A Finite Element Solver for 3–D Compressible Viscous Flows

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1. Introduction

Computation of the flow field inside a space shuttle main engine (SSME) requires the application of the state-of-the-art CFD technology. Several computer codes are under development to solve three dimensional Navier–Stokes equations for analyzing the SSME internal flow, such as the flow through the hot gas manifold. The computational methods used in the Navier–Stokes codes fall into two major categories: finite difference and finite element methods. Some of the algorithms are designed to solve the unsteady compressible Navier–Stokes equations, either by explicit or by implicit factorization methods, using several hundred or thousands of time steps to reach a steady-state solution asymptotically. Other algorithms attempt to solve the steady-state equations by relaxation methods. All of them require body-fitting curvilinear grids with sufficient resolution. Grid requirements, however, differ greatly with the region being modelled and the algorithm used. Implicit factorization based on finite differences typically uses global numerical transformations whereby the transformed grid in the computational space is uniform and rectilinear. This requires the grid to have indices which are separable in the three directions for three dimensional problems, and also be reasonably smooth. However, such requirements may introduce grid singularities when complicated domains are discretized. Flow solver algorithm will have to deal with such grid singularities. Explicit schemes and finite element algorithms have less stringent requirements on the grid structure. However, explicit schemes are slow to converge because of the stability limitations on time step, particularly for large scale viscous problems.

The finite element method is characterized by three basic features which are credited for the enormous success the method has enjoyed in the solution of practical engineering problems. The first feature is that every computational domain is viewed as a collection of simple subdomains, called finite elements. This feature allows us to represent complicated geometries as assemblages of simple parts. It is a desirable feature in the solution of flow problems in complex configurations, not only to describe the complex geometry but also to choose the most suitable computational grid for a particular flow. This feature also allows us to place or remove any obstructions routinely into the flow field. The second feature is that over each element the solution is represented by polynomials of desired degree. This allows us to compute the solution as a continuous function of position instead of at selected few points. The third feature is that the relationship (i.e., the algebraic equations) between the solution and its dual variables is developed using a variational method, such as the Galerkin method. The boundary conditions are then applied on the algebraic equations directly before solving. The three features of the finite element method also allow the easy development and interfacing of pre– and post–processors, and user-defined subroutines for equations for state and turbulence models.

The Galerkin finite element method (i.e., the weight functions are the same as the approximation functions) applied to flow problems always results in implicit schemes. The weighted-residual (or Petrov–Galerkin) method, in which the weight functions are different from the approximation functions, can be used in conjunction with explicit schemes to obtain explicit final equations. For example, by selecting the weight functions to be orthogonal to the approximation functions, the mass matrix can be diagonalized. However, such considerations are entirely in the interest of obtaining explicit schemes and not necessarily in the interest of accuracy or even computational efficiency. In the current project an implicit finite element scheme with suitable dissipation terms for stability is developed. A relaxation procedure, known as the locally implicit scheme is developed to solve the coupled set of algebraic equations efficiently.

Allowing the possibility of unstructured grids is important for discretizing complex flow domains efficiently and also for adding the features of solution-adaptive grids. For grids with large numbers of nodes, direct solution procedures for the finite element equations become impractical. Thus we have undertaken the development of a new iterative algorithm for the solution of implicit finite element equations without assembling global matrices. It is an efficient iteration scheme based on a modified non-linear Gauss–Seidel iteration with symmetric sweeps. This algorithm is analyzed for a model equation and is shown to be unconditionally stable. This analysis is reported in the next Section.

The locally implicit scheme is unconditionally stable based on local linearized analysis. However, for strongly convective flows there is a possibility of non-linear numerical instabilities occurring in some parts of the flow domain and eventually destabilizing the entire flow domain. We have added adaptive artificial dissipation terms of third order to the finite element approximations similar to Jameson and others[1]. These are designed to suppress non-linear instabilities if they appear and at the same time be much smaller than the real viscosity terms in viscous zones.

In numerical schemes for solving fluid flow equations, there is some degree of uncertainty as to the imposition of boundary conditions on some of the variables at different types of boundaries, particularly at the inflow and outflow boundaries. In the current finite element code we have developed special procedures to compute the required flux terms at the boundary surfaces to the same degrees of accuracy as in the interior. We expect that our technique of computing the required surface fluxes iteratively, together with the interior flow variables, should minimize the uncertainties in the imposition of boundary conditions.

The locally implicit scheme is tested on a variety of problems. It has been shown to be efficient with multi-grid acceleration procedures for elliptic problems by Reddy and Nayani[2] and for inviscid compressible flows from transonic to supersonic Mach numbers by Reddy and Jacocks[3]. Reddy, Reddy and Nayani[4] have developed this scheme for viscous flow problems. We developed a 2–D test code for solving unsteady compressible Navier–Stokes equations with finite volume approximation, which is a special case of the finite element approximation. This code has been used to check various features of the locally implicit solution algorithm. We have also added an algebraic turbulence model developed by Baldwin and Lomax[5].

Results for a series of test problems are presented in this report. The finite element code has been tested for Couette flow, described in Schlichting[6], which is a flow under a pressure gradient between two parallel plates in relative motion. Another problem that has been solved is viscous laminar flow over a flat plate. As a test case for the locally implicit scheme, the 2–D finite volume code has been applied to compute subsonic and transonic viscous flows over airfoils for both laminar and turbulent cases. The general 3–D finite element code has been used to compute the flow in an axisymmetric turnaround duct at low Mach numbers.

2. Locally Implicit Scheme for a Model Equation

Locally implicit scheme is a relaxation method for solving the non-linear finite element equations approximating the Navier–Stokes equations. It is a point iteration method at each time step. However, it is not necessary for the iteration to converge fully at each time step if we are interested in computing the time asymptotic steady-state solutions. The analysis of the consistency, stability and hence convergence of the scheme is presented for a model equation for the Navier–Stokes equations.

Consider a one-dimensional convection-diffusion equation,

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2} \tag{2.1}$$

Finite element approximation at a node j on a uniform mesh for equation (2.1) can be written as

$$\frac{\partial}{\partial t}\int u\phi_j dx + \int\left(-au + \nu\frac{\partial u}{\partial x}\right)\frac{\partial \phi_j}{\partial x}dx = 0 \tag{2.2}$$

where ϕ_j is a global test function corresponding to the node j. For a linear element approximation, equation (2.2) gives

$$\begin{gathered}\frac{\partial}{\partial t}\left\{\frac{1}{6}u_{j-1} + \frac{2}{3}u_j + \frac{1}{6}u_{j+1}\right\} + \left(\frac{a}{2\Delta x}\right)(u_{j+1} - u_{j-1}) \\ - \left(\frac{\nu}{\Delta x^2}\right)(u_{j-1} - 2u_j + u_{j+1}) = 0\end{gathered} \tag{2.3}$$

Implicit time integration gives

$$\begin{gathered}\frac{1}{6}\Delta u_{j-1} + \frac{2}{3}\Delta u_j + \frac{1}{6}\Delta u_{j+1} + \frac{C}{2}\left(u_{j+1}^{n+1} - u_{j-1}^{n+1}\right) \\ - R\left(u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}\right) = 0\end{gathered} \tag{2.4}$$

where $\Delta u_j = u_j^{n+1} - u_j^n$

$$C = a\Delta t/\Delta x, \qquad R = \nu\Delta t/\Delta x^2$$

Equation (2.4), together with appropriate boundary conditions, gives a system of linear equations which can be solved easily in one-dimension and this scheme is unconditionally stable. However, the system of equations becomes too large in multi-dimensions and various types of sparse matrix solvers are developed in the literature, but they are usable only with a modest number of nodes. Alternately, we develop a relaxation scheme to solve (2.4) approximately at each time step. The scheme is a modification of the symmetric Gauss–Seidel iteration. The basic Gauss–Seidel iteration, even with symmetric sweeps, is unstable for a whole range of Courant number C in equation (2.4). The present modification makes it unconditionally stable. Rewrite the equation (2.4) in delta form as

$$\begin{aligned}\frac{1}{6}\Delta u_{j-1} + \frac{2}{3}\Delta u_j + \frac{1}{6}\Delta u_{j+1} + \frac{C}{2}\left(\Delta u_{j+1} - \Delta u_{j-1}\right) \\ - R(\Delta u_{j-1} - 2\Delta u_j + \Delta u_{j+1}) = Res_j^n\end{aligned} \tag{2.5}$$

where

$$Res_j^n = -\frac{C}{2}\left(u_{j+1}^n - u_{j-1}^n\right) + R\left(u_{j-1}^n - 2u_j^n + u_{j+1}^n\right) \tag{2.6}$$

As $\Delta u_j = u_j^{n+1} - u_j^n \to 0$ as $n \to \infty$, we obtain the asymptotic steady-state solution as the Res_j function is driven to zero. This process may be speeded up and made more robust by choosing a value for R on the left side of equation (2.5) larger than the value of R on the right side of equation (2.5). To analyze this process we use the notation $\overline{R}$ for R on the left side of equation (2.5). It may be noted that we can always obtain time accurate solution, if that is required, by choosing $\overline{R} = R$. We solve for Δu_j at each time step by a modified Gauss–Seidel iteration:

$$\Delta u_j^{(m+1)} = \Delta u_j^{(m)} + du_j, \qquad \Delta u_j^{(0)} = 0 \tag{2.7}$$

Left-to-right sweep yields

$$\begin{aligned}\frac{2}{3}du_j + \frac{1}{6}du_{j+1} + \frac{C}{2}du_{j+1} \\ - \overline{R}(-2du_j + du_{j+1}) = RHS\end{aligned} \tag{2.8}$$

where

$$\begin{aligned}RHS = & Res_j^n - \left[\frac{1}{6}\Delta u_{j-1}^{(m+1)} + \frac{2}{3}\Delta u_j^{(m)} + \frac{1}{6}\Delta u_{j+1}^{(m)} \right. \\ & \left. + \frac{C}{2}\left(\Delta u_{j+1}^{(m)} - \Delta u_{j-1}^{(m+1)}\right) - \overline{R}\left(\Delta u_{j-1}^{(m+1)} - 2\Delta u_j^{(m)} + \Delta u_{j+1}^{(m)}\right)\right]\end{aligned} \tag{2.9}$$

Now we approximate $du_{j+1} \simeq du_j$ and replace C by its absolute value $|C|$ on the left side of equation (2.8), to accommodate convection velocity direction either in or opposite to the iteration sweep direction. This leads to an explicit expression for du_j:

$$\left(\frac{5}{6} + \frac{|C|}{2} + \overline{R}\right) du_j = RHS \tag{2.10}$$

Right-to-left sweep is defined similarly. A symmetric iteration sweep consists of a left-to-right sweep followed by a right-to-left sweep. It may be noted that du_j is the iterative correction to the time change iterates $\Delta u_j^{(m)}$ and if the iteration process is convergent,

$RHS \to 0$ and the equation (2.5) can be satisfied as accurately as we wish by carrying out the necessary number of symmetric iteration sweeps. The approximations made in the iteration do not affect the actual solution itself. Thus the iteration equations are consistent with the basic equations. One or two symmetric sweeps per time step are usually sufficient for obtaining steady-state solutions. Local stability analysis can be carried out by computing the amplification factor of discrete Fourier modal solutions per time step. In this analysis, we seek modal solutions of the equations (2.9) and (2.10) in the form

$$\begin{aligned} u_j^n &= v^n e^{ij\xi}, \quad 0 \le \xi = \alpha\Delta x \le \pi \\ \Delta u_j^{(m)} &= \Delta v^{(m)} e^{ij\xi}, \quad m = 0, 1, \ldots \\ u_j^{n+1} &= v^{n+1} e^{ij\xi} \end{aligned}$$

For a single symmetric sweep per time step ($m = 0, 1$),

$$v^{n+1} = v^n + \Delta v^{(2)} = g(\xi) v^n$$

where $g(\xi)$ is known as the amplification factor from one time step to the next and is given by

$$\begin{aligned} g(\xi) &= 1 + \frac{r}{h_3}\left[1 + \frac{h_2}{h_1}\right], \quad 0 \le \xi \le \pi \\ r &= -Ci \ \sin\ \xi + 2R(\cos\ \xi - 1) \\ h_1 &= b - e^{-i\xi}\left(\frac{C}{2} + \overline{R} - \frac{1}{6}\right) \\ h_2 &= b - \frac{2}{3} - 2\overline{R} + e^{-i\xi}\left(\frac{C}{2} + \overline{R} - \frac{1}{6}\right) \\ h_3 &= b + e^{i\xi}\left(\frac{C}{2} - \overline{R} + \frac{1}{6}\right) \\ b &= \frac{5}{6} + \frac{|C|}{2} + \overline{R} \end{aligned} \tag{2.11}$$

A necessary condition for stability is $|g(\xi)| \le 1$. It can be shown that $|g(\xi)|$ is indeed ≤ 1 unconditionally. It is also desirable to have $|g(\xi)| < 1$ as much as possible for ξ closer to π which represents the range of high frequency modes of the solution. Figure 1 shows plots of $|g(\xi)|$ versus ξ for different Courant numbers for $R = \overline{R} = \frac{C}{64}$. Figure 2 shows plots of $|g|$ versus ξ for $C = 10$, $\overline{R} = R$ and R takes different values. Figure 3 shows the plots for $C = 10$, $\overline{R} = 2R$ and R takes different values. Numerical plots of $|g|$ against ξ confirm that the scheme is unconditionally stable. However, very large Courant numbers are not necessarily the best. Courant number $C \simeq 10$ and $\overline{R} = 2R \to 4R$ seem desirable ranges. Amplification factors corresponding to two or more symmetric modified Gauss–Seidel iterations have similar behavior. Thus we establish unconditional stability for the modified Gauss–Seidel iteration scheme for the convection-diffusion equation. Similar stability can be shown when the diffusion term is replaced by a 4th difference term of the type that is used as artificial viscosity term of third order for suppressing non-linear instabilities for convection dominated flows. It is possible to use artificial viscosity terms which are smaller than the truncation terms of the second order accurate finite element approximations. In the present Navier–Stokes finite element code where we compute all terms to full second order accuracy, artificial dissipation terms, which are an order of magnitude smaller then truncation error, are included to suppress non-linear instabilities. Stability analysis of the model equation indicates that the locally implicit scheme is unconditionally stable in a local linearized sense.

3. Locally Implicit Scheme for Navier–Stokes Equations

Many algorithms designed to solve the unsteady compressible Navier-Stokes equations use either explicit methods or implicit factorization methods. Finite element approximations usually yield implicit equations. These are solved by explicit time integration methods after making additional approximations. Explicit methods may take thousands of time steps to converge. Solving them implicitly with Newton iteration is possible, but the matrix storage requirements for the resulting algebraic equations and the solution process make it prohibitive even for modest size three dimensional flow problems. There are other algorithms based on relaxation methods. We have developed a locally implicit method for solving the non-linear finite element approximations for 3–D Navier-Stokes equations at each time step.

The method is based on a relaxation procedure for solving the finite element equations corresponding to each node iteratively. The equations for the elements surrounding a particular node are evaluated based on the latest iterates for the flow variables at the nodes around it and the solution is updated at that node by a modified Gauss–Seidel iteration. This procedure does not require the assembly of a global matrix, in contrast to the standard finite element algorithms. It does not require the solution of a system of large number of equations. Thus it is a matrix-free implicit finite element algorithm. An additional feature of the algorithm is that while it uses tri-linear approximations for the flow variables in quadilateral (brick) elements, all the non-linear fluxes in the Navier-Stokes equations are evaluated without any further linear approximation. The fluxes are non-linear and are computed accordingly. This assures the second order spatial accuracy of the scheme even for unstructured grids.

3.1 Finite Element Approximations

The unsteady, compressible Navier-Stokes equations are written in conservation form as

$$\left\{\frac{\partial U}{\partial t}\right\} + \vec{\nabla}\cdot\{\vec{F}^v\} + \vec{\nabla}\cdot\{\vec{F}^I\} = \{0\} \tag{3.1}$$

where

$$\{U\} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho\varepsilon \end{Bmatrix}, \quad \{\vec{F}^v\} = \begin{Bmatrix} \vec{0} \\ -\underline{\tau} \\ -\underline{\tau}\cdot\vec{v} + \underline{q} \end{Bmatrix}, \quad \{\vec{F}^I\} = \begin{Bmatrix} \rho\vec{v} \\ \rho\vec{v}\vec{v} + p\vec{I} \\ \vec{v}(\rho\varepsilon + p) \end{Bmatrix}$$

$\{\vec{F}^I\}$ and $\{\vec{F}^v\}$ represent the inviscid and viscous fluxes respectively. Details of these equations are given in Appendix I.

The variational form (weak form) of equation (3.1) over an element Ω^e is written as

$$0 = \int_{\Omega^e} \left(\{\Phi\}^T \left\{ \frac{\partial U}{\partial t} \right\} - \{\vec{\nabla}\Phi\}^T \cdot \{\vec{F}^v + \vec{F}^I\} \right) dV + \oint_{S^e} \{\Phi\}^T \{F_n\} dS \qquad (3.2)$$

where $\{\Phi\}$ are test functions. They are tri-linear functions for linear finite element approximation and piecewise constants for finite volume approximations. $F_n = (\vec{F}^v + \vec{F}^I) \cdot \vec{n}$ where $\vec{n}$ is the outward drawn unit normal to the surface S^e of the element Ω^e. The conservation variables $\vec{U} = (U_\alpha, \alpha = 1, \cdots 5)$ are approximated by the interpolation functions Ψ_j as

$$U_\alpha = \sum_{j=1}^{N} \hat{U}_\alpha^j \Psi_j(x, y, z) \equiv \{\Psi\}\{\hat{U}_\alpha\} \qquad (3.3)$$

where

$$\{\Psi\} = \{\Psi_1 \Psi_2 \cdots \Psi_N\}, \ \{\hat{U}_\alpha\} = \left(\hat{U}_\alpha^1, \hat{U}_\alpha^2, \ \cdots \ \hat{U}_\alpha^N\right)^T$$

$\hat{U}_\alpha^j$ is the numerical value of the αth component of the conservation flow variable U at jth local node of the element Ω^e. The interpolation functions Ψ and test functions Φ are chosen to be the same for compressible flow equations. $N = 8$ for tri-linear approximations on quadrilateral brick elements. These approximations are done according to the standard finite element approximations (Ref. 7).

Define the total nodal vector of the conservation variables at the nodes of an element as

$$\underset{5N \times 1}{\{\hat{U}\}} = \begin{Bmatrix} \{\hat{U}_1\} \\ \{\hat{U}_2\} \\ \vdots \\ \{\hat{U}_5\} \end{Bmatrix}; \ \underset{5 \times 5N}{[\Psi]^e} = \begin{bmatrix} \{\Psi\} & \{0\} & \{0\} & \{0\} & \{0\} \\ \{0\} & \{\Psi\} & \{0\} & \{0\} & \{0\} \\ \{0\} & \{0\} & \{\Psi\} & \{0\} & \{0\} \\ \{0\} & \{0\} & \{0\} & \{\Psi\} & \{0\} \\ \{0\} & \{0\} & \{0\} & \{0\} & \{\Psi\} \end{bmatrix} \qquad (3.4)$$

Then

$$\underset{5 \times 1}{\{U\}} = \begin{Bmatrix} U_1 \\ U_2 \\ \vdots \\ U_5 \end{Bmatrix} = [\Psi]^e \{\hat{U}\}^e$$

Now the variational statement (2) can be written as

$$\{0\} = \int_{\Omega^e} \left([\Psi]^T [\Psi] \{\dot{\hat{U}}\} - [\vec{\nabla}\Psi]^T \cdot \{\vec{F}\} \right) dV + \oint_{S^e} [\Psi]^T \{F_n\} dS \qquad (3.5)$$

It should be noted at this point that $\vec{F}$ and F_n are non-linear functions of $\vec{U}$ and thus the integrals involving them can be expressed analytically in terms of the components of $\hat{U}$. These expressions are long but they can be programmed into the computer code efficiently. The coupled non-linear differential equations (3.5) are discretized in time by the Euler implicit scheme as follows:

$$\frac{1}{\Delta t}[M^e]\{\Delta\hat{U}^e\} + \{\mathcal{R}^e\}^{m+1} = \{0\} \tag{3.6}$$

where

$$\Delta\hat{U}^e \equiv (\hat{U}^e)^{m+1} - (\hat{U}^e)^m, \qquad m - \text{ time level}$$

$$[M^e] = \int_{\Omega^e} [\Psi]^T[\Psi]dV \tag{3.7}$$

$$\{\mathcal{R}^e\} = -\int_{\Omega^e} [\vec{\nabla}\Psi]^T \cdot \{\vec{F}\}dV + \oint_{S^e} [\Psi]^T\{F_n\}dS \tag{3.8}$$

Details of the expression $\{\mathcal{R}^e\}$ in equation (3.8) are given in Appendix II. In the standard finite element algorithms, the element equations (3.6) are linearized, usually by Newton method, and all the element equations are assembled to derive a global system of linear equations which are solved by sparse matrix solvers. For large scale problems the matrices involved become too big to be practical. Here we develop a matrix-free relaxation method to solve the non-linear equations directly by a modified Gauss-Seidel iteration.

3.2 Locally Implicit Scheme

We wish to solve the non-linear finite element equations iteratively at a node i. We assume the nodal values of the solution at all the surrounding nodes from their latest iterates. The test function Ψi, corresponding to the node i, in equation (3.6) gives the contribution of element Ω^e to the node i in the finite element approximation. Adding similar equations from all the elements surrounding a node *ND* yields the nodal finite element equation. Thus the equations corresponding to a single node, *ND* are

$$\sum_e \left(\frac{1}{\Delta t}[M^e]\{\Delta U^e\} + \{\mathcal{R}^e\}^{n+1}\right)_{ND} = 0 \tag{3.9}$$

where $\hat{U}^e$ is replaced by U^e for convenience. Thus U^e is the conservation variable vector at all the nodes of the element e, and the summation in equation (3.9) is over all elements e surrounding the node *ND*. Equation (3.9) represents 5 equations at *ND* corresponding to each of the 5 conservation equations. The αth conservation equation at *ND* can be written as

$$\left[\sum_e \frac{1}{\Delta t}\int_{\Omega^e}\left(\sum_{j=1}^{8}\Delta U_{\alpha,j}\Psi_j\right)^e \Psi^e_{(ND)}dV - \int_{\Omega^e}\vec{\nabla}\Psi^e_{(ND)}\cdot\vec{F}^{\alpha^{(n+1)}}dV \right. \\ \left. + \oint_{\partial\Omega^e}\Psi^e_{(ND)}\vec{F}^{\alpha^{(n+1)}}\cdot\vec{n}dS\right] = 0 \tag{3.10}$$

where $\Psi^e_{(ND)} = \Psi^e_i$ with i corresponding to the local index of the global node ND in element e. For all interior nodes ND, the surface flux integral in equation (3.10) vanishes. This equation couples U at all the nodes surrounding the node ND. We develop a modified symmetric non-linear Gauss–Seidel iteration to solve the coupled system of non-linear equations directly without linearization. This leads to a matrix-free algorithm for the solution.

