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Computerized Inspection of Real Surfaces and Minimization of Their Deviations

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SUMMARY

A method is developed for the minimization of gear tooth surface deviations between theoretical and real surfaces to improve the precision of surface manufacture. Coordinate measurement machinery is used to determine a grid of surface coordinates. Theoretical calculations are made for the grid points. A least-square method is used to minimize the deviations between real and theoretical surfaces by altering the manufacturing machine-tool settings. An example is given for a hypoid gear.

INTRODUCTION

The Gleason Works were pioneers in the application of coordinate measurements to improve the precision manufacturing of hypoid and spiral bevel gears (ref. 1). In aerospace applications, duplication of flight-qualified master gears is very important, and coordinate measurement has now become part of the normal production process. Methods to enhance and extend the use of this machinery can be very valuable to aerospace gear manufacturers.

The approach developed in this paper enables one do determine deviations of a real surface from the known theoretical surface. This is accomplished by using coordinate measurements and minimizing the deviations to correct the previously applied machine-tool settings. The surface deviations are represented in the direction of the normal to the theoretical surface. The coordinate measurements are performed by a machine with 4 or 5 degrees of freedom. In the case of 4 degrees of freedom, the probe performs three translational motions (fig. 1); the fourth motion, rotation, is performed by turning the table with the workpiece. The axis of rotational motion coincides with the axis of the workpiece. In the case of a 5-degree-of-freedom machine, the fifth degree of freedom is used to provide the probe deflections in the direction of the normal to the theoretical surface. The probe is provided with a changeable spherical surface whose diameter can be chosen from a wide range.

The motions of the probe and the workpiece for coordinate measurements are computer controlled and for this purpose a grid, the set of points on the surface to be measured, must be chosen. There is a reference point, one point on the grid, that is necessary for the initial installments of the probe. There are two orientations of the probe installment that are applied for measurements of a gear (fig. 1(a)) and a pinion (fig. 1(b)), depending on the angle of the pitch cone.

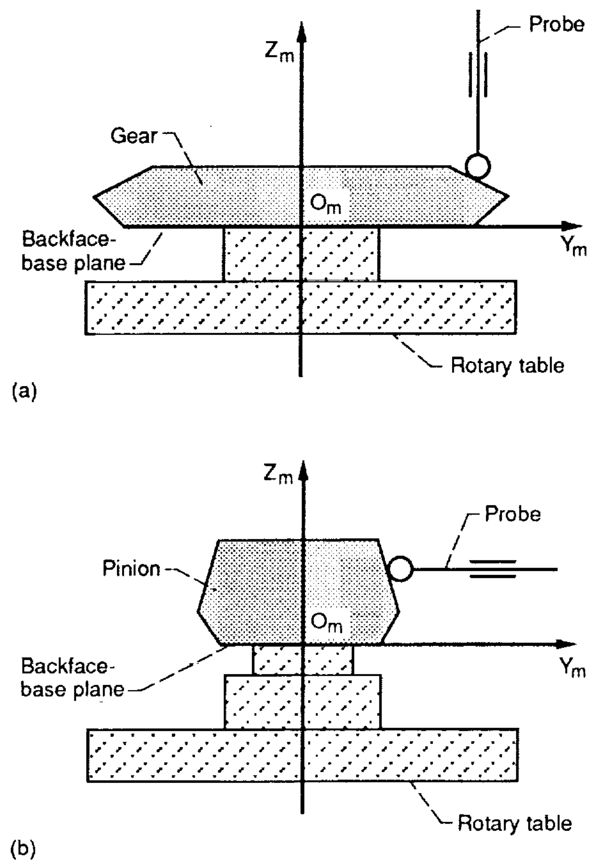

Figure 1.—Surface measurement orientation for (a) gear and (b) pinion.

The mathematical aspects of coordinate measurements will now be described (ref. 2): First, it is necessary to derive the equations for the theoretical surface. In many cases, this surface can be derived as the envelope to the family of generating surfaces, namely the tool surfaces. Next, the results of coordinate measurements must be transformed into deviations of the real surface represented in the direction of the surface normal. Here, the surface variations are represented in terms of the corrections to the machine-tool settings. The surface deviations obtained from coordinate measurements and the surface variations determined by the corrections of machine-tool settings can be represented by an overdetermined system of linear equations. The number k of these equations is equal to the number of grid points, and the number of unknowns m is equal to the number of corrections of machine-tool settings ($m << k$). The optimal solution to such a system of linear equations enables one to determine the sought-for corrections of machine-tool settings.

