# Expert System Training and Control Based on the Fuzzy Relation Matrix

## Final Report



Jie Ren
T. B. Sheridan

Man-Machine Systems Laboratory/MIT

June 1, 1991

Cooperative Agreement NCC 9-16
Research Activity No. AI.04

NASA Johnson Space Center
Information Systems Directorate
Information Technology Division

Research Institute for Computing and Information Systems
University of Houston - Clear Lake

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# The RICIS Concept

The University of Houston-Clear Lake established the Research Institute for Computing and Information systems in 1986 to encourage NASA Johnson Space Center and local industry to actively support research in the computing and information sciences. As part of this endeavor, UH-Clear Lake proposed a partnership with JSC to jointly define and manage an integrated program of research in advanced data processing technology needed for JSC's main missions, including administrative, engineering and science responsibilities. JSC agreed and entered into a three-year cooperative agreement with UH-Clear Lake beginning in May, 1986, to jointly plan and execute such research through RICIS. Additionally, under Cooperative Agreement NCC 9-16, computing and educational facilities are shared by the two institutions to conduct the research.

The mission of RICIS is to conduct, coordinate and disseminate research on computing and information systems among researchers, sponsors and users from UH-Clear Lake, NASA/JSC, and other research organizations. Within UH-Clear Lake, the mission is being implemented through interdisciplinary involvement of faculty and students from each of the four schools: Business, Education, Human Sciences and Humanities, and Natural and Applied Sciences.

Other research organizations are involved via the "gateway" concept. UH-Clear Lake establishes relationships with other universities and research organizations, having common research interests, to provide additional sources of expertise to conduct needed research.

A major role of RICIS is to find the best match of sponsors, researchers and research objectives to advance knowledge in the computing and information sciences. Working jointly with NASA/JSC, RICIS advises on research needs, recommends principals for conducting the research, provides technical and administrative support to coordinate the research, and integrates technical results into the cooperative goals of UH-Clear Lake and NASA/JSC.

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## Preface

This research was conducted under auspices of the Research Institute for Computing and Information Systems by Jie Ren and T. B. Sheridan of the Man-Machine Systems Laboratory at the Massachusetts Institute of Technology. Dr. Terry Feagin served as RICIS research coordinator.

Funding has been provided by Information Technology Division, Information Systems Directorate, NASA/JSC through Cooperative Agreement NCC 9-16 between NASA Johnson Space Center and the University of Houston-Clear Lake. The NASA technical monitor for this activity was Timothy Cleghorn, of the Software Technology Branch, Information Technology Division, Information Systems Directorate, NASA/JSC.

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Man-Machine
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# *Abstract*

*Fuzzy knowledge, that for which the terms of reference are not crisp but overlapped, seems to characterize human expertise. This can be shown from the fact that an experienced human operator can control some complex plants better than a computer can. This report proposes the fuzzy theory to build a fuzzy expert relation matrix (FERM) from given rules or/and examples, either in linguistic terms or in numerical values to mimic human processes of perception and decision making. The knowledge base is codified in terms of many implicit fuzzy rules. Fuzzy knowledge thus codified may also be compared to explicit rules specified by a human expert. It can also provide a basis for modeling the human operator, and allow comparison of what a human expert says to what he or she does in practice.*

*Two experiments were performed. One, control of liquid in a tank, demonstrates how the FERM knowledge base is elicited and trained. The other shows how to use a FERM, built up from linguistic rules, to control an inverted pendulum without a dynamic model.*

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# 1 Introduction to Fuzzy Set Theory

*Perfect notions* or *exact concepts* correspond to the sorts of things envisaged in pure mathematics, while *inexactness* or *fuzziness* prevails in real life. A human operator or an expert's knowledge about variables or their relationship tend to be fuzzy. i.e.,the observations and thoughts of most people most of the time may be said to be mentally modeled and communicated to other persons in terms of natural language such as low, very low, high, very high, etc. Often the outputs from a simulation model are also *fuzzy*, because of the *inexactness* of model parameters and process and sensor noise. The fuzzy approach of expert system is based on the premise that the key elements in human thinking are not *crisply* defined but are more approximately defined. In other words, classes of objects in which the transition from non-membership to membership in set theory is gradual rather than abrupt as in that of a crisp set. It appears that much of the logic behind human reasoning is not the traditional two-valued or even multivalued logic, but logic with fuzzy truth, fuzzy connectives and fuzzy rules of inference.

By relying on the use of fuzzy linguistic variables and fuzzy algorithms, this new approach provides an approximate, yet effective and more flexible means of describing the behavior of systems which are too complex or to ill-defined to admit precise mathematical analysis by classical methods and tools. L.A.Zadeh[1], the founder of fuzzy set theory, modified a mathematical cornerstone, common set theory, and proposed the concept of fuzzy mathematics. His proposal was to absorb the features by which human thinking could distinguish and judge complicated phenomenon. Fuzzy sets, which map the logic of true and false into several ranges, are much more suitable to describe such large, complex systems having interfaces with human experts.

## 1.1 Basic Set Theory

A set is defined as a collection or aggregate of objects. The objects that belong to the set are termed the *elements* of the set. The term *universal set* is applied to the set that contains all the elements which one wishes to consider. The symbol $\mathcal{U}$ represents a general universe. A subset is the set that contains only certain elements from the universal set. For example, letting $A$ represent the letters in the English alphabet and $B$ represent the letters in the word *failure*, $A = \{ a,b,c, \ldots, x,y,z \}$ is a set and $B = \{ f,a,i,l,u,r,e \}$ is a subset. It is clear that whether a collection of objects is called set or subset is determined by the definition of the *universe*. A subset is sometimes called *set under known context.*

For any crisp set $A$, a characteristic function which determines, for any element of the universe, whether that element is a member of $A$, is defined as:

$$\mu_{A(x)} = \begin{cases} 1 & iff \quad x \in A \\ 0 & otherwise \end{cases} \tag{1}$$

It is clear that the characteristic function $\mu_A$ of a classical or crisp set takes a unique value in the two element set $\{0, 1\}$.

One may ask: what if an element is not completely in a set and also not completely out of a set? For example, consider

$$A = \{x | x \text{ is the Safe Temperature of an electric motor } \} \tag{2}$$

Assume it is understood that for the given system, the normal temperature is around $200F$. It seems safe to say that $100F$, $130F$, $160F$ and $180F$ are all elements of set $A$ and it seems equally safe to say that $800F$, $900F$ are not elements of set $A$. But what about $240F$ and $280F$? Intuitively, it is more plausible that $220F$ is an element of $A$ than that $250F$ is an element of $A$. This plausibility leads to the generalization of the degree of membership in a set, which forms the basis of fuzzy set theory.

A fuzzy subset $\tilde{A}$ of some universe $\mathcal{U}$ is a collection of objects from $\mathcal{U}$ such that the characteristic function $\mu_{\tilde{A}}$ takes any value in the interval [0,1]. Fuzzy set $\tilde{A}$ is symbolically denoted

$$\tilde{A} = \{\mu_{\tilde{A}(x)\#x)} | x \in \mathcal{U}\} \tag{3}$$

or

$$\tilde{A} = \int_{x \in \mathcal{U}} \mu_{\tilde{A}(x)} \# x \tag{4}$$

when $\mathcal{U}$ is a continuum, and

$$\tilde{A} = \{\mu_{\tilde{A}}(x_1)\#x_1, \mu_{\tilde{A}}(x_2)\#x_2, \cdots, \mu_{\tilde{A}}(x_n)\#x_n\} \tag{5}$$

when $\mathcal{U}$ has $n$ elements.

The symbol # is employed to link the elements of the support with their grade of membership, and the support of a fuzzy set is the crisp set that contains all the elements that have nonzero membership grade.