For a particular time step n, the iteration is carried out as follows. During the iteration process, we assume that all U's in the αth equation other than U_α are known from the previous step of the iteration. We solve for ΔU_α at node ND approximately by a modified Gauss–Seidel iteration.

$$\Delta U^{(m+1)}_{\alpha,j} = \Delta U^{(m)}_{\alpha,j} + dU_{\alpha,j} \tag{3.11}$$

for all nodes j where $(m+1)$th iterates are not available.

$$\vec{F}^{\alpha^{(n+1)}} \simeq \vec{F}^{\alpha}\left(U^n + \Delta U^{(m+1)}\right) \tag{3.12}$$

at nodes where $\Delta U^{(m+1)}$ is available. At other nodes where only $\Delta U^{(m)}$ is available,

$$\begin{aligned} \vec{F}^{\alpha(n+1)} &\simeq \vec{F}^{\alpha}\left(U^n + \Delta U^{(m)} + dU\right) \\ &\simeq \vec{F}^{\alpha}\left(U^n + \Delta U^{(m)}\right) + \frac{\partial \vec{F}}{\partial U} dU \end{aligned} \tag{3.13}$$

The Jacobian matrices $\frac{\partial \vec{F}}{\partial U}$ have inviscid and viscous parts $\frac{\partial \vec{F}^{Invis}}{\partial U}$, $\frac{\partial \vec{F}^{Vis}}{\partial U}$ respectively. The inviscid part is approximated by the spectral radii of the Jacobian matrices multiplied by identity matrices.

$$\frac{\partial \vec{F}^{Invis}}{\partial U} \longrightarrow (|u| + a,\ |v| + a,\ |w| + a)\, I = \vec{SR} \tag{3.14}$$

where u, v, w are velocity components and a is the speed of sound. The viscous parts of the Jacobian matrices are not altered. For the iterative corrections dU's we make the approximation,

$$dU_{\alpha,j} \simeq dU_{\alpha,(ND)} \tag{3.15}$$

for all the nodes j at which the latest iterates are not available. $dU_{\alpha,(ND)} = dU_{\alpha,i}$ where i is the local index corresponding to the global node ND. With this approximation, we obtain explicit scalar expression for the iterative correction at the node ND, $dU_{\alpha,(ND)}$.

$$C\ dU_{\alpha,ND} = -Res^{(*)}_{\alpha,ND} \tag{3.16}$$

where

$$Res_{\alpha,ND}^{(*)} = \sum_e \frac{1}{\Delta t} \int_{\Omega^e} \left(\sum_{j=1}^{8} \Delta U_{\alpha,j}^{(*)} \Psi_j \right)^e \Psi_{(ND)}^e dV$$
$$- \int_{\Omega^e} \vec{\nabla}\Psi_{(ND)}^e \cdot \vec{F}^{\alpha^{(*)}} dV + \oint_{\partial\Omega^e} \Psi_{(ND)}^e \vec{F}^{\alpha^{(*)}} \cdot \vec{n} dS \tag{3.17}$$

The superscript (*) corresponds to the iteration level (m) or $(m+1)$ which ever is available at the nodes surrounding the node (ND).

$$C = \sum_e \left[\frac{1}{\Delta t} \int_{\Omega^e} \sum_j \Psi_j^e \Psi_{(ND)}^e IND(j) dV \right]$$
$$+ \sum_e \int_{\Omega^e} \left| \vec{\nabla}\Psi_{(ND)}^e \right| \cdot \vec{SR}\ \Psi_{(ND)}^e dV + \sum_e \left[\int_{\Omega^e} \vec{\nabla}\Psi_{(ND)}^e \cdot \sum_j IND(j) \frac{\partial \vec{F}^\alpha Vis}{\partial U_{\alpha,j}} dV \right] \tag{3.18}$$

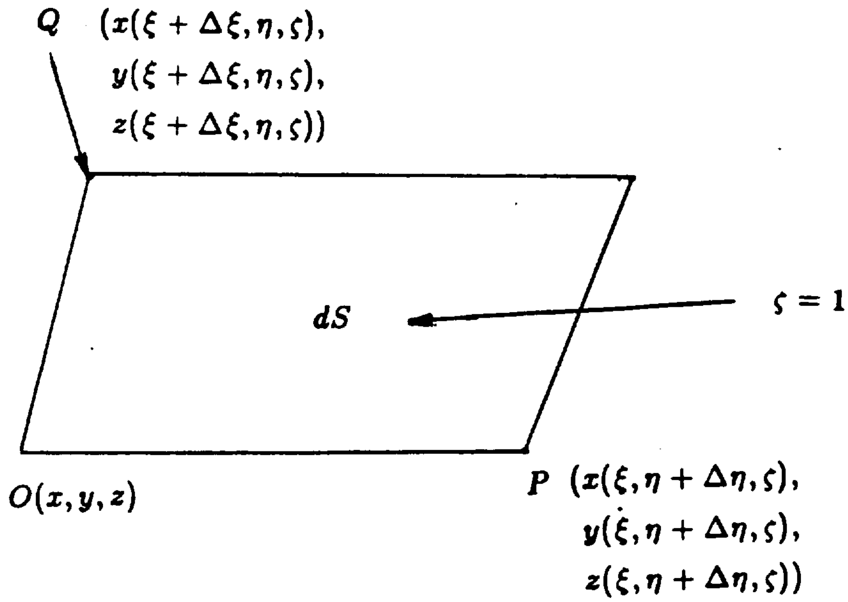
$$IND(j) = \begin{cases} 1 & \text{, for nodes } j \text{ at iteration level } m \\ 0 & \text{, for nodes } j \text{ at iteration level } m+1 \end{cases} \tag{3.19}$$

The absolute value sign $|\cdot|$ in the middle integral indicates the absolute values of each of its components. In defining the coefficient C, contributions of surface integrals do not exist for all interior nodes and they are ignored for boundary nodes for simplicity. Approximations made in C to simplify the algorithm while preserving numerical stability for large Courant numbers, do not affect the solution which is obtained by driving Res function to zero. One iteration sweep starting from the initial node to the final node followed by a reverse sweep makes one symmetric sweep. Typically two symmetric sweeps per time step are sufficient for obtaining time asymptotic solutions.

3.3 Surface Flux Computation

Volume integrals over quadrilateral brick elements are computed by isoparametric transformations to a standard cube and by the use of two point Gaussian integration in each direction. The details of such computations are available in many books on finite element methods. Surface flux computation, however, is less known and the basic idea is outlined below.

Suppose ξ, η, ζ are the local coordinates and x, y, z are global coordinates and we wish to compute the surface flux on the surface $\zeta = 1$ of an element.



$$\oint_{\zeta=1} \vec{F} \cdot \vec{n} dS = \oint_{\zeta=1} \vec{F} \cdot d\vec{S} \tag{3.20}$$

$$\begin{aligned} d\vec{S} &= \vec{n} dS = \vec{OP} \times \vec{OQ} \\ &= (x_\xi \Delta\xi, y_\xi \Delta\xi, z_\xi \Delta\xi) \times (x_\eta \Delta\eta, y_\eta \Delta\eta, z_\eta \Delta\eta) \\ &= \left(\frac{\partial(y,z)}{\partial(\xi,\eta)}, \frac{\partial(z,x)}{\partial(\xi,\eta}, \frac{\partial(x,y)}{\partial(\xi,\eta)} \right) d\xi \, d\eta \end{aligned} \tag{3.21}$$

$\oint_{\zeta=1} \vec{F} \cdot d\vec{S}$ can now be computed with Gaussian integration in ξ and η directions, at $\zeta = 1$. The values of $\vec{F}$ and the surface Jacobians are evaluated at the Gaussian points on the surfaces of the elements, in contrast to the interior evaluation of volume integral computations.

3.4 Artificial Dissipation

Though the scheme is linearly stable, non-linear numerical instabilities could arise in strongly convective flows. Various artificial dissipation terms have been developed in the literature to suppress the numerical instabilities. The features we seek for artificial dissipation terms are that they only suppress numerical instabilities, they be smaller than the real viscous terms, they are of higher order than the truncation terms and finally they should be implementable in the code without excessive computation. For this purpose, we choose the adaptive artificial dissipation terms of third order similar to those developed by Jameson(1) and others. These terms are included in the finite element code. A listing of the code is given in Appendix III.

4. Test Calculations

4.1 Couette Flow

The first test problem is the simulation of a one dimensional shear flow under pressure gradient. It has been computed with a uniform mesh of 2 x 6 x 2 linear (eight-node) elements with the following boundary conditions.

$$
\begin{aligned}
u = v = w = 0 \text{ at } & \; y = 0 \text{ plane} \\
u = U_0,\ v = w = 0 \text{ at } & \; y = 6 \text{ plane} \\
w = 0 \text{ at } z = 0 \text{ and } & \; z = 2 \text{ plane} \\
v = 0 \text{ at } x = 0 \text{ and } & \; x = 2 \text{ plane}
\end{aligned}
$$

A favorable pressure gradient of $\frac{\partial p}{\partial x} = -1$ is imposed. Fig. 4 shows the computed solution with wall velocity $U_0 = 3$. This problem has a simple exact solution as given in Schliching[6]. The computed solution agrees with the exact solution and the two are indistinguishable on the plot. For this simple problem, it takes very few time steps to reach a steady state solution starting from uniform flow conditions. The table of global and local correspondence of nodes, typical of finite element codes is also shown in Fig. 4.

4.2 Laminar Boundary Layer Over a Flat Plate

As another check case, laminar boundary layer over a flat plate has been computed with a stretched mesh of 4 x 6 x 1 linear elements. In this problem the convective terms are of the same order as some of the viscous terms. The finite element solution for a Reynolds number of $Re = 10^4$, along with the boundary conditions and the mesh used are shown in Fig. 5. The computed solution agrees qualitatively with the exact solution even with a very coarse mesh. A converged solution can also be obtained for $Re = 10^5$.

4.3 Flow Over an Airfoil

The locally implicit scheme for two dimensional Navier–Stokes equations with finite volume discretization is applied to compute airfoil flows. Calculations have been carried out with the code and comparisons have been made with experimental results. High Reynolds number viscous flows over an RAE 2822 airfoil have been computed from subsonic to transonic Mach numbers. An algebraic turbulence model developed by Baldwin and Lomax[5] has been incorporated into the code. A body conforming C–grid (128 x 32) for an RAE 2822 airfoil is shown in Fig. 6. The mesh spacing normal to the airfoil is highly stretched to resolve turbulent viscous layer. The spacing ranges from .00005 to 3 chord lengths from inner to outer grid lines. Mach contours for turbulent flow at Mach number, $M = 0.6$, angle of attack, $\alpha = 2.57$ and Reynolds number, $Re = 6.3 \times 10^6$ are shown in Fig. 7a. Fig. 7b shows the corresponding C_p plot where numerical results are compared with experimental values published by Cook, McDonald and Firmin[8]. The agreement of numerical and experimental values for C_p is reasonable for a relatively coarse grid. Similar Mach contour and C_p plots are presented for transonic flow case with $M = 0.725, \alpha = 2.92$ and $Re = 6.5 \times 10^6$ in Figs. 8a and 8b.

4.4 Flow in a Turn-around Duct

As a test for the 3–D finite element code, flow in an axisymmetric turnaround duct is computed at Mach number = 0.1. The schematic sketch of the turnaround duct is shown in Fig. 9. The geometry used corresponds to a test rig at Rockwell International which is shown in Fig. 10. A relatively coarse grid of 8 x 15 x 2 elements are chosen. Since the flow is axisymmetric, 3 sectional planes with 2 elements in the circumferential direction are chosen and flow is set to be the same in each of the planes in the boundary conditions. The grid in one of the constant-angle planes and the computed velocity vectors are shown in Fig. 11 and a more detailed view of the velocity vectors in the bend region are shown in Fig. 12. The flow features are qualitatively correct. But a finer grid computation is necessary for quantitative comparisons with experimental results and it will be carried out later.

5. References

1. Jameson, A., Baker, T. J., Weatherill, N. P., "Calculation of Inviscid Transonic Flow Over a Complete Aircraft", AIAA-86-0103, AIAA 24th Aerospace Sciences Meeting, January 1986.

2. Reddy, K. C., Nayani, S. N., "A Locally Implicit Scheme for Elliptic Partial Differential Equations", presented at the SSME/CFD Working Group Meeting, NASA Marshall Space Flight Center, April 8-11, 1986.

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5. Baldwin, B. S., Lomax, H., "Thin Layer Approximation and Algebraic Model for Seperated Turbulent Flows", AIAA Paper 78-257, January 1978.

6. Schlichting, H., *Boundary Layer Theory*, Pergamon Press, 1955.

7. Reddy, J. N., *An Introduction to the Finite Element Method*, McGraw-Hill Book Company, 1984.

8. Cook, P. H., McDonald, M. A., Firmin, M. C. P., "Airfoil RAE 2822 - Pressure Distributions, and Boundary Layer and Wake Measurements", AGARD-AR-138, 1979.

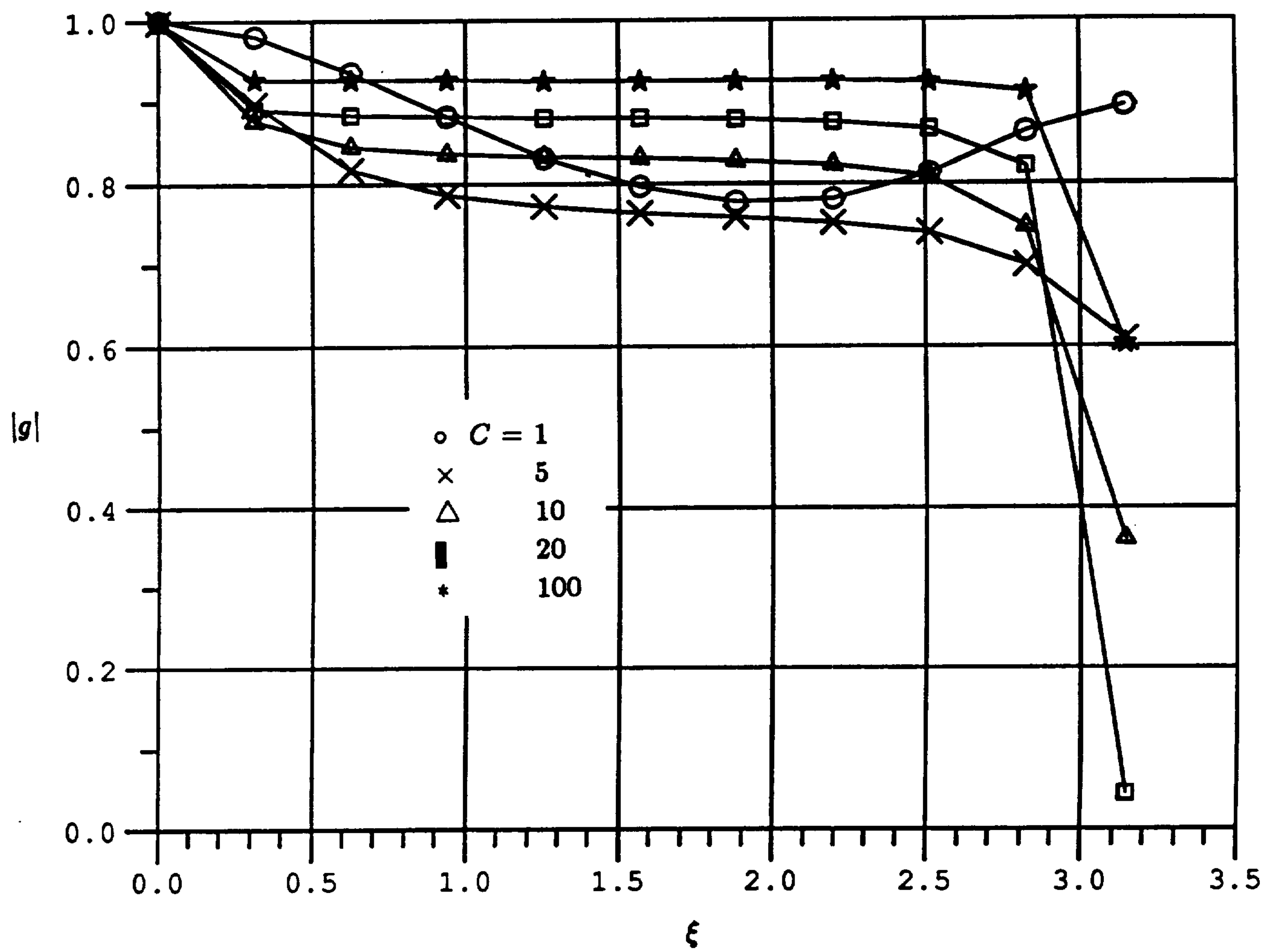


Fig. 1 Amplification Factor for Different Courant Numbers ($\overline{R} = R = C/64$)

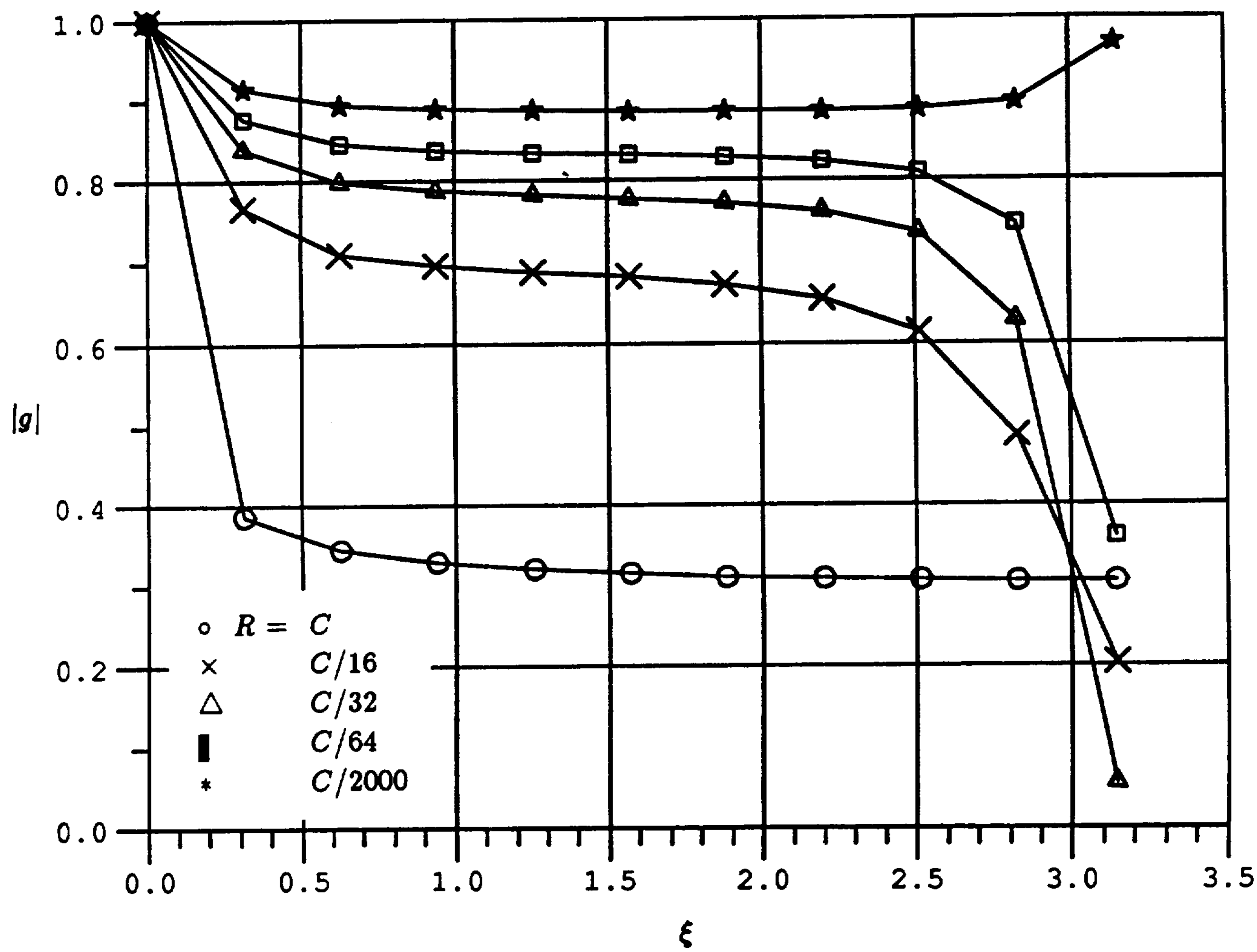


Fig. 2 Amplification Factor for Different Dissipation Parameters ($C = 10$, $\overline{R} = R$)

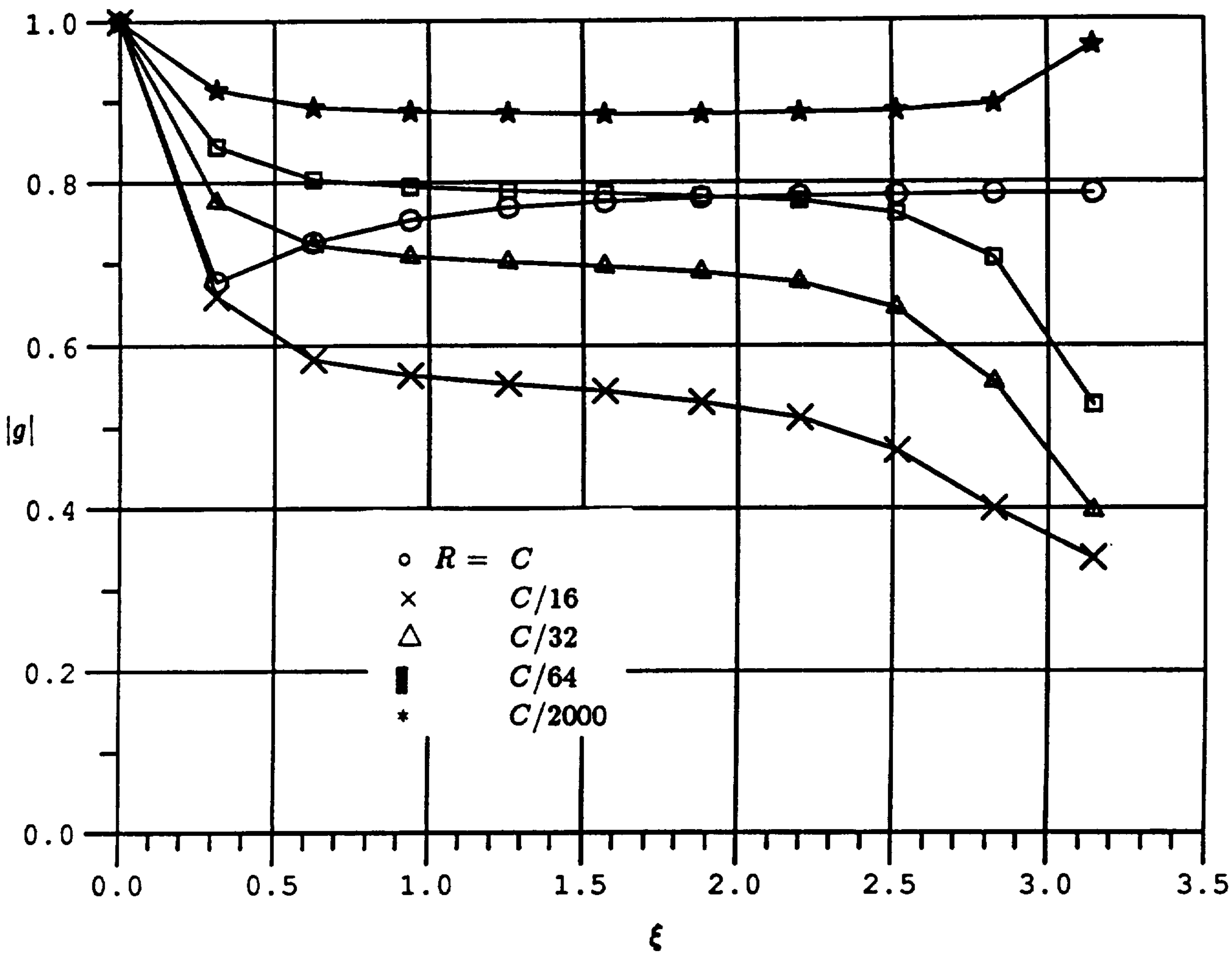
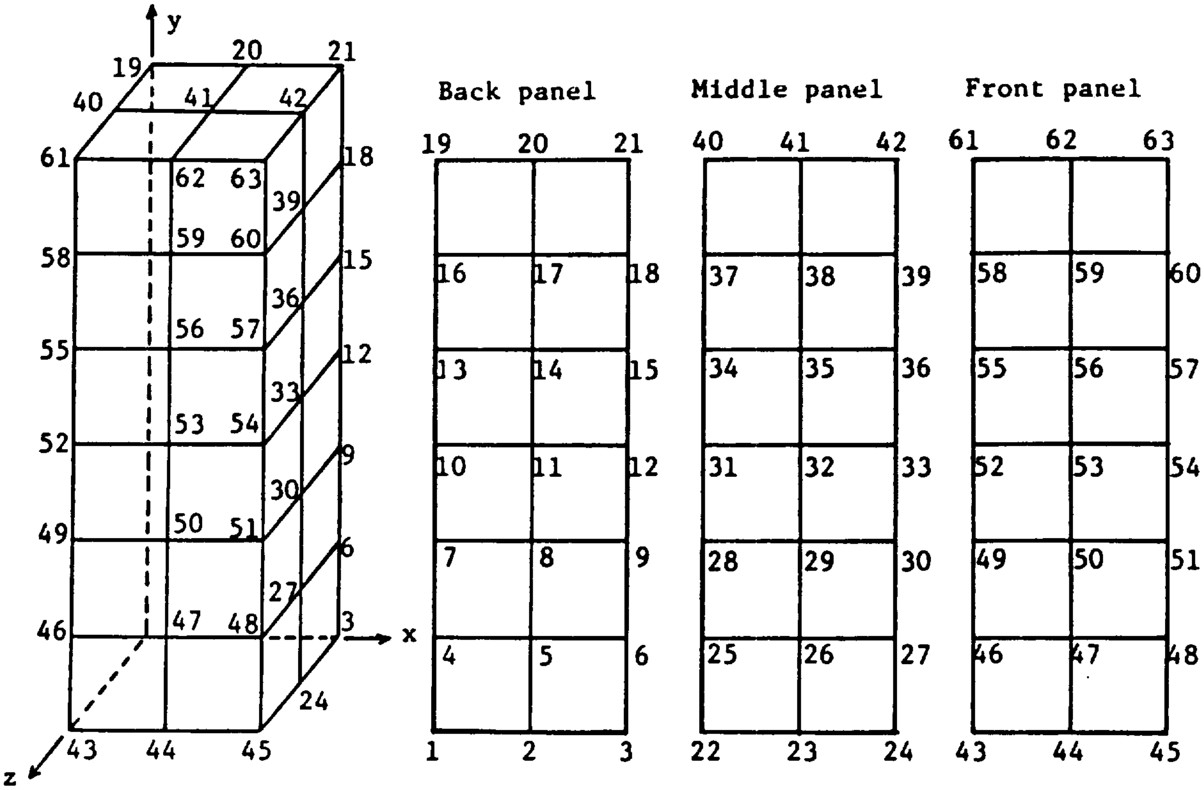
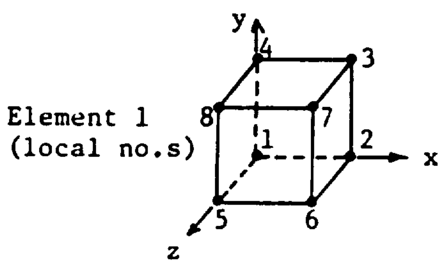


Fig. 3 Amplification Factor for Different Dissipation Parameters ($C = 10$, $\overline{R} = 2R$)





Correspondence of nodes:

Local	Global
1	1
2	2
3	5
4	4
5	22
6	23
7	26
8	25

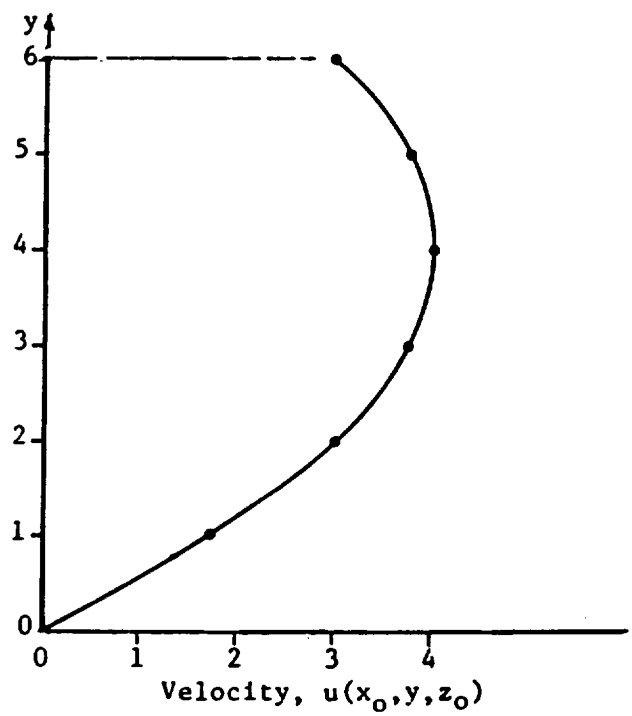


Fig. 4 Couette Flow

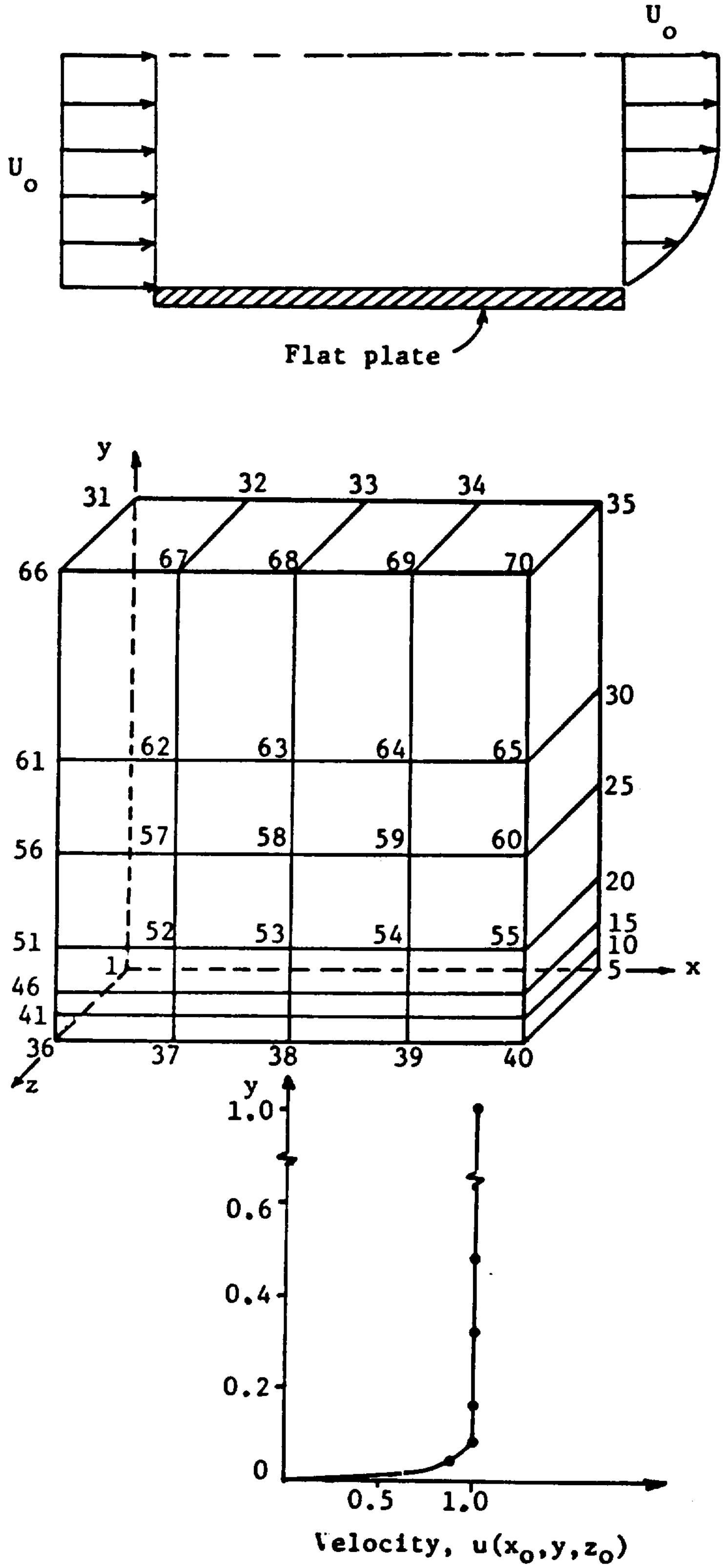


Fig. 5 Flat Plate Boundary Layer Flow

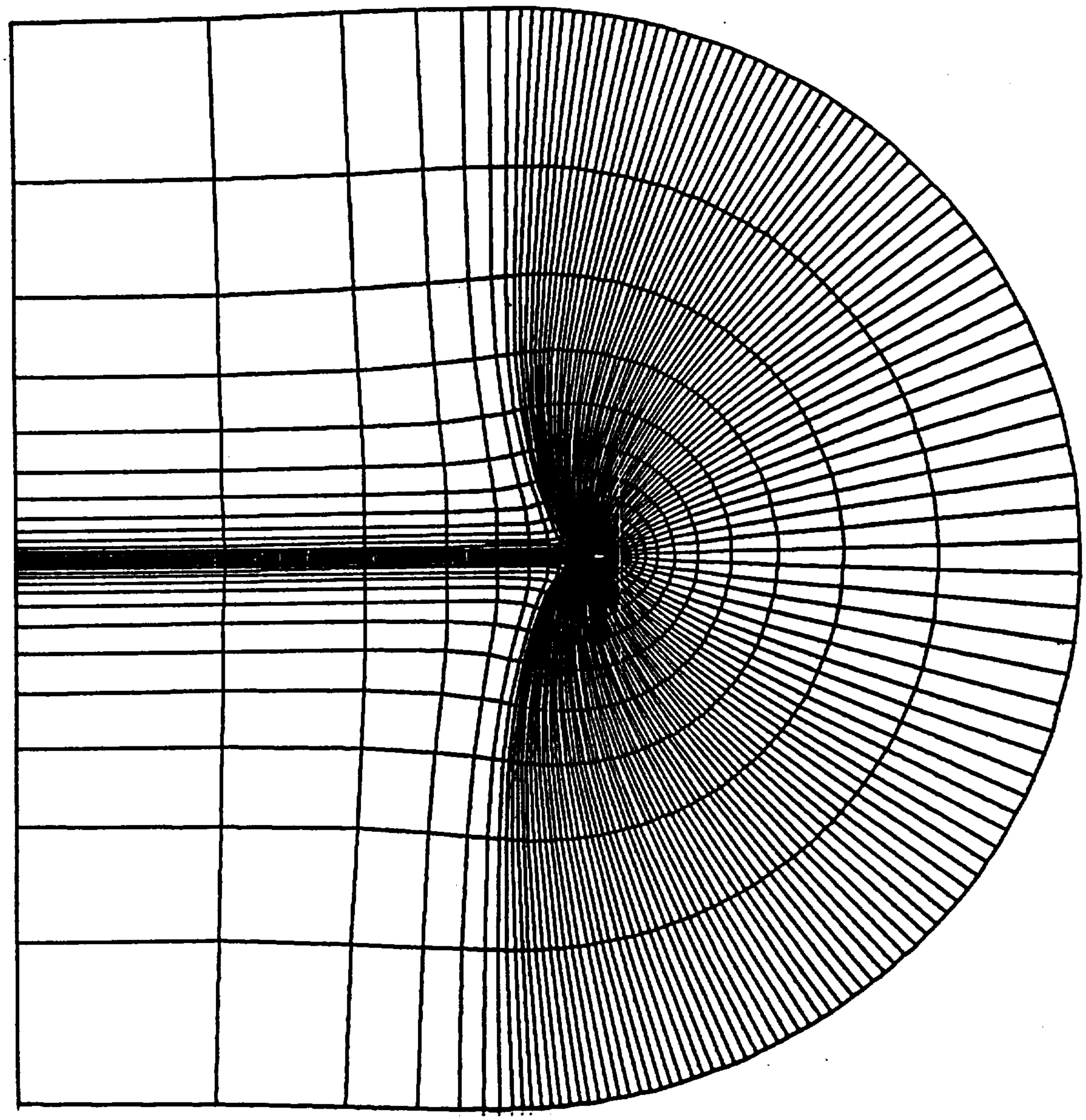


Fig. 6 Computational Grid for Viscous Flows
RAE 2822 Airfoil - C grid (128 x 32)

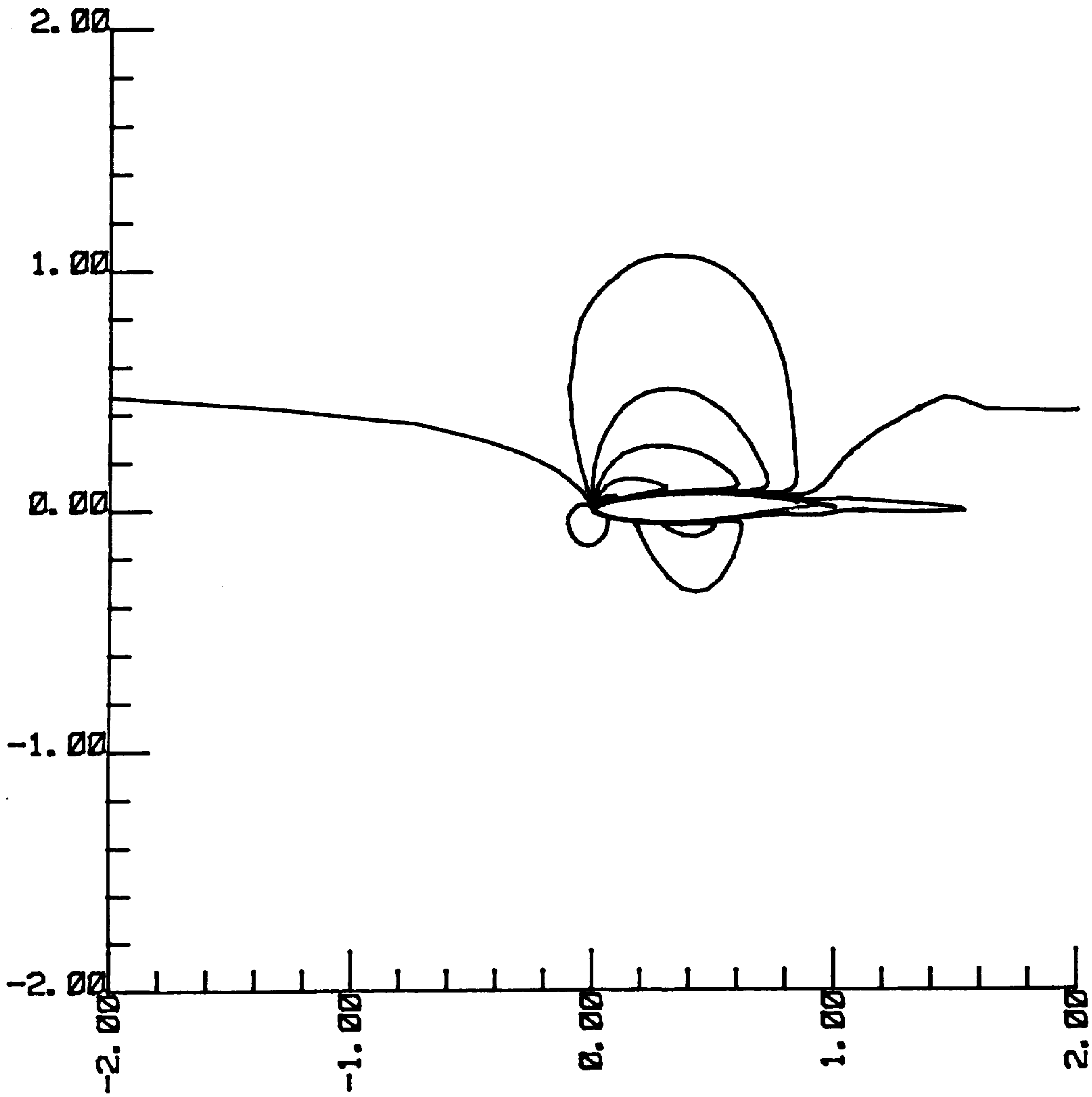


Fig. 7a Mach Number Contours for Viscous Flow
RAE 2822 Airfoil – $M_\infty = 0.6$, $\alpha = 2.57^o$, $Re = 6.3 \times 10^6$

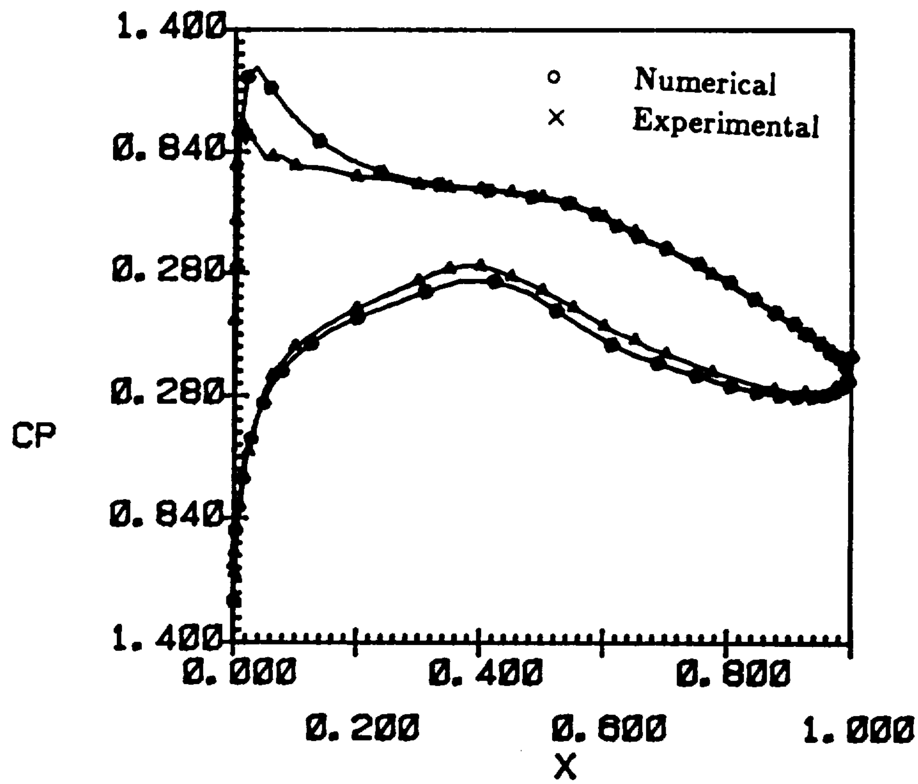


Fig. 7b Numerical and Experimental Pressure Coefficients
RAE 2822 Airfoil – $M_\infty = 0.6$, $\alpha = 2.57^o$, $Re = 6.3 \times 10^6$

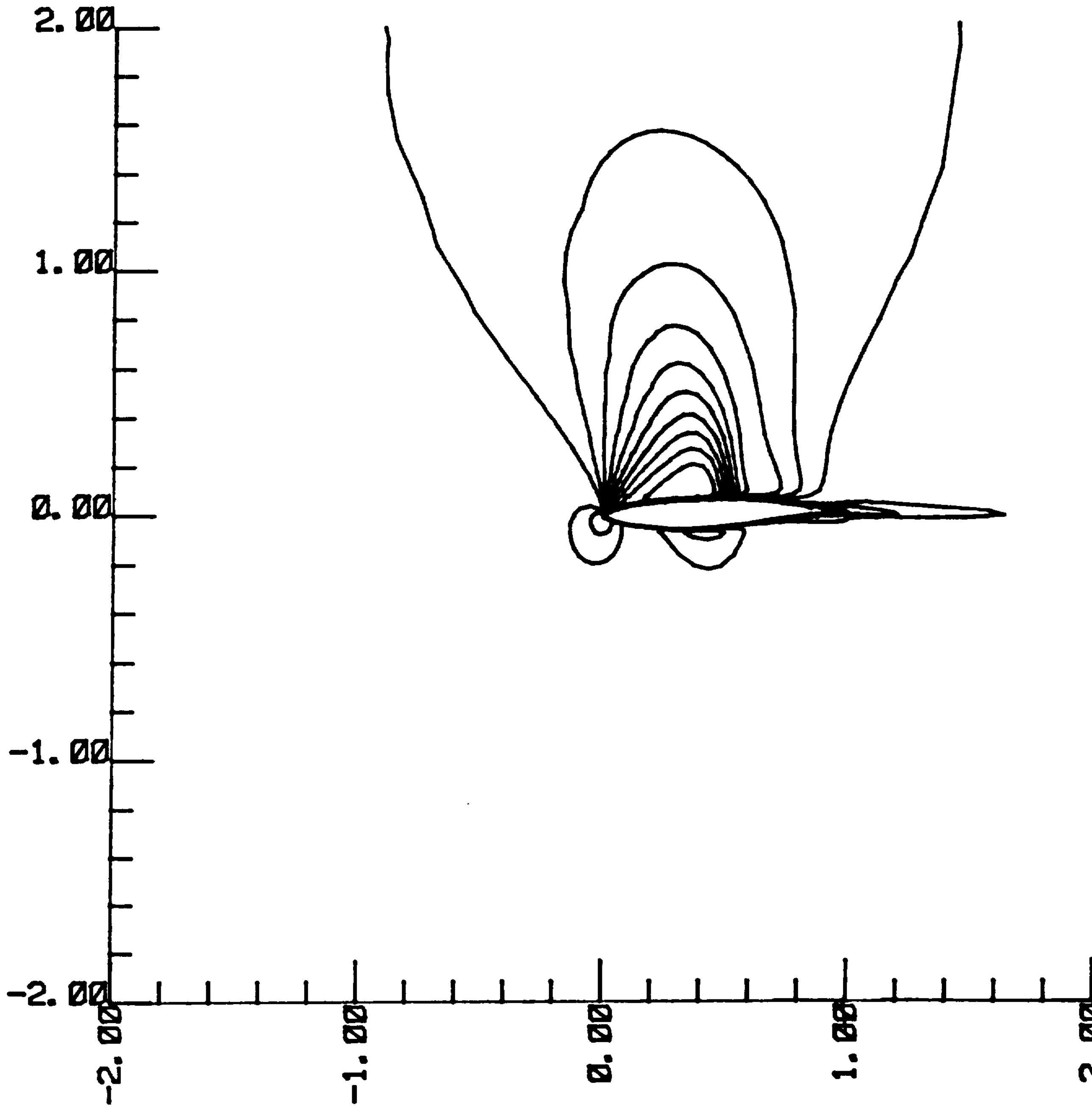


Fig. 8a Mach Number Contours for Viscous Flow
RAE 2822 Airfoil – $M_\infty = 0.7.25$, $\alpha = 2.92^o$, $Re = 6.5 \times 10^6$

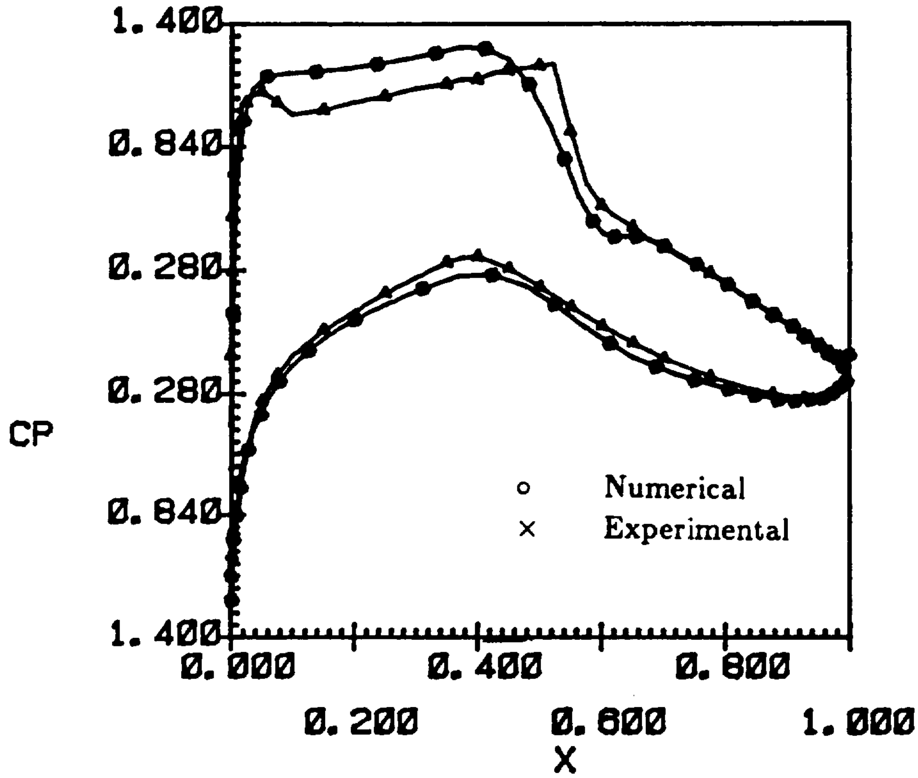


Fig. 8b Numerical and Experimental Pressure Coefficients
RAE 2822 Airfoil - $M_\infty = 0.725$, $\alpha = 2.92^o$, $Re = 6.5 \times 10^6$