EQUATIONS OF THEORETICAL TOOTH SURFACE Σ_t

Considering that the theoretical surface can be determined directly, we represent it in coordinate system S_t in two-parametric form as

$$\mathbf{r}_t(u,\theta), \qquad \mathbf{n}_t(u,\theta) \tag{1}$$

where $\mathbf{r}_t$ and $\mathbf{n}_t$ are the position vector and unit normal to the surface, respectively, and (u,θ) are the Gaussian coordinates (surface coordinates).

For the case when surface Σ_t is the envelope to the family of generating surface Σ_c, we represent in S_t surface Σ_t and the unit normal $\mathbf{n}_t$ to Σ_t as (ref. 3)

$$\mathbf{r}_t = [M_{tc}]\mathbf{r}_c(u_c,\theta_c), f(u_c,\theta_c,\phi) = 0 \tag{2}$$

$$\mathbf{n}_t = [L_{tc}]\mathbf{n}_c(u_c,\theta_c), f(u_c,\theta_c,\phi) = 0 \tag{3}$$

where (u_c,θ_c) are the Gaussian coordinates of the generating surface Σ_c, and ϕ is the generalized parameter of motion in the process for generation. The equation of meshing is

$$f(u_c,\theta_c,\phi) = \mathbf{N}^{(c)} \cdot \mathbf{v}^{(ct)} = 0 \tag{4}$$

where $N^{(c)}$ is the normal to Σ_c, and $\mathbf{v}^{(ct)}$ is the relative motion for a point of contact of Σ_c and Σ_t. Matrices $[M_{tc}]_{4x4}$ and $[L_{tc}]_{3x3}$ describe the coordinate transformation from S_c to S_t for a position vector and surface normal, respectively. Position vectors in three-dimensional space are represented with homogeneous coordinates.

COORDINATE SYSTEMS USED FOR COORDINATE MEASUREMENTS

Coordinate systems S_m and S_t are rigidly connected to the coordinate measuring machine (CMM) and the workpiece being measured, respectively (fig. 2). The

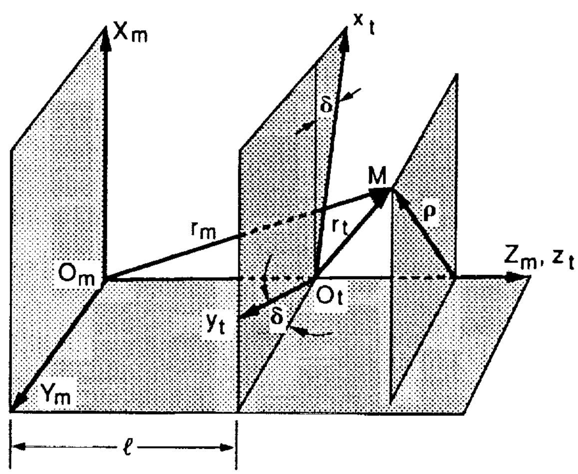

Figure 2.—Relationship between theoretical and measurement coordinate systems. (ρ, radial distance to point from axis of rotation.)

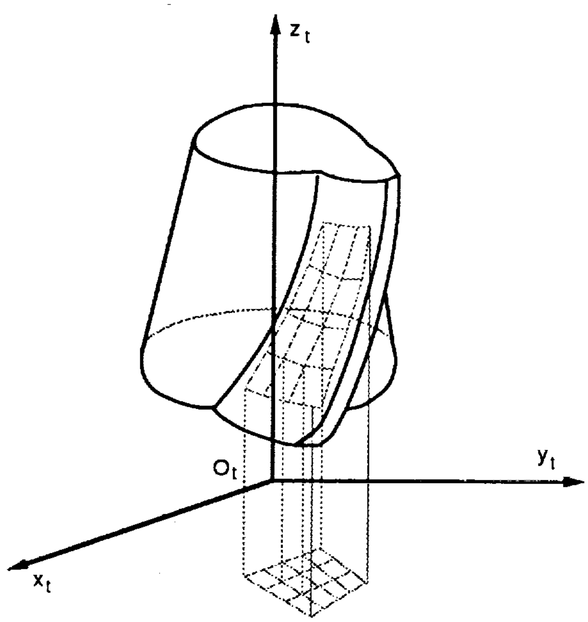

Figure 3.—Measurement grid on tooth surface.