Consider equation (2) again and redefine it as a fuzzy set

$$\tilde{A} = \{\mu_{\tilde{A}}(x)\#x | \ x \text{ is the Safe Temperature of an electric motor } \} \tag{6}$$

where the characteristic function

$$\mu_{\tilde{A}}(x)\#x = \begin{cases} 1.0 & for \quad 0 < x < 220 \\ \frac{1}{1+((x-220)/50)^2} & for \quad 220 \leq x \leq 350 \\ 0.0 & for \quad x > 350 \end{cases} \tag{7}$$

$\mu_{\tilde{A}}(x)\#x$ is conventionally called *membership function.*

The fuzzy set $\tilde{A}$ describes the imprecise term *Safe Temperature* . Clearly, the temperature below $220F$ is considered safe, and the degree of membership is 1.0. It is not so clear what happens when temperature reaches $270F$. In this particular definitions of the fuzzy set *Safe Temperature*, the degree of safety of a temperature $270F$ is 0.5. In this way, the imprecision connected with the concept *Safe Temperature* can be captured mathematically and dealt with in an algorithmic fashion.

In a fuzzy set, it is noticed that the transition between membership and non-membership is gradual rather then abrupt, and that universe $\mathcal{U}$ itself is not fuzzy. If the membership is restricted in two values 0 and 1, a fuzzy set is reduced to a crisp set.

## 1.2 Fuzzy Union and Intersection

The definition of basic operations on sets must be modified for use in fuzzy set theory. Several notable structures can be defined on $\mu_{\tilde{A}}$ in the interval [0,1], each of which introduces the union and intersection operations, and which coincide with the classical ones. The widely accepted max/min definition is given by Zadeh [2]

Union

$$\mu_{\tilde{A}\cup\tilde{B}} = max(\mu_{\tilde{A}(x)}, \mu_{\tilde{B}(x)}) \tag{8}$$

Intersection

$$\mu_{\tilde{A}\cap\tilde{B}} = min(\mu_{\tilde{A}(x)}, \mu_{\tilde{B}(x)}) \tag{9}$$

# 2 Fuzzy Relation and Knowledge Base

Any decision making process involves input sates and output actions. For instance, system control is the decision and action in response to observed present and past system states in order to improve system performance according to some given criterion; the decision for failure diagnosis is based on observed symptoms (input states). Figure (1) shows the proposed framework of fuzzy knowledge base. The fuzzy sets $\tilde{S}_1, \tilde{S}_2, \cdots, \tilde{S}_i, \cdots, \tilde{S}_m$ are observed fuzzy states of a system, and the fuzzy sets $\tilde{C}_1, \tilde{C}_2, \cdots, \tilde{C}_j, \cdots, \tilde{C}_n$ are possible control actions corresponding to these states. It is clear that each $\tilde{C}_j$ may have some degree of relevance to each $\tilde{S}_i$. These relations form basis of the proposed fuzzy expert relation matrix ($FERM$).

## 2.1 Crisp Relation and Fuzzy Relation

A crisp relation shows the presence or absence of the association and interaction between elements of two or more sets. The relations, or the strength of ties, is either one or zero. The Cartesian product of two crisp sets X and Y is defined as:

$$X \times Y = \{(x, y)|_{x \in X \ \ and \ \ y \in Y}\} \tag{10}$$

It is not associative, that is,

$$X \times Y \neq Y \times X \qquad if \quad X \neq Y$$

For a family of crisp sets, the Cartesian product is generalized as [3]

$$X_1 \times X_2 \times \cdots \times X_n = \{(x_1, x_2, \cdots, x_n)|_{x_i \in X_i} \quad i = 1, 2, \cdots N\} \tag{11}$$

A relation among crisp sets $X_1, X_2, \cdots X_n$ is a subset of the Cartesian product $X_1 \times X_2 \times \cdots \times X_n$, and is denoted as $R(X_1, X_2, \cdots X_n)$. Therefore

$$R(X_1, X_2, \cdots X_n) \ \subset \ X_1, X_2, \cdots X_n \tag{12}$$

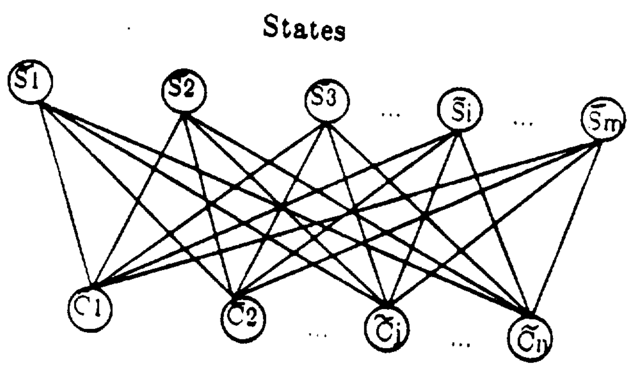


Figure 1: Relations of System States and Actions

For relations among sets $X_1, X_2, \cdots X_n$, the Cartesian product $X_1 \times X_2 \times \cdots \times X_n$ represents the universal set.

Because a relation itself is also a set, the basic set concepts such as containment or subset, union, intersection and complement can be applied without modification to relations.

Each crisp relation $R$ can be defined by a characteristic function that assigns a value of 1 to every pair of the universal set belonging to the relation, and a 0 to every pair that does not belong. For a given relation, this function assigns a value $\mu_R$ to every tuple $(X_1, X_2, \cdots X_n)$ such that

$$\mu_R(X_1, X_2, \cdots X_n) = \begin{cases} 1 & \text{iff} \quad (X_1, X_2, \cdots X_n) \in R \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

A fuzzy relation is an extension of a crisp relation [4]. The concept of crisp relation can be generalized to allow for various degrees, strength of relation, or interaction between elements. The values of the characteristic function are no longer only zero or one in fuzzy relations, they can take any values between zero and one:

$$\mu_{\tilde{R}}(\tilde{X}_1, \tilde{X}_2, \cdots \tilde{X}_n) = \begin{cases} (0,1] & \text{iff} \quad (\tilde{X}_1, \tilde{X}_2, \cdots \tilde{X}_n) \in \tilde{R} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

With the fuzzy characteristic function in mind, we can extend most theorems derived from crisp set theory, because the crisp relation can be viewed as a restricted case of the more general fuzzy relation.

## 2.2 Binary Relation

To show one property of a fuzzy relation, let's restrict ourselves to a relation between two sets $\tilde{X}$ and $\tilde{Y}$ which is called a *binary relation.*

In the real world, we can regard $\tilde{X}$ as an observed fuzzy state of a system which might be couched in terms of natural language, and the fuzzy set $\tilde{Y}$ as the possible control actions. The fuzzy relation defines the system in a fashion similar to the transfer function in control theory.

A binary fuzzy relation defined in the Cartesian set $\tilde{X} \times \tilde{Y}$ is a mapping from $\tilde{X}(x)$ and $\tilde{Y}(y)$ to $\tilde{R}(x, y)$,

$$\begin{aligned} R: \quad & \tilde{X} \times \tilde{Y} \Rightarrow \quad [0,1] \\ & \forall x \in \tilde{X}(x), \qquad y \in \tilde{Y}(x) \end{aligned}$$

For each pair of elements $(x_i, y_i) \in (\tilde{X}, \tilde{Y})$, there exists a $r_{ij} \in [0,1]$ which expresses the strength of ties between the pair $x_i$ and $y_j$. $\tilde{R}$ is a actually a membership function of the input states and the output decisions.

When the universe of discourse is infinite, the relation is in a continuous form. For instance, the word *similar* can be mathematically expressed in the form

$$\tilde{R}(x, y) = \begin{cases} \frac{1}{1+(x-y)^4} & \text{if } |x - y| \leq 5 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

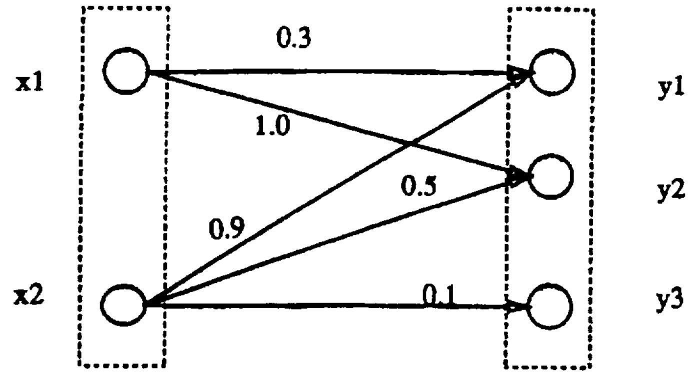


Figure 2: Example of a Binary Fuzzy Relation

When the universe of discourse if finite, the relation is in the discrete form, and is treated as a *fuzzy relation matrix.*

$$\tilde{R}(\tilde{X},\tilde{Y}) = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \tag{16}$$

where $\quad r_{ij} \in [0,1] \qquad i = 1,2,\cdots m; \quad j = 1,2,\cdots n;$

Here is an example of fuzzy relation matrix $\tilde{R}$ defined in $\tilde{X} = \{x_1, x_2\}$ and $\tilde{Y} = \{y_1, y_2, y_3\}$

$$\tilde{R}(\tilde{X},\tilde{Y}) = \begin{bmatrix} 0.3 & 1.0 & 0.0 \\ 0.9 & 0.5 & 0.1 \end{bmatrix} \tag{17}$$

which can be displayed as a directed graph shown in figure (2).
The number between two nodes represents the strength of ties between tuples.