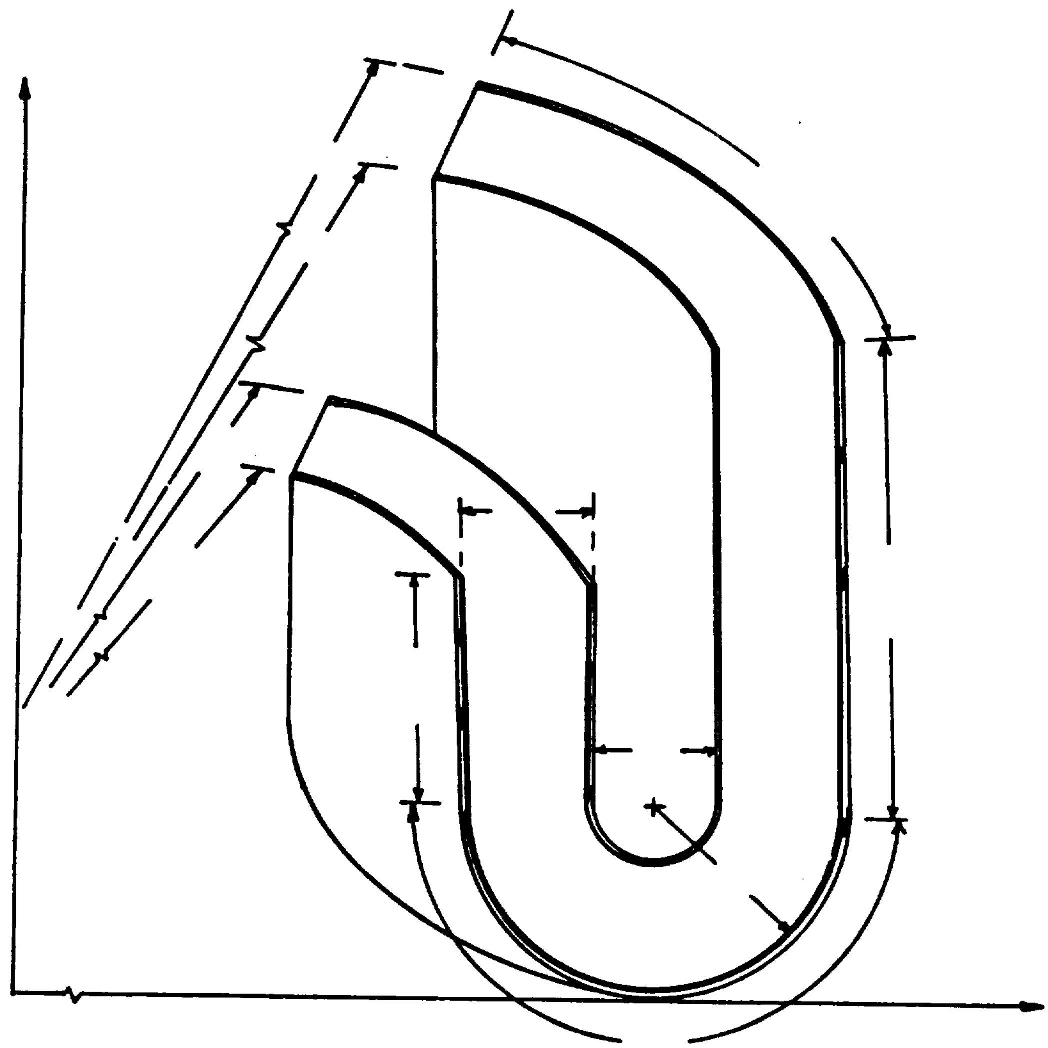


Fig. 9 Sketch of a Section of a Turnaround Duct

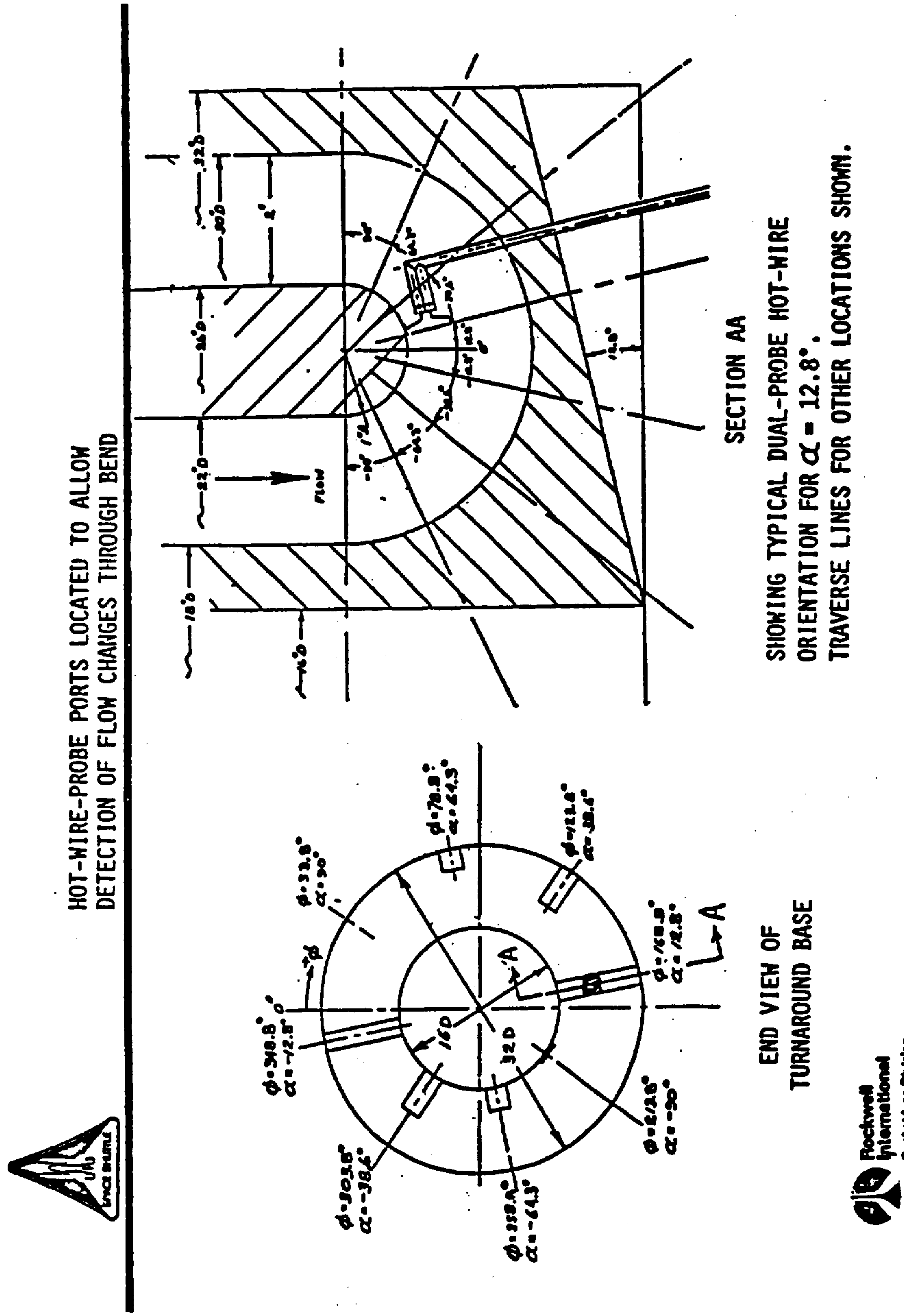


Fig. 10 Geometry of a Test Rig for a Turnaround Duct

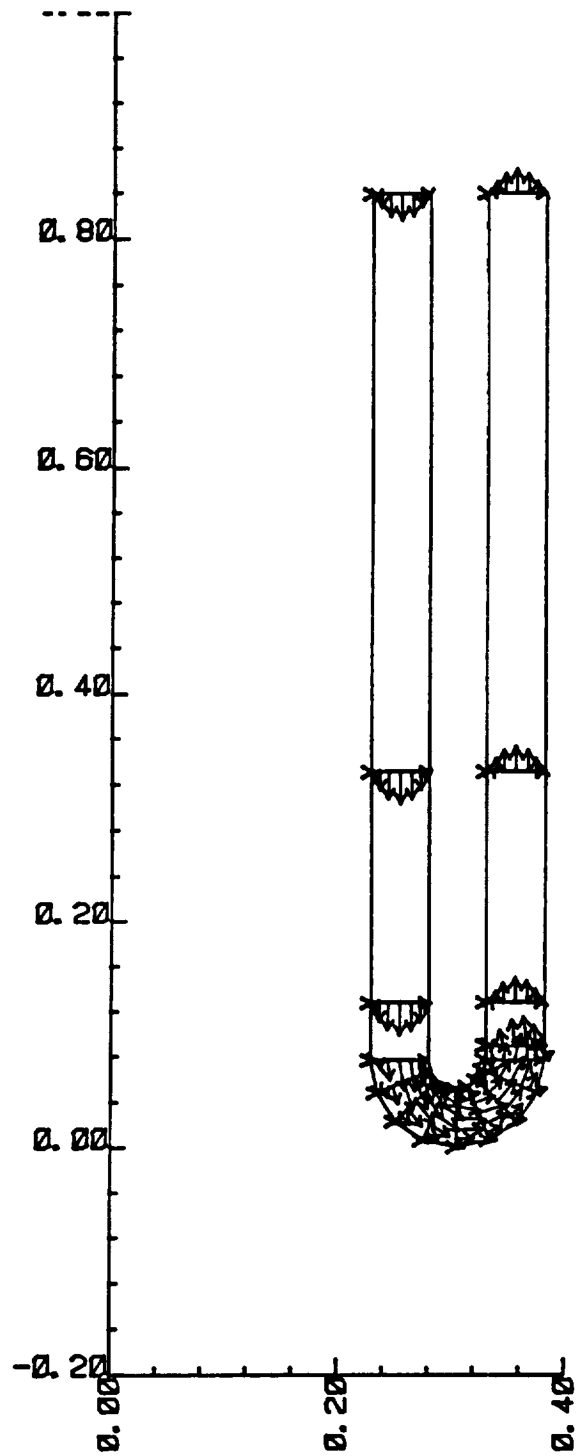


Fig. 11 Computational Grid and Velocity Vectors in a Cross Section of the Turnaround Duct

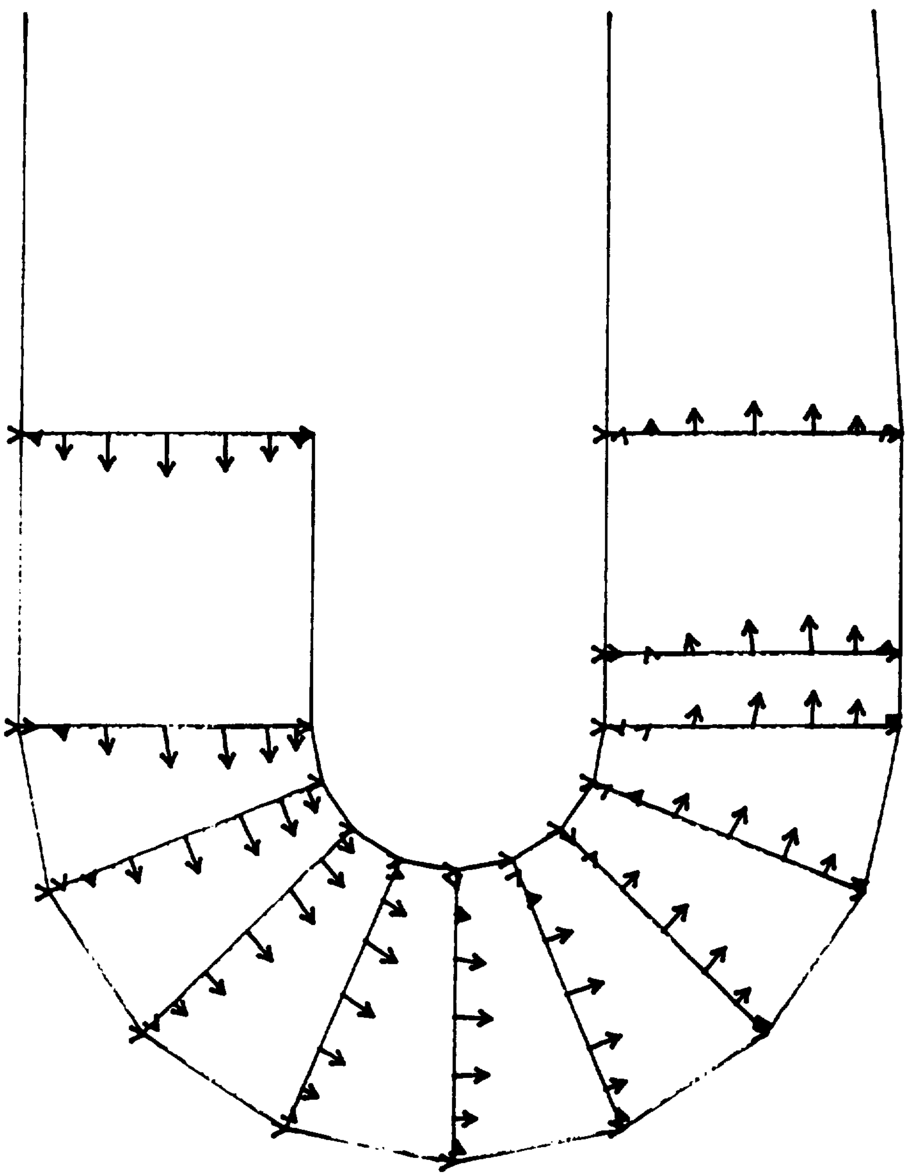


Fig. 12 Velocity Vectors in the *Re* Bend Region of the Turnaround Duct

APPENDIX I

The details of the Unsteady Compressible Navier–Stokes equations, which are used in the finite element code are given below. The equations are written in conservation form as

$$\left\{\frac{\partial U}{\partial t}\right\} + \vec{\nabla}\cdot\{\vec{F}^v\} + \vec{\nabla}\cdot\{\vec{F}^I\} = \{0\}$$

where

$$\{U\} = \left\{\begin{matrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho\varepsilon \end{matrix}\right\}, \ \{\vec{F}^v\} = \left\{\begin{matrix} \vec{0} \\ -\underline{\tau} \\ -\underline{\tau}\cdot\vec{v} + \underline{q} \end{matrix}\right\}, \ \{\vec{F}^I\} = \left\{\begin{matrix} \rho\vec{v} \\ \rho\vec{v}\vec{v} + p\vec{I} \\ \vec{v}(\rho\varepsilon + p) \end{matrix}\right\}$$

$$\underline{q} = -k\vec{\nabla}T, \quad \tau_{ij} = -\frac{2}{3}\mu\delta_{ij}e_{kk} + 2\mu e_{ij}$$

$$p = (\gamma - 1)\left[\rho\varepsilon - \frac{\rho}{2}\left(u^2 + v^2 + w^2\right)\right] \qquad e_{ij} = \frac{1}{2}\left(u_{i,j} + v_{j,i}\right)$$

The viscous and inviscid fluxes are given by

$$\underset{(5\times 3)}{\vec{F}^v} = \left\{\begin{matrix} 0 & 0 & 0 \\ \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \\ D_1 & D_2 & D_3 \end{matrix}\right\}, \ \underset{(5\times 3)}{\vec{F}^I} = \left\{\begin{matrix} \rho u & \rho v & \rho w \\ \rho u^2 + p & \rho uv & \rho uw \\ \rho vu & \rho v^2 + p & \rho vw \\ \rho wu & \rho wv & \rho w^2 + p \\ u(\rho\varepsilon + p) & v(\rho\varepsilon + p) & w(\rho\varepsilon + p) \end{matrix}\right\}$$

$$p = (\gamma - 1)\left[e - \frac{\rho}{2}(u^2 + v^2 + w^2)\right] \quad (p = \rho RT), \quad e = \rho\varepsilon$$

- <u>Sutherland's theory of viscosity</u>:

$$\mu = \mu_0\left(\frac{T}{T_0}\right)^{\frac{3}{2}}\left(\frac{T_0 + S_1}{T + S_1}\right)$$

S_1 = constant (= 110 °K for air)

- <u>Properties of air</u> at 20 ℃$(= T_0)$ and atmospheric pressure ($p_1 = 1\ atm$)

$$\rho_0 = 1.205 Kg/m^3$$
$$p_0 = 0.101325 \times 10^6 N/m^2$$
$$T_0 = 20\ ^\circ C = 293\ ^\circ K$$
$$R = \left(\frac{p_0}{\rho_0 T_0}\right) = 287 \left(\frac{N \cdot m}{Kg \cdot K} \text{ or } \frac{m^2}{Sec^2 - \ ^\circ K}\right)$$
$$\mu_0 = 17.9 \times 10^{-6} (Pa - Sec)$$
$$k = 2.5 \times 10^{-2} (W/m - \ ^\circ K)$$
$$P_r = 0.72$$
$$\alpha = 0.208$$
$$\gamma = 1.402$$

AUXILIARY RELATIONS

$p =$ Pressure (N/m^2)

$T =$ Temperature ($^\circ K$)

$$\gamma = \frac{C_p}{C_v}$$

$C_p =$ Specific heat at constant pressure

$C_v =$ Specific heat at constant volume

$R =$ Gas constant $(N \cdot m/Kg - \ ^\circ K)$

$k =$ Thermal conductivity $(W/m - \ ^\circ K)$

$\mu_0 =$ Reference viscosity $(Pa - Sec.)$

$T_0 =$ Reference temperature ($^\circ K$)

$\rho_0 =$ Reference density (Kg/m^3)

$$p = \rho R T$$
$$C_p = \frac{\gamma R}{\gamma - 1}$$
$$C_v = \frac{R}{\gamma - 1}$$

$\alpha =$ Thermal diffusitivity, $= \dfrac{k}{\rho C_p}$

$P_r =$ Prandtl number $= \dfrac{\mu C_p}{k}$

$M_\infty =$ Mach number $= \dfrac{U_\infty}{C_\infty}$

APPENDIX II
Details of Finite Element Equations

The details of finite element equations which approximate the Navier–Stokes equations are given below. In equation (3.8) the residual $\{\mathcal{R}^e\}$ has two parts. One is a volume integral, $\mathcal{R}_v$ and the other is a surface integral, $\mathcal{R}_s$.

$$\{\mathcal{R}^e\} = \{\mathcal{R}_v\} + \{\mathcal{R}_s\}$$

where

$$\{\mathcal{R}_v\} = -\int_{\Omega^e} [\vec{\nabla}\Psi]^T\{\vec{F}\}dV$$
$$\{\mathcal{R}_s\} = \oint_{\partial\Omega^e} [\Psi]^T\{F_n\}dS$$

The components of $\{\mathcal{R}_v\}$ for Ψ_I which corresponds to a node I are given by

$$\mathcal{R}_v^1 = -\int_{\Omega^e}\left(\frac{\partial\Psi_I}{\partial x}U_2 + \frac{\partial\Psi_I}{\partial y}U_3 + \frac{\partial\Psi_I}{\partial z}U_4\right)dV$$

$$\begin{aligned}\mathcal{R}_v^2 = -\int_{\Omega^e}\Bigg\{&\left(\frac{U_2^2}{U_1}+p\right)\frac{\partial\Psi_I}{\partial x} + \frac{U_2U_3}{U_1}\frac{\partial\Psi_I}{\partial y} + \frac{U_2U_4}{U_1}\frac{\partial\Psi_I}{\partial z}\\
&+\frac{\partial\Psi_I}{\partial x}\left[\frac{2}{3}\mu\left(-2\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right)+\frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right)+\frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right)\right]\\
&+\frac{\partial\Psi_I}{\partial y}\left[-\mu\left(\frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right)+\frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right)\right)\right]\\
&+\frac{\partial\Psi_I}{\partial z}\left[-\mu\left(\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right)+\frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right)\right]\Bigg\}dV\end{aligned}$$

where

$$\frac{\partial}{\partial x_i}\left(\frac{U_\alpha}{U_1}\right) = \frac{1}{U_1}\left(\frac{\partial U_\alpha}{\partial x_i} - \frac{U_\alpha}{U_1}\frac{\partial U_1}{\partial x_i}\right)$$

$$\begin{aligned}\mathcal{R}_v^3 = -\int_{\Omega^e}\Bigg\{&\frac{\partial\Psi_I}{\partial x}\cdot\frac{U_2U_3}{U_1} + \left(\frac{U_3^2}{U_1}+p\right)\frac{\partial\Psi_I}{\partial y} + \frac{U_3U_4}{U_1}\frac{\partial\Psi_I}{\partial z}\\
&+\frac{\partial\Psi_I}{\partial x}\left[-\mu\frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right)-\mu\frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right)\right]\\
&+\frac{\partial\Psi_I}{\partial y}\left[\frac{2}{3}\mu\left(\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right)+\frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)-2\frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right)\right)\right]\\
&+\frac{\partial\Psi_I}{\partial z}\left[-\mu\frac{\partial}{\partial z}\left(\frac{U_3}{U_1}\right)-\mu\frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right]\Bigg\}dV\end{aligned}$$

$$
\begin{aligned}
\mathcal{R}_v^4 = -\int_{\Omega^e} \Bigg\{ & \frac{\partial \Psi_I}{\partial x}\frac{U_2 U_4}{U_1} + \frac{\partial \Psi_I}{\partial y}\frac{U_3 U_4}{U_1} + \left(\frac{U_4^2}{U_1} + p\right)\frac{\partial \Psi_I}{\partial z} \\
& + \frac{\partial \Psi_I}{\partial x}\left[-\mu\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) - \mu\frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right] \\
& + \frac{\partial \Psi_I}{\partial y}\left[-\mu\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) - \mu\frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right] \\
& + \frac{\partial \Psi_I}{\partial z}\left[\frac{2}{3}\mu\left(\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right) - 2\frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right)\right]\Bigg\} dV
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_v^5 = -\int_{\Omega^e} \Bigg\{ & \frac{U_2}{U_1}(U_5 + p)\frac{\partial \Psi_I}{\partial x} + \frac{U_3}{U_1}(U_5 + p)\frac{\partial \Psi_I}{\partial y} + \frac{U_4}{U_1}(U_5 + p)\frac{\partial \Psi_I}{\partial z} \\
& - \frac{2}{3}\mu\frac{U_2}{U_1}\frac{\partial \Psi_I}{\partial x}\left[2\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) - \frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right) - \frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_3}{U_1}\frac{\partial \Psi_I}{\partial x}\left[\frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right)\right] \\
& - \mu\frac{U_4}{U_1}\frac{\partial \Psi_I}{\partial x}\left[\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_2}{U_1}\frac{\partial \Psi_I}{\partial y}\left[\frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right)\right] \\
& - \frac{2}{3}\mu\frac{U_3}{U_1}\frac{\partial \Psi_I}{\partial y}\left[2\frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right) - \frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) - \frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_4}{U_1}\frac{\partial \Psi_I}{\partial y}\left[\frac{\partial}{\partial z}\left(\frac{U_3}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_2}{U_1}\frac{\partial \Psi_I}{\partial z}\left[\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_3}{U_1}\frac{\partial \Psi_I}{\partial z}\left[\frac{\partial}{\partial z}\left(\frac{U_3}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right] \\
& - \frac{2}{3}\mu\frac{U_4}{U_1}\frac{\partial \Psi_I}{\partial z}\left[2\frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right) - \frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) - \frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right)\right] \\
& - \widehat{k}\left[\frac{\partial \Psi_I}{\partial x}\frac{\partial Q}{\partial x} + \frac{\partial \Psi_I}{\partial y}\frac{\partial Q}{\partial y} + \frac{\partial \Psi_I}{\partial z}\frac{\partial Q}{\partial z}\right]\Bigg\} dV
\end{aligned}
$$

where

$$
Q = \frac{1}{U_1}\left[U_5 - \frac{1}{2U_1}(U_2^3 + U_3^2 + U_4^2)\right]
$$

For defining the components of $\{\mathcal{R}_s\}$ we write

$$
\begin{aligned}
F_n dS &= \vec{F} \cdot \vec{n} dS = \vec{F} \cdot d\vec{S} \\
&= \vec{F} \cdot \left(\frac{\partial(y,z)}{\partial(\xi,\eta)}, \frac{\partial(z,x)}{\partial(\xi,\eta)}, \frac{\partial(x,y)}{\partial(\xi,\eta)}\right) d\xi \; d\eta
\end{aligned}
$$

as derived in equation (11) of the last report[3], for a typical surface, say $\zeta = 1$ of an element.

Denote

$$(V_1, V_2, V_3) = \left(\frac{\partial(y,z)}{\partial(\xi,\eta)}, \frac{\partial(z,x)}{\partial(\xi,\eta)}, \frac{\partial(x,y)}{\partial(\xi,\eta)}\right)$$

Now the components of $\{\mathcal{R}_s\}$ for Ψ_I which corresponds to a node I, for a typical surface $\zeta = 1$ of an element can be written as

$$\begin{aligned}
\mathcal{R}_s^1 &= \oint_{\partial\Omega^e} (V_1 U_2 + V_2 U_3 + V_3 U_4)\, \Psi_I d\xi\, d\eta \\
\mathcal{R}_s^2 &= \oint_{\partial\Omega^e} \left\{ \left(\frac{U_2^2}{U_1} + p\right) V_1 + \frac{U_2 U_3}{U_1} V_2 + \frac{U_2 U_4}{U_1} V_3 \right. \\
&\quad + V_1 \left[\frac{2}{3}\mu \left(-2\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right) + \frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right)\right] \\
&\quad + V_2 \left[-\mu \left(\frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right)\right)\right] \\
&\quad \left. + V_3 \left[-\mu \left(\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right)\right] \right\} \Psi_I d\xi\, d\eta
\end{aligned}$$

where

$$\frac{\partial}{\partial x_i}\left(\frac{U_\alpha}{U_1}\right) = \frac{1}{U_1}\left(\frac{\partial U_\alpha}{\partial x_i} - \frac{U_\alpha}{U_1}\frac{\partial U_1}{\partial x_i}\right)$$

$$\begin{aligned}
\mathcal{R}_s^3 &= \oint_{\partial\Omega^e} \left\{ \frac{U_2 U_3}{U_1} V_1 + \left(\frac{U_3^2}{U_1} + p\right) V_2 + \frac{U_3 U_4}{U_1} V_3 \right. \\
&\quad + V_1 \left[-\mu\frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right) - \mu\frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right)\right] \\
&\quad + V_2 \left[\frac{2}{3}\mu \left(\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right) - 2\frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right)\right)\right] \\
&\quad \left. + V_3 \left[-\mu\frac{\partial}{\partial z}\left(\frac{U_3}{U_1}\right) - \mu\frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right] \right\} \Psi_I d\xi\, d\eta \\
\mathcal{R}_s^4 &= \oint_{\partial\Omega^e} \left\{ \frac{U_2 U_4}{U_1} V_1 + \frac{U_3 U_4}{U_1} V_2 + \left(\frac{U_4^2}{U_1} + p\right) V_3 \right. \\
&\quad + V_1 \left[-\mu\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) - \mu\frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right] \\
&\quad + V_2 \left[-\mu\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) - \mu\frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right] \\
&\quad \left. + V_3 \left[\frac{2}{3}\mu \left(\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right) - 2\frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right)\right] \right\} \Psi_I d\xi\, d\eta
\end{aligned}$$

$$
\begin{aligned}
\mathcal{R}_s^5 = \oint_{\partial\Omega^e} \Bigg\{ & \frac{U_2}{U_1}(U_5+p)V_1 + \frac{U_3}{U_1}(U_5+p)V_2 + \frac{U_4}{U_1}(U_5+p)V_3 \\
& - \frac{2}{3}\mu\frac{U_2}{U_1}V_1\left[2\frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) - \frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right) - \frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_3}{U_1}V_1\left[\frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right)\right] \\
& - \mu\frac{U_4}{U_1}V_1\left[\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_2}{U_1}V_2\left[\frac{\partial}{\partial y}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_3}{U_1}\right)\right] \\
& - \frac{2}{3}\mu\frac{U_3}{U_1}V_2\left[2\frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right) - \frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) - \frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_4}{U_1}V_2\left[\frac{\partial}{\partial z}\left(\frac{U_3}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_2}{U_1}V_3\left[\frac{\partial}{\partial z}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial x}\left(\frac{U_4}{U_1}\right)\right] \\
& - \mu\frac{U_3}{U_1}V_3\left[\frac{\partial}{\partial z}\left(\frac{U_3}{U_1}\right) + \frac{\partial}{\partial y}\left(\frac{U_4}{U_1}\right)\right] \\
& - \frac{2}{3}\mu\frac{U_4}{U_1}V_3\left[2\frac{\partial}{\partial z}\left(\frac{U_4}{U_1}\right) - \frac{\partial}{\partial x}\left(\frac{U_2}{U_1}\right) - \frac{\partial}{\partial y}\left(\frac{U_3}{U_1}\right)\right] \\
& - \widehat{k}\left[\frac{\partial\Psi_I}{\partial x}\frac{\partial Q}{\partial x} + \frac{\partial\Psi_I}{\partial y}\frac{\partial Q}{\partial y} + \frac{\partial\Psi_I}{\partial z}\frac{\partial Q}{\partial z}\right]\Bigg\}\Psi_I d\xi\, d\eta
\end{aligned}
$$

where

$$
Q = \frac{1}{U_1}\left[U_5 - \frac{1}{2U_1}(U_2^3 + U_3^2 + U_4^2)\right]
$$

Components of $\{\mathcal{R}_s\}$ for other surfaces of an element can be written similarly.