backface of the gear is installed flush with the base plane of CMM. The distance l between the origins O_m and O_t is assumed to be known, but the parameter δ of orientation must be determined (see the following section). The coordinate transformation from S_t to S_m is represented by the matrix equation

$$\mathbf{r}_m = [M_{mt}]\mathbf{r}_t \tag{5}$$

MEASUREMENT GRID AND ESTABLISHMENT OF THE REFERENCE POINT

The grid is a set of points on Σ_t chosen as points of contact between the probe and Σ_t (fig. 3). Fixing the value of z_t for the point of the grid and the value of, say, y_t (or x_t), we can obtain the following equations:

$$y_t(u_i,\theta_i) = h_i, \qquad z_t(u_i,\theta_i) = l_i \qquad (i = 1,\ldots,k) \tag{6}$$

where k is the number of grid points.

We consider h_i and l_i as given and solve equations (6) for (u_i,θ_i). Then we can determine the position vectors and the unit normals for k points of the grid using the equations

$$\mathbf{r}_t^{(i)} = [x_t(u_i,\theta_i)y_t(u_i,\theta_i)z_t(u_i\theta_i)]^T, \qquad (i = 1,\ldots,k) \tag{7}$$

$$\mathbf{n}_t^{(i)} = [n_{xt}(u_i,\theta_i)n_{yt}(u_i,\theta_i)n_{xt}(u_i,\theta_i)]^T, \qquad (i = 1,\ldots,k) \tag{8}$$

The position vector for the center of the probe, if the deviations are zero, is represented by

$$\mathbf{R}_t^{(i)} = \mathbf{r}_t^{(i)} + \rho \mathbf{n}_t^{(i)} \qquad (i = 1,\ldots,k) \tag{9}$$

where ρ is the radius of the probe sphere.

The reference point

$$\mathbf{r}_t^{(o)} = [x_t(u^{(o)},\theta^{(o)}) y_t(u^{(o)},\theta^{(o)}) z_t(u^{(o)})]^T \tag{10}$$

is usually chosen as the mean point of the grid.

The center of the probe that corresponds to the reference point on Σ_t is determined from equation (9) as

$$\mathbf{R}_t^{(o)} = [X_t(u^{(o)},\theta^{(o)}) Y_t(u^{(o)},\theta^{o}) Z_t(u^{(o)},\theta^{(o)})]^T \tag{11}$$

where $(u^{(o)},\theta^{(o)})$ are known values.

The coordinates of the reference center of the probe are represented in coordinate system S_m of the measuring machine by the matrix equation

$$\mathbf{R}_m^{(o)} = [M_{mt}(\delta)]\mathbf{R}_t^{(o)} \tag{12}$$

Equation (12) yields

$$\left.\begin{aligned} x_m^{(o)} &= x_m^{(o)}(\delta,u^{(o)},\theta^{(o)}) \\ y_m^{(o)} &= y_m^{(o)}(\delta,u^{(o)},\theta^{(o)}) \\ z_m^{(o)} &= z_m^{(o)}(\delta,u^{(o)},\theta^{(o)}) \end{aligned}\right\} \tag{13}$$

Three equations (13) contain four unknowns: δ, $x_m^{(o)}$, $y_m^{(o)}$, $z_m^{(o)}$. To solve these equations, we may consider that one of the coordinates of the reference point of the probe center, say, $y_m^{(o)}$, may be chosen equal to zero. This is accomplished by requiring the reference point to lie in the $x_m - z_m$ plane. The orientation of angle δ is now established to satisfy this requirement, and all measurements are referenced from this location. Then equation system (13) allows one to determine δ, $x_m^{(o)}$ and $z_m^{(o)}$ (ref. 2). Coordinates $x_m^{(o)}, y_m^{(o)} = 0$, $z_m^{(o)}$ are necessary for the initial installment of the center of the probe.

MEASUREMENT OF THE DEVIATIONS OF THE REAL SURFACE

The deviations of the real surface are caused by manufacturing errors, heat treatment, etc. Vector positions of the center of the probe for the theoretical surface and the real surface can be represented as follows:

$$\mathbf{R}_m = \mathbf{r}_m(u,\theta) + \rho\mathbf{n}_m(u,\theta) \tag{14}$$

$$\mathbf{R}_m^* = \mathbf{r}_m(u,\theta) + \lambda\mathbf{n}_m(u,\theta) \tag{15}$$

where $\mathbf{r}_m$ and $\mathbf{n}_m$ are the position vector and the unit normal to the theoretical surface and are represented in coordinate system S_m of the measuring machine; λ determines the real location of the probe center and is considered along the normal to the theoretical surface; $\mathbf{R}_m$ and $\mathbf{R}_m^*$ represent in S_m the position vector of the probe center for the theoretical and real surfaces, respectively. Equations (14) and (15) yield