## 2.3 T-Norms and T-Conorms

T-norms, or triangular norms, and T-conorms, or triangular conorms, are general operators used to deal with data which fall in the interval [0,1]. Statisticians have used this concept for a long time [5]. Now this concept has been adapted to fuzzy set theory, especially in the fields of fuzzy logic and fuzzy expert systems [6].

Definition

T-norm is a mapping from two arguments $L \in [0,1]$ to $T \in [0,1]$. That is, $L \times L \Rightarrow T$ and

T-conorm is a mapping from two argument $L \in [0,1]$ to $T_{co} \in [0,1]$. That is, $L \times L \Rightarrow T_{co}$ such that it has the following characteristics:

1. Monotonicity

$$\begin{aligned} if \quad x \le y,\ w \le z, \quad then \quad & T(x,w) \le T(y,z) \\ & T_{co}(x,w) \le T_{co}(y,z) \end{aligned} \tag{18}$$

2. Commutative

$$T(x,y) = T(y,x) \qquad T_{co}(x,y) = T_{co}(y,x) \tag{19}$$

3. Associative

$$T(T(x,y),z) = T(x,T(y,z)) \qquad T_{co}(T_{co}(x,y),z) = T_{co}(x,T_{co}(y,z)) \tag{20}$$

4. Boundary conditions

$$\begin{aligned} T(x,0) &= 0 \quad and \quad T(x,1) = x \quad for\ T-norm \\ T_{co}(x,0) &= x \quad and \quad T_{co}(x,1) = 1 \quad for\ T-conorm \end{aligned} \tag{21}$$

$\forall\ x,y,z,w \in [0,1]$

A method for generating a T-norm and a T-conorm is summarized as follows: Suppose g(s) and h(s) are strictly monotonic in a segment of R, and

$$G(t) = g^{-1}(s), \qquad H(t) = h^{-1}(s) \tag{22}$$

If $F(a,b)$ is generated by g(s) , where g(0) = 0 and g(1) = 1, then

$$F(a,b) = G[1 \wedge g(a) + g(b)] \tag{23}$$

is a T-conorm and

$$F(a,b) = G[0 \vee (g(a) + g(b) - 1)] \tag{24}$$

is a T-norm.

<u>Some T-norms and T-conorms</u>

$$\begin{aligned} & T(x,y) = min(x,y) = x \wedge y && T_{co}(x,y) = max(x,y) = x \vee y \\ & T(x,y) = x \cdot y && T_{co} = x + y - x \cdot y \\ & T(x,y) = \frac{xy}{1+(1-x)(1-y)} && T_{co}(x,y) = \frac{x+y}{1+xy} \\ & T(x,y) = \frac{xy}{v+(1-v)(x+y-xy)} && T_{co}(x,y) = \frac{x+y-xy-(1-v)xy}{v+(1-v)(1-xy)} \\ & && v \in [0,\infty] \\ & T(x,y) = 1 - \{1 \wedge [(1-x)^p + (1-y)^p]^{1/p}\} && T_{co}(x,y) = \vee(x^p + y^p)^{1/p} \\ & && p \in [1,\infty] \end{aligned} \tag{25}$$

All T-norms and T-conorms satisfy DeMorgan's law, but only the first set of T-norm and T-conorm (max-min) satisfies

$$\begin{array}{ll} T(x,x) = x & T_{co}(x,x) = x \\ T(x, T_{co}(y,z)) = T_{co}(T(x,y), T(x,z)) & T_{co}(T(x,T(y,z)) = T(T_{co}(x,y), T_{co}(x,z)) \end{array} \tag{26}$$

Therefore, the max-min operators are widely used in fuzzy set theory as fuzzy operators. But in the fuzzy relation operation, other T-norms and T-conorms can be used and compared under certain circumstances.

## 2.4 Representing Knowledge by Fuzzy Relations

Suppose we have a set of rules expressed in linguistic terms.

$$\begin{array}{llllllll} IF & \tilde{X}_1^1 & and & \tilde{X}_2^1 & and \cdots and & \tilde{X}_n^1 & THEN & do \;\; \tilde{Y}_1 \\ IF & \tilde{X}_1^2 & and & \tilde{X}_2^2 & and \cdots and & \tilde{X}_n^2 & THEN & do \;\; \tilde{Y}_2 \\ IF & \vdots & & & \cdots & & & do \;\; \vdots \\ IF & \vdots & & & \cdots & & & do \;\; \vdots \\ IF & \tilde{X}_1^m & and & \tilde{X}_2^m & and \cdots and & \tilde{X}_n^m & THEN & do \;\; \tilde{Y}_m \end{array} \tag{27}$$

where the logic *or* can also be used in the place of, or in combination with logic *and*. The above $m$ rules constitute $m$ fuzzy relations.

$$\tilde{R}_i(\tilde{X}_1, \tilde{X}_2, \cdots \tilde{X}_n) = \tilde{X}_{1i} \times \tilde{X}_{2i} \times \cdots \times \tilde{X}_{ni} \times \tilde{Y}_i \tag{28}$$

$$i = 1, 2, \cdots m$$

The overall relation matrix $\tilde{R}$ obtained from the fuzzy rules is calculated as the union of m individual relation matrices [7]

$$\tilde{R} = \tilde{R}_1 \cup \tilde{R}_2 \cup \cdots \tilde{R}_m = \bigcup_{i=1}^{m} \tilde{R}_i \tag{29}$$

This fuzzy relation matrix $\tilde{R}$ functions as a knowledge base which will trigger an output action $\tilde{Y}$ when a set of fuzzy states $\tilde{X}_1^i, \tilde{X}_2^i, \cdots, \tilde{X}_n^i$ is given.

An example is a linguistic rule concerning identification of a broken bearing in a servo-motor by using vibration measurement:

IF *the vibration is high* and *the period is proportional to motor speed*
THEN *the likelihood of a broken bearing is high*