The coefficient C of equation (3.13) has volume integrals of the derivatives of viscous flux terms. The details of those integrals are given below.

Denote

$$
\int_{\Omega^e} \vec{\nabla}\Psi^e_{(ND)} \cdot \frac{\partial \vec{F}^{\alpha}\, Vis}{\partial U_{\alpha,j}} dV = N^{\alpha}_{(ND),j}
$$

Subscript (ND) corresponds to the local index i of the global node ND in element e. These integrals can be written as

$$
N^1_{ij} = 0
$$

$$
N^2_{ij} = \mu\int_{\Omega^e}\left[\frac{4}{3}\frac{\partial\Psi_i}{\partial x}\frac{\partial}{\partial x}\left(\frac{\Psi_j}{U_1}\right) + \frac{\partial\Psi_i}{\partial y}\frac{\partial}{\partial y}\left(\frac{\Psi_j}{U_1}\right) + \frac{\partial\Psi_i}{\partial z}\frac{\partial}{\partial z}\left(\frac{\Psi_j}{U_1}\right)\right] dV
$$

where

$$
\frac{\partial}{\partial x}\left(\frac{\Psi_j}{U_1}\right) = \frac{1}{U_1}\left[\frac{\partial\Psi_j}{\partial x} - \Psi_j \cdot \frac{\partial U_1}{\partial x}\frac{1}{U_1}\right], \text{ etc.,}
$$

$$N_{ij}^{3} = \mu \int_{\Omega^e} \left[\frac{\partial \Psi_i}{\partial x} \frac{\partial}{\partial x} \left(\frac{\Psi_j}{U_1} \right) + \frac{4}{3} \frac{\partial \Psi_i}{\partial y} \frac{\partial}{\partial y} \left(\frac{\Psi_j}{U_1} \right) + \frac{\partial \Psi_i}{\partial z} \frac{\partial}{\partial z} \left(\frac{\Psi_j}{U_1} \right) \right] dV$$

$$N_{ij}^{4} = \mu \int_{\Omega^e} \left[\frac{\partial \Psi_i}{\partial x} \frac{\partial}{\partial x} \left(\frac{\Psi_j}{U_1} \right) + \frac{\partial \Psi_i}{\partial y} \frac{\partial}{\partial y} \left(\frac{\Psi_j}{U_1} \right) + \frac{4}{3} \frac{\partial \Psi_i}{\partial z} \frac{\partial}{\partial z} \left(\frac{\Psi_j}{U_1} \right) \right] dV$$

$$N_{ij}^{5} = \widehat{k} \int_{\Omega^e} \left[\frac{\partial \Psi_i}{\partial x} \frac{\partial}{\partial x} \left(\frac{\Psi_j}{U_1} \right) + \frac{\partial \Psi_i}{\partial y} \frac{\partial}{\partial y} \left(\frac{\Psi_j}{U_1} \right) + \frac{\partial \Psi_i}{\partial z} \frac{\partial}{\partial z} \left(\frac{\Psi_j}{U_1} \right) \right] dV$$