$$\mathbf{R}_m^* - \mathbf{R}_m = (\lambda - \rho)\mathbf{n}_m = \Delta n\mathbf{n}_m \tag{16}$$

and

$$\Delta n = (\mathbf{R}_m^* - \mathbf{R}_m) \cdot \mathbf{n}_m \tag{17}$$

The position vector $\mathbf{R}_m^*$ is determined by coordinate measurements for points of the grid. Equation (17) determines numerically the function

$$\Delta n_i = \Delta n_i(u_i,\theta_i) \qquad (i = 1,\dots,k) \tag{18}$$

that represents the deviations of the real surface for each point of the grid.

MACHINE TOOL SETTINGS TO MINIMIZE DEVIATIONS

The procedure used to minimize the deviations can be represented in two stages: (1) determination of variations of theoretical surface caused by changes of applied machine-tool settings, and (2) minimization of deviations of real surface by appropriate correction of machine-tool settings.

We consider that the theoretical surface is represented in S_t as

$$\mathbf{r}_t = \mathbf{r}_t(u,\theta,d_j) \qquad (j = 1,\dots,m) \tag{19}$$

where parameters d_j are the machine-tool settings. The surface variation is represented by

$$\delta\mathbf{r}_t = \frac{\partial\mathbf{r}_t}{\partial u}\delta u + \frac{\partial\mathbf{r}_t}{\partial\theta}\delta\theta + \sum_{j=1}^{m}\frac{\partial\mathbf{r}_t}{\partial d_j}\delta d_j \tag{20}$$

We multiply both sides of equation (20) by the surface unit normal $\mathbf{n}_t$ and take into account that $\partial\mathbf{r}_t/\partial\theta \cdot \mathbf{n}_t = \partial\mathbf{r}_t/\partial u \cdot \mathbf{n}_t = 0$, since $\partial\mathbf{r}_t/\partial\theta$ and $\partial\mathbf{r}_t/\partial u$ lie in the plane that is tangent to the surface. Then we obtain

$$\delta\mathbf{r}_t \cdot \mathbf{n}_t = \sum_{j=1}^{m}\left(\frac{\partial\mathbf{r}_t}{\partial d_j} \cdot \mathbf{n}_t\right)\delta d_j = \sum_{j=1}^{m} a\delta d_j \tag{21}$$

We can now consider a system of k linear equations in m unknowns ($m \ll k$) of the following structure:

$$\left.\begin{array}{l} a_{11}\delta d_1 + a_{12}\delta d_2 + \dots + a_{1m}\delta d_m = b_1 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{k1}\delta d_1 + a_{k2}\delta d_2 + \dots + a_{km}\delta d_m = b_k \end{array}\right\} \tag{22}$$

Here

$$b_i = \Delta n_i = (\mathbf{R}^*_{mi} - \mathbf{R}_{mi}) \cdot \mathbf{n}_{mi} \tag{23}$$

where i designates the number of grid points; a_{sj} ($s = 1,\dots,k$; $j = 1,\dots,m$) represents the dot product of partial derivatives $\partial\mathbf{r}_t/\partial d_j$ and unit normal $\mathbf{n}_t$. The system (22) of linear equations is overdetermined since $m \ll k$. The essence of the procedure for minimization of deviations is determining unknowns δd_j ($j = 1,\dots,m$) that will minimize the difference between the left and right sides of equations (22). The solution employed the least-square method. The subroutine DLSQRR of IMSL MATH/LIBRARY (ref. 4) was used to computerize the procedure.

APPLICATION OF METHOD TO THE INSPECTION OF FORMATE HYPOID GEAR

Each tooth side of a formate face-hobbed gear is generated by a cone, and the gear tooth surface is the surface of the generating cone. Two cones that are shown in figure 4(a) represent both sides of the gear space. The following equations represent in coordinate system S_c gear surfaces for both sides and the unit normal to such surfaces (fig. 4(b)):

$$\mathbf{r}_c = \begin{bmatrix} -s_G\cos\alpha_G \\ (r - s_G\sin\alpha_G)\sin\theta_G \\ (r - s_G\sin\alpha_G)\cos\theta_G \\ 1 \end{bmatrix} \tag{24}$$