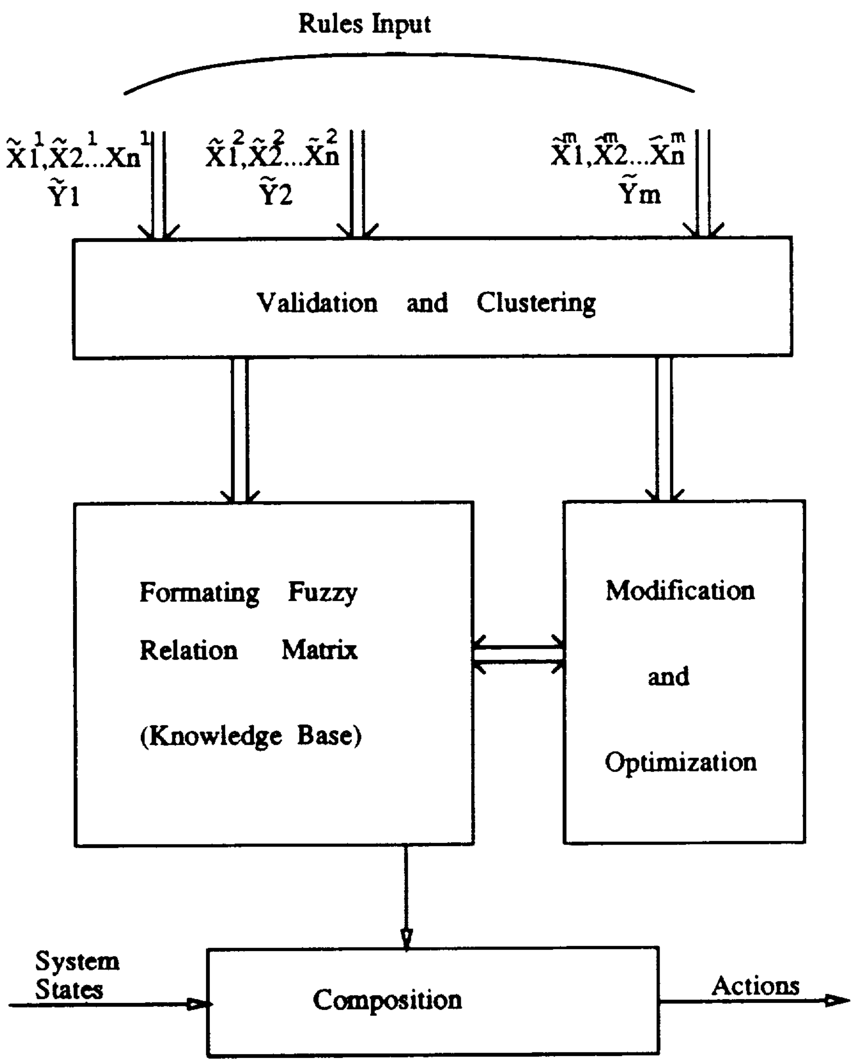


Figure 3: Fuzzy Knowledge Base

## 2.5 Fuzzy Clustering

It often happens that the rules collected from experts are incomplete or even contradictory. When two experts observe the same, or almost the same system states, but give out completely different conclusions, the knowledge base so constructed could form a misleading inference chain.

When we are given $M$ patterns, $A_1, A_2, \cdots, A_p$, contained in the pattern space S, the process of clustering can be formally stated as: seek the regions $S_1, S_2, \cdots, S_K$ such that every $A_i$, $i = 1, 2, \cdots, M$ falls into one of these regions and no $A_i$ falls in two regions [8] ; that is,

$$S_1 \cup S_2 \cup \cdots \cup S_K = S \tag{30}$$

$$S_i \neq S_j \qquad \forall \ i \neq j \tag{31}$$

Therefore, the linguistic rules collected are first passed through a prefilter, for rule validation (filting out the contradicted rules), and for clustering (putting the similar rules together).

## 2.6 Inference with Fuzzy Relation Matrix

The inference engine forms one of the important functional blocks of a knowledge-based system. Equipped with the fuzzy relation matrix constructed with an inference engine, the system is capable of inferring a useful conclusion with given data input. We call it a *Fuzzy Expert Relation Matrix*, or *FERM*. Fuzzy composition [9] [10] is used for fuzzy the inference engine.

As inference engine of fuzzy composition implemented with a fuzzy relation matrix is *parallel inherited.* That means, in contrast to the inference engine of backward or forward reasoning in a symbolic rule-base expert system, this method allows all the rules be actived in parallel, and therefore makes possible much faster computation.

Suppose we have two fuzzy relations $\tilde{R} \in \tilde{F}(\tilde{X} \times \tilde{Z})$ and $\tilde{S} \in \tilde{F}(\tilde{Z} \times \tilde{Y})$. A fuzzy relation $\tilde{R} \circ \tilde{S}$ defined in $\tilde{X} \times \tilde{Y}$ is the composition of

$$(\tilde{R} \circ \tilde{S})|_{x,y} = Sup_{z \in Z} \left\{ T[\tilde{R}(\tilde{X}, \tilde{Z}), \tilde{S}(\tilde{Z}, \tilde{Y})] \right\} \tag{32}$$

or

$$(\tilde{R} \otimes \tilde{S})|_{x,y} = Sup_{z \in Z} \left\{ T_{co}[\tilde{R}(\tilde{X}, \tilde{Z}), \tilde{S}(\tilde{Z}, \tilde{Y})] \right\} \tag{33}$$

where $\circ$ and $\otimes$ are operators for fuzzy composition.

They are associated together by the following relationship.

$$\overline{\tilde{R} \circ \tilde{S}} = \overline{\tilde{R} \otimes \tilde{S}} \tag{34}$$

In the previous example of fuzzy relations, the inference composition was

$$\tilde{Y} = \tilde{R} \circ (\tilde{X}_1 \times \tilde{X}_2 \times \cdots \times \tilde{X}_n) \tag{35}$$

or

$$\tilde{Y} = \tilde{R} \otimes (\tilde{X}_1 \times \tilde{X}_2 \times \cdots \times \tilde{X}_n) \tag{36}$$

and in more details

$$\tilde{Y} = Sup_{x_i \in \tilde{X}, i=1,2,\cdots,n} \left\{ T(\prod_{i=1}^{n} \tilde{X}_i, \tilde{R}) \right\} \tag{37}$$

$$\tilde{Y} = Proj_{\tilde{Y}} \left\{ interception(\prod_{i=1}^{n} \tilde{X}_i, \tilde{R}) \right\} \tag{38}$$

For simplicity, consider the binary fuzzy relation

$$\begin{array}{llll} Antecedents & \tilde{X}_i \Rightarrow & Consequences \quad \tilde{Y}_i & i = 1, 2, \cdots, m \\ Premise & \tilde{X} & & \\ \hline & & & \\ & & Conclusion \quad \tilde{Y} = ? & \end{array}$$

To infer the conclusion $\tilde{Y}$ for any premise $\tilde{X}$, perform max-min composition of $\tilde{X}$ and $\tilde{Y}$

$$\tilde{Y} = \tilde{R} \circ \tilde{X} = \left\{ \bigvee_{i=1}^{m} (\tilde{X}_i \times \tilde{Y}_i) \right\} \circ \tilde{X} \tag{39}$$

It is easy to verify that the fuzzy set $\tilde{Y}$ can be computed as the union of fuzzy set $\tilde{Y}_i$ intersected by $\tilde{A}_i$ with a constant membership function that plays the role of rule firing.

$$\tilde{Y}(y) = \bigvee_{i=1}^{m} [\tilde{A}_i(y) \wedge \tilde{Y}_i(y)] \qquad \forall y \in \tilde{Y} \tag{40}$$

and

$$\tilde{A}_i(y) = \lambda_i \qquad \forall y \in \tilde{Y} \tag{41}$$

and

$$\lambda_i = Sup_{x \in \tilde{X}} \left[ \tilde{X}(x) \wedge \tilde{X}_i(x) \right] \qquad i = 1, 2, \cdots, m \tag{42}$$

The calculation is a three-step scheme which works as follows:

1. Matching step
   The fuzzy data $\tilde{X}$ is matched against $\tilde{X}_i$ and the value of the possibility measure is obtained. It returns with a degree to which the fuzzy quantities $\tilde{X}$ and $\tilde{X}_i$ overlap.

2. Active step
   The fuzzy consequence $\tilde{Y}_i$ is intersected by the degree of overlap $\lambda_i$. The higher $\lambda_i$, the more the rule $\tilde{Y}_i$ is fired.

3. Combination step
   All the results coming from different rules are put together by means of the union composition.

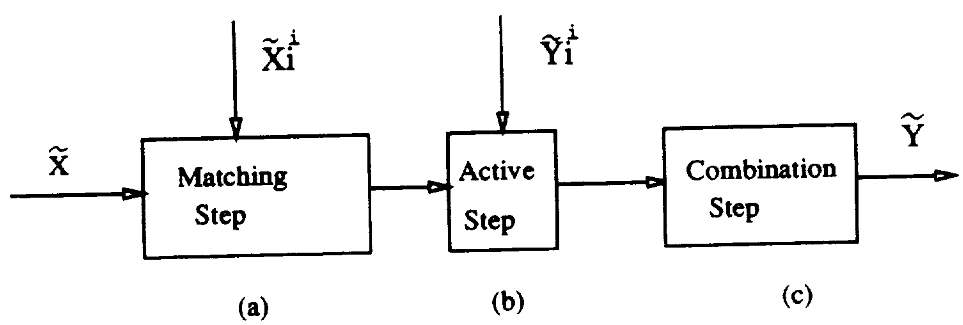


Figure 4: Rule Firing

# 3 Fuzzy Learning

The idea of a fuzzy expert relation matrix ($FERM$) has been expressed in the previous sections. We are going to use this idea to capture the human operator's expertise and store it in the form of a $FERM$. A computer fluid level control simulation has been developed for experiment and demonstration. The fluid tank (figure 5) is a nonlinear system whose model is unknown. Its inputs and outputs are fuzzified to natural language.