APPENDIX III

```
C     FILE NAME: COMPR3D VERS3;  DATE: FEB. 22, 1988;  LINES:  2066
C
C
C   ________________________________________________________________
C  |                                                                |
C  | FINITE-ELEMENT ANALYSIS OF FLOWS OF VISCOUS, COMPRESSIBLE     |
C  |          FLUIDS IN THREE-DIMENSIONAL ENCLOSURES.              |
C  |________________________________________________________________|
C
C     ____________________________________________________________
C    |                                                            |
C    | THIS PROGRAM IS DEVELOPED BY PROFESSORS  J. N. REDDY  OF   |
C    | VIRGINIA  POLYTECHNIC INSTITUTE  AND  K. C. REDDY OF THE   |
C    | UNIVERSITY OF TENNESSEE SPACE INSTITUTE.  THE PROGRAM IS   |
C    | UNDER CONTINUOUS DEVELOPMENT DURING APRIL '86 TO PRESENT.  |
C    | UNAUTHORIZED USE OF THE PROGRAM IS PROHIBITED.             |
C    |                                                            |
C    | DEVELOPED: APRIL 1986 - PRESENT                            |
C    |____________________________________________________________|
C
C
C     ____________________________________________________________
C    |                                                            |
C    | D E S C R I P T I O N   O F   T H E   V A R I A B L E S    |
C    |                                                            |
C    |                                                            |
C    | CFL.......THE COURANT-FRIEDRICHS-LEVY NUMBER                |
C    | ELXYZ.....ARRAY OF ELEMENT COORDINATES OF NODES            |
C    | IBNDC.....ARRAY OF BOUNDARY NODES FOR DIFFERENT            |
C    |           VARIABLES                                        |
C    | IORDER....ORDER OF THE EQUATIONS TO BE SOLVED              |
C    |                                                            |
C    | ISTART....RESTART INDEX (1=RESTART; 0=NEW START)           |
C    |                                                            |
C    | KELSUR....A TWO-DIMENSIONAL ARRAY THAT CONTAINS ELEMENT    |
C    |           NUMBER AND LOCAL NUMBER OF ITS SURFACE THAT      |
C    |           REQUIRES FLUX COMPUTATION:                       |
C    |                                                            |
C    |           KELSUR(I,1)=GLOBAL ELEMENT NUMBER OF THE         |
C    |                       GLOBAL I-TH SURFACE                  |
C    |           KELSUR(I,2)=LOCAL SURFACE NUMBER OF THE          |
C    |                       GLOBAL I-TH SURFACE                  |
C    |                                                            |
C    | KNDSUR....A TWO-DIMENSIONAL (M BY 4) ARRAY WHICH CONTAINS  |
C    |           GLOBAL SURFACE NUMBERS SURROUNDING A NODE THAT   |
C    |           REQUIRES FLUX COMPUTATION.  HERE M DENOTES THE   |
C    |           NUMBER OF NODES REQUIRING FLUX COMPUTATION:      |
C    |                                                            |
C    |           KNDSUR(I,J)=GLOBAL NUMBER OF THE LOCAL  J-TH     |
C    |                       SURFACE ASSOCIATED WITH THE I-TH     |
C    |                       BOUNDARY NODE THAT REQUIRES FLUX     |
C    |                       COMPUTATION.                         |
C    |                                                            |
C    | MEN.......MAXIMUM NUMBER OF ELEMENTS AT A NODE             |
C    | MNE.......MAXIMUM NUMBER OF NODES PER ELEMENT              |
C    | NDF.......NO. OF UNKNOWNS AT EACH NODE                     |
C    |                                                            |
C    | NDSURF....ARRAY CONTAINING THE SEQUENTIAL NUMBER OF THE    |
C    |           BOUNDARY NODES WHICH REQUIRE FLUX COMPUTATION    |
C    |           OR CONTAINING ZERO:                              |
C    |                                                            |
C    |           NDSURF(I)=0, IF NO SURFACES AROUND THE  I-TH     |
C    |           NODE REQUIRES FLUX COMPUTATION.                  |
C    |           NDSURF(I)=J, IF THE I-TH NODE REQUIRES FLUX      |
C    |           COMPUTATION; HERE J DENOTES THE SEQUENTIAL       |
C    |           NUMBER OF NODE I IN THE LIST OF SURFACES THAT    |
C    |           REQUIRE FLUX COMPUTATION.                        |
C    |                                                            |
C    | NELEM.....CONNECTIVITY MATRIX RELATING GLOBAL NODE TO      |
```

```
C     |            ELEMENTS AROUND THE NODE:                      |
C     |                                                           |
C     |            NELEM(I,M)=GLOBAL ELEMENT NUMBER CORRESPONDING |
C     |            TO THE M-TH LOCAL ELEMENT SURROUNDING GLOBAL   |
C     |            NODE I (MAXIMUM VALUE OF M IS 8).              |
C     |                                                           |
C     |  NEM.......NUMBER OF ELEMENTS IN THE MESH                 |
C     |  NGP.......NUMBER OF GAUSSIAN POINTS                      |
C     |                                                           |
C     |  NMSH......INDICATOR FOR GENERATING MESH:                 |
C     |                                                           |
C     |            NMSH=0, MESH INFORMATION IS TO BE READ         |
C     |            NMSH>0, MESH IS GENERATED BY THE PROGRAM       |
C     |                    (ONLY FOR PRISMATIC AND TAD DOMAINS)   |
C     |                                                           |
C     |  NNM.......NUMBER OF NODES IN THE MESH                    |
C     |                                                           |
C     |  NODES.....BOOLEAN MATRIX RELATING LOCAL NODES TO  GLOBAL |
C     |            NODES OF ELEMENTS:                             |
C     |                                                           |
C     |            NODES(N,J)=GLOBAL NODE NUMBER CORRESPONDING TO |
C     |            THE J-TH LOCAL NODE OF ELEMENT N.              |
C     |                                                           |
C     |  NSURF.....TOTAL NUMBER OF SURFACES THAT REQUIRE          |
C     |            FLUX COMPUTATION                               |
C     |  NTMSTP....NO. OF TIME STEPS                              |
C     |                                                           |
C     |  U.........ARRAY OF FIVE PRIMARY UNKNOWNS:                |
C     |            RHO, RHO*U, RHO*V, RHO*W, RHO*E                |
C     |                                                           |
C     |  X,Y,Z.....GLOBAL COORDINATES OF THE NODES                |
C     |                                                           |
C     |___________________________________________________________|
C
C
C      ___________________________________________________________
C     |                                                           |
C     |            S U B R O U T I N E S   U S E D                |
C     |                                                           |
C     |  BCUPDT....UPDATES THE BOUNDARY CONDITIONS AT THE END OF  |
C     |            EACH ITERATION OR TIME STEP.                   |
C     |                                                           |
C     |  BNDRY.....GENERATES ARRAY 'KNDSUR', CONTAINING  SURFACES |
C     |            REQUIRING FLUX COMPUTATION.                    |
C     |                                                           |
C     |  COEFNT....GENERATES THE COEFFICIENT VALUES OF EACH       |
C     |            VARIABLE AT EACH NODE OF THE MESH.             |
C     |                                                           |
C     |  DISPTN....COMPUTES THE DISSIPATION MODEL.                |
C     |                                                           |
C     |  DSFSUR....COMPUTES THE DERIVATIVES OF THE SHAPE FUNCTIONS|
C     |            AT GAUSS POINTS OF A SURFACE.                  |
C     |                                                           |
C     |  FLUXES....COMPUTES FLUX FOR EACH VARIABLE AT EACH  NODE  |
C     |            OF THE MESH.                                   |
C     |                                                           |
C     |  GCSURF....GENERATES ARRAY 'GC', WHICH CONTAINS THE       |
C     |            DERIVATIVE OF X(I) W.R.T. XI(J).               |
C     |                                                           |
C     |  GMETRY....GENERATES ARRAYS 'SF', 'CNST', 'GDSF' AND 'VOL'|
C     |            GLOBALLY.                                      |
C     |                                                           |
C     |  INTIAL....GENERATES INITIAL CONDITIONS ON BOUNDARY FACES.|
C     |                                                           |
C     |  INVDET....COMPUTES THE INVERSE OF THE JACOBIAN MATRIX.   |
C     |                                                           |
C     |  MATMUL....COMPUTES THE PRODUCT OF TWO MATRICES.          |
```

```
C       |                                                               |
C       |  SHAPEL....EVALUATES THE SHAPE FUNCTIONS AND THEIR DERIVA-   |
C       |            TIVES AT THE GUASS POINTS.                         |
C       |                                                               |
C       |  SURFGM....COMPUTES COMPONENTS OF THE UNIT NORMAL  AT         |
C       |            GAUSS POINTS OF EACH BOUNDARY SURFACE.             |
C       |                                                               |
C       |  TADMSH....GENERATES THE MESH ( X, Y AND Z COORDINATES AND    |
C       |            ARRAY 'NODES') FOR THE TURN-AROUND-DUCT (TAD).     |
C       |                                                               |
C       |_______________________________________________________________|
C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER (NNM=432,NEM=240,MXE=8,NGP=2,NDIM=3,NPE=8,NDF=5,
     1           NBS=600)
      DIMENSION X(NNM),Y(NNM),Z(NNM),TITLE(20),UOLD(NNM,6),U(NNM,6),
     2          NODES(NEM,NPE),NELEM(NNM,MXE),ELXYZ(NPE,NDIM),E0(NNM),
     3          IORDER(NDF),DIS4(NNM,6),DC4(NNM),DELU(NPE,6),AMU(NNM),
     4          GDSF(MXE,NPE,NGP,NGP,NGP,NDIM),GNORM(NDIM,NBS,NGP,NGP),
     5          SF(NPE,NGP,NGP,NGP),CNST(MXE,NGP,NGP,NGP),EMU(NPE),
     6          VOLND(NNM),VOL(MXE),DSURF(NDIM,NPE,6,NGP,NGP),
     7          ELU(NPE,6),IEL(MXE),IBNDC(NNM,NDF),MINDX(NPE),
     8          KELSUR(NBS,2),KNDSUR(NBS,4),NDSURF(NNM)
      COMMON/GMT/SN22(8,8),SN33(8,8),SN44(8,8),SN55(8,8)
      COMMON/DTA/GAMA,AMU0,TEMP0,S1,R0,GPR,GAM1,CFL
      DATA IORDER/1,2,3,4,5/
      DATA IN,IT/5,6/
C
C        _______________________________________________________________
C       |                                                               |
C       |  P   R   E   P   R   O   C   E   S   S   O   R                |
C       |_______________________________________________________________|
C
      READ(5,2000) TITLE
      READ(5,*) ISTART,NMSH,ITER,NTMSTP,CFL,RLXOUT,RLXIN
      READ(5,*) AMU0,TEMP0,S1,R0,GAMA,PR,AMACH,DNST0
      IF(NMSH.EQ.0)GOTO 5
C     ------------------------------------------------------------
      CALL TADMSH(X,Y,Z,IBNDC,KELSUR,NODES,NSURF,NNM,NBS,NDF,NEM,NPE)
C     ------------------------------------------------------------
      GOTO 10
    5 READ(5,*)  ((NODES(I,J),J=1,8),I=1,NEM)
      READ(5,*)  ((NELEM(I,J),J=1,MXE),I=1,NNM)
      READ(5,*)  (X(I),Y(I),Z(I),I=1,NNM)
      READ(5,*)  ((U(I,J),J=1,NDF),I=1,NNM)
      READ(5,*) NSURF
      IF(NSURF.EQ.0)GOTO 10
      READ(5,*)  ((KELSUR(I,J),J=1,2),I=1,NSURF)
      READ(5,*)  ((IBNDC(I,J),J=1,5),I=1,NNM)
C
C     E N D    O F    T H E    I N P U T    D A T A
C
C
C     OPEN THE OUTPUT FILE IN WHICH THE DATA IS TO BE STORED.
C     THE NAME OF THE FILE IS 'TEST' AND THE DATA IS STORED IN THE FORM
C     OF BINARY NUMBERS.
C
   10 CONTINUE
      IREC=30000
      OPEN(UNIT=08,FILE='TEST',STATUS='NEW',ACCESS='DIRECT',
     #        FORM='UNFORMATTED',RECL=IREC,ACTION='READWRITE')
      IF(ISTART.EQ.1)THEN
      OPEN(UNIT=07,FILE='RSTART',STATUS='OLD',ACCESS='DIRECT',
     #        FORM='UNFORMATTED',RECL=IREC,ACTION='READWRITE')
```

```
      ENDIF
C
C     GENERATE ARRAY 'NELEM' USING ARRAY 'NODES'
C
      DO 40 I=1,NNM
      DO 15 L=1,MXE
   15 NELEM(I,L)=0
      ICNT=0
      DO 30 J=1,NEM
      DO 20 K=1,8
      JK=NODES(J,K)
      IF(I.NE.JK)GOTO 20
      ICNT=ICNT+1
      NELEM(I,ICNT)=J
      IF(ICNT.EQ.MXE)GOTO 40
      GOTO 30
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
C
C     DEFINE FIXED PARAMETERS
C
      NGPT=NGP*NGP*NGP
      GAM1=GAMA-1.0
      GPR=GAMA/PR
C
C     INITIALIZE THE FLOW FIELD
C
      NINIT=0
      IF(ISTART .EQ. 0) THEN
C     ---------------------------------------------------------------
      CALL INTIAL(NDF,NNM,AMACH,AMU0,TEMP0,S1,R0,GAMA,PR,U,DNST0)
C     ---------------------------------------------------------------
C     --------------------------------------------------------------
      CALL BCUPDT(NNM,GAMA,R0,TEMP0,U,DNST0)
C     --------------------------------------------------------------
      ELSE
      READ(07,REC=1) NINIT,U
      END IF
      NTMSTP = NTMSTP + NINIT
      NINIT=NINIT+1
      DO 50 II=1,6
      DO 50 JJ=1,NNM
   50 UOLD(JJ,II)=U(JJ,II)
C
C     WRITE OUT INPUT DATA
C
      WRITE(IT,2600)
      WRITE(IT,2500)
      WRITE(IT,2600)
      WRITE(IT,3000) TITLE
      WRITE(IT,2100)AMU0,TEMP0,S1,R0,GAMA,PR,DNST0
      WRITE(IT,2200)ITER,NTMSTP,CFL,RLXOUT,RLXIN
      WRITE(IT,741)AMACH
  741 FORMAT(10X,'FREE STREAM MACH NUMBER  =',E10.4)
      WRITE(IT,3500)
      DO 70 I = 1, NEM
   70 WRITE(IT,4000) I,(NODES(I,J),J=1,8)
      WRITE(IT,4500)
      DO 80 I = 1, NNM
   80 WRITE(IT,4000) I,(NELEM(I,J),J=1,MXE)
      WRITE(IT,5500)
      DO 90 I = 1, NNM
   90 WRITE(IT,5000) I,X(I),Y(I),Z(I)
      WRITE(IT,6100)
      DO 100 I=1,NNM
```

```
  100 WRITE(IT,6500)I,(U(I,J),J=1,5)
      WRITE(IT,6200)
      DO 110 I=1,NNM
  110 WRITE(IT,4000) I,(IBNDC(I,J),J=1,5)
      WRITE(IT,6300)
      WRITE(IT,4000)((KELSUR(I,J),J=1,2),I=1,NSURF)
C
C     FIND MAX. NO. OF NODES PER EACH ELEMENT,  COMPUTE ELEMENTAL
C     VOLUMES, SHAPE FUNCTIONS AND THEIR GLOBAL DERIVATIVES, AND
C     THE PRODUCT OF THE WEIGHTS AND THE DETERMINANT OF THE JACOBIAN
C     MATRIX FOR EACH GAUSS POINT OF EACH ELEMENT.
C
      DO 155 ND=1,NNM
C
C     COMPUTE THE NUMBER OF ELEMENTS AROUND NODE 'ND'
C
      DO 115 J=1,MXE
      IF(NELEM(ND,J).EQ.0)GOTO 120
  115 CONTINUE
      J=MXE+1
  120 NUMEL=J-1
C
C     INITIALIZE THE ARRAYS
C
      VOLND(ND)=0.0
      DC4(ND)=7*NUMEL
C
C     COMPUTE ARRAY 'IEL' WHICH CONTAINS LOCAL NODE CORR TO NODE ND
C
      DO 150 N=1,NUMEL
      NEL=NELEM(ND,N)
      DO 140 I=1,NPE
      NI=NODES(NEL,I)
      IF(NI.EQ.ND)IEL(N)=I
      ELXYZ(I,1)=X(NI)
      ELXYZ(I,2)=Y(NI)
  140 ELXYZ(I,3)=Z(NI)
C     ---------------------------------------------------------------
      CALL GMETRY(NNM,NEM,MXE,N,NPE,NGP,ELXYZ,SF,GDSF,CNST,VOL,
     1             NDIM,IEL(N))
C     ---------------------------------------------------------------
  150 VOLND(ND)=VOLND(ND)+VOL(N)
      WRITE(08, REC=ND) ND, CNST, GDSF, VOL,NUMEL,IEL,SN22,SN33,
     1                   SN44,SN55
*     PRINT*, ND, CNST(1,1,1,1), GDSF(1,1,1,1,1,1), VOL(1)
  155 CONTINUE
C*    WRITE(IT,8000)(VOL(I),I=1,NEM)
C     ---------------------------------------------------------------
      CALL BNDRY(NBS,NEM,NNM,NPE,NSURF,NODES,KELSUR,NDSURF,KNDSUR)
      CALL DSFSUR(DSURF,NGP,NPE,NDIM)
C     ---------------------------------------------------------------
*    WRITE(IT,1000)
*    WRITE(IT,4000)((KELSUR(I,J),J=1,2),I=1,NSURF)
*    WRITE(IT,4000)(NDSURF(I),I=1,16)
*    WRITE(IT,4000)((KNDSUR(I,J),J=1,4),I=1,NSURF)
C
      DO 180 NDS=1,NSURF
      KE=KELSUR(NDS,1)
      K1=KELSUR(NDS,2)
      DO 160 I=1,NPE
      NI=NODES(KE,I)
      ELXYZ(I,1)=X(NI)
      ELXYZ(I,2)=Y(NI)
  160 ELXYZ(I,3)=Z(NI)
C     ---------------------------------------------------------------
  180 CALL SURFGM(K1,NDS,ELXYZ,DSURF,GNORM,NBS,NGP,NPE,NDIM)
```

```
C     ------------------------------------------------------------------
C
C
C     |---------------------------------------------------------------|
C     |          P    R    O    C    E    S    S    O    R            |
C     |---------------------------------------------------------------|
C
C     BEGIN THE DO-LOOP ON THE NUMBER OF TIME STEPS TO COMPUTE THE SOLN
C
      ERROR=0.0
      DO 800 ITMSTP=NINIT,NTMSTP
      WRITE(IT,6000) ITMSTP
      DO 190 I=1,NNM
      TEMP=U(I,6)/R0/U(I,1)
  190 AMU(I)=AMU0*((TEMP/TEMP0)**1.5)*((TEMP0+S1)/(TEMP+S1))
C
C     CALL SUBROUTINE 'DISPTN' TO COMPUTE GLOBAL ARTIFICIAL DISSIPATION
C
C     ------------------------------------------------------------------
      CALL DISPTN(NNM,NEM,MXE,X,Y,Z,U,DC4,NODES,NELEM,DIS4,NPE,
     *            E0,VOLND)
C     ------------------------------------------------------------------
C
C     SYMMETRIC NONLINEAR GAUSS-SEIDEL ITERATION LOOP BEGINS HERE
C
      ITMAX=2*ITER
      DO 700 ITR=1,ITMAX
      IF(MOD(ITR,2).EQ.1)THEN
        NBEGIN=1
        NEND=NNM
        NINC=1
      ELSE
        NBEGIN=NNM
        NEND=1
        NINC=-1
      ENDIF
*     WRITE(IT,4007)ITR,ITMAX
C
C     BEGIN THE DO-LOOP ON THE NUMBER OF NODES TO COMPUTE THE SOLUTION
C
      DO 600 ND=NBEGIN,NEND,NINC
*     WRITE(IT,4006)NBEGIN,NEND,NINC,ND
C
C     COMPUTE THE NUMBER OF ELEMENTS (NUMEL) SURROUNDING A NODE
C
      READ(08, REC=ND) ID, CNST, GDSF, VOL,NUMEL,IEL,SN22,SN33,
     1                 SN44,SN55
      IF(ID.NE.ND) THEN
      PRINT *,'ERROR IN THE READ OF FILES'
      STOP
      ENDIF
C
      NSTART=1
      NLAST=5
      INCR=1
      DO 500 LOOP=1,1
C
C     DO-LOOP ON THE NUMBER OF CONSERVATION EQUATIONS BEGINS HERE
C
      DO 400 NEQ=NSTART,NLAST,INCR
C     WRITE(IT,4004)NSTART,NLAST,INCR,NEQ,LOOP
      LEQ=IORDER(NEQ)
      IF(IBNDC(ND,LEQ).EQ.0)GOTO 400
C
C     DO-LOOP ON NUMBER OF ELEMENTS SURROUNDING NODE 'ND' BEGINS HERE
C
```

```
      GCM=0.0
      GCKVIS=0.0
      GCKINV=0.0
      TCOEF=0.0
      TRES=0.0
      TFLX=0.0
      DO 300 N=1,NUMEL
      WRITE(IT,4003)NUMEL,N
      NEL=NELEM(ND,N)
C
C     TRANSFER GLOBAL INFORMATION TO ELEMENT 'NEL'
C
      DO 260 I=1,NPE
      MINDX(I)=0
      NI=NODES(NEL,I)
      EMU(I)=AMU(NI)
      IF(NINC.EQ.1 .AND. NI.GE.ND)MINDX(I)=1
      IF(NINC.EQ.-1 .AND. NI.LE.ND)MINDX(I)=1
      DO 260 II=1,6
      DELU(I,II)=U(NI,II)-UOLD(NI,II)
  260 ELU(I,II)=U(NI,II)
C
C     CALL SUBROUTINE 'COEFNT' TO COMPUTE THE COEFFICIENTS FOR THE EQN
C
C     ------------------------------------------------------------
      CALL COEFNT(IEL(N),LEQ,N,NPE,NEM,NGP,ELU,SF,GDSF,CNST,VOL,RES,
     *                     CM,EMU,DELU,MINDX,CKINV,NDF,NDIM,NGPT,MXE)
C     ------------------------------------------------------------
      GOTO(271,272,273,274,275),LEQ
  271 GCKVIS=0.0
      GOTO 276
  272 DO 282 J1=1,NPE
  282 GCKVIS=GCKVIS+SN22(N,J1)*MINDX(J1)
      GOTO 276
  273 DO 283 J1=1,NPE
  283 GCKVIS=GCKVIS+SN33(N,J1)*MINDX(J1)
      GOTO 276
  274 DO 284 J1=1,NPE
  284 GCKVIS=GCKVIS+SN44(N,J1)*MINDX(J1)
      GOTO 276
  275 DO 285 J1=1,NPE
  285 GCKVIS=GCKVIS+SN55(N,J1)*MINDX(J1)
  276 CONTINUE
      GCM=GCM+CM
      GCKINV=GCKINV+CKINV
  300 TRES=TRES+RES
      GCKINV=GCKINV*8.0/NUMEL
      GCKVIS=GCKVIS*AMU(ND)/U(ND,1)
      IF(LEQ.EQ.5) GCKVIS=GCKVIS*GPR
      TCOEF=GCM+DABS(GCKINV)+GCKVIS
      TCOEF=TCOEF+DC4(ND)
      IF(NDSURF(ND).EQ.0)GOTO 340
      DO 335 J=1,4
      KG1=KNDSUR(NDSURF(ND),J)
      IF(KG1.EQ.0)GOTO 340
      K1=KELSUR(KG1,2)
      KL=KELSUR(KG1,1)
      DO 310 II=1,NPE
      IF(NELEM(ND,II).EQ.KL)THEN
      NI=II
      GOTO 315
      ENDIF
  310 CONTINUE
  315 DO 330 I1=1,NPE
      EMU(I1)=AMU(NODES(KL,I1))
      DO 320 J1=1,NDF
```

```
  320 ELU(I1,J1)=U(NODES(KL,I1),J1)
  330 IF(NODES(KL,I1).EQ.ND)LI=I1
C     ------------------------------------------------------------
      CALL FLUXES(LI,LEQ,NI,NPE,NGP,ELU,SF,GDSF,GNORM,K1,KG1,FLX,
     1            EMU,MXE,NBS,NDF,NDIM)
  335 TFLX=TFLX+FLX
  340 CONTINUE
      IF(LEQ.NE.2)GOTO 350
      ERROR0=ERROR
      ERROR=DMAX1(ERROR0,DABS(TRES+TFLX))
      IF(ERROR.GT.ERROR0)MAXND=ND
  350 CONTINUE
C     DIS4(ND,LEQ)=0.0
      DU=-(TRES+TFLX-DIS4(ND,LEQ))/TCOEF
      U(ND,LEQ)=U(ND,LEQ)+DU*RLXIN
      U(ND,6)=GAM1*(U(ND,5)-0.5*(U(ND,2)*U(ND,2)+U(ND,3)*U(ND,3)+
     *         U(ND,4)*U(ND,4))/U(ND,1))
      WRITE(IT,7500)LEQ,ND,TRES,TFLX,TCOEF,U(ND,LEQ)
  400 CONTINUE
      NTEMP=NSTART
      NSTART=NLAST
      NLAST=NTEMP
      INCR=-1*INCR
  500 CONTINUE
*     WRITE(6,9999) ND, (U(ND,LI),LI=1,6)
*9999  FORMAT(I5,6E15.7)
  600 CONTINUE
C
C     END OF THE COMPUTATION FOR ALL NODES IN THE SWEEP
C
      NTEMP=NBEGIN
      NBEGIN=NEND
      NEND=NTEMP
      NINC=-1*NINC
C
C     RESET THE VALUES AT INFLOW, OUTFLOW AND RADIAL SYMMETRY PLANES
C
C     ------------------------------------------------------------
      CALL BCUPDT(NNM,GAMA,R0,TEMP0,U,DNST0)
C     ------------------------------------------------------------
  700 CONTINUE
C
C     RELAXATION OF THE UPDATED SOLUTION  AND COMPUTATION OF PRESSURE
C
      DO 720 II=1,5
      DO 720 JJ=1,NNM
      U(JJ,II)=UOLD(JJ,II)+RLXOUT*(U(JJ,II)-UOLD(JJ,II))
  720 UOLD(JJ,II)=U(JJ,II)
      DO 730 J1=1,NNM
      U(J1,6)=GAM1*(U(J1,5)-0.5*(U(J1,2)*U(J1,2)+U(J1,3)*U(J1,3)+
     *         U(J1,4)*U(J1,4))/U(J1,1))
  730 UOLD(J1,6)=U(J1,6)
C*    WRITE(IT,7000)ERROR,MAXND
      DO 750 I=1,NNM
  750 WRITE(IT,6500)I,(U(I,J),J=1,6)
  800 CONTINUE
      OPEN(UNIT=09,FILE='ROLD',STATUS='NEW',ACCESS='DIRECT',
     *     FORM='UNFORMATTED',RECL=IREC,ACTION='READWRITE')
      WRITE(09,REC=1)NTMSTP,U
C
      STOP
C
C     |------------------------------------------------------------|
C     |            F   O   R   M   A   T   S                       |
C     |------------------------------------------------------------|
C
```

```
 1000 FORMAT (5X,'ARRAYS: KELSUR, NDSURF AND KNDSUR:',/)
 2000 FORMAT (20A4)
 2100 FORMAT (/,2X,'P R O B L E M   D A T A:',/
     2       /,5X,'REFERENCE VISCOSITY (AMU0)............=',E12.4,
     3       /,5X,'REFERENCE TEMPERATURE (TEMP0).........=',E12.4,
     4       /,5X,'SUTHERLANDS CONSTANT (S1).............=',E12.4,
     5       /,5X,'GAS CONSTANT (R0).....................=',E12.4,
     6       /,5X,'RATIO OF SPECIFIC HEATS (GAMA)........=',E12.4,
     7       /,5X,'PRANDTL NUMBER (PR)...................=',E12.4,
     8       /,5X,'REFERENCE DENSITY (DNST0).............=',E12.4,/)
 2200 FORMAT (/,2X,'P A R A M E T E R S   O F   A P P R O X. :',/,
     2       /,5X,'NUMBER OF ITERATIONS PER TIME STEP....=',I3,
     3       /,5X,'NUMBER OF TIME STEPS (NTMSTP).........=',I3,
     4       /,5X,'THE  C F L  NUMBER (CFL)..............=',E12.4,
     5       /,5X,'OUTER RELAXATION PARAMETER (RLXOUT)...=',E12.4,
     6       /,5X,'INNER RELAXATION PARAMETER (RLXIN)....=',E12.4,/)
 2500 FORMAT (/,15X,'O U T P U T   F R O M   P R O G R A M   COMPR3D',/)
 2600 FORMAT (80('-'))
 3000 FORMAT (1H1,20A4)