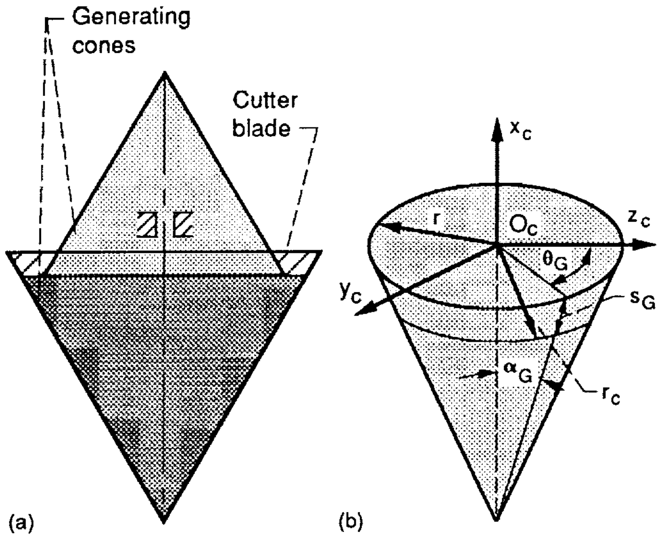

Figure 4.—Generating cones representing cutter blades.

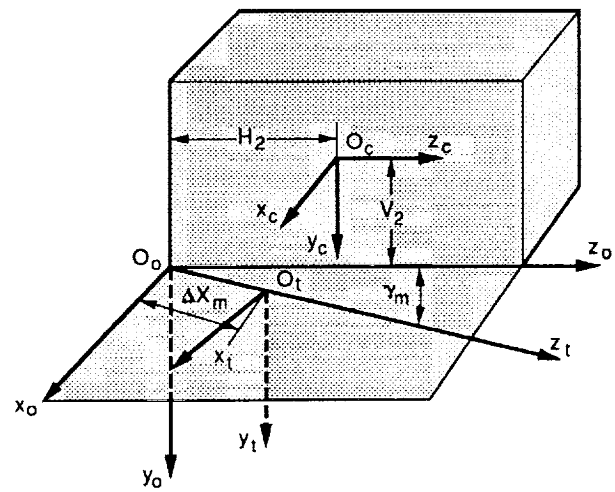

Figure 5.—Coordinate system orientation and machine-tool settings for hypoid gear.

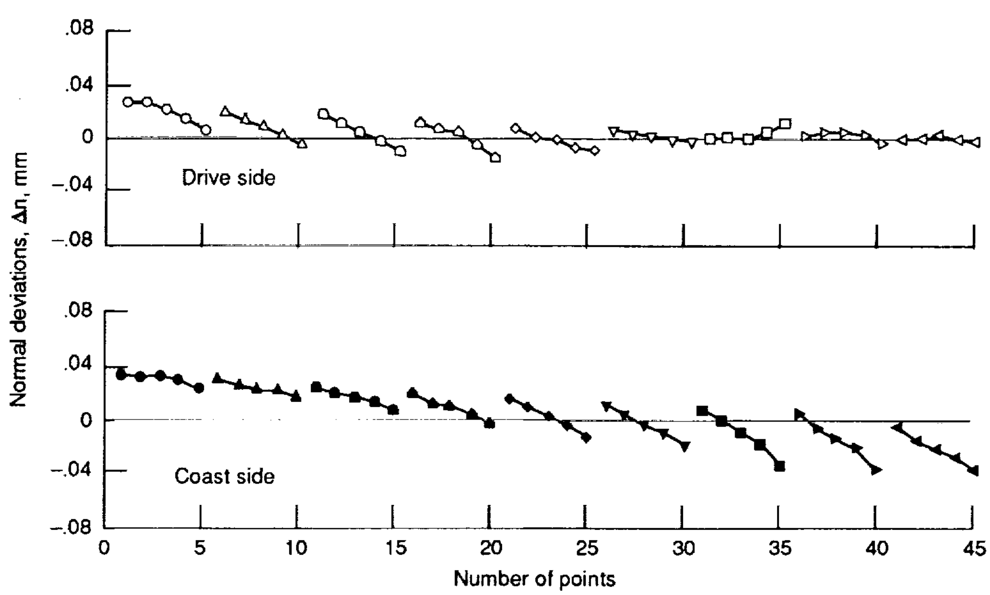

Figure 6.—Deviations of gear real tooth surface.

$$\mathbf{n}_c = \begin{bmatrix} \sin \alpha_G \\ -\cos \alpha_G \sin \theta_G \\ -\cos \alpha_G \cos \theta_G \end{bmatrix} \tag{25}$$

where, $\mathbf{r}_c$ is the position vector and $\mathbf{n}_c$ the unit normal; r is the cutter tip radius; α_G is the cutter blade angle ($\alpha_G > 0$ for the concave side and $\alpha_G < 0$ for the convex side).