A human operator observes a series of errors between the liquid set level and the current level, and give out a series of control actions. Our purpose is to capture the human operator's control strategy based on his/her observations and control outputs.

The observed system states and operator's control actions are recorded. These records are then to be used for building the $FERM$ by means of fuzzy learning.

## 3.1 Fluid Level Control System Example

The fluid tank has a pipe for flow into the tank and a pipe for flow out from the tank. Each flow is controlled by its corresponding valve. The shape of the tank can be changed easily as a set of parameters. This simulated tank system has two functional inputs: two valve openings which are controlled by a human operator. The functional output is fluid level. The goal of the operation is to keep the fluid level as close to the set point as possible. There are random noises associated with the valve openings and the measurement of the fluid level. For simplicity, the functional inputs and output are fuzzified by linear membership functions.

The relation matrix $\tilde{R}$ is given by the Cartesian product:

$$\tilde{R} = \tilde{E}_k \times \tilde{CE}_k \times \tilde{U}_k \tag{43}$$

where

$$\tilde{E}_k = \{\mu_{E_1}\#1, \mu_{E_2}\#2, ..., \mu_{E_{24}}\#24\} \tag{44}$$

is the fuzzy variable of error between the measurement and set point.

$$\tilde{CE}_k = \{\mu_{CE_1}\#1, \mu_{CE_2}\#2, ..., \mu_{CE_{24}}\#24\} \tag{45}$$

is the fuzzy variable of error rate of change between the measurement and set point.

$$\tilde{U}_k = \{\mu_{U_1}\#1, \mu_{U_2}\#2, ..., \mu_{U_{24}}\#24\} \tag{46}$$

is the fuzzy variable of control valve openings, a function of the two valve opening $V_1$ and $V_2$.

The overall relation matrix obtained from the fuzzy rules is calculated as the union of N individual relation matrices:

$$\tilde{R} = \tilde{R}_1 \cup \tilde{R}_2 \cup \cdots \cup \tilde{R}_k \cdots = \bigcup_{k=1}^{N} \tilde{R}_k \tag{47}$$

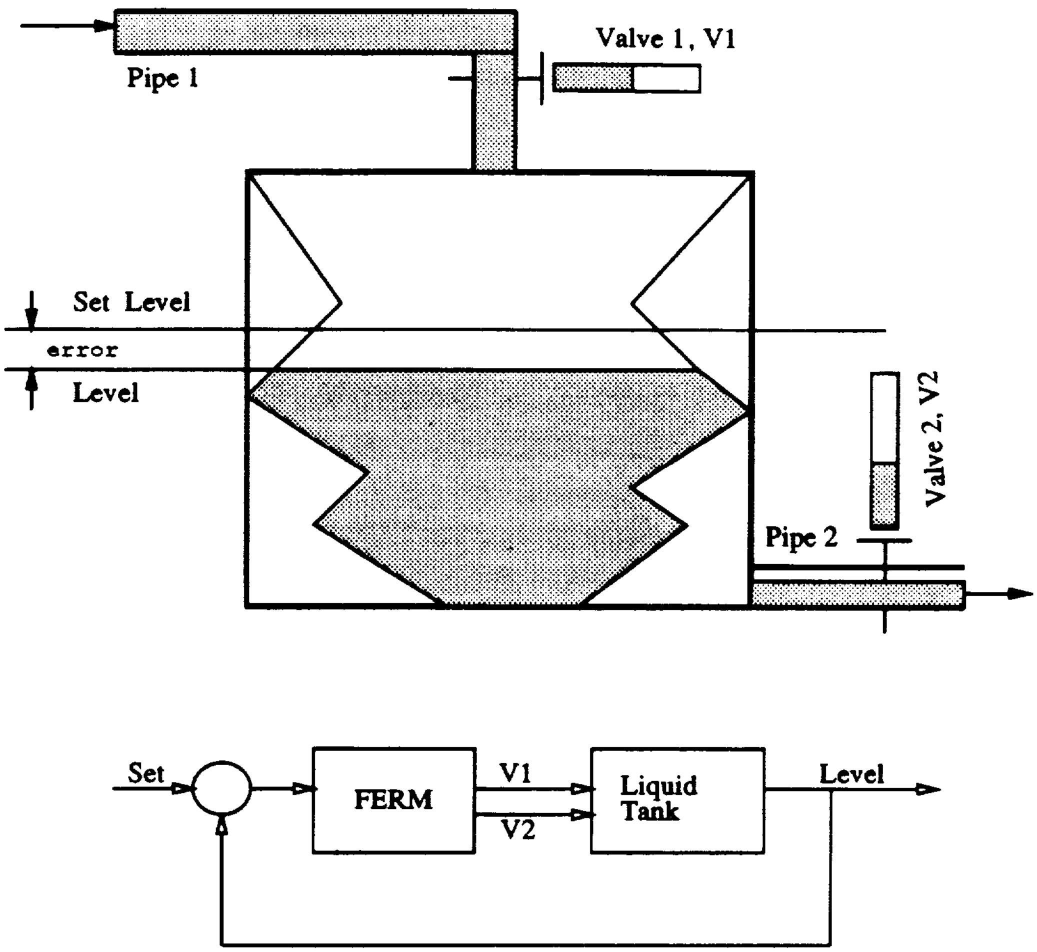


Figure 5: Water Tank Simulation

The output $\tilde{U}$ from the fuzzy controller can therefore be obtained by its inputs $\tilde{E}$ and $\tilde{CE}$ using the composition rules of inference.

$$\tilde{U} = (\tilde{E} \times \tilde{CE}) \circ \tilde{R} \tag{48}$$

It can be expressed at each sampling instant as follows:

$$\tilde{U}(nT) = (\tilde{E}(nT) \times \tilde{CE}(nT)) \circ \tilde{R} \tag{49}$$

A fuzzy decision (de-fuzzification ) is required to obtain the crisp control action $u$ of the process.

Building of the fuzzy rule starts with a completely empty relation matrix model from which it is not possible to make any conclusions. As the iterations proceed, various entries (43) and (47) are added to the model. These are essentially accurate but only cover a portion of the input-output space. Any assumption that is made to allow extra entries to be added to the relation matrix need not to be very accurate.

The element of the relation matrix can be said to be a rule of the form

$$If \quad \tilde{E}^i \quad and \quad \tilde{CE}^j \quad then \quad \tilde{U}^k \; with \; membership \; \mu_{ijk} \tag{50}$$

where the antecedents and consequences are base fuzzy sets.

The fuzzy model, as currently programmed, is described by a relation matrix with $24 \times 24 \times 24 = 13824$ elements, where each variable is defined to have 24 base fuzzy sets. Each rule of the form (50 ) where $\mu_{ijk}$ is greater than an arbitrary cutoff level ( say 0.1 ) is called a *simple* rule. Any pair $\tilde{E}^i$ and $\tilde{CE}^i$ will be the antecedents for a number of simple rules. This set of simple rules is called a *compound rule* for inputs $\tilde{E}_i$ and $\tilde{CE}_i$. For this model a compound rule can consist of a maximum of 24 simple rules, one for each fuzzy output set. The model can contain a maximum of 576 compound rules, consisting of a maximum of 13824 rules. The relation matrix model can be described in terms of the number of simple and compound rules it contains.