 3500 FORMAT (/,2X,'C O N N E C T I V I T Y   M A T R I X:',/,
     *        2X,'(ELEMENT-TO-NODES)',/)
 4000 FORMAT (I5,2X,11I5)
 4002 FORMAT (5X,'DO-LOOP 200 :',/,9I5)
 4003 FORMAT (5X,'DO-LOOP 300 :',/,9I5)
 4004 FORMAT (5X,'DO-LOOP 400 :',/,9I5)
 4005 FORMAT (5X,'DO-LOOP 500 :',/,9I5)
 4006 FORMAT (5X,'DO-LOOP 600 :',/,9I5)
 4007 FORMAT (5X,'DO-LOOP 700 :',/,9I5)
 4008 FORMAT (5X,'DO-LOOP 800 :',/,9I5)
 4500 FORMAT (/,2X,'C O N N E C T I V I T Y   A R R A Y :',/,
     *        2X,'(NODE-TO-ELEMENTS)',/)
 5000 FORMAT (I5,3(2X,E12.4))
 5500 FORMAT (/,2X,'(X,Y,Z)-C O O R D I N A T E S   O F   N O D E S:',/)
 6000 FORMAT (/,2X,'T I M E   S T E P =',I5,/)
 6100 FORMAT (/,2X,'I N I T I A L   F I E L D   V A L U E S:',/)
 6200 FORMAT (/,2X,'SPECIFIED NODAL QUANTITIES (=0, SPECIFIED):',/)
 6300 FORMAT (/,2X,'ELEMENT NUMBERS AND THEIR SURFACES THAT REQUIRE FLUX
     * COMPUTATION:',/)
 6500 FORMAT (I5,6E12.4)
 7000 FORMAT (/,5X,'MAX. ERROR =',E12.4,/,5X,'NODE NUMBER =',I5,/)
 7500 FORMAT (/,5X,'LEQ =',I2,2X,'NODE =',I4,2X,'RESIDUAL=',E12.4,2X,
     *        'FLUX=',E12.4,2X,'TCOEF=',E12.4,2X,'SOLN.=',E12.4)
 8000 FORMAT (5X,'VOLUME OF EACH ELEMENT:',/,5X,6E12.4)
      END


      SUBROUTINE BCUPDT(NNM,GAMA,R0,TEMP0,U,DNST0)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/MSH/ARCANG,NX,NY,NZ,NX1,NX2,NX3
      DIMENSION  U(NNM,6)
C
C
C     DEFINE FIXED PARAMETERS
C
      ANX=0.0
      ANY=DSIN(0.5*ARCANG)
      ANZ=DCOS(0.5*ARCANG)
      GAM1=GAMA-1.0
      NXX=NX+1
      NYY=NY+1
      NZZ=NZ+1
C
C     SET THE NORMAL VELOCITY TO ZERO AT THE MIDPLANE
C
      DO 30 IX=1,NXX
```

```
      DO 30 IY=1,NYY
      ND=(IX-1)*NYY*NZZ+NYY+IY
      U(ND,3)=U(ND,3)*(1.0-ANY*ANY)-U(ND,4)*ANY*ANZ
      U(ND,4)=-U(ND,3)*ANY*ANZ+U(ND,4)*(1.0-ANZ*ANZ)
      U(ND,5)=U(ND,6)/GAM1+0.5*(U(ND,2)*U(ND,2)+U(ND,3)*U(ND,3)+
     *                          U(ND,4)*U(ND,4))/U(ND,1)
C
C     RESET THE VALUES ON PARALLEL PLANES TO THOSE ON THE MIDPLANE
C
      ND1=ND-NYY
      ND2=ND+NYY
      U(ND1,1)=U(ND,1)
      U(ND1,2)=U(ND,2)
      U(ND1,3)=U(ND,3)*ANZ-U(ND,4)*ANY
      U(ND1,4)=U(ND,3)*ANY+U(ND,4)*ANZ
      U(ND1,5)=U(ND,5)
      U(ND1,6)=U(ND,6)
      U(ND2,1)=U(ND,1)
      U(ND2,2)=U(ND,2)
      U(ND2,3)=U(ND,3)*ANZ+U(ND,4)*ANY
      U(ND2,4)=-U(ND,3)*ANY+U(ND,4)*ANZ
      U(ND2,5)=U(ND,5)
      U(ND2,6)=U(ND,6)
   30 CONTINUE
C
C     RESET THE VALUES AT OUTFLOW BOUNDARY
C
      DO 40 IZ=1,NZZ
      DO 40 IY=1,NYY
      ND = IY + (IZ-1)*NYY + NX*NYY*NZZ
      U(ND,6)=DNST0*R0*TEMP0*0.98
      U(ND,5)=U(ND,6)/GAM1+0.5*(U(ND,2)*U(ND,2)+U(ND,3)*U(ND,3)+
     *                          U(ND,4)*U(ND,4))/U(ND,1)
   40 CONTINUE
C
C     SET CONSTANT TEMPERATURE ON THE WALLS
C
      DO 60 KD = 1, NX-1
      ND1 = (NYY*NZZ)*KD + 1
      DO 50 JZ = 1, NZZ
      ND = ND1 + (JZ-1)*NYY
      U(ND,6)=U(ND,5)*GAM1
      U(ND,1)=U(ND,6)/(R0*TEMP0)
C
      NN = ND + NY
      U(NN,6)=U(NN,5)*GAM1
      U(NN,1)=U(NN,6)/(R0*TEMP0)
   50 CONTINUE
C
   60 CONTINUE
      RETURN
      END



      SUBROUTINE BNDRY(NBS,NEM,NNM,NPE,NSURF,NODES,KELSUR,NDSURF,KNDSUR)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION NODES(NEM,NPE),KELSUR(NBS,2),KNDSUR(NBS,4),NDSURF(NNM),
     *          K(4)
      NCOUNT=0
      DO 10 I=1,NNM
   10 NDSURF(I)=0
      DO 20 L=1,4
      DO 20 J=1,NSURF
   20 KNDSUR(J,L)=0
C
```

```
      DO 150 I=1,NSURF
      KEL=KELSUR(I,1)
      KSRF=KELSUR(I,2)
      GOTO(30,40,50,60,70,80),KSRF
   30 K(1)=NODES(KEL,1)
      K(2)=NODES(KEL,4)
      K(3)=NODES(KEL,8)
      K(4)=NODES(KEL,5)
      GOTO 90
   40 K(1)=NODES(KEL,2)
      K(2)=NODES(KEL,3)
      K(3)=NODES(KEL,7)
      K(4)=NODES(KEL,6)
      GOTO 90
   50 K(1)=NODES(KEL,1)
      K(2)=NODES(KEL,5)
      K(3)=NODES(KEL,6)
      K(4)=NODES(KEL,2)
      GOTO 90
   60 K(1)=NODES(KEL,4)
      K(2)=NODES(KEL,8)
      K(3)=NODES(KEL,7)
      K(4)=NODES(KEL,3)
      GOTO 90
   70 K(1)=NODES(KEL,1)
      K(2)=NODES(KEL,2)
      K(3)=NODES(KEL,3)
      K(4)=NODES(KEL,4)
      GOTO 90
   80 K(1)=NODES(KEL,5)
      K(2)=NODES(KEL,6)
      K(3)=NODES(KEL,7)
      K(4)=NODES(KEL,8)
   90 CONTINUE
      DO 120 J=1,4
      IF(NDSURF(K(J)).EQ.0)THEN
      NCOUNT=NCOUNT+1
      NDSURF(K(J))=NCOUNT
      KNDSUR(NCOUNT,1)=I
      ELSE
      NC=NDSURF(K(J))
      DO 100 JJ=2,4
      IF(KNDSUR(NC,JJ).EQ.0)THEN
      KNDSUR(NC,JJ)=I
      GOTO 110
      ENDIF
  100 CONTINUE
  110 CONTINUE
      ENDIF
  120 CONTINUE
  150 CONTINUE
      RETURN
      END


      SUBROUTINE COEFNT(IEL,LEQ,N,NPE,NEM,NGP,ELU,SF,GDSF,CNST,VOL,RES,
     *                         CM,EMU,DELU,MINDX,CKINV,NDF,NDIM,NGPT,MXE)
C     ----------------------------------------------------------------
C
C     ELU(I,J)......ELEMENT SOLUTION VECTOR (J-TH COMPO. AT I-TH NODE)
C     SF(I,...).....SHAPE FUNCTION ASSOCIATED WITH THE I-TH NODE
C     GDSF(N,J,..I).GLOBAL DERIVATIVE OF J-TH SHAPE FUNCTION
C                   WITH RESPECT TO X(I) COORDINATE
C     ----------------------------------------------------------------
C
```

```
C      THIS IS A VECTORIZED VERSION OF THE SUBROUTINE COEFNT
C
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION SF(NPE,NGP,NGP,NGP),CNST(MXE,NGP,NGP,NGP),VOL(MXE),
     2           GDSF(MXE,NPE,NGP,NGP,NGP,NDIM),ELU(NPE,6),EMU(NPE),
     3           U(6,8),DU(7,3,8),DU1(7,3,8),U1(6,8),DELU(NPE,6),
     4           III(8),JJJ(8),KKK(8),F(8,8),DF(9,9,3),MINDX(NPE),
     5           DQ1(3),C(8),GMU(8)
       COMMON/DTA/GAMA,AMU0,TEMP0,S1,R0,GPR,GAM1,CFL
C
       DATA III/1,2,1,2,1,2,1,2/
       DATA JJJ/1,1,2,2,1,1,2,2/
       DATA KKK/1,1,1,1,2,2,2,2/
C
       CM=0.0
       CK=0.0
       CKINV=0.0
       DLNGTH=0.0
       RES=0.0
       FMAS=0.0
       SPEED=DSQRT(ELU(IEL,6)*GAMA/ELU(IEL,1))
C
       DO 10 L=1,NGPT
       C(L) = CNST(N,III(L),JJJ(L),KKK(L))
       DO 10 I=1,NPE
       F(L,I) = SF(I,III(L),JJJ(L),KKK(L))
       DF(L,I,1) = GDSF(N,I,III(L),JJJ(L),KKK(L),1)
       DF(L,I,2) = GDSF(N,I,III(L),JJJ(L),KKK(L),2)
    10 DF(L,I,3) = GDSF(N,I,III(L),JJJ(L),KKK(L),3)
       TSPEED=SPEED+(DABS(ELU(IEL,2))+DABS(ELU(IEL,3))+DABS(ELU(IEL,4)))/
     *               ELU(IEL,1)
       DT=CFL*(VOL(N)**(1./3.))/TSPEED
C
C      EVALUATE THE SOLUTION AND ITS DERIVATIVES AT THE GAUSS POINT
C
       DO 40 J=1,NDF
       DO 40 L=1,NGPT
       SUM1=0.0
       SUM2=0.0
       SUM3=0.0
       SUM4=0.0
       DO 30 I=1,NPE
       SUM1=SUM1+DF(L,I,1)*ELU(I,J)
       SUM2=SUM2+DF(L,I,2)*ELU(I,J)
       SUM3=SUM3+DF(L,I,3)*ELU(I,J)
    30 SUM4=SUM4+F(L,I)*ELU(I,J)
       DU(J,1,L)=SUM1
       DU(J,2,L)=SUM2
       DU(J,3,L)=SUM3
    40 U(J,L)=SUM4
       DO 50 J=2,4
       DO 50 L=1,NGPT
       U1(J,L)=U(J,L)/U(1,L)
       DU1(J,1,L)=(DU(J,1,L)-U1(J,L)*DU(1,1,L))
       DU1(J,2,L)=(DU(J,2,L)-U1(J,L)*DU(1,2,L))
    50 DU1(J,3,L)=(DU(J,3,L)-U1(J,L)*DU(1,3,L))
C
C      COMPUTE MASS MATRIX TIMES DELU TERM
C
       DO 70 J1=1,NPE
       DO 60 L=1,NGPT
       PROD=F(L,IEL)*F(L,J1)*C(L)
       CM=CM+PROD*MINDX(J1)
    60 FMAS=FMAS+PROD*DELU(J1,LEQ)
    70 CONTINUE
C
```

```
C     COMPUTE INVISCID COEFFICIENT FOR INNER ITERATION
C
      DO 90 L=1,NGPT
      CKINV=CKINV+(DABS(DF(L,IEL,1)*(DABS(U1(2,L))+SPEED))
     1           + DABS(DF(L,IEL,2)*(DABS(U1(3,L))+SPEED))
     2           + DABS(DF(L,IEL,3)*(DABS(U1(4,L))+SPEED)))*C(L)
     3           *F(L,IEL)
   90 CONTINUE
C
C     COMPUTE RESIDUES ETC FOR A CONSERVATION EQUATION
C
      GOTO(100,200,300,400,500),LEQ
C
  100 DO 110 L=1,NGPT
      RES=RES-(DF(L,IEL,1)*U(2,L)+DF(L,IEL,2)*U(3,L)
     1        +DF(L,IEL,3)*U(4,L))*C(L)
  110 CONTINUE
      GOTO 600
C
  200 DO 240 L=1,NGPT
      SUM=0.0
      DO 220 I=1,NPE
  220 SUM=SUM+EMU(I)*F(L,I)
  240 GMU(L)=SUM
      DO 260 L=1,NGPT
      U22=U(2,L)*U(2,L)
      U23=U(2,L)*U(3,L)
      U24=U(2,L)*U(4,L)
      U33=U(3,L)*U(3,L)
      U44=U(4,L)*U(4,L)
      PRES=GAM1*(U(5,L)-0.5*(U22+U33+U44)/U(1,L))
      AMU23=2.0*GMU(L)/3.0
      AMU43=2.0*AMU23
      RES=RES-C(L)*((U22+PRES*U(1,L)+AMU23*(-2.0*DU1(2,1,L)
     1        +DU1(3,2,L)+DU1(4,3,L)))*DF(L,IEL,1)
     2        +(U23-GMU(L)*(DU1(3,1,L)+DU1(2,2,L)))*DF(L,IEL,2)
     3        +(U24-GMU(L)*(DU1(4,1,L)+DU1(2,3,L)))*DF(L,IEL,3))/U(1,L)
  260 CONTINUE
      GOTO 600
C
  300 DO 340 L=1,NGPT
      SUM=0.0
      DO 320 I=1,NPE
  320 SUM=SUM+EMU(I)*F(L,I)
  340 GMU(L)=SUM
      DO 360 L=1,NGPT
      U22=U(2,L)*U(2,L)
      U23=U(2,L)*U(3,L)
      U33=U(3,L)*U(3,L)
      U34=U(3,L)*U(4,L)
      U44=U(4,L)*U(4,L)
      PRES=GAM1*(U(5,L)-0.5*(U22+U33+U44)/U(1,L))
      AMU23=2.0*GMU(L)/3.0
      AMU43=2.0*AMU23
      RES=RES-C(L)*((U33+PRES*U(1,L)+AMU23*(-2.0*DU1(3,2,L)
     1        +DU1(4,3,L)+DU1(2,1,L)))*DF(L,IEL,2)
     2        +(U34-GMU(L)*(DU1(4,2,L)+DU1(3,3,L)))*DF(L,IEL,3)
     3        +(U23-GMU(L)*(DU1(2,2,L)+DU1(3,1,L)))*DF(L,IEL,1))/U(1,L)
  360 CONTINUE
      GOTO 600
C
  400 DO 440 L=1,NGPT
      SUM=0.0
      DO 420 I=1,NPE
  420 SUM=SUM+EMU(I)*F(L,I)
  440 GMU(L)=SUM
```

```fortran
      DO 460 L=1,NGPT
      U22=U(2,L)*U(2,L)
      U24=U(2,L)*U(4,L)
      U33=U(3,L)*U(3,L)
      U34=U(3,L)*U(4,L)
      U44=U(4,L)*U(4,L)
      PRES=GAM1*(U(5,L)-0.5*(U22+U33+U44)/U(1,L))
      AMU23=2.0*GMU(L)/3.0
      AMU43=2.0*AMU23
      RES=RES-C(L)*((U44+PRES*U(1,L)+AMU23*(-2.0*DU1(4,3,L)
     1         +DU1(2,1,L)+DU1(3,2,L)))*DF(L,IEL,3)
     2         +(U24-GMU(L)*(DU1(2,3,L)+DU1(4,1,L)))*DF(L,IEL,1)
     3         +(U34-GMU(L)*(DU1(3,3,L)+DU1(4,2,L)))*DF(L,IEL,2))/U(1,L)
  460 CONTINUE
      GOTO 600
C
  500 DO 540 L=1,NGPT
      SUM=0.0
      DO 520 I=1,NPE
  520 SUM=SUM+EMU(I)*F(L,I)
  540 GMU(L)=SUM
      DO 560 L=1,NGPT
      U22=U(2,L)*U(2,L)
      U33=U(3,L)*U(3,L)
      U44=U(4,L)*U(4,L)
      PRES=GAM1*(U(5,L)-0.5*(U22+U33+U44)/U(1,L))
      AKH=GMU(L)*GPR
      AMU23=2.0*GMU(L)/3.0
      AMU43=2.0*AMU23
      DQ1(1)=DU(5,1,L)-U1(2,L)*DU(2,1,L)-U1(3,L)*DU(3,1,L)
     2        -U1(4,L)*DU(4,1,L)+DU(1,1,L)*(-U(5,L)/U(1,L)
     3        +U1(2,L)*U1(2,L)+U1(3,L)*U1(3,L)+U1(4,L)*U1(4,L))
      DQ1(2)=DU(5,2,L)-U1(2,L)*DU(2,2,L)-U1(3,L)*DU(3,2,L)
     2        -U1(4,L)*DU(4,2,L)+DU(1,2,L)*(-U(5,L)/U(1,L)
     3        +U1(2,L)*U1(2,L)+U1(3,L)*U1(3,L)+U1(4,L)*U1(4,L))
      DQ1(3)=DU(5,3,L)-U1(2,L)*DU(2,3,L)-U1(3,L)*DU(3,3,L)
     2        -U1(4,L)*DU(4,3,L)+DU(1,3,L)*(-U(5,L)/U(1,L)
     3        +U1(2,L)*U1(2,L)+U1(3,L)*U1(3,L)+U1(4,L)*U1(4,L))
C
      RES1 = (U(2,L)*(U(5,L)+PRES)-AMU23*U1(2,L)*(2.0*DU1(2,1,L)
     2        -DU1(3,2,L)-DU1(4,3,L))-GMU(L)*(U1(3,L)*(DU1(2,2,L)
     3        +DU1(3,1,L))+U1(4,L)*(DU1(2,3,L)+DU1(4,1,L)))
     4        -AKH*DQ1(1))*DF(L,IEL,1)
      RES2 = (U(3,L)*(U(5,L)+PRES)-AMU23*U1(3,L)*(2.0*DU1(3,2,L)
     2        -DU1(4,3,L)-DU1(2,1,L))-GMU(L)*(U1(4,L)*(DU1(3,3,L)
     3        +DU1(4,2,L))+U1(2,L)*(DU1(3,1,L)+DU1(2,2,L)))
     4        -AKH*DQ1(2))*DF(L,IEL,2)
      RES3 = (U(4,L)*(U(5,L)+PRES)-AMU23*U1(4,L)*(2.0*DU1(4,3,L)
     2        -DU1(2,1,L)-DU1(3,2,L))-GMU(L)*(U1(2,L)*(DU1(4,1,L)
     3        +DU1(2,3,L))+U1(3,L)*(DU1(4,2,L)+DU1(3,3,L)))
     4        -AKH*DQ1(3))*DF(L,IEL,3)
      RES = RES - (RES1+RES2+RES3)*C(L)/U(1,L)
  560 CONTINUE
  600 CONTINUE
      RES=RES+FMAS/DT
      CM=CM/DT
      RETURN
      END



      SUBROUTINE DISPTN(NNM,NEM,MXE,X,Y,Z,U,DC4,NODES,NELEM,DIS4,
     *                  NPE,E0,VOLND)
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
      DIMENSION  X(NNM),Y(NNM),Z(NNM),U(NNM,6),NODES(NEM,8),E0(NNM),
     2           NELEM(NNM,MXE),DIS4(NNM,6),VOLND(NNM),DC4(NNM)
C
      DATA KAPA2,KAPA4/0.1,0.01/
      DO 50 IE=1,6
      DO 40 ND=1,NNM
      SUME0=0.0
      DO 20 NE=1,MXE
      IF(NELEM(ND,NE).EQ.0)GOTO 30
      NEL=NELEM(ND,NE)
      DO 20 NP=1,NPE
      NI=NODES(NEL,NP)
   20 SUME0=SUME0+U(NI,IE)-U(ND,IE)
      NE=MXE+1
   30 CONTINUE
      DC4(ND)=7*(NE-1)

   40 DIS4(ND,IE)=SUME0
   50 CONTINUE
      DO 60 ND=1,NNM
      DIS4(ND,5)=DIS4(ND,5)+DIS4(ND,6)
   60 DIS4(ND,6)=ABS(DIS4(ND,6))/U(ND,6)*KAPA2
C
C     COMPUTE THE FOURTH-ORDER DISSIPATION
C
      DO 150 IE=1,5
      DO 140 ND=1,NNM
      SUMDC=0.0
      E0(ND)=0.0
      SUMD0=0.0
      ISW=1
      IF(DIS4(ND,6).GT.KAPA4) ISW=0
      DO 120 NE=1,MXE
      NEL=NELEM(ND,NE)
      IF(NEL.EQ.0)GOTO 130
      DO 100 NP=1,NPE
      NI=NODES(NEL,NP)
      IF(NI.EQ.ND)GOTO 100
      XL=X(NI)-X(ND)
      YL=Y(NI)-Y(ND)
      ZL=Z(NI)-Z(ND)
      EDGE =DSQRT(XL*XL+YL*YL+ZL*ZL)
      EPSLN=(VOLND(ND)+VOLND(NI))*0.5/EDGE
      IF(IE.EQ.5)SUMDC=SUMDC+EPSLN*((DC4(ND)-1.0)*KAPA4*ISW+DIS4(ND,6))
      SUMD0=SUMD0-(DIS4(NI,IE)-DIS4(ND,IE))*EPSLN*KAPA4*ISW
  100 CONTINUE
  120 CONTINUE
  130 CONTINUE
      IF(IE.EQ.5)DC4(ND)=SUMDC
  140 E0(ND)=SUMD0
      DO 150 ND = 1,NNM
  150 DIS4(ND,IE)=E0(ND)+DIS4(ND,IE)*DIS4(ND,6)
      RETURN
      END


      SUBROUTINE DSFSUR(DSURF,NGP,NPE,NDIM)
C     ---------------------------------------------------------------
C     THIS SUBROUTINE EVALUATES THE DERIVATIVES OF THE SHAPE FUNCTIONS
C     AT THE GAUSS POINTS OF THE SURFACES OF AN ELEMENT
C     ---------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION XNODE(8,3),XYZ(3),DSURF(NDIM,NPE,6,NGP,NGP),GAUSS(2)
      DATA XNODE/-1.0D0,2*1.0D0,2*-1.0D0,2*1.0D0,-1.0D0,2*-1.0D0,2*1.0D0
```

```
     1,2*-1.0D0,2*1.0D0,4*-1.0D0,4*1.0D0/
C
      FCK(A,B,C)=0.125*A*B*C
      SQRT3=DSQRT(3.0D0)
      GAUSS(1)=-1.0D0/SQRT3
      GAUSS(2)=-GAUSS(1)
      DO 80 K1=1,6
      DO 60 NGPI=1,NGP
      DO 60 NGPK=1,NGP
C
      GOTO(10,10,20,20,30,30),K1
   10 XYZ(1)=(-1)**K1
      XYZ(2)=GAUSS(NGPI)
      XYZ(3)=GAUSS(NGPK)
      GOTO 40
   20 XYZ(2)=(-1)**K1
      XYZ(3)=GAUSS(NGPI)
      XYZ(1)=GAUSS(NGPK)
      GOTO 40
   30 XYZ(3)=(-1)**K1
      XYZ(1)=GAUSS(NGPI)
      XYZ(2)=GAUSS(NGPK)
   40 DO 50 I=1,NPE
      XNP1=XYZ(1)*XNODE(I,1)+1.0
      YNP1=XYZ(2)*XNODE(I,2)+1.0
      ZNP1=XYZ(3)*XNODE(I,3)+1.0
      DSURF(1,I,K1,NGPI,NGPK)=FCK(XNODE(I,1),YNP1,ZNP1)
      DSURF(2,I,K1,NGPI,NGPK)=FCK(XNP1,XNODE(I,2),ZNP1)
   50 DSURF(3,I,K1,NGPI,NGPK)=FCK(XNP1,YNP1,XNODE(I,3))
   60 CONTINUE
   80 CONTINUE
      RETURN
      END


      SUBROUTINE FLUXES(IEL,LEQ,N,NPE,NGP,ELU,SF,GDSF,GNORM,K1,KG1,FLX,
     1                   EMU,MXE,NBS,NDF,NDIM)
C     ---------------------------------------------------------------
C
C     ELU(I,J)......ELEMENT SOLUTION VECTOR (J-TH COMPO. AT I-TH NODE)
C     SF(I,...).....SHAPE FUNCTION ASSOCIATED WITH THE I-TH NODE
C     GDSF(N,J,..I).GLOBAL DERIVATIVE OF J-TH SHAPE FUNCTION
C                   WITH RESPECT TO X(I) COORDINATE OF THE N-TH ELEMENT
C     GDINT(I,J)....INTERPOLATED GDSF ON SURFACE OF AN ELEMENT
C     SFINT(I)......INTERPOLATED SF ON SURFACE OF AN ELEMENT
C     ---------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION SF(NPE,NGP,NGP,NGP),GDSF(MXE,NPE,NGP,NGP,NGP,NDIM),
     2          GDINT(8,3),SFINT(8),GNORM(NDIM,NBS,NGP,NGP),EMU(NPE),
     3          ELU(NPE,6),DU(6,3),U(6),U1(6),DU1(6,3),DQ1(3),VECTR(3)
      COMMON/DTA/GAMA,AMU0,TEMP0,S1,R0,GPR,GAM1,CFL
C
      K0=(K1+1)/2
      FLX=0.0
      SQRT3=DSQRT(3.0D0)
C
C     DO-LOOP ON GAUSS INTEGRATION BEGINS HERE
C
      DO 200 JJ=1,NGP
      DO 200 KK=1,NGP
      AMU=0.0
C
C     EVALUATE THE COMPONENTS OF THE SURFACE NORMAL AT THE GAUSS POINTS
C
```

```
      IF(K0-2)30,40,50
   30 NI=1
      NI1=2
      NJ=JJ
      NJ1=NJ
      NK=KK
      NK1=NK
      GOTO 60
   40 NJ=1
      NJ1=2
      NK=JJ
      NK1=NK
      NI=KK
      NI1=NI
      GOTO 60
   50 NK=1
      NK1=2
      NI=JJ
      NI1=NI
      NJ=KK
      NJ1=NJ
C
   60 DO 70 I=1,NPE
      F1=SF(I,NI,NJ,NK)
      F2=SF(I,NI1,NJ1,NK1)
      SFINT(I)=((-1)**K1*SQRT3*(F2-F1)+F2+F1)/2.0
      F3=GDSF(N,I,NI,NJ,NK,1)
      F4=GDSF(N,I,NI1,NJ1,NK1,1)
      GDINT(I,1)=((-1)**K1*SQRT3*(F4-F3)+F4+F3)/2.0
      F3=GDSF(N,I,NI,NJ,NK,2)
      F4=GDSF(N,I,NI1,NJ1,NK1,2)
      GDINT(I,2)=((-1)**K1*SQRT3*(F4-F3)+F4+F3)/2.0
      F3=GDSF(N,I,NI,NJ,NK,3)
      F4=GDSF(N,I,NI1,NJ1,NK1,3)
      GDINT(I,3)=((-1)**K1*SQRT3*(F4-F3)+F4+F3)/2.0
   70 AMU=AMU+SFINT(I)*EMU(I)
      DO 100 J=1,NDF
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      DO 80 I=1,NPE
      SUM1=SUM1+GDINT(I,1)*ELU(I,J)
      SUM2=SUM2+GDINT(I,2)*ELU(I,J)
      SUM3=SUM3+GDINT(I,3)*ELU(I,J)
   80 SUM4=SUM4+SFINT(I)*ELU(I,J)
      DU(J,1)=SUM1
      DU(J,2)=SUM2
      DU(J,3)=SUM3
  100 U(J)=SUM4
      U1(2)=U(2)/U(1)
      U1(3)=U(3)/U(1)
      U1(4)=U(4)/U(1)
      DO 110 J=2,4
      DU1(J,1)=(DU(J,1)-U1(J)*DU(1,1))
      DU1(J,2)=(DU(J,2)-U1(J)*DU(1,2))
  110 DU1(J,3)=(DU(J,3)-U1(J)*DU(1,3))
      VECTR(1)=GNORM(1,KG1,JJ,KK)
      VECTR(2)=GNORM(2,KG1,JJ,KK)
      VECTR(3)=GNORM(3,KG1,JJ,KK)
C
C     COMPUTE PRESSURE, TEMPERATURE, VISCOSITY USING THE SUTHERLAND'S
C     LAW, AND THE DIFFUSION CONSTANT AT THE GAUSS POINTS
C
      U22=U(2)*U(2)
      U23=U(2)*U(3)
```

```
      U24=U(2)*U(4)
      U33=U(3)*U(3)
      U34=U(3)*U(4)
      U44=U(4)*U(4)
      PRES=GAM1*(U(5)-0.5*(U22+U33+U44)/U(1))
      AKH=AMU*GPR
      AMU23=2.0*AMU/3.0
      AMU43=2.0*AMU23
C
C     COMPUTE THE FLUX FOR EACH CONSERVATION EQUATION AT THE NODE
C
      GOTO(140,150,160,170,180),LEQ
  140 FLX=FLX+(U(2)*VECTR(1)+U(3)*VECTR(2)+U(4)*VECTR(3))*SFINT(IEL)
      GOTO 200
  150 FLX=FLX+((U22+PRES*U(1)+AMU23*(-2.0*DU1(2,1)+DU1(3,2)+DU1(4,3)))
     1         *VECTR(1)+(U23-AMU*(DU1(3,1)+DU1(2,2)))*VECTR(2)
     2         +(U24-AMU*(DU1(4,1)+DU1(2,3)))*VECTR(3))*SFINT(IEL)/U(1)
      GOTO 200
  160 FLX=FLX+((U33+PRES*U(1)+AMU23*(-2.0*DU1(3,2)+DU1(4,3)+DU1(2,1)))
     1         *VECTR(2)+(U34-AMU*(DU1(4,2)+DU1(3,3)))*VECTR(3)
     2         +(U23-AMU*(DU1(2,2)+DU1(3,1)))*VECTR(1))*SFINT(IEL)/U(1)
      GOTO 200
  170 FLX=FLX+((U44+PRES*U(1)+AMU23*(-2.0*DU1(4,3)+DU1(2,1)+DU1(3,2)))
     1         *VECTR(3)+(U24-AMU*(DU1(2,3)+DU1(4,1)))*VECTR(1)
     2         +(U34-AMU*(DU1(3,3)+DU1(4,2)))*VECTR(2))*SFINT(IEL)/U(1)
      GOTO 200
  180 DQ1(1)=DU(5,1)-U1(2)*DU(2,1)-U1(3)*DU(3,1)-U1(4)*DU(4,1)
     2        +DU(1,1)*(-U(5)/U(1)+U1(2)*U1(2)+U1(3)*U1(3)+U1(4)*U1(4))
      DQ1(2)=DU(5,2)-U1(2)*DU(2,2)-U1(3)*DU(3,2)-U1(4)*DU(4,2)
     2        +DU(1,2)*(-U(5)/U(1)+U1(2)*U1(2)+U1(3)*U1(3)+U1(4)*U1(4))
      DQ1(3)=DU(5,3)-U1(2)*DU(2,3)-U1(3)*DU(3,3)-U1(4)*DU(4,3)
     2        +DU(1,3)*(-U(5)/U(1)+U1(2)*U1(2)+U1(3)*U1(3)+U1(4)*U1(4))
      FLX=FLX+((U(2)*(U(5)+PRES)-AMU23*U1(2)*(2.0*DU1(2,1)-DU1(3,2)
     2         -DU1(4,3))-AMU*(U1(3)*(DU1(2,2)+DU1(3,1))+U1(4)*(DU1(2,3)
     3         +DU1(4,1)))-AKH*DQ1(1))*VECTR(1)
     4         +(U(3)*(U(5)+PRES)-AMU23*U1(3)*(2.0*DU1(3,2)-DU1(4,3)
     5         -DU1(2,1))-AMU*(U1(4)*(DU1(3,3)+DU1(4,2))+U1(2)*(DU1(3,1)
     6         +DU1(2,2)))-AKH*DQ1(2))*VECTR(2)
     7         +(U(4)*(U(5)+PRES)-AMU23*U1(4)*(2.0*DU1(4,3)-DU1(2,1)
     8         -DU1(3,2))-AMU*(U1(2)*(DU1(4,1)+DU1(2,3))+U1(3)*(DU1(4,2)
     9         +DU1(3,3)))-AKH*DQ1(3))*VECTR(3))*SFINT(IEL)/U(1)
  200 CONTINUE
C*    WRITE(6,300)LEQ,FLX
C 300 FORMAT (5X,'LEQ =',I2,5X,'FLUX =',E12.4)
      RETURN
      END


      SUBROUTINE GCSURF(GC,DSURF,ELXYZ,NGPI,NGPK,NGP,K1,NDIM,NPE)
C
C     GC(I,J)......DERIVATIVE OF X(I) W.R.T. XI(J)
C     DSURF(I,J,K..DERIVATIVE OF PSI(J) W.R.T. XI(I), J=1,...,NPE,
C                  ON K-TH SURFACE OF MASTER ELEMENT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ELXYZ(NPE,NDIM),DSURF(NDIM,NPE,6,NGP,NGP),GC(NDIM,NDIM)
C
      DO 200 I=1,NDIM
      DO 200 K=1,NDIM
      SUM=0.0
      DO 100 J=1,NPE
  100 SUM=SUM+DSURF(K,J,K1,NGPI,NGPK)*ELXYZ(J,I)
  200 GC(I,K)=SUM
      RETURN
      END
```

```
      SUBROUTINE GMETRY(NNM,NEM,MXE,N,NPE,NGP,ELXYZ,SF,GDSF,CNST,VOL,
     1                  NDIM,IEL)
C     ---------------------------------------------------------------
C
C     SF(I,II,JJ,KK)........I-TH SHAPE FUNCTION AT THE (II,JJ,KK)-TH
C                           GAUSS POINT
C     GDSF(N,I,II,JJ,KK,J)..GLOBAL DERIVATIVE OF I-TH SHAPE FUNCTION
C                           WITH RESPECT TO THE X(J) COORDINATE
C                           FOR ELEMENT N
C     DSF(I,J)..............LOCAL DERIVATIVE OF I-TH SHAPE FUNCTION
C                           WITH RESPECT TO J-TH LOCAL COORDINATE
C     ELXYZ(I,J)............J-TH GLOBAL COORDINATE OF I-TH NODE
C     XYZ(II)...............II-TH GAUSSIAN POINT
C     ---------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION SF(NPE,NGP,NGP,NGP),CNST(MXE,NGP,NGP,NGP),VOL(MXE),
     2          GDSF(MXE,NPE,NGP,NGP,NGP,NDIM),ELXYZ(NPE,NDIM),WT(2),
     3          GAUSS(2),GJ(3,3),XYZ(3),GJINV(3,3),DSF(3,8),GDSFL(3,8),
     4          SFL(8)
      COMMON/GMT/SN22(8,8),SN33(8,8),SN44(8,8),SN55(8,8)
      DATA NCOUNT/0/
C
      SQRT3=DSQRT(3.0D0)
      GAUSS(1)=-1.0D0/SQRT3
      GAUSS(2)=-GAUSS(1)
      WT(1)=1.0D0
      WT(2)=1.0D0
C
C     DO-LOOP ON GAUSS INTEGRATION BEGINS HERE
C
      VOL(N)=0.0
      DO 50 J=1,NPE
      SN22(N,J)=0.0
      SN33(N,J)=0.0
      SN44(N,J)=0.0
   50 SN55(N,J)=0.0
      DO 200 II=1,NGP
      DO 200 JJ=1,NGP
      DO 200 KK=1,NGP
      XYZ(1)=GAUSS(II)
      XYZ(2)=GAUSS(JJ)
      XYZ(3)=GAUSS(KK)
C     ***************************************************************
      CALL SHAPEL(XYZ,SFL,DSF,NDIM,NPE)
      CALL MATMUL(DSF,ELXYZ,GJ,NDIM,NPE,NDIM)
      CALL INVDET(GJ,GJINV,DET)
      CALL MATMUL(GJINV,DSF,GDSFL,NDIM,NDIM,NPE)
C     ***************************************************************
      CNST(N,II,JJ,KK)=DET*WT(II)*WT(JJ)*WT(KK)
      DO 150 I=1,NPE
      SN22(N,I)=SN22(N,I)+(4.0/3.0*GDSFL(1,IEL)*GDSFL(1,I)+
     1          GDSFL(2,IEL)*GDSFL(2,I)+GDSFL(3,IEL)*GDSFL(3,I))*
     2          CNST(N,II,JJ,KK)
      SN33(N,I)=SN33(N,I)+(4.0/3.0*GDSFL(2,IEL)*GDSFL(2,I)+
     1          GDSFL(3,IEL)*GDSFL(3,I)+GDSFL(1,IEL)*GDSFL(1,I))*
     2          CNST(N,II,JJ,KK)
      SN44(N,I)=SN44(N,I)+(4.0/3.0*GDSFL(3,IEL)*GDSFL(3,I)+
     1          GDSFL(1,IEL)*GDSFL(1,I)+GDSFL(2,IEL)*GDSFL(2,I))*
     2          CNST(N,II,JJ,KK)
      SN55(N,I)=SN55(N,I)+(GDSFL(1,IEL)*GDSFL(1,I)+
     1          GDSFL(2,IEL)*GDSFL(2,I)+GDSFL(3,IEL)*GDSFL(3,I))*
     2          CNST(N,II,JJ,KK)
```

```
      IF(NCOUNT.GT.0)GOTO 100
      SF(I,II,JJ,KK)=SFL(I)
  100 GDSF(N,I,II,JJ,KK,1)=GDSFL(1,I)
      GDSF(N,I,II,JJ,KK,2)=GDSFL(2,I)
  150 GDSF(N,I,II,JJ,KK,3)=GDSFL(3,I)
      VOL(N)=VOL(N)+CNST(N,II,JJ,KK)
  200 CONTINUE
      NCOUNT=1
      RETURN
      END


      SUBROUTINE INTIAL(NDF,NNM,AMACH,AMU0,TEMP0,S1,R0,GAMA,PR,U,DNST0)
C
C     INITIAL CONDITIONS FOR THE TURN-AROUND-DUCT PROBLEM
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/MSH/ARCANG,NX,NY,NZ,NX1,NX2,NX3
      DIMENSION  U(NNM,6)
C
C
C     DEFINE FIXED PARAMETERS
C
      GAM1=GAMA-1.0
      NYY=NY+1
      NZZ=NZ+1
C
C     INITIALIZE THE FLOW FIELD
C
      DO 10 J=2,4
      DO 10 I=1,NNM
   10 U(I,J)=0.0
C
      DO 20 IZ=1,NZZ
      DO 20 IY=2,NY
      ND = IY + (IZ-1)*NYY
      U(ND,2)=-DSQRT(GAMA*R0*TEMP0)*AMACH
      IF(IY.EQ.2.OR.IY.EQ.8)U(ND,2)=U(ND,2)*0.1885
      IF(IY.EQ.3.OR.IY.EQ.7)U(ND,2)=U(ND,2)*0.5066
      IF(IY.EQ.4.OR.IY.EQ.6)U(ND,2)=U(ND,2)*0.8393
