=B
      AA(03)=BELU
      AA(04)=BAZU
      AA(05)=BH
      AA(06)=BELHU
      AA(07)=BAZHU
      AA(08)=BS1
      AA(09)=BS2
      AA(10)=BS3
      AA(11)=BS1Q
      AA(12)=BS2Q
      AA(13)=BS3Q
      AA(14)=BS1QE
      AA(15)=BS2QE
      AA(16)=BS3QE
      AA(17)=BS0P
      AA(18)=BS1P
      AA(19)=BS2P
      AA(20)=BS3P
      AA(21)=BS0PF
      AA(22)=BS1PF
      AA(23)=BS2PF
      AA(24)=BS3PF
      AA(25)=BS0PFD
      AA(26)=BS1PFD
      AA(27)=BS2PFD
      AA(28)=BS3PFD
      AA(29)=BS1CH
      AA(30)=BS2CH
      AA(31)=BS3CH
      AA(32)=AS3H
      AA(33)=ASR1
      AA(34)=ASR2
      AA(35)=AP1
      AA(36)=AP2
      AA(37)=AS1
      AA(38)=AS2
      WRITE(7)AA
      NP=0
   30 CONTINUE
C UPDATE FILTER OUTPUTS, ORBIT/EARTH ANGLES, TIME, AND COUNTER FOR
C STORING DATA TO PLOT----------------------------------------
      CALL LPF2(DT,WNB,ZETAB,BS0P,BS0PF,BS0PFD)
      CALL LPF2(DT,WNF,ZETAF,BS1P,BS1PF,BS1PFD)
      CALL LPF2(DT,WNF,ZETAF,BS2P,BS2PF,BS2PFD)
      CALL LPF2(DT,WNF,ZETAF,BS3P,BS3PF,BS3PFD)
      GA2=GA2+OMG*DT
      LA2=LA2+OMA*DT
      NU2=NU2+OMO*DT
      IF(NU2.GT.TWOPI)NU2=NU2-TWOPI
      T=T+DT
```

```
      NP=NP+1
      GO TO 10
  100 CONTINUE
      STOP
      END
      SUBROUTINE LPF2(DT,WN,ZETA,U,X1,X2)
C THIS SUBROUTINE UPDATES STATES FOR 2ND ORDER LOW PASS FILTER USING
C 4TH ORDER RUNGA-KUTTA------------------------------------------------
      X1P=X1
      X2P=X2
      SUM1=0.0
      SUM2=0.0
      DO 5 I=1,4
      WFACT=1.0
      DTP=DT/2.0
      IF(I.EQ.2.OR.I.EQ.3)WFACT=2.0
      IF(I.EQ.3)DTP=DT
      X1D=X2P
      X2D=WN*(WN*(U-X1P)-2.0*ZETA*X2P)
      SUM1=SUM1+WFACT*X1D
      SUM2=SUM2+WFACT*X2D
      X1P=X1+DTP*X1D
      X2P=X2+DTP*X2D
    5 CONTINUE
      X1=X1+SUM1*DT/6.0
      X2=X2+SUM2*DT/6.0
      RETURN
      END
      SUBROUTINE BFIELD(RV,START,BMAG)
C THIS IS 6-DISPLACED-DIPOLE MODEL OF THE EARTH'S MAG FIELD
C FLUX DENSITY IS GIVEN IN GAUSS---------------------------------------
      DIMENSION RV(3),RO(12,3),R(3),BMAG(3),EM(12,3)
      IF(START.NE.1.) GO TO 4
      RO(1,1)=500.805
      RO(2,1)=-292.9318
      RO(3,1)=455.075
      RO(4,1)=414.636
      RO(5,1)=-1611.292
      RO(6,1)=14.958
C
      RO(1,2)=146.800
      RO(2,2)=-818.6409
      RO(3,2)=-902.802
      RO(4,2)=1540.2948
      RO(5,2)=1095.094
      RO(6,2)=1243.2935
C
      RO(1,3)=718.994
      RO(2,3)=583.55
      RO(3,3)=791.483
      RO(4,3)=-65.878
      RO(5,3)=-282.390
      RO(6,3)=-849.084
C
      EM(1,1)=1.423489E6
```

```
      EM(2,1)=4.558039E6
      EM(3,1)=-7.045997E6
      EM(4,1)=-.555610E6
      EM(5,1)=.154356E6
      EM(6,1)=2.920314E6
C
      EM(1,2)=-18.247848E6
      EM(2,2)=3.575713E6
      EM(3,2)=3.744393E6
      EM(4,2)=-.696502E6
      EM(5,2)=-.593411E6
      EM(6,2)=4.470056E6
C
      EM(1,3)=2.104301E6
      EM(2,3)=-3.079872E6
      EM(3,3)=-4.175109E6
      EM(4,3)=6.801960E6
      EM(5,3)=-.856652E6
      EM(6,3)=-1.303223E6
C
      START=0.0
C
  4   DO 1 J=1,3
  1   BMAG(J)=0.
      DO 3 I=1,6
      DO 2 J=1,3
  2   R(J)=RV(J)-RO(I,J)
      RM=R(1)*R(1)+R(2)*R(2)+R(3)*R(3)
      B=3.*(R(1)*EM(I,1)+R(2)*EM(I,2)+R(3)*EM(I,3))/RM
      RM=RM**1.5
      DO 3 J=1,3
  3   BMAG(J)=BMAG(J)-(EM(I,J)-B*R(J))/RM
C CONVERT FLUX DENSITY FROM WEBERS/(METERS**2) TO GAUSS
      DO 5 J=1,3
  5   BMAG(J)=BMAG(J)*1.0E+04
      RETURN
      END
      SUBROUTINE QUANT(IQ,IQFST,NBITS,XMAX,X,XQ)
      IF(IQFST.NE.1)GO TO 10
      XLSB=(2.0*XMAX)/(2.0**NBITS)
      XMAXP=XMAX-XLSB
      XMIN=-XMAX
      PRINT*,NBITS,XMAX,XLSB,XMAXP,XMIN
      IQFST=0
   10 CONTINUE
      XQ=X
      IF(IQ.EQ.0)GO TO 20
      XDUM=XQ
      IF(XDUM.LT.XMIN)XDUM=XMIN
      IF(XDUM.GT.XMAXP)XDUM=XMAXP
      XDUM=XDUM+XMAX+XLSB/2.0
      XDUM=XDUM/XLSB
      IXDUM=XDUM
      XDUM=IXDUM
      XDUM=XDUM*XLSB
```

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      XQ=XDUM-XMAX
   20 CONTINUE
      RETURN
      END
```

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16. Abstract

This paper presents a unique scheme for reconstructing tethered satellite skiprope motion by ground processing of satellite magnetometer measurements. The measurements are modified based on ground knowledge of the Earth's magnetic field and passed through bandpass filters tuned to the skiprope frequency. Simulation results are presented which verify the scheme and show it to be quite robust. The concept is not just limited to tethered satellites. Indeed, it can be applied wherever there is a need to reconstruct the coning motion of a body about a known axis, given measurements of a known vector in body-fixed axes.

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