## 3.2 Learning Runs

A series of experiments was performed to investigate the learning properties of the proposed fuzzy identification and control algorithm. We had two kind of experiments: one with human operator, and another with a *PID* controller on an approximate linear plant in the place of human operator. In the first case, we assumed after some training time, that the human operator would always give correct control just as an experienced human operator. In the second case, we can assume the *PID operator* was always doing the most desirable control.

Each run consisted of 100 point samples, with the relation matrix obtained at the end of each run used as the initial matrix of the next run. Noise levels were set to 10% of the input valve opening and 5% of the fluid level measurement. Each run was started at the same initial fluid level in the tank, which is approximately half way between the bottom and the set point level of the tank. Learning was judged by the number of simple and compound rules in the relation matrix at the end of each run, and by the variance of the fluid level around the setpoint during a run. The variance is a measure of the controller's ability to control.

### 3.3 Results of Nonlinear Liquid Tank Experiment

Sufficient learning runs were performed to ensure a sub-convergence of the process model identification. That is, the learning run sequence was stopped when the number of compound rules had converged, and the number of simple rules was at least increasing only slowly. The identified model was much fuzzier than the predefined model (a decision table built artificially) in the sense that the ratio of compound rules to simple rules was much lower for the identified model (0.12 - 0.2 for identified model, 0.5 for predefined model). However, the test runs did not show that the predefined model was much better than that of the identified model. This means some rules are not sensitive to the system performance. On the other hand, the number of simple rules increased slowly for many runs after the number of compound rules had converged. This had little effect on the quality of control. Hence the learning runs were stopped before convergence of the simple rules.

In the simulation test, the number of simple rules was around 3230 and the number of compound rules around 120, after about 800 samples.

After the $FERM$ model was built, the data recorded was put into this model, and the outputs from the model were compared to the actual control measurements. For the relation matrix established after different iteration entries using (43) and (47), the results are shown in figures (6), (7) and (8).

In these figures, the solid line represents the data output from the identified model, while the dotted line represents the data by measurement. It is seen that after a certain number of learning trials, the $FERM$ model captures the human operator's control process quite well.

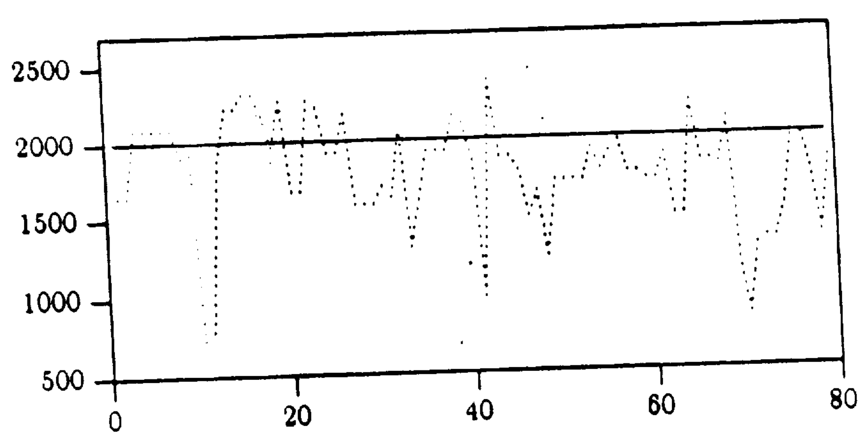


Figure 6: Sample Points: 10

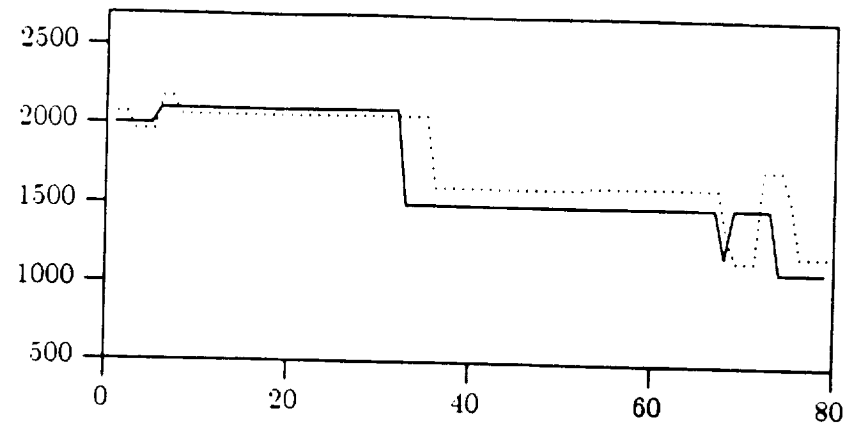


Figure 7: Sample Points: 400

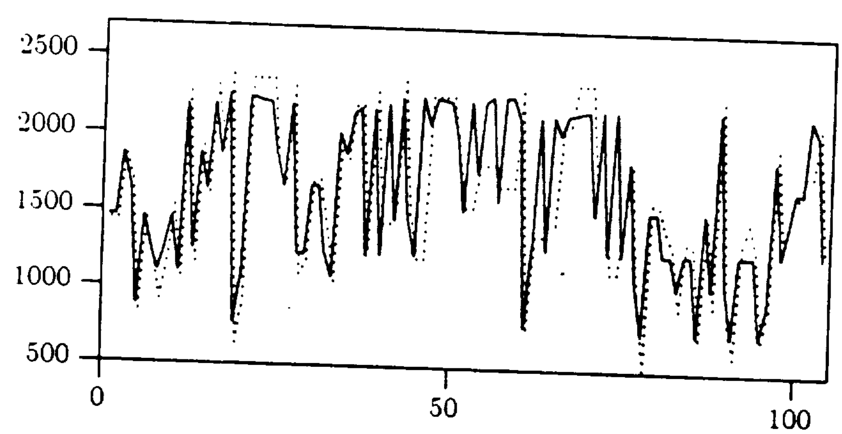


Figure 8: Sample Points: 800

## 3.4 Conclusions

From these experiments, it is concluded:

1. The *FERM* model is capable of capturing and identifying the human operator's control capability and knowledge about a given process control task.

2. The *FERM* model can be built up from some approximation model, for example, a linearized model of a system. Then so constructed, the *FERM* model can be used to control the nonlinear system.

3. The *FERM* model is not very sensitive to noise.

4. The knowledge collected in the *FERM* form is very natural to a human operator's ways of thinking. The *FERM*'s way of watching and learning makes it easier to elicit an operator's expertise. Once acquired, such expertise may be used for direct control, failure diagnosis, or design and reconfiguration.

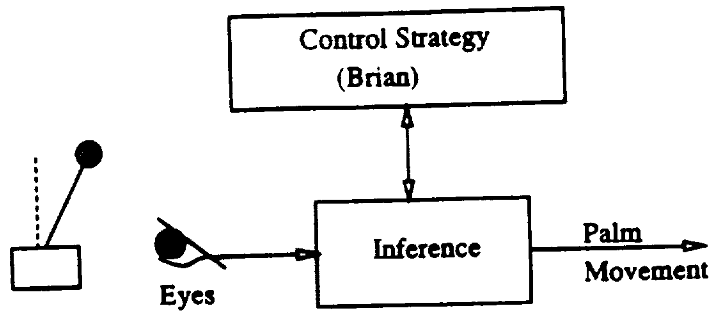


Figure 9: Inverted Pendulum Fuzzy Control by a Human

# 4 Control Based on Given Fuzzy Rules

In the previous section, we showed how to elicit a human operator's knowledge and capture it into a *FERM*. In this section, as an example of application, we will use the *FERM* idea to control an inverted pendulum.

## 4.1 Fuzzy Control of an Inverted Pendulum

The inverted pendulum is a classic example of an inherently unstable system. Its dynamics is a classical example of systems involving balance maintenance such as rocket thruster control. Many inverted pendulum control designs have been investigated. So far, most controller designs are limited in the linerization of the dynamic model[11]. It takes a long time to figure out workable *PID* gains or full-state-feedback gains. When either the size or the rod length of the pendulum is changed, the controller should be redesigned.