Figure 5 shows the installment of the generating cone on the cutting machine. Coordinate systems S_o and S_t are rigidly connected to the cutting machine and the gear being generated, respectively. Systems S_c, S_o, and S_t are rigidly connected to each other since the gear is formate cut. To represent in S_t the theoretical gear tooth surface Σ_t and the unit normal to Σ_t, we use the following matrix equations:

$$\mathbf{r}_t(s_G,\theta_G,d_j) = [M_{tc}]\mathbf{r}_c(s_G,\theta_G) \tag{26}$$

$$\mathbf{n}_t(s_G,\theta_G,d_j) = [L_{tc}]\mathbf{n}_c(s_G,\theta_G) \tag{27}$$

where

$$[M_{tc}] = [M_{to}][M_{oc}]$$
$$= \begin{bmatrix} \cos \gamma_m & 0 & -\sin \gamma_m & 0 \\ 0 & 1 & 0 & 0 \\ \sin \gamma_m & 0 & \cos \gamma_m & -\Delta X_m \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -V_2 \\ 0 & 0 & 1 & H_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

The surface Gaussian coordinates are s_G and θ_G, and $d_j(\gamma_m, V_2, H_2,$ and $\Delta X_m)$ are the machine-tool settings.

The numerical example presented in this paper is based on the experiment that has been performed at the Dana Corporation (Fort Wayne, IN, U.S.A.). The initial deviations Δn for each side of the real tooth surface have been obtained by measurements on a coordinate measuring machine (fig. 6). The grid for the measurement is formed by nine sections along the tooth length, each section having five points. The number of grid points k is therefore 45, and the reference point is at the middle of the grid, i.e., the third point of the fifth section. In the measurement, the coordinate $y_m^{(o)}$ of the reference point is chosen to be zero and the alignment angle δ is determined from solving equation system (13).

The minimization of deviations was performed in accordance with the algorithm described in MACHINE TOOL SETINGS TO MINIMIZE DEVIATIONS, and the results are illustrated in figure 7 and table I.

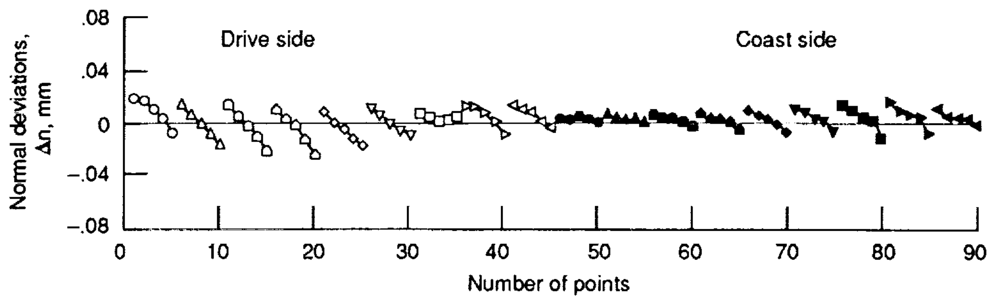

Figure 7.—Minimized deviations after corrections made to machine-tool settings.

TABLE I. - RESULTS OF MINIMIZATION

[Pressure angle, α_G = 21.25°; cutter diameters = 9 in.; point width of cutters = 0.08 in.]

Machine-tool settings	Machine-tool setting parameters			
	V_2, mm	H_2, mm	γ_m, rad	ΔX_m, mm
Initial	103.252550	27.466600	1.059816	0.009677
Corrected	103.25220	27.21603	1.06437	-0.53343

CONCLUSION

A general approach for a computerized determination of deviations of a real surface from the theoretical one based on coordinate measurements has been proposed. An algorithm for computerized minimization of deviations by corrections of initially applied machine-tool settings has been developed. The approach is illustrated with the example of the tooth surface of a hypoid formate gear.

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16. Abstract
A method is developed for the minimization of gear tooth surface deviations between theoretical and real surfaces for the improvement of precision of surface manufacture. Coordinate measurement machinery is used to determine a grid of surface coordinates. Theoretical calculations are made for the grid points. A least-square method is used to minimize the deviations between real and theoretical surfaces by altering the manufacturing machine-tool settings. An example is given for a hypoid gear.

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