   20 CONTINUE
C
C     INITIALIZE THE MID PLANE
C
      DO 30 IX = 2,NX1+1
      DO 30 IY = 2,NY
      ND = (IX-1)*NYY*NZZ+NYY+IY
      NDI= NYY+IY
   30 U(ND,2) = U(NDI,2)
      PI = ATAN(1.0)*4.0
      DO 40 IX = NX1+2,NX1+NX2
      DO 40 IY = 2,NY
      ND = (IX-1)*NYY*NZZ+NYY+IY
      NDI= NYY+IY
      U(ND,2) = U(NDI,2)*COS((IX-NX1-1)*PI/NX2)
   40 U(ND,3) =-U(NDI,2)*SIN((IX-NX1-1)*PI/NX2)
      DO 45 IX = NX1+NX2+1,NX+1
      DO 45 IY = 2,NY
      ND = (IX-1)*NYY*NZZ+NYY+IY
      NDI= NYY+IY
      U(ND,2) =-U(NDI,2)
   45 CONTINUE
C
C     U(ND,3) AND U(ND,4) ARE ZERO (HENCE, U(ND,5) IS AS DEFINED BELOW)
C
```

```
      DO 50 ND=1,NNM
      U(ND,1)=DNST0
      U(ND,2)=U(ND,2)*U(ND,1)
      U(ND,6)=U(ND,1)*R0*TEMP0
      U(ND,5)=U(ND,6)/GAM1+0.5*U(ND,2)*U(ND,2)/U(ND,1)
   50 CONTINUE
      RETURN
      END


      SUBROUTINE INVDET(A,B,DET)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3,3),B(3,3)
C
      G(Z1,Z2,Z3,Z4)=Z1*Z2-Z3*Z4
      F(Z1,Z2,Z3,Z4)=G(Z1,Z2,Z3,Z4)/DET
C
      C1=G(A(2,2),A(3,3),A(2,3),A(3,2))
      C2=G(A(2,3),A(3,1),A(2,1),A(3,3))
      C3=G(A(2,1),A(3,2),A(2,2),A(3,1))
      DET=A(1,1)*C1+A(1,2)*C2+A(1,3)*C3
      B(1,1)=F(A(2,2),A(3,3),A(3,2),A(2,3))
      B(1,2)=-F(A(1,2),A(3,3),A(1,3),A(3,2))
      B(1,3)=F(A(1,2),A(2,3),A(1,3),A(2,2))
      B(2,1)=-F(A(2,1),A(3,3),A(2,3),A(3,1))
      B(2,2)=F(A(1,1),A(3,3),A(3,1),A(1,3))
      B(2,3)=-F(A(1,1),A(2,3),A(1,3),A(2,1))
      B(3,1)=F(A(2,1),A(3,2),A(3,1),A(2,2))
      B(3,2)=-F(A(1,1),A(3,2),A(1,2),A(3,1))
      B(3,3)=F(A(1,1),A(2,2),A(2,1),A(1,2))
      RETURN
      END



      SUBROUTINE MATMUL(A,B,C,M,N,L)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(M,N),B(N,L),C(M,L)
      DO 20 I=1,M
      DO 20 J=1,L
      SUM=0.0
      DO 10 K=1,N
   10 SUM=SUM+A(I,K)*B(K,J)
   20 C(I,J)=SUM
      RETURN
      END


      SUBROUTINE SHAPEL(XYZ,SF,DF,NDIM,NPE)
C     ---------------------------------------------------------------
C     SHAPE FUNCTIONS FOR LINEAR, ISOPARAMETRIC 3-DIMENSIONAL ELEMENT
C     THIS SUBROUTINE EVALUATES THE SHAPE FUNCTIONS  AND THEIR FIRST
C     DERIVATIVES AT THE GAUSSIAN POINT XYZ
C     ---------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION XNODE(8,3),XYZ(NDIM),SF(NPE),DF(NDIM,NPE)
      DATA XNODE/-1.0D0,2*1.0D0,2*-1.0D0,2*1.0D0,-1.0D0,2*-1.0D0,2*1.0D0
     1,2*-1.0D0,2*1.0D0,4*-1.0D0,4*1.0D0/
C
      FCK(A,B,C)=0.125*A*B*C
      DO 20 I=1,NPE
      XNP1=XYZ(1)*XNODE(I,1)+1.0
      YNP1=XYZ(2)*XNODE(I,2)+1.0
      ZNP1=XYZ(3)*XNODE(I,3)+1.0
```

```
      SF(I)=FCK(XNP1,YNP1,ZNP1)
      DF(1,I)=FCK(XNODE(I,1),YNP1,ZNP1)
      DF(2,I)=FCK(XNP1,XNODE(I,2),ZNP1)
   20 DF(3,I)=FCK(XNP1,YNP1,XNODE(I,3))
      RETURN
      END


      SUBROUTINE SURFGM(K1,KG1,ELXYZ,DSURF,GNORM,NBS,NGP,NPE,NDIM)
C
C     GNORM(I,J,K,L)....I-TH COMPONENT OF 'NORMAL*DS' ON J-TH BOUNDARY
C                       SURFACE AT (K,L) GAUSS POINT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ELXYZ(NPE,NDIM),DSURF(NDIM,NPE,6,NGP,NGP),GC(3,3),
     *          GNORM(NDIM,NBS,NGP,NGP)
C
      K0=(K1+1)/2
      K2=K0+1
      IF(K2.EQ.4)K2=1
      K3=K2+1
      IF(K2.EQ.3)K3=1
      DO 200 NGPI=1,NGP
      DO 200 NGPK=1,NGP
C
      CALL GCSURF(GC,DSURF,ELXYZ,NGPI,NGPK,NGP,K1,NDIM,NPE)
      DO 100 I=1,NDIM
      I1=I+1
      IF(I1.EQ.4)I1=1
      I2=I1+1
      IF(I1.EQ.3)I2=1
  100 GNORM(I,KG1,NGPI,NGPK)=(GC(I1,K2)*GC(I2,K3)-GC(I1,K3)*GC(I2,K2))
     1          *(-1)**K1
  200 CONTINUE
      RETURN
      END

************************************************************************
*
      SUBROUTINE TADMSH(X,Y,Z,IBNDC,KELSUR,NOD,NSURF,NNM,NBS,NDF,NEM,
     *                  NPE)
*
************************************************************************
*
*                   MESH GENERATOR FOR TURN AROUND DUCT.
*                   -----------------------------------
*
*
*     PURPOSE :   TO GENERATE A THREE DIMENSIONAL MESH FOR A TURN AROUND
*                 DUCT. THE ELEMENT LIBRARY HAS THREE TYPES OF ELEMENTS
*                 VIZ. 8-NODED, 20 NODED, AND 27 NODED BRICK ELEMENTS.
*
*
*                                  FACE 5 (BACK)
*                                 /
*                     1 0-----/---------0 4
*                      /|    /         /|
*                     / |             / |
*                    /  | FACE 1     /  |                  .______ ETA
*     FACE 3 -----/     | (TOP)     / <-+-- FACE 4       /|
*     (L. SIDE)  /      |          /    |              /  |
*             5 0-----|---------0 8     | (R. SIDE)  ZETA  XI
*               |   2 0---------+-----0 3
*               |    /          |    /
*               |   /           |.../... FACE 2 (BOTTOM)
*               |  / FACE 6     |  /
```

```
*                   | / (FRONT)      | /
*                   |/               |/
*                 6 0--------------0 7
*
*                                          LINEAR RECTANGULAR ELEMENT
*
*     LIST OF VARIABLES :
*     -------------------
*
*     NX1            = NUMBER OF DIVISIONS IN FLOW DIRN. IN PART 1(INLET)
*     NX2            = NUMBER OF DIVISIONS IN FLOW DIRN. IN PART 2(CURVE)
*     NX3            = NUMBER OF DIVISIONS IN FLOW DIRN. IN PART 3(OUTLET)
*     NY             = NUMBER OF DIVISIONS IN RADIAL DIRECTION:
*     NZ             = NUMBER OF DIVISIONS IN Z-DIRECTION:
*     NPE            = NODES PER ELEMENT.(8 OR. 20 OR 27)
*     NOD(NNM,NPE)   = CONNECTIVITY MATRIX
*     IEL            = ELEMENT TYPE (1 = LINEAR(8 NODED) ; 2 = QUADRATIC)
*     R1             = INNER RADIUS OF THE CURVE.
*     R2             = OUTER RADIUS OF THE CURVE.
*     X0             = X - COORDINARE OF FIRST NODE IN X-Y-Z PLANE.
*     Y0             = Y - COORDINARE OF FIRST NODE IN X-Y-Z PLANE.
*     Z0             = Z - COORDINATE OF FIRST NODE IN X-Y-Z PLANE
*     X(NNM)         = ARRAY CONTAINING X-COORDINATES OF NODES.
*     Y(NNM)         = ARRAY CONTAINING Y-COORDINATES OF NODES.
*     Z(NNM)         = ARRAY CONTAINING Z-COORDINATES OF NODES.
*
**********************************************************************
*
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/MSH/ARCANG,NX,NY,NZ,NX1,NX2,NX3
      DIMENSION DX1(10),DX3(10),DY(20),DZ(5),X(NNM),Y(NNM),Z(NNM),
     #          IBNDC(NNM,NDF),KELSUR(NBS,2),NOD(NEM,NPE)
*
      READ(5,*) NX1, NX2, NX3, NY, NZ, IEL, NPE, R1, R2, X0, Y0, Z0,
     #          ARCANG
*
*     COMPUTE THE NUMBER OF ELEMENTS AND NODES IN THE MESH:
*
      PI = 3.141592654
      NELM = (NX1+NX2+NX3)*NY*NZ
      NXX1 = IEL*NX1
      NXX3 = IEL*NX3
      NYY  = IEL*NY
      NZZ = IEL*NZ
*
      IF(Z0 .LE. 1.0E-10) THEN
           PHI = 0.0D0
      ELSE
           PHI = ATAN(Z0/X0)
      END IF
      ARCANG = ARCANG*PI/180
      ANGINC = ARCANG/NZZ
*     RZ = DSQRT(Y0*Y0 + Z0*Z0)
      RZ = Y0
*
      READ(5,*)  (DX1(I),I=1,NXX1)
      READ(5,*)  (DX3(I),I=1,NXX3)
      READ(5,*)  (DY(I),I=1,NYY)
*
      NXX1 = IEL*NX1 + 1
      NXX2 = IEL*NX2
      NXX3 = IEL*NX3
      NYY = IEL*NY + 1
      NZZ = IEL*NZ + 1
*
      IF(NPE .EQ. 20) THEN
```

```
          NDS = NYY*((NX1 + NX2 + NX3 + 1)*(NZ+1)) +
     #          (NY+1)*((NX1+NX2+NX3+1) + (NZ+1)*(NX1+NX2+NX3))
      ELSE
          NDS = NYY*(NXX1 + NXX2 + NXX3)*NZZ
      END IF
*
      IF(NDS.NE.NNM .OR. NEM.NE.NELM)THEN
      WRITE(6,999)NNM,NDS,NEM,NELM
      STOP
      ENDIF
*
      NTX   = IEL*NX1 + 1
      NTXX  = IEL*NX2
      NTXXX = IEL*NX3
      NTXT  = NTX + NTXX
      NTXTT = NTXT + NTXXX
*
*     COMPUTE THE NODAL COORDINATES IN SECTION 1 (STRAIGHT INLET)
*
      NTY = IEL*NY + 1
      NTZ = IEL*NZ + 1
      NY1 = (IEL-1)*NY + 1
*
      IIX = 0
      L = 0
      DO 1050 IX = 1, NTX
          IF(NPE .EQ. 20) THEN
              MODY = MOD(IX,2)
          ELSE
              MODY = 1
          END IF
*
          ZC = Z0
          ANGLE = PHI
*
          IF(MODY .EQ. 1) THEN
              IF(NPE .EQ. 20) THEN
                  I = (NYY*(NZ+1) + (NY+1)*(NZZ))*IIX
              ELSE
                  I = NYY*(IX - 1)*NZZ
              END IF
*
              DO 1020 IZ = 1, NTZ
                  IF(NPE .EQ. 20) THEN
                      MODZ = MOD(IZ,2)
                  ELSE
                      MODZ = 1
                  END IF
*
                  IF(MODZ .EQ. 1) THEN
                      I = I + 1
                      X(I) = X0
                      Y(I) = RZ*COS(ANGLE)
                      Z(I) = RZ*SIN(ANGLE)
*
                      DO 1000 IY = 1, NTY-1
                          I = I + 1
                          X(I) = X0
                          Y(I) = (Y(I-1) + DY(IY))*COS(ANGLE)
                          Z(I) = (Y(I-1) + DY(IY))*SIN(ANGLE)
 1000                 CONTINUE
*
                  ELSE
                      I = I + 1
                      X(I) = X0
                      Y(I) = Y0*COS(ANGLE)
```

```
                        Z(I) = ZC*SIN(ANGLE)
*
                        DO 1010 IY = 1, (NTY-NY1)
                             I = I + 1
                             K = 2*IY - 1
                             X(I) = X0
                             Y(I) = (RZ + DY(K) + DY(K+1))*COS(ANGLE)
                             Z(I) = (RZ + DY(K) + DY(K+1))*SIN(ANGLE)
1010                    CONTINUE
*
                   END IF
*
                   IF(IZ .LT. NTZ) ANGLE = ANGLE + ANGINC
1020          CONTINUE
              IIX = IIX + 1
*
         ELSE
*
              DO 1040 IZ = 1, (NZ+1)
                   I = I + 1
                   M = 2*IZ - 1
                   X(I) = X0
                   Y(I) = RZ*COS(ANGLE)
                   Z(I) = RZ*SIN(ANGLE)
*
                   DO 1030 IY = 1, (NTY-NY1)
                        I = I + 1
                        K = 2*IX - 1
                        X(I) = X0
                        Y(I) = (RZ + DY(K) + DY(K+1))*COS(ANGLE)
                        Z(I) = (RZ + DY(K) + DY(K+1))*SIN(ANGLE)
1030               CONTINUE
                   ANGLE = ANGLE + ANGINC
1040          CONTINUE
*
         END IF
         IF(IX .LT. NTX) X0 = X0 - DX1(IX)
*
1050  CONTINUE
*
*     COMPUTE THE NODAL COORDINATES IN THE CURVED SECTION:
*
      NXPT1 = NTX + 1
      THINC = PI/NXX2
      THETA = PI + THINC
      YC = Y0 + R2
*
      DO 1110 IX = NXPT1, NTXT
         IF(NPE .EQ. 20) THEN
              MODY = MOD(IX,2)
         ELSE
              MODY = 1
         END IF
*
         ZC = Z0
         ANGLE = PHI
         IF(MODY .EQ. 1) THEN
              DO 1080 IZ = 1, NTZ
                   IF(NPE .EQ. 20) THEN
                        MODZ = MOD(IZ,2)
                   ELSE
                        MODZ = 1
                   END IF
*
                   IF(MODZ .EQ. 1) THEN
                        I = I + 1
```

```
                        X(I) = X0 + R2*SIN(THETA)
                        Y(I) = (YC + R2*COS(THETA))*COS(ANGLE)
                        Z(I) = (YC + R2*COS(THETA))*SIN(ANGLE)
*
                        DYY = 0.0D0
                        DO 1060 IY = 1, NTY-1
                           I = I + 1
                           DYY = DYY + DY(IY)
                           X(I) = X0 + (R2 - DYY)*SIN(THETA)
                           Y(I) =(YC+(R2-DYY)*COS(THETA))*COS(ANGLE)
                           Z(I) =(YC+(R2-DYY)*COS(THETA))*SIN(ANGLE)
 1060                   CONTINUE
                     ELSE
                        I = I + 1
                        X(I) = X0 + R2*SIN(THETA)
                        Y(I) = (YC + R2*COS(THETA))*COS(ANGLE)
                        Z(I) = (YC + R2*COS(THETA))*SIN(ANGLE)
*
                        DYY = 0.0D0
                        DO 1070 IY = 1, (NTY-NY1)
                           I = I + 1
                           K = 2*IY - 1
                           DYY = DYY + DY(K) + DY(K+1)
                           X(I) = X0 + (R2 - DYY)*SIN(THETA)
                           Y(I) = (YC+(R2-DYY)*COS(THETA))*COS(ANGLE)
                           Z(I) = (YC+(R2-DYY)*COS(THETA))*COS(ANGLE)
 1070                   CONTINUE
                     END IF
                     IF(IZ .LT. NTZ) ANGLE = ANGLE + ANGINC
 1080             CONTINUE
                  IIX = IIX + 1
*
            ELSE
*
                  DO 1100 IZ = 1, (NZ+1)
                     I = I + 1
                     M = 2*IZ - 1
                     X(I) = X0 + R2*SIN(THETA)
                     Y(I) = (YC + R2*COS(THETA))*COS(ANGLE)
                     Z(I) = (YC + R2*COS(THETA))*SIN(ANGLE)
*
                     DYY = 0.0D0
                     DO 1090 IY = 1, (NTY-NY1)
                        I = I + 1
                        K = 2*IY - 1
                        DYY = DYY + DY(K) + DY(K+1)
                        X(I) = X0 + (R2 - DYY)*SIN(THETA)
                        Y(I) =(YC+(R2-DYY)*COS(THETA))*COS(ANGLE)
                        Z(I) =(YC+(R2-DYY)*COS(THETA))*SIN(ANGLE)
 1090                CONTINUE
                     ANGLE = ANGLE + 2.0*ANGINC
 1100             CONTINUE
*
            END IF
            THETA = THETA + THINC
*
 1110 CONTINUE
*
*
*     COMPUTE THE NODAL COORDINATES IN SECTION 3 (STRAIGHT OUTLET)
*
      NTXP11 = NTXT + 1
      Y0 = Y0 + 2.0*R2*COS(PHI)
      J = 0
*
      DO 1170 IX = NTXP11, NTXTT
```

```
            IF(NPE .EQ. 20) THEN
                 MODY = MOD(IX,2)
            ELSE
                 MODY = 1
            END IF
*
            J = J + 1
            X0 = X0 + DX3(J)
            ZC = Z0 + 2.0*R2*SIN(PHI)
            ANGLE = PHI
*
            IF(MODY .EQ. 1) THEN
                 DO 1140 IZ = 1, NTZ
*
                      IF(NPE .EQ. 20) THEN
                           MODZ = MOD(IZ,2)
                      ELSE
                           MODZ = 1
                      END IF
*
                      IF(MODZ .EQ. 1) THEN
                           I = I + 1
                           X(I) = X0
                           Y(I) = Y0*COS(ANGLE)
                           Z(I) = Y0*SIN(ANGLE)
*
                           DYY = 0.0D0
                           DO 1120 IY = 1, NTY-1
                                DYY = DYY + DY(IY)
                                I = I + 1
                                X(I) = X0
                                Y(I) = (RZ + 2*R2 -DYY)*COS(ANGLE)
                                Z(I) = (RZ + 2*R2 - DYY)*SIN(ANGLE)
1120                       CONTINUE
*
                      ELSE
                           I = I + 1
                           X(I) = X0
                           Y(I) = Y0*COS(ANGLE)
                           Z(I) = Y0*SIN(ANGLE)
*
                           DO 1130 IY = 1, (NTY-NY1)
                                I = I + 1
                                K = 2*IY - 1
                                X(I) = X0
                                Y(I) = (RZ+2*R2-DY(K)-DY(K+1))*COS(ANGLE)
                                Z(I) = (RZ+2*R2-DY(K)-DY(K+1))*SIN(ANGLE)
1130                       CONTINUE
*
                      END IF
*
                      IF(IZ .LT. NTZ) ANGLE = ANGLE + ANGINC
1140             CONTINUE
                 IIX = IIX + 1
*
            ELSE
*
                 DO 1160 IZ = 1, (NZ+1)
                      I = I + 1
                      M = 2*IZ - 1
                      X(I) = X0*COS(ANGLE)
                      Y(I) = Y0
                      Z(I) = X0*SIN(ANGLE)
*
                      DO 1150 IY = 1, (NTY-NY1)
                           I = I + 1
```

```
                        K = 2*IY - 1
                        X(I) = X0
                        Y(I) = (RZ+2*R2-DY(K)-DY(K+1))*COS(ANGLE)
                        Z(I) = (RZ+2*R2-DY(K)-DY(K+1))*SIN(ANGLE)
 1150                CONTINUE
                     ANGLE = ANGLE + 2.*ANGINC
 1160           CONTINUE
*
           END IF
*
 1170 CONTINUE
*
      DO 1175 I=1,NNM
      X(I)=0.0254*X(I)
      Y(I)=0.0254*Y(I)
 1175 Z(I)=-0.0254*Z(I)
*
*     DETERMINE THE CONNECTIVITY MATRIX:
*
      NX = NX1 + NX2 + NX3
      IF(NPE .EQ. 20) NTY = 3*NY + 2
*
      DO 1200 IX = 1, NX
           DO 1190 IZ = 1, NZ
                DO 1180 IY = 1, NY
*
                     I = IY + (IX-1)*NY*NZ + (IZ-1)*NY
*
                     IF(NPE .EQ. 20) THEN
                          NOD(I,1) = IEL*IY - (IEL-1) + (NYY*(NZ+1) +
     #                               (NY+1)*NZZ)*(IX-1)+(IZ-1)*(NYY+NY)
                          NOD(I,2) = NYY*(NZ+1)+(NY+1)*NZZ + NOD(I,1)
                     ELSE
                          NOD(I,1) = IEL*IY - (IEL-1) + (IX-1)*
     #                               (NYY*NZZ)*IEL+(IZ-1)*IEL*NYY
                          NOD(I,2) = NYY*NZZ*IEL + NOD(I,1)
                     END IF
*
                     NOD(I,3) = NOD(I,2) + IEL
                     NOD(I,4) = NOD(I,1) + IEL
*
                     IF(NPE .EQ. 20) THEN
                          NOD(I,5) = NTY + NOD(I,1)
                          NOD(I,6) = NYY*(NZ+1) + (NY+1)*NZZ + NOD(I,5)
                     ELSE
                          NOD(I,5) = NYY + NOD(I,1)
                          NOD(I,6) = NYY*NZZ*IEL + NOD(I,5)
                     END IF
*
                     NOD(I,7) = NOD(I,6) + IEL
                     NOD(I,8) = NOD(I,5) + IEL
C                    IF(NPE .EQ. 20) THEN
C                         NOD(I,9) = NOD(I,1) + NYY*(NZ+1) + (NY+1)*NZ
C    #                              + (1-IY)
C                         NOD(I,10) = NOD(I,2) + 1
C                         NOD(I,11) = NOD(I,9) + 1
C                         NOD(I,12) = NOD(I,1) + 1
C                         NOD(I,13) = NYY + NOD(I,1)
C                         NOD(I,14) = NYY + NOD(I,2)
C                         NOD(I,15) = NOD(I,14) + 1
C                         NOD(I,16) = NOD(I,13) + 1
C                         NOD(I,17) = NOD(I,5) + (NYY + NY + 1)*NZ +
C    #                               (1-IY)
C                         NOD(I,18) = NOD(I,6) + 1
C                         NOD(I,19) = NOD(I,17) + 1
C                         NOD(I,20) = NOD(I,5) + 1
```

```
C                       ELSE IF(NPE .EQ. 27) THEN
C                            NOD(I,9) =  NOD(I,5) + NYY
C                            NOD(I,10) = NOD(I,9) + NYY*NZZ*IEL
C                            NOD(I,11) = NOD(I,10) + IEL
C                            NOD(I,12) = NOD(I,9) + IEL
C                            NOD(I,13) = NOD(I,1) + NYY*NZZ
C                            NOD(I,14) = NOD(I,2) + 1
C                            NOD(I,15) = NOD(I,13) + 2
C                            NOD(I,16) = NOD(I,1) + 1
C                            NOD(I,17) = NOD(I,5) + NYY*NZZ
C                            NOD(I,18) = NOD(I,6) + 1
C                            NOD(I,19) = NOD(I,17) + 2
C                            NOD(I,20) = NOD(I,5) + 1
C                            NOD(I,21) = NOD(I,9) + NYY*NZZ
C                            NOD(I,22) = NOD(I,10) + 1
C                            NOD(I,23) = NOD(I,21) + 2
C                            NOD(I,24) = NOD(I,9) + 1
C                            NOD(I,25) = NOD(I,13) + 1
C                            NOD(I,26) = NOD(I,17) + 1
C                            NOD(I,27) = NOD(I,21) + 1
C                       END IF
1180              CONTINUE
1190         CONTINUE
1200  CONTINUE
*
*
*     COMPUTE THE NUMBER OF BOUNDARY SURFACES AND DETERMINE SURFACE
*     INDICES
*
      NSURF = 2*NX*(NY+NZ) +2*NY*NZ
*
*     ELEMENT FLUX SURFACES AT THE INLET OF THE DUCT:
*
      I = 0
      NYZ = NY*NZ
      DO 1210 IYZ = 1, NYZ
           I = I + 1
           KELSUR(I,1) = IYZ
           KELSUR(I,2) = 1
1210  CONTINUE
*
*     ELEMENT FLUX SURFACES AT THE SOLID SURFACE OF THE DUCT (OUTER):
*
      DO 1220 IX = 1, NX
           DO 1220 IZ = 1, NZ
           I = I + 1
           ILL = (IX-1)*NY*NZ + (IZ-1)*NY + 1
           KELSUR(I,1) = ILL
           KELSUR(I,2) = 3
1220  CONTINUE
*
*     ELEMENT FLUX SURFACES AT THE SOLID SURFACE OF THE DUCT (INNER):
*
      DO 1230 IX = 1, NX
           DO 1230 IZ = 1, NZ
           I = I + 1
           ILL = (IX-1)*NY*NZ + IZ*NY
           KELSUR(I,1) = ILL
           KELSUR(I,2) = 4
1230  CONTINUE
*
*     ELEMENT FLUX SURFACES AT SYMMETRY SURFACE OF THE DUCT (IZ = 1):
*
      DO 1240 IX = 1, NX
           DO 1240 IY = 1, NY
           I = I + 1
```

```
            ILL = IY + (IX-1)*NY*NZ
            KELSUR(I,1) = ILL
            KELSUR(I,2) = 5
 1240  CONTINUE
*
*      ELEMENT FLUX SURFACES AT SYMMETRY SURFACE OF THE DUCT (IZ = NZ):
*
       DO 1250 IX = 1, NX
            DO 1250 IY = 1, NY
            I = I + 1
            ILL = IY + (IX-1)*NY*NZ + NY*(NZ-1)
            KELSUR(I,1) = ILL
            KELSUR(I,2) = 6
 1250  CONTINUE
*
*      ELEMENT FLUX SURFACES AT THE INLET OF THE DUCT:
*
       J = 0
       DO 1260 IZ = 1, NZ
            DO 1260 IY = 1, NY
            J = J + 1
            I = I + 1
            ILL = (NX-1)*NY*NZ + J
            KELSUR(I,1) = ILL
            KELSUR(I,2) = 2
 1260  CONTINUE
*
*
*      DETERMINE THE BOUNDARY CONDITIONS:
*
       NBNDC = 0
       ND = 0
       NXX = NX + 1
       NYY = NY + 1
       NZZ = NZ + 1
*
       DO 1212 I = 1, NDS
            DO 1212 J = 1, 5
            IBNDC(I,J) = 1
 1212  CONTINUE
*
*      SPECIFY THE   I N L E T   BOUNDARY DEGREES OF FREEDOM
*
       DO 1280 ID = 1, NYY
            DO 1270  JD = 1, NZZ
                 ND = ND + 1
                 NBNDC = NBNDC + 1
                 IBNDC(ND,2) = 0
                 IBNDC(ND,3) = 0
                 IBNDC(ND,4) = 0
                 IBNDC(ND,5) = 0
 1270       CONTINUE
 1280  CONTINUE
*
*      SPECIFY THE   S O L I D - W A L L   BOUNDARY DEGREES OF FREEDOM
*
       DO 1300 KD = 1, NX
            ND1 = (NYY*NZZ)*KD + 1
            DO 1290 JZ = 1, NZZ
                 ND = ND1 + (JZ-1)*NYY
                 NBNDC = NBNDC + 1
                 IBNDC(ND,2) = 0
                 IBNDC(ND,3) = 0
                 IBNDC(ND,4) = 0
CC               IBNDC(ND,5) = 0
*
```

```
                  NBNDC = NBNDC + 1
                  IBNDC(ND+NY,2) = 0
                  IBNDC(ND+NY,3) = 0
                  IBNDC(ND+NY,4) = 0
CC                IBNDC(ND+NY,5) = 0
*
1290        CONTINUE
*
1300  CONTINUE
*
*     SPECIFY THE   E X I T   BOUNDARY DEGREES OF FREEDOM
*
      NBD1 = NYY*NZZ*NX
      DO 1320 I = 1, NZZ
           NBD = NBD1 + (I-1)*NYY
           DO 1310 J = 1, NYY
                NBD = NBD + 1
                NBNDC = NBNDC + 1
                IBNDC(NBD,5) = 0
C               IBNDC(NBD,3) = 0
C               IBNDC(NBD,4) = 0
1310       CONTINUE
1320  CONTINUE
C
      RETURN
  999 FORMAT(/,5X,'***** THE PARAMETERS  NNM AND NEM  SENT FROM THE MAIN
     # DO NOT COINCIDE WITH THOSE GENERATED IN TADMSH *****',/,5X,'*****
     # THE PROGRAM IS TERMINATED *****',/,5X,'NNM,NDS,NEM,NELM =',4I5)
      END
 FLOW IN A TURN-AROUND-DUCT  (15X8X2 MESH)
 1  1 02   100 05. 1.0 0.8
 1.79E-03 293.0  110.0  287.0  1.402  0.72  0.1 1.205
 3  8  4   8   2   1   8   1.0   3.0   33.0  9.0   0.0   2.0
 20.0   8.0   2.0
 0.5  1.5   8.0  20.0
 0.1  0.2  0.3   0.4  0.4  0.3  0.2  0.1
```