On the other hand, a child can stand a rod on his palm without gripping it after a little practice, even though he does not understand anything about dynamics or control theory. He gains the strategy of moving his palm in order to keep the rod vertically stable by successes and failures. The strategy is not represented by any *PID* or state-feedback controller based on differential equations, but by gathering some relations between the pendulum angles and the corresponding palm movements. The angle measured by human eyes is not so precise, or rather is fuzzy, and so is the palm movement. But the fuzzy inputs and fuzzy output, together with the fuzzy relation, gives satisfactory control action. In our example, we capture these fuzzy relations in a computer simulation.

Figure (9) shows the inverted pendulum controlled by a human, and figure (10) illustrates the same control from a *FERM*-based controller.

Similar to the above control by a human, we can summarize some control rules in fuzzy language terms, and then construct a fuzzy relation in the place of a control strategy.

This investigation consists of two stages:

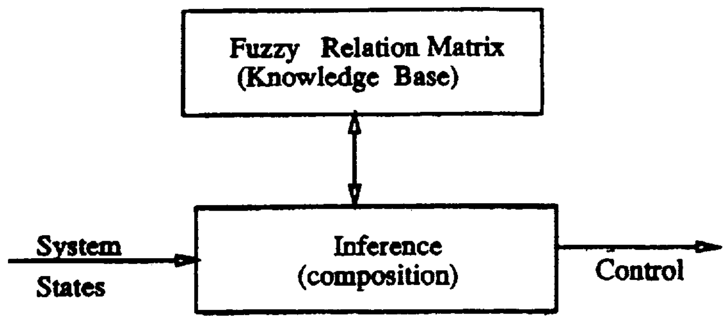


Figure 10: Fuzzy Relation Control

1. Obtain fuzzy control rules, establish the fuzzy relation matrix and optimize the relation matrix.
2. Run computer simulation. (This program was written in C++, and an executable file for IBM personal computer with EGA or VGA monitor is included in the report).

## 4.2 The Pendulum Setup

The pendulum system consisted of a rod mounted on a shaft on top of a cart that was free to move in the horizontal plane. There was no actuator at the base of the pendulum, and the cart was driven by a pulley connected to a servomotor. An optical encoder on the motor measured the cart position while another optical encoder on the pivot of the pendulum measured the angle of the pendulum. The whole setup is illustrated as in figure (11), and the *FERM* consists of the rules shown in (12).

where $e_k = \theta - \theta_o$, and $\dot{e_k} = e_k - e_{k-1}$ and the natural language terms

$$\begin{array}{ll} NL & : negative \;\; large \\ NS & : negative \;\; small \\ ZE & : zero \\ PS & : positive \;\; large \\ PL & : positive \;\; small \end{array}$$

The membership function for rod angle error is shown in figure(13), the membership function for rod angle error change rate is shown in figure(14), and the membership function for control output is shown in figure(15),

## 4.3 Results of the Inverted Pendulum Experiment

Figure (17) shows the response of the inverted pendulum to fuzzy control. and Figure (17) shows the response of full state feedback control. The pendulum has an offset of about six

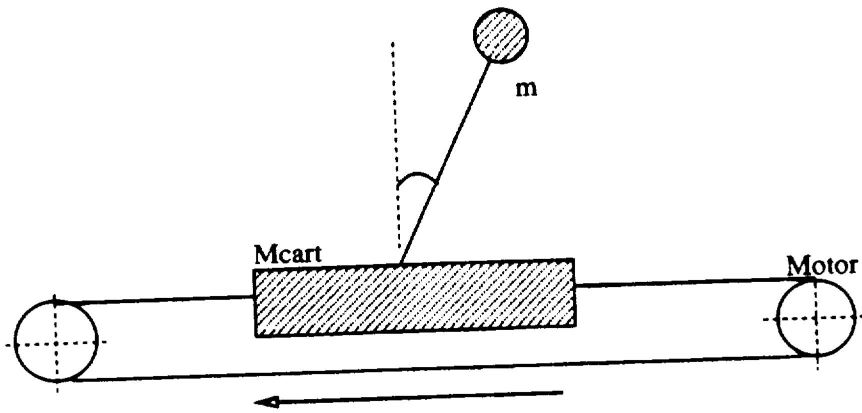


Figure 11: Inverted Pendulum Setup

| | $\dot{e}_k$ | | | | |
|---|---|---|---|---|---|
| $e_k$ | NL | NS | ZE | PS | PB |
| NL | NL | NL | NL | NS | PB |
| NS | NL | NL | NS | PS | PB |
| ZE | NL | NS | ZE | PS | PB |
| PS | NL | NS | PS | PB | PB |
| PL | NL | PS | PL | PB | PB |

Figure 12: Linguistic Rules

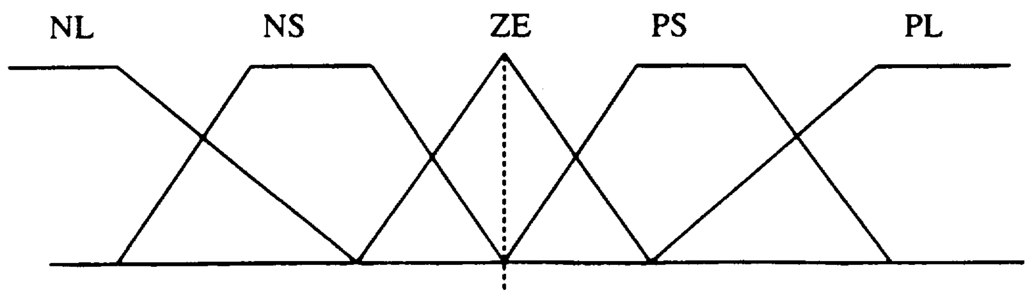


Figure 13: Error $e_k$

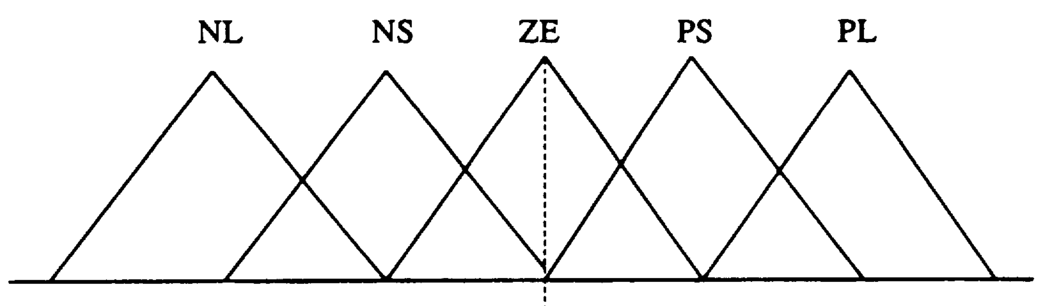


Figure 14: Error Change Rate $\dot{e_k} = e_k - e_{k-1}$

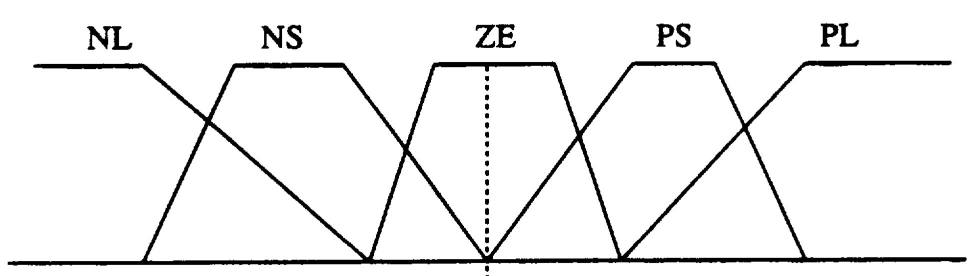


Figure 15: Control Voltage

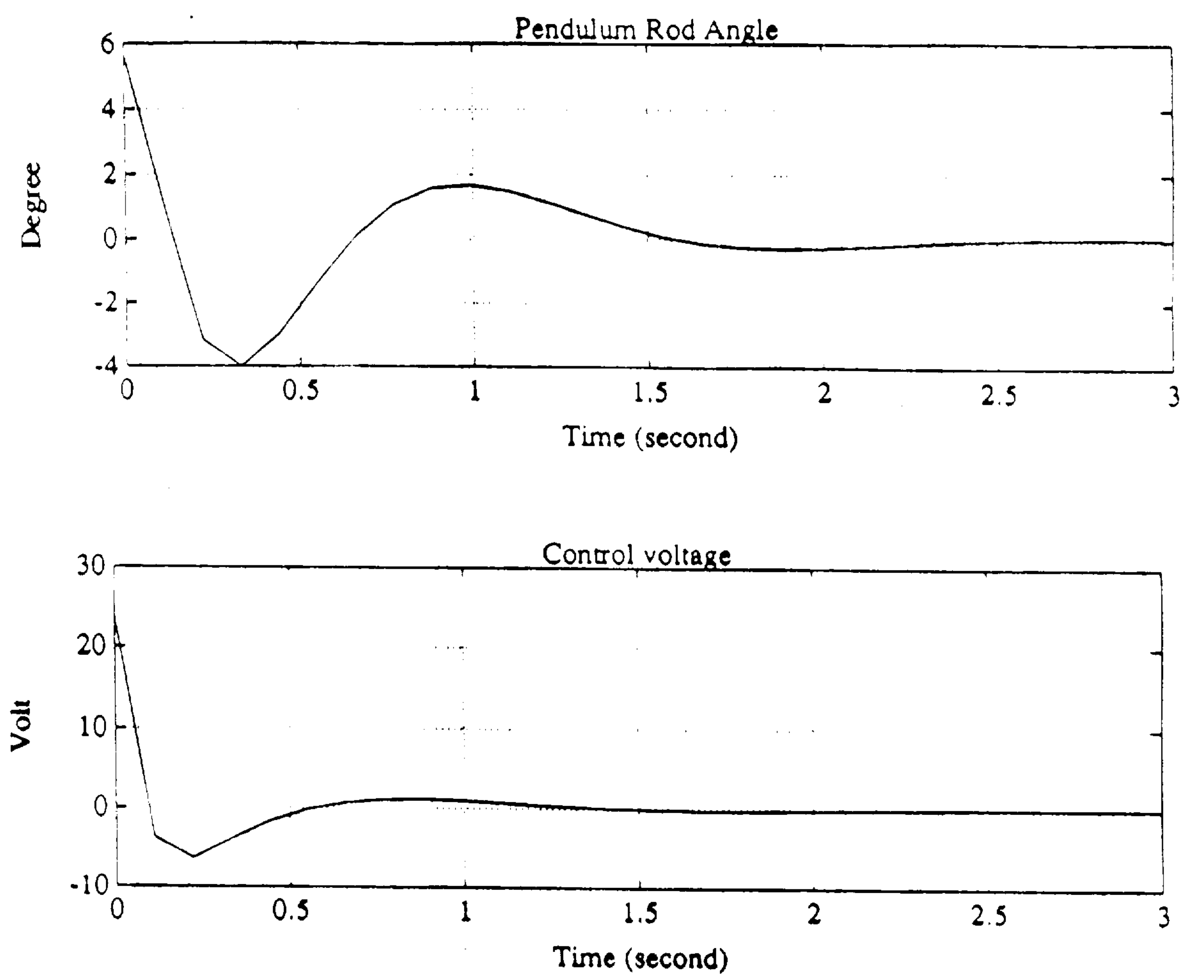


Figure 16: State Feedback Control

degrees in both experiments.

Compared with the full state feedback controller(16) , the fuzzy controller has faster response and smaller overshoot.

## 4.4 Conclusions from the Inverted Pendulum Experiment

1. The simulation result shows that the $FERM$ controller can be used in the place of human to control an inverted pendulum. This example generalizes to a class of control problems involving balance.

2. The $FERM$ approach does not require a detailed mathematical model to formulate the algorithm. It is more tolerable to system parameter change and noise than that

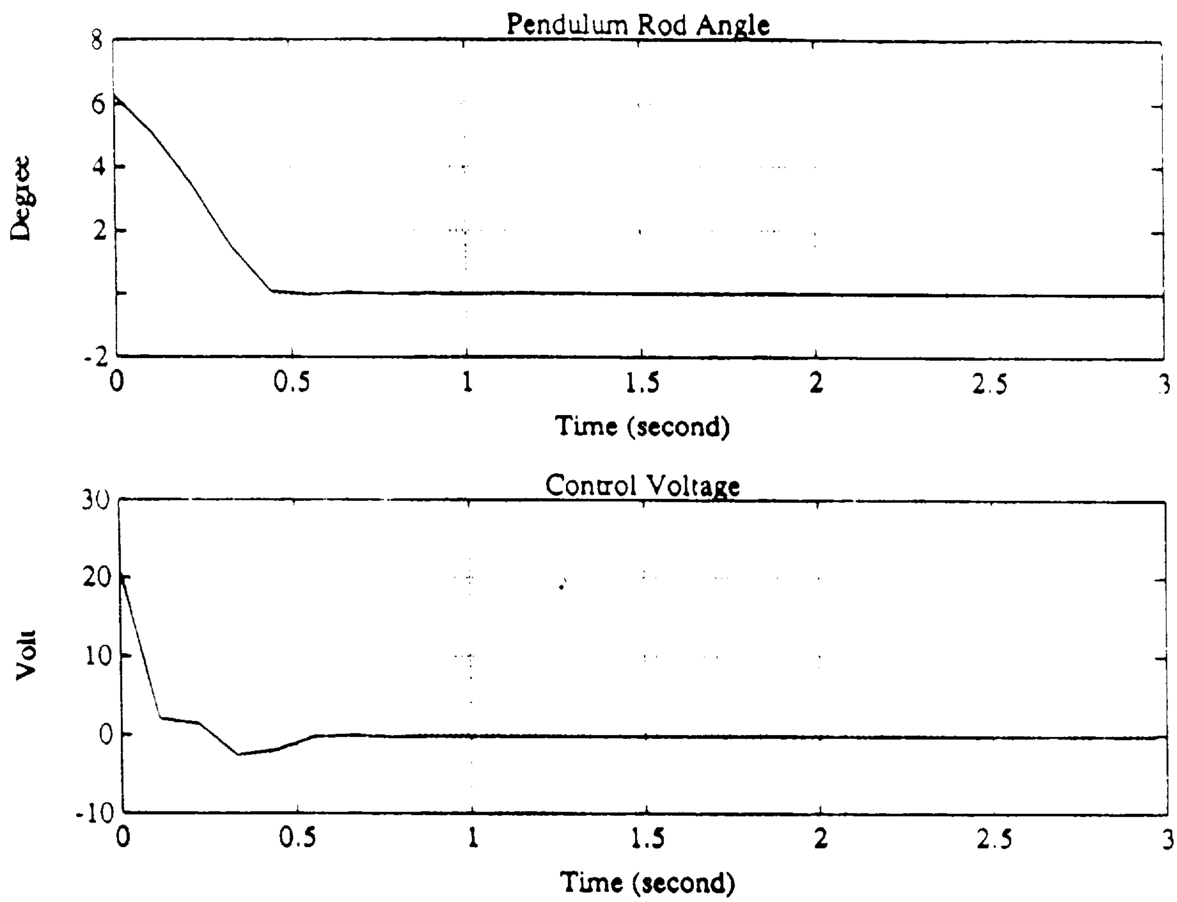


Figure 17: Fuzzy Relation Matrix Control

designed from traditional control theory. Therefore, in the sense of parameter changes due to timing or environment change, the fuzzy control approach is more reliable.

# 5 Conclusions and Recommendations

Conclusions

1. A general theory was developed for the fuzzy expert relation matrix technique for expert system development and use.
2. A fuzzy learning experiment was done to demonstrate learning from a human operator in the context of controlling a nonlinear process.
3. A control experiment was done to demonstrate the application of the fuzzy expert relation matrix technique to control of an inverted pendulum.

Recommendations

1. Further research is needed on how to optimize the fuzzy relation matrix approach. When compared with neural network approach, it is outwardly similar in terms of system learning and identification. But inside, it is different in terms of knowledge storage and the exact mechanism of learning. Research on the relation between the two approaches may produce a more efficient way of learning and identification from imprecise data.
2. Further experiments with human operators need be done to find a more efficient way to cluster linguistic rules.

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